# Activity Analysis

Thijs ten Raa

## Contents



**Abstract**

This chapter opens with the historical root of activity analysis. The framework of activity analysis admits multiple techniques to produce a commodity. Substitution theorems investigate when the market mechanism singles out a best technique for each product and if the best techniques vary with the data of an economy, such as resource availabilities. Houthakker's Theorem initiated a literature on the relationship between the distribution of activities and the form of the aggregate production function. Activity analysis is connected to modern input-output analysis, where the numbers of products and industries differ, which facilitates the measurement of the efficiency of the production units of an economy and of the economy.

T. ten Raa (✉)
Utrecht School of Economics, Utrecht University, Utrecht, The Netherlands
e-mail: tenraa@uvt.nl



S. C. Ray et al. (eds.), *Handbook of Production Economics*,
https://doi.org/10.1007/978-981-10-3450-3_25-2

**Keywords**

Activity analysis · Efficiency · Houthakker's theorem · Input-output analysis · Substitution theorem

## Introduction

This chapter shows how activity analysis is a basis of production functions. One of the advantages of activity analysis is that it offers a framework for the measurement of efficiency. Given the ensemble of all activities, one may determine the frontier and then measure how close activities are to the frontier. There are two problems: First, in which direction should one go to measure the gap between an activity and a best practice frontier element? Second, how should one aggregate the efficiencies of the individual activities? Both issues are solved in the recent literature, and these results neatly complement neoclassical production analysis with its explicit or implicit assumption of cost minimization.

In the section "The Origin of Activity Analysis," I discuss the historical root of activity analysis. The framework of activity analysis admits multiple techniques to produce a commodity. An important economic question is: Does the market mechanism single out a best technique for each product? If so, does the best technique vary with the data of an economy, such as resource availabilities? Substitution theorems are discussed in the section "Substitution." Section "Activity Foundation of the Production Function" discusses in detail the relationship between the distribution of activities and the form of the production function. The classic result is Houthakker's Theorem. Section "Variants of Houthakker's Theorem" discusses the further literature on this relationship. Section "Activity Foundation of Input-Output Analysis" connects activity analysis to modern input-output analysis, where the numbers of products and industries differ. Section "Efficiency" analyzes the measurement of the efficiency of the production units of an economy and of the economy.

## The Origin of Activity Analysis

Activity analysis originates from Tjalling Koopmans' [10] *Activity Analysis of Production and Allocation*, a conference volume. The conference was on linear programming, and indeed, activity analysis is still considered a practical tool for the analysis of production and allocation. The conference volume is an icon in the history of economic thought. In 1975, his research on this topic earned Dutch American Koopmans, jointly with Russian Leonid Kantorovich, the Nobel Prize in economics. Incidentally, Koopmans was named by the then young William Baumol, who sadly died during the writing of this chapter. Baumol's [1] *Economic Theory and Operations Analysis* popularized the use of operations research in economics and was a leading microeconomic textbook until well into the 1980s.

Koopmans' *Activity Analysis of Production and Allocation* consists of four parts: theory (ten papers), applications (six papers), convex analysis (four papers), and algorithms (five papers). The third and fourth parts are now standard fare in applied mathematics and software, greatly facilitated by recent computer power. The second part is a collection of assorted papers. The first part is the most important one, at least for our objective, an overview of activity analysis in the context of production economics.

The first part of *Activity Analysis of Production and Allocation* begins with Dantzig's mathematical programming, and Koopmans' "Analysis of Production as an Efficient Combination of Activities," continues with Von Neumann and Leontief's dynamic models and completes with five (!) papers on the static Leontief model. Indeed, there is a close connection between input-output analysis and activity analysis, both at the level of concept and of application. To begin with the latter, mind that activity analysis blossomed shortly after World War II. Planning was an important policy tool, not only in Russia, but also in the United States. Warfare prompted a change in demand (toward aircraft and other equipment), and this had to be reconciled with resource scarcity.

## Substitution

At the conceptual level, an *activity* is a pair of an *input* vector and an *output* vector. Leontief, who won the Nobel Prize 2 years earlier, was ahead of Koopmans and activity analysis, but the latter, not surprisingly, offers a more general framework for economic analysis. By the same token, input-output analysis is a special, albeit important, case of activity analysis. This is particularly true of static input-output analysis. Here an activity is a pair of an input vector and a pure output vector, where "pure" means that only one component is nonzero. Joint production is ruled out.

