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Explaining large electromagnetic logarithms from loops of inflationary gravitons

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Abstract: Recent progress on nonlinear sigma models on de Sitter background has permitted the resummation of large inflationary logarithms by combining a variant of Starobinsky's stochastic formalism with a variant of the renormalization group. We reconsider single graviton loop corrections to the photon wave function, and to the Coulomb potential, in light of these developments. Neither of the two 1-loop results have a stochastic explanation, however, the flow of a curvature-dependent field strength renormalization explains their factors of $\ln(a)$. We speculate that the factor of $\ln(Hr)$ in the Coulomb potential should not be considered as a leading logarithm effect.

Keywords: de Sitter space, Renormalization Group

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1 Introduction

Cosmology is characterized by scale factor $a(t)$, Hubble parameter $H(t)$ and first slow roll parameter $\epsilon(t)$,

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \qquad \Longrightarrow \qquad H(t) \equiv \frac{\dot{a}}{a} \,, \quad \epsilon(t) \equiv -\frac{\dot{H}}{H^2} \,. \tag{1.1}$$

Inflation is the special case for which both the first and second time derivatives of the scale factor are positive ($H(t) > 0$ and $0 \leq \epsilon(t) < 1$). It is the accelerated expansion of inflation which produces the primordial spectra of scalars [1] and gravitons [2] by ripping these quanta out of the vacuum.

At some level these quanta must interact with themselves and with other particles. These interactions can change single particle kinematics and long range forces, and one might expect that the changes grow because more and more quanta are ripped out of the vacuum as time progresses. For example, a single loop of gravitons on de Sitter background ($\epsilon(t) = 0$) corrects the electric field strength of a plane wave photon [3] and the Coulomb potential of a point charge [4] to,

$$F^{0i}(t,\vec{x}) = F^{0i}_{\rm tree}(t,\vec{x}) \left\{ 1 + \frac{2GH^2}{\pi} \ln(a) + O(G^2) \right\} \,, \tag{1.2}$$

$$\Phi(t,r) = \frac{Q}{4\pi a r} \left\{ 1 + \frac{2G}{3\pi a^2 r^2} + \frac{2GH^2}{\pi} \ln(aHr) + O(G^2) \right\} \,. \tag{1.3}$$

Similar results have been reported for fermions [5], for massless, minimally coupled scalars [6], and for gravitons [7, 8].

A fascinating aspect of these results is that they continue to grow for as long as inflaton persists. For sufficient inflation, the factors of $\ln[a(t)]$ must eventually overwhelm the loop-counting parameter GH^2 causing perturbation theory to break down. Evolving past this point requires a nonperturbative resummation technique of the sort recently developed for nonlinear sigma models on de Sitter background [9–11]. The technique combines a variant of Starobinsky's stochastic formalism [12, 13], based on curvature-dependent effective potentials, with a variant of the renormalization group, based on the subset of counterterms which can be viewed as curvature-dependent renormalizations of parameters in the bare theory. The latter part of the technique is not encountered in renormalizable matter theories, where the curvature-independent renormalization group explains large secular logarithms [14]. Even better, the technique can be generalized to a arbitrary cosmological background (1.1) which has undergone primordial inflation [15], and applying it transmits inflationary effects to late times [16].

It seems entirely possible to generalize this technique from nonlinear sigma models to quantum gravity. The first step has been taken by using a variant of the renormalization group to explain the large logarithm in the 1-graviton loop correction to the exchange potential of a massless, minimally coupled scalar [6]. The purpose of this paper is to do the same for the 1-graviton loop corrections (1.2)–(1.3) to electrodynamics. In section 2 we review the exact calculation. Section 3 uses the renormalization group to explain the factors of $\ln[a(t)]$ in both results. We do not believe there is any curvature-dependent effective potential for this system, and we suspect that the factor of $\ln(Hr)$ in (1.3) is not a leading logarithm effect. The case for that is made in section 4. Our conclusions comprise section 5.

2 The exact calculation

The purpose of this section is to review the exact calculation of the 1-graviton loop contribution to the vacuum polarization $i[^\mu\Pi^\nu](x;x')$ [17] from which the results (1.2)–(1.3) were derived [3, 4]. These results were obtained by perturbatively solving the quantum-corrected Maxwell equation,

$$\partial_\nu \left[\sqrt{-g}\, g^{\nu\rho} g^{\sigma\mu} F_{\rho\sigma}(x)\right] + \int d^4x' \left[^\mu\Pi^\nu\right](x;x') A_\nu(x') = J^\mu(x)\,, \tag{2.1}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor and J^μ is the current density. We begin by explaining how the vacuum polarization is represented and why de Sitter breaking is unavoidable. Next the counterterms are given. The section closes by giving the structure functions and isolating those terms which are responsible for the large logarithms in (1.2)–(1.3). Throughout we employ conformal coordinates (based on $d\eta \equiv dt/a(t)$) so that the de Sitter metric $g_{\mu\nu} = a^2\eta_{\mu\nu}$ is proportional to Minkowski metric of flat space.

