Three complementary frameworks to capture student peer discussion in problem solving

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Peer discussions play a major role in students' collaborative problem-solving activity. These discussions provide researchers and teachers with a wealth of information about the students' reasoning. To analyse such discussions, different theoretical lenses are available, such as Schoenfeld's problem solving model, the Florida Taxonomy of Cognitive Behaviour, and the Scheme for Educational Dialogue Analysis. The question is, however, how these three perspectives can complement each other. To investigate this, the discussion between four students was analysed through the three lenses. Results indicate that these frameworks are both complementary and connected. This connection allows an in-depth analysis of the discussion and reveals possibilities and limitations for an integration of the three models, which will guide future discussions' analyses in our study.

Keywords: Peer discussion, problem solving, Schoenfeld, Florida taxonomy of cognitive behaviour, scheme for educational dialogue analysis.

Introduction

In mathematics classrooms, students work together to solve mathematics problems. During these collaborations, students talk over their peers' arguments or ask for clarification, and so on. Many studies have shown the effectiveness of peer discussion for students' conceptual development (Barth-Cohen et al., 2016; Crouch & Mazur, 2001; Wang & Murota, 2016). Mathematical discussions have been analysed from different perspectives. Some authors focus on the cognitive and the sociocultural perspective (Sfard & Kieran, 2001). Others look at these discussions from a dialogic perspective (Kazak et al., 2015) and mainly study the form of the student contributions. However, the issue is that most analyses in the literature are fragmented because each framework has a different perspective. There is no integrative framework that can tell us what factors affect the success or failure of a peer discussion. Additionally, guidance is needed to improve discussions in mathematical problem solving. Our short-term goal is to develop a framework to analyse peer discussions integrally; the long-term goal is to design learning activities in which mathematically productive discussions are promoted, including higher-order thinking skills. The pilot study described here is the first step towards a more comprehensive model. An extensive review of the literature revealed that three frameworks capture crucial aspects of peer discussion in problem solving best: Schoenfeld's problem-solving model, the Florida Taxonomy of Cognitive Behaviour (FTCB) and the Scheme for Educational Dialogue Analysis (SEDA). These frameworks are used to analyse a discussion between students. The research question addressed is as follows: Which aspects from the three different frameworks co-emerge in the analysis of a peer discussion?

Theoretical framework

The study's theoretical framework consists of the three frameworks mentioned above. First, Schoenfeld's (1985) framework for analysing problem-solving skills presents four categories of knowledge and behaviour for an adequate characterization of mathematical problem solving performance. These categories are necessary and sufficient for the analysis of the success or failure of someone’s problem-solving attempt: (a) *Resources*: Mathematical knowledge possessed by the individual; (b) *Heuristics*: Strategies and techniques for making progress on unfamiliar or nonstandard problems; (c) *Control*: The individual’s monitoring and self-regulation; global decisions regarding the selection and implementation of resources and strategies; (d) The individual’s *belief systems* and their origins in the student’s mathematical experiences. It is important to highlight that with good control, problem solvers can make the most of their resources and solve rather difficult problems with some efficiency. Without it, they can waste their resources and fail to solve problems within their grasp (Schoenfeld, 1985). Therefore, in this article we focus on the category control. We have split this category into two sub-aspects called resources control (RC) and heuristics control (HC) because control cannot exist by itself, but is always related to resources or heuristics.

The second framework (FTCB) can be used to measure the cognitive level of students’ input on all aspects of problem solving, regardless of content type. In this pilot study, we focus on the level of analysis students demonstrate when justifying their choice of mathematical content or strategy. FTCB is based on Bloom’s Taxonomy and is used as a tool to assign Bloom’s Taxonomy levels to utterances from the target audience. It was designed by Brown et al. (1966) and has been used frequently in different contexts, for example to investigate the level of cognitive behaviour exhibited by secondary agriculture teachers (Ulmer, 2005). While Bloom’s Taxonomy distinguishes six cognitive levels, FTCB uses seven. It is a hierarchical model, with each level of intellectual skills building on the previous. FTCB includes 55 behaviour descriptions, 21 of which are related to the three higher-order thinking codes: analysis, synthesis and evaluation. These descriptions indicate the observable behaviours that characterize each thinking level and help to classify the input in the dialogue.

As a third framework, we use SEDA to determine the type of input from individual students in the discussion. Many articles have been written about dialogue in education, and there is emerging consensus about the types of educational dialogue that seem to be productive for learning. These focus on atonement to others’ perspectives and the continuous co-construction of knowledge through sharing, critiquing, and gradually reconciling contrasting ideas (Littleton & Mercer, 2013). In SEDA the essence of dialogic interactions is operationalized as systematic indicators for these productive forms of educational dialogue (Hennessy et al., 2016). SEDA contains 33 codes, and the authors later developed a shorter version, named T-SEDA framework (Teacher Scheme for Educational Dialogue Analysis). T-SEDA focuses on key dialogue features, highlighting those that are known to be productive for learning (Kershner et al., 2020). We opted for the short version (T-SEDA) because it simplifies coding and allows us to see if further refinement is needed when coding the student discussion. Each category can be identified by words or short sentences that are uttered during the dialogue.