Dynamic input-output analysis is more general. It features two types of inputs, namely, absorbed inputs and capital, and the output is basically still pure, but accompanied by the same capital. There may be depreciation, but that is modeled by including it in the absorbed inputs. The main simplification of input-output analysis is the implicit assumption that for every output there is only one activity that produces it. Each product has a unique input structure. Economists, particularly students of Leontief, starting with Paul Samuelson, were intrigued by this simplifying, implicit assumption and theorized about it. Samuelson, Koopmans, and Arrow each contribute a paper on this subject to Koopmans [10].

The result of their analysis is the substitution theorem. If activities are pure, but different ones coexist for the production of commodities, and if there is only one nonproduced input, called "labor," then there exists a collection of activities, one for each product, that minimizes the labor input of the net output vector produced, irrespective of the proportions of the net output vector. Except for labor, commodities feature in output vectors and input vectors, and the difference between the two is net output, available for household consumption. Thus, the substitution theorem states that there is a dominant technology to produce net output, whatever its commodity composition. An activity is also called a “technique,” and a collection of available activities or techniques is also called a “technology.” For an obvious reason, the substitution theorem – industries can substitute techniques but do not do so when minimizing the resource use – is also called the nonsubstitution theorem.

Activity analysis sticks to the input-output assumption of constant returns to scale. Consequently, the minimization of the resource input for a given level of net output is equivalent to the maximization of the level of net output given a resource input. This observation is obvious in the context of the substitution theorem (a single nonproduced input and pure output vectors) and remains valid when net outputs are not pure but feature multiple nonzero components. For example, if a given net output vector is producible with only 4/5th or 80% of the actual use of the observed amount of resource, then, under constant returns to scale, the actual amount of resource could produce 5/4th or 125% of the given net output vector. We say the efficiency of the economy is 80%. Equivalently, potential output is 125% of actual output. In determining these performance measures, the collection of available activities is considered to be given, but the intensities with which each activity is run are to be determined by the mathematical program that minimizes the resource requirement or maximizes the level of net output. There are two constraints in either program. Commodity balances require that the activity intensities are large enough so that the supply of net output is at least equal to household demand, but small enough so that production demand for the resource does not exceed the available stock.

The constraints pick up Lagrange multipliers: a factor reward for the resource and also shadow prices for the produced commodities. The shadow prices, including the one for the factor input, fulfill the so-called dual constraints. The dual constraints are such that all activities have nonpositive profit and the activities running with positive intensity break even. Therefore, the shadow prices are competitive prices. Competitive prices would signal to entrepreneurs which activities to undertake. The competitive prices are also a useful analytical tool. An advantage is that in the general activity model, there are more activities than commodities, thus allowing substitution. Therefore, the dual variables (the prices) have lower dimension than the primal variables (the activity intensities). Johansen [8] approached the substitution theorem using the competitive prices, and ten Raa [21] filled the gaps.

The substitution theorem has the striking result that prices are independent of demand, hence determined by supply, more precisely, by technology. This is classical economics. It rests on the classical assumptions of a single nonproduced input and no joint production. The classical assumptions have a built-in tension. The assumption of a single nonproduced input, labor, suggests that capital is a produced commodity. We have no difficulty with this view. Capital is buildings, machinery, equipment, and infrastructure, and these are produced commodities indeed. However, the essence of productive capital is that at least some of it (after correcting for depreciation) remains when an activity has been completed. But then, the activity has at least two outputs: the commodity produced and the remaining capital. In other words, there is joint production. There are two approaches to deal with this issue. The first, going back to Von Neumann [25], is to accommodate multiple positive output components in the activity vectors. However, Von Neumann trades off this generality on the supply side against more specificity on the demand side, assuming that labor services are the output of the household activity with fixed consumption coefficients, which essentially is a classical, Marxian assumption. The second is to accommodate joint production and to analyze to which extent substitution emerges [15].