Of the ten graviton loops which have so far been evaluated on de Sitter background [17–26],[1] all but one of them [24] used the simplest gauge [29, 30]. The great thing about the dimensionally regulated (spacetime dimension D) propagator in this gauge is that it consists of three scalar propagators (with masses $M_A^2 = 0$, $M_B^2 = (D-2)H^2$ and $M_C^2 = 2(D-3)H^2$)

[1] See also the computation of graviton corrections to massless, conformally coupled scalars [27, 28] which disagrees with our result [26].

multiplied by constant tensor factors which are formed using the Minkowski metric $\eta_{\mu\nu}$ and $\delta^0{}_\mu$ (with $\overline{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0{}_\mu \delta^0{}_\nu$),

$$i\left[{}_{\mu\nu}\Delta_{\rho\sigma}\right](x;x') = \sum_{I=A,B,C} i\Delta_I(x;x') \times \left[{}_{\mu\nu}T^I_{\rho\sigma}\right] , \tag{2.2}$$

$$\left[{}_{\mu\nu}T^A_{\rho\sigma}\right] = 2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} - \frac{2\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}}{D-3} , \qquad \left[{}_{\mu\nu}T^B_{\rho\sigma}\right] = -4\delta^0{}_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0{}_{\sigma)} , \tag{2.3}$$

$$\left[{}_{\mu\nu}T^C_{\rho\sigma}\right] = \frac{2[\overline{\eta}_{\mu\nu} + (D-3)\delta^0{}_\mu\delta^0{}_\nu][\overline{\eta}_{\rho\sigma} + (D-3)\delta^0{}_\rho\delta^0{}_\sigma]}{(D-3)(D-2)} . \tag{2.4}$$

Another huge advantage of this gauge is that the $D = 4$ dimensional limits of the three scalar propagators are simple,

$$i\Delta_A(x;x') \longrightarrow \frac{1}{4\pi^2}\left[\frac{1}{aa'\Delta x^2} - \frac{H^2}{2}\ln\left(\frac{1}{4}H^2\Delta x^2\right)\right] , \tag{2.5}$$

$$i\Delta_B(x;x') \longrightarrow i\Delta_C(x;x') \longrightarrow \frac{1}{4\pi^2}\frac{1}{aa'\Delta x^2} , \tag{2.6}$$

where $\Delta x^2 = -(|\eta - \eta'| - i\epsilon)^2 + \|\vec{x} - \vec{x}'\|^2$, with $\epsilon > 0$ infinitesimal.

Although the propagator (2.2)–(2.4) is the easiest to use, this gauge does break de Sitter invariance, which means that noninvariant counterterms can and do occur. The inevitability of de Sitter breaking for the graviton propagator on de Sitter background has been a contentious issue for decades [31–39]. However, the presence of noninvariant counterterms seems to have been settled by the computation of the vacuum polarization in a general class of de Sitter invariant gauges [40]. In spite of the de Sitter invariant gauge, noninvariant counterterms still arise due to the unavoidable breaking in the time-ordered interactions [24]. So we will just go ahead with the result [17] derived in the simplest gauge.

General relativity plus Maxwell is not perturbatively renormalizable [41, 42], however, the 1PI (one-particle-irreducible) n-point functions of any quantum field theory can be renormalized, order-by-order in perturbation theory, using BPHZ (Bogoliubov, Parasiuk [43], Hepp [44] and Zimmermann [45, 46]) counterterms. The ones needed to renormalize the 1-loop vacuum polarization on de Sitter background are [17, 24],

$$\begin{aligned}\Delta\mathcal{L} = {} & \Delta C H^2 F_{ij}F_{k\ell}g^{ik}g^{j\ell}\sqrt{-g} + \overline{C}H^2 F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} \\ & + C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} ,\end{aligned} \tag{2.7}$$

where D_α represents the covariant derivative operator. In the simplest gauge the divergent coefficients are [17],

$$\Delta C = -1 \times \frac{\kappa^2\mu^{D-4}}{16\pi^2(D-4)} , \quad \overline{C} = \frac{7}{6} \times \frac{\kappa^2\mu^{D-4}}{16\pi^2(D-4)} , \quad C_4 = \frac{1}{6} \times \frac{\kappa^2\mu^{D-4}}{16\pi^2(D-4)} , \tag{2.8}$$

where $\kappa^2 \equiv 16\pi G$ is the loop-counting parameter of quantum gravity and μ is the mass scale of dimensional regularization.

Owing to the unavoidable breaking of de Sitter invariance, the vacuum polarization requires two structure functions [47]. Various representations are possible [48], of which we chose the one first employed for the vacuum polarization induced by scalar quantum electrodynamics [49, 50],

$$i\left[{}^{\mu}\Pi^{\nu}\right](x;x') = \left[\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right]\partial_\rho\partial'_\sigma F(x;x') + \left[\overline{\eta}^{\mu\nu}\overline{\eta}^{\rho\sigma} - \overline{\eta}^{\mu\sigma}\overline{\eta}^{\nu\rho}\right]\partial_\rho\partial'_\sigma G(x;x')\,. \tag{2.9}$$