Methods

The set-up

Four first-year in-service student teachers (P, Q, R, S) from the Bachelor Mathematics Teacher Training Program at Utrecht University of Applied Sciences worked together to solve a mathematics problem from the Mathematics Olympiad training. Their curriculum included a course called problem-solving, in which the students were faced with mathematics problems in which various heuristics had to be applied. They are used to explaining things and it was expected that they would not give up easily since they had practiced enough with heuristics in the aforementioned course. We aimed to elicit a discussion with much interaction at different levels that could be analysed using the previously mentioned frameworks. The students were asked to solve the problem together and to interact with each other as much as possible. In the present paper, a fragment of the discussion that happened while the students worked on the problem is analysed.

The students worked on the following problem (Stichting Nederlandse Wiskunde Olympiade, 2022): *Figure 1 consists of a square ABCD and a semicircle with diameter AD outside the square. The sides of the square have length 1. What is the radius of the figure's circumscribed circle?*

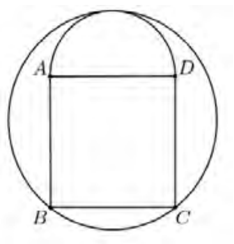

Figure 1: Problem situation

The students were alone in a room with no teacher or researcher nearby. They were videotaped and they handed in their written work afterwards.

The coding

Table 1: The three frameworks with their associated codes

Schoenfeld	FTCB	T-SEDA
resources control (RC); heuristics control (HC)	Knowledge (K); translation (T); interpretation (I); application (AP); analysis (A); synthesis (S); evaluation (E)	invite to build on ideas (IB); build on ideas (B); challenge (CH); invite reasoning (IRE); make reasoning explicit (R); coordination of ideas and agreement (CA); connect (C); reflect on dialogue or activity (RD); guide direction of dialogue or activity (G), express or invite ideas (EI)

The recording was transferred to Atlas TI for data analysis. Three researchers individually coded a fragment of this video using the three coding schemas, and then discussed their findings. The developers of T-SEDA were consulted to clarify the background and the choices made regarding their codes. Additionally, the three frameworks were presented to a focus group within the research group

'Mathematical and analytical skills of professionals' of the first author's institute to search for potential theoretical connections between the codes and to identify typical examples of these codes in practice. These activities contributed to the validity of our application of the frameworks and the reliability of the coding of this fragment, which was subsequently coded by one of the authors.

In the solution process, the students chose particular mathematical approaches or heuristics that determined the path to a possible solution. We describe these episodes as critical moments. Subsequently, three researchers separately determined the critical moments in the recording of the problem. In this paper we analyse one of these moments, which was identified by all three coders as particularly critical, since it sent the students on the wrong track for 80 percent of the time spent on this problem. The chosen episode of 16 minutes and 20 seconds was divided into clips. Each clip consists of one contribution in the form of a sentence or sentences per person.

Results

The mathematical approach and some examples of coding

The problem was not solved, but several attempts were made. In the chosen episode a mathematically wrong approach was proposed by student P. The entire group engaged in this approach during this episode. Figure 2 shows that P draws the centre of the circumscribed circle and then makes a triangle by connecting this point to point B and the intersection of the vertical centreline of the circle and the circle itself. P assumes that angle B is 90°. P also chooses an unknown x which she then tries to calculate using the Pythagorean theorem. This results in three equations with four unknowns. P then looks for a fourth equation to solve the problem. It does not work. The episode ends when P arrives at an, considering Figure 2, unlikely conclusion that $y = x/4$.

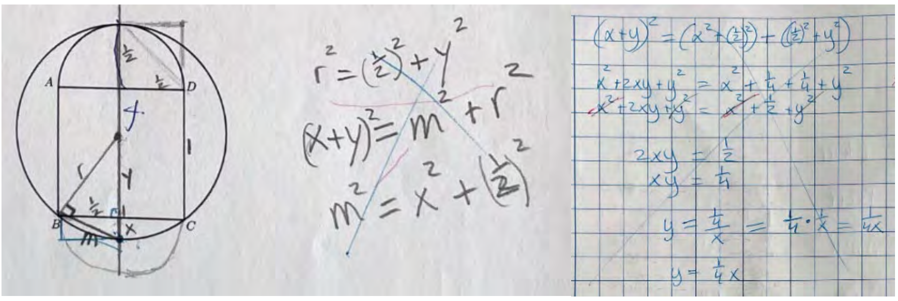

Figure 2: Student P's scratch paper

Student P is thinking aloud during the solution process. P says that angle B is 90° because of a mathematical rule that she cannot remember the name of. Q asks: *Is that 90°?*(RC; CH). Student P replies: *Yes, that is 90°* (A; R). Q does not continue questioning, and then this approach is worked out by P while Q, R and S watch. No one takes action to verify P's claim. Then P said: "Maybe my approach is too complicated, I don't know if I can solve this" (HC; RD) The rest of the group did not react to this. When P seemed to get stuck, Q and R suggested a different mathematical approach. P rejected these ideas through correct substantive arguments (RC; A; CH; R). Ten minutes later S mumbles "we are making things too complicated"; here again no response from the group (HC; RD).