A more blatant violation of the substitution theory assumptions occurs when there are multiple nonproduced inputs, for example, labor and land, or, as in many neoclassical economic models, labor and capital, where at any point of time the latter is considered to be given by the past. Then the choice of techniques will depend on the composition of the resources. A relatively more labor-endowed economy will employ more labor-intensive techniques when maximizing the level of output. The argument is simple, particularly when the relative factor intensities range from very low to very high values compared to the endowment ratio. Then both endowments can be fully employed, and the average factor intensity in production will be equal to the endowment ratio. When East Germany was absorbed, labor became less scarce and capital more so. The shadow price of labor became smaller, supporting more efficient labor-intensive production. This reasoning pertains to the potential output of the economy. The actual economy may have followed a different, less efficient path.

## Activity Foundation of the Production Function

The economy has a supply side, populated with producers, and a demand side, populated with consumers. Center pieces of the supply side are production functions. Production functions have different functional forms. Implicit are alternative degrees of substitution and scale economies [5]. In activity analysis, however, the situation is more basic. An activity is like a recipe. There are input requirements per unit of output. Output may be multidimensional as well, for example, juice may be a byproduct when cooking. Differences in output are accompanied by differences in inputs. And even when there are no differences in outputs, such as in an industry producing a homogeneous product, there may be differences in inputs. Moreover, alternative activities may produce the same output. Local conditions vary, alternative production techniques compete, and some production units are simply less efficient than others, a phenomenon which shows in a different (higher) input structure.

Activities replace each other. For example, when a new supply of some resource is discovered, activities which make relatively intensive use of this resource will expand. The economy will use a different mix of inputs. This, indeed, may be described by a production function, but it is an interesting question how differences in activities translate into alternative functional forms of production.

In activity analysis, alternative techniques to produce commodities coexist, and the efficient ones are determined using a mathematical program in which intensities, one for each activity, that is, technique, are the variables. How can we reconcile this framework with a neoclassical production function, such as the Cobb-Douglas function, $Y = AK\alpha L\beta$? Here $K$ and $L$ are inputs, $Y$ is output, and $A$, $\alpha$, and $\beta$ are parameters. There are decreasing/constant/increasing returns to scale if $\alpha + \beta < / = / > 1$, respectively.

A simplistic link would be as follows, for the case of constant returns to scale. An activity would be a pair of variable inputs and an output, which can be normalized, ($k$, $l$; 1), with $Ak^{\alpha\, l\, \beta} = 1$. The activities can be parameterized by one input, for example, $k$. Then $l = (Ak\alpha)^{-1/}\beta$, and therefore the technology set of activities is $\{(k, (Ak\alpha)^{-1/}\beta; 1) | \, k > 0\}$. Each activity can be run with intensity $s\ k$. Total output will be $\int s\, k\, dk$, where the integral is taken over the positive numbers. The constraints are $\int s\, k\, k dk \leq K$ and $\int s\, k\, l dl \leq L$, where $K$ and $L$ are the factor endowments. Because of the convexity of the technology set and the assumption of constant returns to scale, $\alpha + \beta = 1$, running different activities with positive intensity can be improved upon, in terms of output, by replacement of the activities by their intensity-weighted average. Hence, output is maximized by running the single activity with the right factor intensity that matches the endowments ratio, $k/l = k(Ak\alpha)^{1/}\beta = K/L$. Solving, using $\alpha + \beta = 1$, $k^* = (K/L)\beta/A$. The intensity is determined by $s\ k^*\ k^* = K$, hence $s\, k_* = K(K/L)^{-}\beta A = AK\alpha L\beta$. All other intensities $s\ k$ are zero. Since activities were normalized by output and returns to scale are constant, output equals the activity intensity, $Y = AK\alpha L\beta$. In other words, the aggregate production function is the same as the underlying microtechnology.

In this simple activity analytic underpinning of the aggregate production function, all production units are free to choose from a continuum of activities, from labor to capital intensive. And all production units would select the same activity. This extreme flexibility, with its concentrated optimal activity pattern, is not very realistic. Capital intensities of production units are fixed once installed and vary across production units. Individual production units cannot access the full menu of activities, technology. In activity analysis, it is customary to assume that production units have given techniques and are represented by their activities. The implicit assumption is that production units cannot substitute inputs. However, at the macrolevel, substitution may take place. When a factor input becomes abundant, such as labor in the time of German unification, its price will go down, making activities with intensive use of the abundant factor input financially feasible. The subpopulation of active production units will shift to the more intensive users of the abundant endowment. In this framework, we better do not assume that production intensities can vary freely from zero to infinity. If so, then a single production unit with the right factor intensity, which matches the endowments ratio, would pick up all activity. The result would be the same as in the simplistic world were all production units to have access to the full technology.