We employed the Schwinger-Keldysh formalism [51–59] in order to keep the effective field equations real and causal. With a convenient choice of the finite parts of ΔC, $\overline{C}$ and C_4, the Schwinger-Keldysh structure functions are [4],

$$\begin{aligned} iF(x;x') =& \frac{\kappa^2 H^2}{8\pi^2}\left\{-\ln\left(\frac{\mu a}{2H}\right) - \frac{\partial_0}{3aH} + \frac{\ln\left(\frac{\mu a}{2H}\right)}{3H^2}\partial_\mu \frac{1}{a^2}\partial^\mu\right\}\delta^4(x-x') \\ &+ \frac{\kappa^2\partial^6}{384\pi^3 aa'}\left\{\theta(\Delta\eta-\Delta r)\left[\ln[H^2(\Delta\eta^2-\Delta r^2)]-1\right]\right\} - \frac{\kappa^2 H^2}{128\pi^3}\left\{\left[\partial^4+4\partial^2\partial_0^2\right]\right. \\ &\left.\times\left[\theta(\Delta\eta-\Delta r)\ln[H^2(\Delta\eta^2-\Delta r^2)]\right] - \left[\partial^4-4\partial^2\partial_0^2\right]\theta(\Delta\eta-\Delta r)\right\}, \end{aligned} \tag{2.10}$$

$$iG(x;x') = \frac{\kappa^2 H^2}{6\pi^2}\ln\left(\frac{\mu a}{2H}\right)\delta^4(x-x') + \frac{\kappa^2 H^2\partial^4}{96\pi^3}\left\{\theta(\Delta\eta-\Delta r)\left[\ln\left(H^2(\Delta\eta^2-\Delta r^2)\right)-1\right]\right\}, \tag{2.11}$$

where $\Delta\eta \equiv \eta - \eta'$ and $\Delta r \equiv \|\vec{x} - \vec{x}'\|$. The flat space result [60] is recovered by the terms which contain no net factors of H. The terms which contain factors of H represent the new, de Sitter corrections which represent inflationary particle production.

It remains to comment on the gauge issue. Any quantity with a graviton propagtor, such as the 1-graviton loop contribution to the vacuum polarization, is liable to depend on the gauge fixing function. This dependence is easy to quantify in the flat space result [60] and it must therefore be present at least in the flat space limit of the de Sitter results we have just presented. Presumably there is also gauge dependence in the new, de Sitter contributions [24]. Eliminating this gauge dependence is an important problem for the physical interpretation of results such as (1.2)–(1.3), and a procedure has been developed for accomplishing this which works in flat space [61, 62] and is being generalized to de Sitter [6, 63]. However, the issue of gauge dependence has no relevance for the study we are making here, of how to explain the large logarithms which occur in a specific gauge.

3 Renormalization group explanation

The purpose of this section is to show how the factors of $\ln(a)$ in expressions (1.2)–(1.3) can be explained as the renormalization group flow of a curvature-dependent renormalization of the electromagnetic field strength. We accordingly identify the appropriate counterterm and compute the associated gamma function. Then the Callan-Symanzik equation for Green's functions is written down.

The structure functions (2.10) and (2.11) include all information about 1-graviton loop corrections to the linearized Maxwell equation (2.1). However, the factors of $\ln(a)$

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and $\ln(Hr)$ evident in expressions (1.2)–(1.3) derive from just two terms in $F(x;x')$,[2]

$$F(x;x') \longrightarrow -\frac{\kappa^2 H^2}{8\pi^2}\ln\left(\frac{\mu a}{2H}\right)\delta^4(x-x') - \frac{\kappa^2 H^2 \partial^4}{128\pi^3}\left\{\theta(\Delta\eta-\Delta r)\ln\left[H^2(\Delta\eta^2-\Delta r^2)\right]\right\}, \tag{3.1}$$

$$G(x;x') \longrightarrow 0\,. \tag{3.2}$$

The factors of $\ln(a)$ in (1.2)–(1.3) come entirely from the local term of (3.1), whereas it is the nonlocal term at the end of (3.1) which produces the factor of $\ln(Hr)$ in the Coulomb potential.

The Renormalization Group is associated with the dependence on the dimensional regularization mass scale μ which enters through the coefficients (2.8) of the counterterms (2.7). To understand how this scale affects the structure functions $F(x;x')$ and $G(x;x')$ we exploit conformal coordinates to exhibit the scale factors, and we expand the covariant derivatives of the C_4 counterterm so that they give ordinary derivatives plus terms which can be combined with the ΔC and $\overline{C}$ counterterms,

$$\begin{aligned}\Delta\mathcal{L} = {} & [\Delta C-(D-6)C_4]\,a^{D-4}H^2 F_{ij}F_{ij} \\ & +\left[\overline{C}-(3D-8)C_4\right]a^{D-4}H^2 F_{\mu\nu}F^{\mu\nu} + C_4 a^{D-6}\partial_\alpha F_{\mu\nu}\partial^\alpha F^{\mu\nu}\,.\end{aligned} \tag{3.3}$$

Note that we use the Minkowski metric to raise indices on the field strength ($F^{\mu\nu} \equiv \eta^{\mu\rho}\eta^{\sigma\nu}F_{\rho\sigma}$) and the partial derivative operator ($\partial^\alpha \equiv \eta^{\alpha\beta}\partial_\beta$).