The overlap between the three frameworks

The tables 2 and 3 and the diagrams 1 and 2 give the co-occurrence between the codes of the three frameworks. This sheds light on the question how the three frameworks complement each other and how they overlap. For this study, we chose to focus on two codes: Schoenfeld's Resources control (RC) and Heuristics control (HC) and their relations with the codes of T-SEDA and FTCB. The number after each code in parentheses in tables 2 and 3 shows how often the code is applied in the entire project. The number in the cell indicates the number of hits, how often the two codes co-occur. For example, the evaluation code was assigned 9 times throughout the project, and 3 of them coincided with the RC code. The row and column entities of the tables are represented in the corresponding charts (Fig 3 and 4) as nodes and edges, showing the strength of co-occurrence between the pairs of nodes.

Table 2: Co-occurrence between the two control codes and five FTCB codes

	Resources control (19)	Heuristics control (3)
Evaluation (9)	3	0
Analysis (22)	6	0
Application (16)	1	0
Translation (1)	0	1
Knowledge (11)	1	0

Figure 3: Co-occurrence between the two control codes and five FTCB codes

Table 3: Co-occurrence between the two control codes and six T-SEDA codes

	Resources control (19)	Heuristics control (3)
Invite to build on ideas (15)	3	0
Build on ideas (4)	1	0
Challenge (15)	6	0
Invite reasoning (7)	3	0
Make reasoning explicit (61)	5	1
Reflect on dialogue/activity (9)	0	2

Figure 4: Co-occurrence between the two control codes and six T-SEDA codes

Conclusion

Three frameworks were used to analyse a discussion between four students. Schoenfeld describes the categories that play a role in mathematical problem-solving. The thinking level of the students was studied through the lens of FTCB and the character of the personal input was classified with T-SEDA. We focused on one of the most important factors that determine the success or failure of mathematical problem-solving (Schoenfeld, 1985), namely: resources control and heuristics control. We observed that resources control in our data was more prevalent than heuristics control. That means that more mathematical knowledge was discussed and thereafter applied or rejected, during the search for the solution, than strategies or heuristics.

It should also be noted that the level of thinking displayed during the moments when students rejected different mathematical approaches was generally high. The expressions that were observed linked to resources control were: points out unstated assumption, shows interaction or relation of elements, points out particulars to justify conclusions, checks hypotheses with given information, distinguishes relevant from irrelevant statements and detects error in thinking. Several moments were observed in which the mathematical results were evaluated from evidence and from criteria. The code translation is connected to heuristics control only once. That is a lower order thinking skill according to FTCB. In this case, the students translate verbalization into graphic form.

The moments in the discussion when doubts about the approach came to light were, unfortunately, not further analysed by the students. The opportunity to take a different, more promising path, was lost as a result of the approach not being evaluated. In this case, these doubts were caused by the fact that the students did not make any progress with the problem because the mathematical fallacy at the start had not been discovered sooner. It appears that if this particular mathematical approach had been evaluated earlier, it might have led to a change in strategy. Whereas when the students focussed their evaluation on resources, they failed to evaluate heuristics, leading to ineffective, time-consuming problem-solving behaviour. This study therefore shows the need to investigate further how we can teach students to assess their problem-solving approach earlier in order to improve their problem-solving skills. The analysis of the connections of resources control and heuristics control to the T-SEDA codes may provide insights to address this issue.

Figure 4 illustrates a striking phenomenon about the relation between T-SEDA and the control codes. The codes: invite to build on ideas, build on ideas, challenge, and invite reasoning were only linked to resources control. The moments of analysis and evaluation described earlier were often started when a student challenged others, asked them to build on ideas, or explained their reasoning more clearly. These invitations to encourage others to actively participate in the discussion were followed by explicit reasoning. Consequently, thinking errors were discovered and hypotheses were tested, only referring, however, to the manipulation of mathematical knowledge but not about the approach. It is remarkable that heuristics control is linked to the T-SEDA code reflection. Indeed, those moments of reflection were individual expressions that provoked no response from the rest of the group. These seem to be missed opportunities to change the approach. Therefore, it seems plausible that moments of reflection (T-SEDA) should naturally cause moments of evaluation (FTCB), but in this case, they did not.

According to the literature (Kershner et al., 2020), T-SEDA offers an opportunity to teach students how to discuss constructively, for example, by challenging others or inviting others to express ideas. This pilot research indicates that the kind of contributions classified by T-SEDA are strongly connected to the level of thinking (FTCB) involved when students respond to the expressions of others. For example, inviting someone to reason implies a certain level of analysis. However, there are also striking missing connections between T-SEDA and FTCB, for example, reflecting on the dialogue is not necessarily accompanied by evaluation. By understanding these connections, it may be possible to improve mathematical problem-solving, in particular Schoenfeld's crucial category heuristics control. Then we conclude that the three frameworks are complementary. We also suspect that they are connected at a deeper level where different aspects of these frameworks may influence each other. More research is needed to explore these connections further.

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