In line with the factor specificity of an activity, it is assumed there is a capacity constraint for each activity. A fixed input causes the capacity constraint. The fixed input is other than the variable inputs, capital, and labor. Houthakker [7] suggests entrepreneurial resources. The distribution of the capacity constraint (of entrepreneurial resources) over activities ($k$, $l$; 1) is considered to be given, $y(k, l)$. This distribution need not be concentrated on a frontier like $\{(k, l) | \, Ak\alpha\, l\beta = 1\}$. Some activities may dominate others, with both components of ($k$, $l$) smaller. Yet a dominated activity may be run, because the superior activity, like all activities, has a capacity constraint. Activities can be run with intensities $0 \leq s(k, l) \leq y(k, l)$. Subject to the factor constraints $\iint s(k, l) k dk dl \leq K$ and $\iint s(k, l) l dk dl \leq K$, we maximize output $\iint s(k, l) dk dl$. This is a linear program with a continuum of variables $s(k, l)$. Denote the shadow prices of the two factor constraints by $r$ and $w$, respectively. By the phenomenon of complementary slackness, unprofitable activities, with unit cost $rk + wl > 1$, are not run, $s(k, l) = 0$. By the same argument, profitable activities, with unit cost $rk + wl < 1$, are run at full capacity, $s(k, l) = y(k, l)$. Activities which break even, $rk + wl = 1$, have activity $0 \leq s(k, l) \leq y(k, l)$, but since the set of such activities has measure zero we may set $s(k, l) = y(k, l)$. It follows that inputs and output are $K = \iint_{rk+wl \leq 1} y(k, l) k dk dl$, $L = \iint_{rk+wl \leq 1} y(k, l) l dk dl$, and $Y = \iint_{rk+wl \leq 1} y(k, l) k dk dl$, respectively. The implicit assumption is that all factor input can be fully employed. There must be activities with factor intensity $k/l$ below endowment ratio $K/L$ and activities with factor intensity above the endowment ratio.

The three expressions, for inputs $K$ and $L$ and output $Y$, are interrelated by the two shadow prices $r$ and $w$. The idea of Houthakker [7] is to use the first to expressions to solve for $r$ and $w$ in terms of $K$ and $L$. Substitution of the results in the third expression yields output as function of the inputs. Houthakker [7] carries out this calculation for the capacity distribution with Pareto density function, $y(k, l) = \mu k \kappa^{-1} l \lambda^{-1}$, where $\mu$, $\kappa$, and $\lambda$ are positive constants. The result is $Y = AK \alpha L \beta$ with $\alpha = \kappa(\kappa + \lambda + 1)$, $\beta = \lambda(\kappa + \lambda + 1)$, and $A$ a positive constant depending on $\mu$, $\kappa$, and $\lambda$. In other words, a Pareto capacity distribution yields a Cobb-Douglas production function. This is Houthakker's Theorem. At the microlevel, activities have fixed input-output ratios – it takes given amounts of labor to operate given machinery and equipment – but a change in resources, such as the inclusion of the East German labor force, is accommodated by the activation of new activities and the deactivation of some incumbent activities. Reallocations of resources across activities manifest as substitutions.

The capacity distribution is not concentrated on a single isoquant in input space. Both $k$ and $l$ can be bigger. In solving the output maximization, smaller input combinations are activated, but only to full capacity. Residual inputs are employed by more input-intensive activities. The capacity constraints thus yield decreasing returns to scale. Indeed, the Cobb-Douglas function has exponents summing to a number less than one. Houthakker's activity foundation of neoclassical production functions works only if returns to scale are decreasing.

## Variants of Houthakker's Theorem

Clearly, different capacity distributions for the activity levels will generate different production functions. Houthakker [7] has generated a stream of theoretical and applied research, to date. The bulk of this literature features a lower dimension, with only one variable input, namely labor, and again one fixed output, which is now capital. In this one fixed-one variable input framework, Levhari [14] found the capital distribution for which total output is a CES function of the total fixed input (capital) and the total variable input (labor) and showed it encompasses the

Cobb-Douglas function. Muysken [16] has consolidated the Cobb-Douglas, CES, and VES functions by showing they are all generated by beta distributions, with alternative parametrizations. Two books on the distribution approach to production are Johansen [9] and Sato [19].