From expression (3.3) we can read off how the coefficient of each counterterm affects the structure functions,

$$\overline{C}-(3D-8)C_4 = +\frac{1}{2}\times\frac{\kappa^2\mu^{D-4}}{16\pi^2(D-4)} \quad\Longrightarrow\quad \Delta F_1 = -\frac{\kappa^2 H^2}{8\pi^2}\ln\left(\frac{\mu a}{2H}\right)\delta^4(x-x')\,, \tag{3.4}$$

$$C_4 = +\frac{1}{6}\times\frac{\kappa^2\mu^{D-4}}{16\pi^2(D-4)} \quad\Longrightarrow\quad \Delta F_2 = +\frac{\kappa^2}{24\pi^2}\ln\left(\frac{\mu a}{2H}\right)\partial_\mu\frac{1}{a^2}\partial^\mu\delta^4(x-x')\,, \tag{3.5}$$

$$\Delta C-(D-6)C_4 = -\frac{2}{3}\times\frac{\kappa^2\mu^{D-4}}{16\pi^2(D-4)} \quad\Longrightarrow\quad \Delta G = +\frac{\kappa^2 H^2}{6\pi^2}\ln\left(\frac{\mu a}{2H}\right)\delta^4(x-x')\,. \tag{3.6}$$

Comparison with (3.1)–(3.2) reveals that neither (3.5) nor (3.6) is responsible for the factors of $\ln(a)$ in (1.2)–(1.3). The factors of $\ln(a)$ all come from (3.4), which can be regarded as the coefficient of a curvature-dependent field strength renormalization,

$$\delta Z \equiv -4\left[\overline{C}-(3D-8)C_4\right]H^2 = -\frac{\kappa^2 H^2}{8\pi^2}\times\frac{\mu^{D-4}}{D-4} + O(\kappa^4 H^4)\,. \tag{3.7}$$

The associated gamma function is,

$$\gamma \equiv \frac{\partial\ln(1+\delta Z)}{\partial\ln(\mu^2)} = -\frac{\kappa^2 H^2}{16\pi^2} + O(\kappa^4 H^4)\,. \tag{3.8}$$

[2] For the Coulomb potential, see equation (30) of [4]; for the photon field strength, see table 1 of [3].

The Callan-Symanzik equation for n-point Green's functions is,[3]

$$\left[\frac{\partial}{\partial \ln(\mu)} + \beta_{\kappa^2}\frac{\partial}{\partial \kappa^2} + n\gamma\right] G_n\left(x_1; x_2; \dots; x_n; \mu; \kappa^2\right) = 0\,. \tag{3.9}$$

The beta function for this theory goes like $\beta_{\kappa^2} \sim \kappa^4 H^2$, so it does not affect 1-loop results. As one can see from (2.10)–(2.11), the factors of $\ln(\mu)$ are always associated with factors $\ln(a)$ in the form $\ln(\mu a)$. This is because primitive divergences produce no D-dependent scale factors, whereas the counterterms which absorb them not only contain a factor of μ^{D-4} but also a factor of a^{D-4},

$$\frac{1}{D-4} - \frac{\mu^{D-4}a^{D-4}}{D-4} = -\ln(\mu a) + O(D-4)\,. \tag{3.10}$$

Hence we can replace the derivative with respect to $\ln(\mu)$ in expression (3.9) with a derivative with respect to $\ln(a)$. If we then regard the photon field strength (1.2) and the Coulomb potential (1.3) as 2-point Green's functions it will be seen that the Callan-Symanzik equation (3.9), with gamma function (3.8), explains the factors of $\ln(a)$ in both results.

4 Search for a stochastic explanation

The previous section demonstrated that the factors of $\ln(a)$ in the photon field strength (1.2) and the Coulomb potential (1.3) can be explained using a variant of the Renormalization Group. The purpose of this section is to explain why there seems to be no compelling variant of the stochastic formalism which explains the factor of $\ln(Hr)$ in the Coulomb potential. We begin by noting the characteristics of the $\ln(Hr)$ term. In particular, it may not even count as a "leading logarithm" effect as the factors of $\ln(a)$ do. We then discuss the problems with developing a compelling stochastic explanation for it.

4.1 Peculiarities of the $\ln(Hr)$ term

We have already mentioned that the factor of $\ln(Hr)$ in the Coulomb potential (1.3) derives from the nonlocal part of the vacuum polarization on the second line of expression (3.1). This descends from the "tail" part of graviton propagator [64]; that is, from the logarithm part of $i\Delta_A(x;x')$ visible in expression (2.5). Its origin from the finite, nonlocal part of the graviton propagator means that the factor of $\ln(Hr)$ is not explainable by the Renormalization Group. If it is to be understood as a "large logarithm" we must seek a stochastic explanation based on a curvature-dependent correction to the electromagnetic field equation, similar to the curvature-dependent effective potentials which served to explain many of the large logarithms in nonlinear sigma models [9].

Before searching for a stochastic explanation we should discuss whether or not the factor of $\ln(Hr)$ qualifies as a "large logarithm" which should appear in the leading logarithm approximation. Many perfectly valid loop corrections are not recovered in this approximation. One example is the fractional correction of $2G/(3\pi a^2 r^2)$ in the Coulomb potential (1.3). This is the de Sitter descendant of a well-known flat space correction which

[3] Change $+n\gamma$ to $-n\gamma$ for one-particle-irreducible n-point functions.

was discovered by Radkowski in 1970 [65]. It has nothing to do with inflationary particle production and clearly does not belong to the leading logarithm approximation.

Because the initial manifold has coordinate radius comparable to the Hubble length [66], we do not have access to the regime of $Hr \gg 1$. Hence the factor of $\ln(Hr)$ can only become large for $Hr \ll 1$. That looks more like an ultraviolet effect than an infrared one. In the same sense, the Radkowski correction only becomes significant for small r. On the other hand, the two effects depend very differently on the physical separation length $a(t)Hr$,

$$\text{Radkowski} \longrightarrow \left[\frac{1}{a(t)Hr}\right]^2 \qquad \text{versus} \qquad \text{Inflation} \longrightarrow \ln\left[a(t)Hr\right] \, . \tag{4.1}$$

The Radkowski effect only becomes large when the physical separation is small, and for $aHr \ll 1$ it overwhelms the logarithm contribution, whereas the inflationary effect is large when the physical separation becomes enormous.

4.2 Problems with a stochastic explanation

To understand our problems in deriving a stochastic formulation of electrodynamics it is good to contrast the Lagrangian of electromagnetism plus gravity,

$$\mathcal{L}_{\rm EMGR} = \frac{(R-2\Lambda)\sqrt{-g}}{16\pi G} - \frac{1}{4} F_{\rho\sigma} F_{\mu\nu} g^{\rho\mu} g^{\sigma\nu} \sqrt{-g} \, , \tag{4.2}$$

with the nonlinear sigma model [9, 10] for which a compelling stochastic formulation exists,

$$\mathcal{L}_{AB} = -\frac{1}{2}\partial_\mu A \partial_\nu A g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\Big(1 + \frac{\lambda}{2}A\Big)^2 \partial_\mu B \partial_\nu B g^{\mu\nu}\sqrt{-g} \, . \tag{4.3}$$

Both theories involve two fields, one of which engenders large logarithms and the other not,

$$h_{\mu\nu} \longrightarrow (\text{Logs}) \, , \qquad\qquad A \longrightarrow (\text{Logs}) \, , \tag{4.4}$$
$$A_\mu \longrightarrow (\text{No Logs}) \, , \qquad\qquad B \longrightarrow (\text{No Logs}) \, . \tag{4.5}$$

(The graviton field $h_{\mu\nu}$ is defined by conformally transforming the metric, $g_{\mu\nu} \equiv a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$.) The stochastic formulation of the nonlinear sigma model (4.3) was derived by integrating out the "No Logs" field B from the equation of the "Logs" field A in the presence of a constant A background,

$$\frac{\delta S[A,B]}{\delta A} = \partial_\mu \left[\sqrt{-g}\, g^{\mu\nu}\partial_\nu A\right] - \frac{\lambda}{2}\left(1 + \frac{\lambda}{2}A\right)\sqrt{-g}\, g^{\mu\nu}\partial_\mu B \partial_\nu B \, , \tag{4.6}$$

$$\longrightarrow \partial_\mu \left[\sqrt{-g}\, g^{\mu\nu}\partial_\nu A\right] - \frac{\lambda}{2}\left(1 + \frac{\lambda}{2}A\right)\sqrt{-g}\, g^{\mu\nu} \times \frac{\partial_\mu \partial'_\nu i\Delta_A(x;x')|_{x'=x}}{\left(1+\frac{\lambda}{2}A\right)^2} \, , \tag{4.7}$$

$$\longrightarrow \partial_\mu \left[\sqrt{-g}\, g^{\mu\nu}\partial_\nu A\right] + \frac{\frac{3\lambda H^4}{16\pi^2}\sqrt{-g}}{1 + \frac{\lambda}{2}A} \, . \tag{4.8}$$

This is a scalar potential model with potential $V_{\rm eff}(A) = -\frac{3H^4}{8\pi^2}\ln\left|1 + \frac{\lambda}{2}A\right|$ and it can be treated using Starobinsky's stochastic formalism [12, 13]. Doing so recovers large logarithms in 1-loop corrections to the scalar mode function and the exchange potential [9], as well as 1-loop and 2-loop contributions to the expectation value of A [9, 10].

The analog of the reduction (4.6)–(4.8) for our model (4.2) would be to integrate out the "No Logs" photon field from the "Logs" metric field equation,

$$\frac{16\pi G}{\sqrt{-g}}\frac{\delta S_{\rm EMGR}}{\delta g^{\mu\nu}} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda - 8\pi G\left[\delta^{\alpha}{}_{\mu}\delta^{\beta}{}_{\nu}g^{\rho\sigma} - \frac{1}{4}g_{\mu\nu}g^{\alpha\beta}g^{\rho\sigma}\right]F_{\alpha\rho}F_{\beta\sigma}\,. \tag{4.9}$$

This might describe large logarithms affecting the graviton field [67], but it cannot capture the large logarithms (1.2)–(1.3) induced by the graviton in the photon field. A stochastic explanation of those logarithms would presumably derive from integrating out the graviton from the photon field equation,

$$\frac{\delta S_{\rm EMGR}}{\delta A_{\mu}} = \partial_{\nu}\left[\sqrt{-g}\,g^{\nu\rho}g^{\mu\sigma}F_{\rho\sigma}\right]\,. \tag{4.10}$$

In the nonlinear sigma model (4.3) this would be like integrating out the A field from the B equation,

$$\frac{\delta S_{AB}}{\delta B} = \partial_{\mu}\left[\left(1+\frac{\lambda}{2}A\right)^2\sqrt{-g}\,g^{\mu\nu}\partial_{\nu}B\right]\,. \tag{4.11}$$

That is exactly what was *not* done. Nor was there any stochastic explanation for the explicit 1-loop and 2-loop results which were obtained for the field B [9]. These results were all explained using the Renormalization Group. Moreover, integrating out the metric field would result in an electromagnetic equation that still has derivative interactions, precluding the stochastic formalism from being applied directly.