In this literature, activities have fixed input-output proportions, and capacity constraints explain the existence of inefficient activities. Increases in levels of inputs prompt the activation of less efficient activities, in Ricardian style. The law of one price yields rents to the more efficient activities. The activation of different activities prompts different proportions between the input totals and the output. Substitution is considered a symptom of the change in the range of active activities (run with positive intensity).

The interrelation of total output to two inputs is a shortcut with a strong macroeconomic flavor (e.g., Lagos [12]). One way to reconcile economy-wide analysis with activities is to aggregate in stages, from activities (production units) via conglomerates (industries) to the economy. In the second stage, one has to aggregate production functions more general than fixed proportions functions, also called Leontief functions. In the one variable-one fixed output framework [18] analyzed how micro-CES functions and an appropriate inefficiency distribution (reflecting capacity constraints) generate a macro-CES function, with a greater elasticity of substitution (for the same reason as capacity, constraints create substitutability when the microfunctions are Leontief).

Growiec [6] generalizes the capacity distribution with Pareto density function, $y(k, l) = \mu k^{\kappa-1} l^{\lambda-1}$. He keeps the multiplicative structure, in other words the independence of the unit factor productivities, $F\,a$ and $F\,b$. For each $(K, L)$, firms maximize CES output $A[\psi(bK)\theta + (1 - \psi)(aL)\theta]^{1/\theta}$ with respect to unit factor productivities $a$ and $b$, subject to $Fa(a)Fb(b) = N$, where $N$ indexes the technology frontier, $0 < N < 1$. If $Fa(a) = c\,a\,a\,\gamma,\ F\,b(b) = c\,b\,b\,\gamma\alpha^{/(1-\alpha)}$, then maximum output is $AK\alpha\,L^{1-\alpha}$. This is the case where a Pareto distribution of unit factor productivities and free choice of technology yields Cobb-Douglas output. However, the mechanism is very different than in Houthakker [7]. In Growiec [6], firms freely choose from a menu which is parametrized by a distribution. The formal similarity – a Pareto distribution translates into a Cobb-Douglas function – is coincidental. In Growiec, the Weibull distribution translates into a CES function, while this is not the case in Levhari's [14] CES analysis of the Houthakker [7] model.

## Activity Foundation of Input-Output Analysis

The two-stage aggregation, from activities via industries to the economy, is a useful framework to accommodate the output differences between industries and to relate their inefficiencies. We return to the definition of an activity: a pair of an *input* vector and an *output* vector. Unlike traditional input-output analysis, modern activity analysis allows for multiple outputs and even different numbers of outputs and production units or industries. The advantage of input-output analysis, the accommodation of intermediate inputs, is preserved though. An input vector consists of $m$ commodities and $l$ factor inputs. An output vector consists of $m$ commodities (the same as in the input vector). There are $n$ production units, that is, activities. The first production unit has produced inputs ($u_{11}, \ldots, u\ m_1$), factor inputs ($f_{11}, \ldots, f\ l_1$), and outputs ($v_{11}, \ldots, v\ m_1$). Here $f$ stands for factor, $u$ stands for use, and $v$, the next letter in the alphabet, stands for output. Writing these vectors as column vectors (as the index notation suggests), and stacking the column vectors representing the other production units next to them, the $n$ activities are represented by the triplet of $l \times n$-dimensional factor input matrix $F$, $m \times n$-dimensional intermediate input matrix $U$, and $m \times n$-dimensional output matrix $V$. If all activities are included, the economy is represented by the triplet ($F$, $U$, $V$) and the nonnegative $l$-dimensional available resource vector $\omega$.

An *allocation* is a nonnegative $n$-dimensional activity vector, $s$, where the $i$-th component is the scale of production of unit $i$. For example, if $s\ i = 1.1$, all inputs and outputs of activity $i$ are 10% greater than observed. An allocation is *feasible* if $Fs \leq \omega$. Intermediate demand is $Us$. Gross output is $V$s. Net output is the difference, $(V - U)\ s$. This is final consumption.