It is nevertheless undeniable that the graviton infrared modes are hugely enhanced, and one might try to apply a perturbative version of the stochastic approximation without integrating out any fields. For scalar potential models this amounts to approximating the real part of the A-type scalar propagator in (2.2) by the corresponding infrared stochastic sum,

$$\begin{aligned}&{\rm Re}\left\{i\Delta_A(x;x')\right\}\longrightarrow\\ &S(x;x')\equiv\int\frac{d^3k}{(2\pi)^3}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\times\theta(\varepsilon Ha-k)\theta(\varepsilon Ha'-k)\theta(k-\delta H)U(\eta,k)U^*(\eta',k)\,,\quad\varepsilon,\delta\ll 1\,,\end{aligned} \tag{4.12}$$

where the Chernikov-Tagirov-Bunch-Davies mode funtion is [68, 69],

$$U(\eta,k) = \frac{H}{\sqrt{2k^3}}\left[1+ik\eta\right]e^{-ik\eta}\xrightarrow{-k\eta\ll 1}\frac{H}{\sqrt{2k^3}}\,. \tag{4.13}$$

This implies that the late time limit of (4.12) is,

$$S(x;x') = \frac{H^2}{4\pi^2}\times\ln(A)\,,\qquad A=\min[a,a']\,. \tag{4.14}$$

The imaginary part of the propagator descends from inverting kinetic operators of the equation of motion and should be kept as is. This approximation is known to capture the leading infrared logarithms in massless scalar potential models to all loops [70–72], while the variant of this approximation adapted for light massive scalars is known to capture leading $H^2/m^2 \gg 1$ corrections to 2-loop order [73].

Applying this approximation to the one-graviton-loop correction to electromagnetism on de Sitter first requires expanding the photon field equation (4.10) in powers of graviton fluctuations,

$$-\eta^{\mu[\rho}\eta^{\sigma]\nu}\partial_\nu F_{\rho\sigma} - \frac{\kappa}{2}V^{\mu\rho\nu\sigma\alpha\beta}\partial_\nu\left[h_{\alpha\beta}F_{\rho\sigma}\right] - \frac{\kappa^2}{2}U^{\mu\rho\nu\sigma\alpha\beta\gamma\delta}\partial_\nu\left[h_{\alpha\beta}h_{\gamma\delta}F_{\rho\sigma}\right] + \mathcal{O}(\kappa^3) = J^\mu\,. \tag{4.15}$$

Here the 3- and 4-point vertex tensor structures are [17],

$$V^{\mu\rho\nu\sigma\alpha\beta} = \eta^{\mu[\rho}\eta^{\sigma]\nu}\eta^{\alpha\beta} + 4\eta^{\alpha)[\mu}\eta^{\nu][\rho}\eta^{\sigma](\beta}\,, \tag{4.16}$$

$$\begin{aligned} U^{\mu\rho\nu\sigma\alpha\beta\gamma\delta} = &\left[\frac{1}{4}\eta^{\alpha\beta}\eta^{\gamma\delta} - \frac{1}{2}\eta^{\alpha(\gamma}\eta^{\delta)\beta}\right]\eta^{\mu[\rho}\eta^{\sigma]\nu} + \eta^{\gamma)[\mu}\eta^{\nu][\rho}\eta^{\sigma](\delta}\eta^{\alpha\beta} \\ &+\eta^{\alpha)[\mu}\eta^{\nu][\rho}\eta^{\sigma](\beta}\eta^{\gamma\delta} + \eta^{\mu(\alpha}\eta^{\beta)[\rho}\eta^{\sigma](\gamma}\eta^{\delta)\nu} + \eta^{\mu(\gamma}\eta^{\delta)[\rho}\eta^{\sigma](\alpha}\eta^{\beta)\nu} + \eta^{\mu[\rho}\eta^{\sigma](\alpha}\eta^{\beta)(\gamma}\eta^{\delta)\nu} \\ &+\eta^{\mu[\rho}\eta^{\sigma](\gamma}\eta^{\delta)(\alpha}\eta^{\beta)\nu} + \eta^{\mu(\delta}\eta^{\gamma)(\alpha}\eta^{\beta)[\rho}\eta^{\sigma]\nu} + \eta^{\mu(\alpha}\eta^{\beta)(\gamma}\eta^{\delta)[\rho}\eta^{\sigma]\nu}\,. \end{aligned} \tag{4.17}$$

We subsequently look for a perturbative solution of the field strength,

$$F_{\mu\nu} = F^{(0)}_{\mu\nu} + \kappa F^{(1)}_{\mu\nu} + \kappa^2 F^{(2)}_{\mu\nu} + \mathcal{O}(\kappa^3)\,. \tag{4.18}$$

This is done by iterating the equation (4.15) to order κ^2,[4]

$$\begin{aligned} -\eta^{\mu[\rho}\eta^{\sigma]\nu}\partial_\nu F^{(2)}_{\rho\sigma}(x) = &\frac{\kappa^2}{2}U^{\mu\rho\nu\sigma\alpha\beta\gamma\delta}\partial_\nu\left[\langle h_{\alpha\beta}(x)h_{\gamma\delta}(x)\rangle F^{(0)}_{\rho\sigma}(x)\right] \\ &-\frac{\kappa^2}{2}\partial_\nu\left\{\int d^4x'\,\partial'_\sigma\partial'_\lambda G(x;x')V^{\mu\rho\nu\sigma\alpha\beta}\eta_{\rho\kappa}V^{\kappa\theta\lambda\phi\gamma\delta}\langle h_{\alpha\beta}(x)h_{\gamma\delta}(x')\rangle F^{(0)}_{\theta\phi}(x')\right\}, \end{aligned} \tag{4.19}$$

where the inverse of the flat space d'Alembertian $\partial^2 = -\partial_0^2 + \nabla^2$ is,

$$G(x;x') = -\frac{\theta(\Delta\eta)}{4\pi}\frac{\delta(\Delta\eta - \|\Delta\vec{x}\|)}{\|\Delta\vec{x}\|}\,. \tag{4.20}$$

The stochastic approximation then affects the graviton 2-point function (2.2), where the only contributing part is the one containing the A-type propagator,

$$\langle h_{\mu\nu}(x)h_{\rho\sigma}(x')\rangle \longrightarrow \left[2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} - 2\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}\right]S(x;x')\,, \tag{4.21}$$

where the stochastic sum $S(x;x')$ is defined in (4.12). Applying this prescription to the plane wave photon and to the Coulomb potential gives the following contributions,

$$F^{0i}_{(2)} = F^{0i}_{(0)} \times \frac{\kappa^2H^2}{2\pi^2}\ln(a)\,, \qquad \Phi_{(2)} = \Phi_{(0)} \times \frac{\kappa^2H^2}{2\pi^2}\ln(a)\,. \tag{4.22}$$

in the limit $\varepsilon \ll 1$. These contributions descend only from the first term on the right-hand-side of eq. (4.19), while the remaining nonlocal term provides no leading order contributions.

[4] Note that in (4.19) we have not included the contribution formally of the same order descending from the Einstein equation (4.9). This contribution corresponds to the gravitational response to the photon, and does not harbor any large logarithms.

Not only does the Coulomb potential contribution in (4.22) fail to capture the $\ln(Hr)$ term, but both contributions overestimate the $\ln(a)$ corrections (1.2)–(1.3) from the full computation, that are completely captured by the RG explanation of section 3. Upon closer examination, this discrepancy can be attributed to the lack of control over the cutoff parameter ε. While for scalar potential models taking the limit $\varepsilon \ll 1$ remarkably works out to capture the leading contributions, in theories with derivative interactions this is not so,[5] and the Hubble scale modes contribute relevant corrections, that for the system at hand have to cancel the contributions in (4.22).

The issue with applying the stochastic sum approximation to the graviton propagator is ultimately tied to derivative interactions, that are ubiquitous in gravity. The issues arising from derivative interactions are well illustrated by the mixed second derivative of the coincident propagator. The dimensionally regulated computation gives,

$$\langle \partial_\mu \phi(x) \partial_\nu \phi(x) \rangle = -\frac{H^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D)}{2\,\Gamma\left(\frac{D+2}{2}\right)} g_{\mu\nu} \xrightarrow{D\to 4} -\frac{3H^4}{32\pi^2} g_{\mu\nu}\,. \tag{4.23}$$

However, when we apply the stochastic sum truncation to this quantity one finds,

$$\langle \partial_\mu \phi(x) \partial_\nu \phi(x) \rangle \xrightarrow{a\to\infty} \frac{H^4}{8\pi^2}\left[\frac{1}{2} a^2 \delta^0_\mu \delta^0_\nu \varepsilon^4 + \frac{1}{3} \overline{g}_{\mu\nu} \varepsilon^2\right], \tag{4.24}$$

where $\overline{g}_{\mu\nu} = g_{\mu\nu} + a^2 \delta^0_\mu \delta^0_\nu$. Whereas the exact result (4.23) has a negative definite $\mu = i$, $\nu = j$ component, any stochastic mode sum such as (4.24) must produce positive definite results for the squares of operators. Derivative interactions prevent the affected fields from carrying infrared logarithms, in which case these fields make nonzero contributions of order one such as (4.23) that come as much from the ultraviolet as from the infrared. No stochastic mode sum can correctly describe these effects.

Another signal of problems in expression (4.24) is its strong dependence on the cutoff. This arises in the stochastic formalism when the approximate scale invariance of the super-Hubble modes is either not present at tree level, or is suppressed by derivative interactions. For example, applying the stochastic formalism to vector fields in axion inflation results in a truncation which is sensitive to the cutoff [75, 76]. Capturing large logarithms in these cases requires a systematic approach such as [9–11, 70–72, 77, 78].

5 Conclusions

The continuous production of gravitons during inflation is responsible for the tensor power spectrum [2] and for secondary effects involving interactions with themselves and other particles. In chronological order there have so far been six secondary, 1-loop effects reported on de Sitter background:

- Enhancement of the fermion field strength [5];
- Growth of the Coulomb potential in space and time [4];

[5] Another example is 1-scalar loop corrections to the photon wave function of scalar quantum electrodynamics [50, 74].

- Enhancement of the photon field strength [3];
- Enhancement of the graviton field strength [7];
- Spatial suppression of the massless, minimally coupled scalar exchange potential [6]; and
- Suppression of the Newtonian potential [8].

Prior to this work only the penultimate result had been given a Renormalization Group interpretation analogous to the stochastic-RG synthesis that was recently developed for nonlinear sigma models [9]. The terrific advantage of such an interpretation is that it permits an all-orders re-summation of the series of leading logarithms. So it is wonderful news that we have here been able to provide a Renormalization Group explanation for the factors of $\ln(a)$ discovered in 1-graviton loop corrections to the Coulomb potential (1.3) and the photon field strength (1.2). This was done in section 3.

We were not able to achieve a similar explanation for the factor of $\ln(Hr)$ in the Coulomb potential (1.3). Because this term derives from the nonlocal part of the vacuum polarization (see the second line of equation expression (3.1) for the structure function $F(x;x')$) the $\ln(Hr)$ does not appear to be associated with the mass scale μ, the way the scale factor $a(t)$ is through relation (3.10). In section 4 we searched for a compelling stochastic explanation for the factor of $\ln(Hr)$. We concluded that none exists. The successful stochastic formulation of nonlinear sigma models [9] was derived by integrating out the derivative interactions (4.6)–(4.8), whereas it is the vector potential which is differentiated in the electromagnetic field equation (4.10). Derivative interactions resist a stochastic interpretation because they mediate order one effects which derive from all parts of the dimensionally regulated mode sum, rather than just from the leading infrared part. On the other hand, we cannot integrate the vector potential out of its own equation (4.10), both because we want the resulting equation to describe electromagnetic effects and because the equation is linear in the vector potential. We suspect that the lack of an explanation for the factor of $\ln(Hr)$ may indicate that it should not be considered a leading logarithm effect.

In nonlinear sigma models, which show both stochastic and RG effects [9–11], there are really three things going on:

- The generation of curvature-dependent, effective forces by integrating out differentiated fields in the presence of an approximately constant background;
- The generation of stochastic jitter in the approximately constant background by the continual redshift of sub-horizon modes to the super-horizon; and
- The generation of secular logarithms through the incomplete cancellation (3.10) between curvature-dependent primitive divergences and counterterms.

As noted above, the first of these receives contributions from both ultraviolet and infrared, whereas the second is a purely infrared effect. We lump them both under the rubric of "stochastic" because the second cannot occur without the first, and we note again that there is no mechanism for producing the first thing in the present analysis. The third thing does happen in our analysis and it is driven by the combination of ultraviolet electromagnetic modes with the ultraviolet "tail" part of graviton modes.

The next step in our program is to attempt a similar explanation for the three remaining 1-graviton loop enhancements: the growing fermion field strength [5], and the effects on gravitational radiation [7] and on the force of gravity [8]. We anticipate that the fermionic effect will have a Renormalization Group explanation, as did the electromagnetic effects we considered here. However, explaining the two gravitational results may well require a stochastic analysis. That is as it should be because the graviton field is analogous to the single field Φ in the nonlinear sigma model analysis, and the factors of $\ln(a)$ in its mode function, exchange potential and expectation value all had a stochastic origin [9].

Another step in our program is deriving the beta function $\beta_{\kappa^2} \equiv \mu \frac{\partial \delta\kappa^2}{\partial \mu}$ so that we can use the Renormalization Group to derive all-orders results. This requires the portion of $\delta\kappa^2$ determined by a single loop of photons. A single matter loop of any sort induces two gravitational counterterms [79, 80],

$$\Delta\mathcal{L}_{\rm GR} = c_1 R^2 \sqrt{-g} + c_2 C^{\alpha\beta\gamma\delta} C_{\alpha\beta\gamma\delta} \sqrt{-g}\,, \tag{5.1}$$

where R is the Ricci scalar and $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor. The counterterm proportional to c_2 makes a higher derivative contribution term of no relevance to leading inflationary logarithms, however, the counterterm proportional to c_1 can be rewritten so that it contains a part proportional to the Einstein-Hilbert Lagrangian,

$$R^2 = \Big[R - D(D-1)H^2\Big]^2 + 2D(D-1)H^2 \Big[R - (D-1)(D-2)H^2\Big] + D(D-1)^2(D-4)H^4. \tag{5.2}$$

Just as we regarded the middle term of (3.3) as a curvature-dependent field strength renormalization so too we can think of the middle term of (5.2) as a curvature-dependent renormalization of Newton's constant,

$$\delta\kappa^2 = -2D(D-1)c_1\kappa^4 H^2\,. \tag{5.3}$$

Because the factors of $\ln(\mu)$ are associated with $\ln(a)$ according to relation (3.10), it should be noted that physical significance of our beta function differs from the usual sense in which a negative sign means that the theory becomes perturbative at high energy scales. For us it is the *positive* sign which betokens a perturbative theory at late times.

A final point is that this analysis has been made in the context of the simplest graviton gauge [29, 30]. We did not resolve the gauge problem, nor must we do so in order to explain the large logarithms generated within a single gauge. Of course we should eventually employ the procedure for purging gauge dependence [61, 62] to establish that the large logarithms are real, and to fix their numerical coefficients. Work on this is far advanced [6, 63] but analyses in quantum gravity are so difficult that it is best to report on one at a time.

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