In traditional input-output analysis, the number of activities equals the number of commodities, i.e., $m = n$. In this literature, gross output Vs is denoted $x$ and net output $(V - U)\ s$ is denoted $y$. It is reasonable to assume that output matrix $V$ has a dominant diagonal. Then $V$ is invertible, and the choice between the allocation variable $s$ of activity analysis and the gross output variable $x$ of input-output analysis is a matter of a change of variable, $x =$ Vs and $s = V^{-1}\ x$. The material balance $y = (V - U)\ s$ can be rewritten as Leontief's [13] basic equation, $y = x - Ax$. Here $A$ is the matrix of input-output equations determined by $A = UV^{-1}$. The latter specification is the so-called commodity technology model, which has superior balance and invariance properties [11]. The upshot is that activity analysis encompasses input-output analysis.

In the System of National Accounts [2], the number of commodities is greater than the number of industries: $m > n$. Standard input-output analysis is problematic, but activity analysis remains doable [24].

When input and output data are used in raw form, at the level of reporting production units, without aggregation to industries, the number of activities is greater than the number of commodities, $m < n$. In this case, there is a wealth of data, and activity analysis facilitates stochastic input-output analysis. The commodity technology model, $A = UV^{-1}$, does not exist when output matrix $V$ is not square, but input-output coefficients matrix $A$ can be estimated by regressing inputs $U$ on outputs $V$: $U = AV + \varepsilon$, including an error term [17].

## Efficiency

An allocation is *efficient* if no other allocation is better to one consumer without being worse to the other consumers. Observed allocations tend to be inefficient. The efficiency of an economy is measured by Debreu's [4] coefficient of resource utilization, $\rho$, a number between 0 and 1. $\rho$ is the lowest number such that all consumers can be equally well off if the endowment is reduced from $\omega$ to $\rho\omega$. The coefficient of resource allocation depends on the preferences of the consumers. If there is much substitutability, there is much scope for reallocations and, therefore, for potential efficiency gains. In this case, the coefficient of resource utilization will be low. Conversely, ten Raa [22] has shown that if the consumers have Leontief preferences (consumption with fixed proportions), the coefficient of resource utilization attains its upper bound. In other words, the assumption of Leontief preferences yields a conservative inefficiency measure. ten Raa [22] coins this measure the Debreu-Diewert coefficient of resource allocation.

The Debreu-Diewert coefficient of resource allocation, by its assumption of fixed consumption bundles, rules out efficiency gains due to consumers' exchanges. If Debreu's coefficient of resource allocation is 0.7 and the Debreu-Diewert coefficient is 0.8, then the difference represents consumer inefficiency. In this example, overall inefficiency is 30%, production inefficiency 20%, and consumer inefficiency 10%. ten Raa [22] shows that microdata of final consumption are not needed to calculate the Debreu-Diewert coefficient of resource utilization.

The calculation of the Debreu-Diewert coefficient of resource allocation is simple in the activity framework of the economy. ten Raa [22] shows that the better set of Pareto noninferior allocations is $\{s \geq 0 | (V - U)s \geq (V - U)e\}$, where $e$ is the unit or summation vector with all components equal to one and the inequalities are commodity constraints. Over this set, one must minimize $\rho$ subject to feasibility condition $Fs \leq \rho\omega$. This is a linear program. The Lagrange multipliers of the commodity and factor constraints are competitive prices. By the phenomenon of complementary slackness, activities with positive activity level break even, and unprofitable activities are shut down. In other words, the principle of profit maximization selects the activities that minimize resource use.

The competitive commodity prices can be used to evaluate the net output growth, competitive factor rewards are used to evaluate the factor input growth, and the difference is *total factor productivity growth* (TFP). A classical result is that for perfectly competitive economies, TFP equals the shift in the production function or *technical change* (TC; see Solow [20]). In general, TFP equals the sum of TC and the change in the Debreu-Diewert coefficient of resource utilization or, briefly, *efficiency change* (EC; see ten Raa [22]). Both components can be decomposed further in numerous ways.

The decomposition of efficiency involves a bias issue. The efficiency of a system of production units is less than the average efficiency of the production units. The reason is that the allocation of resources may be inefficient. This bias issue was first analyzed by Blackorby and Russell [3] who demonstrated that only when production is linear in the sense that marginal rates of substitution and marginal rates of transformation are constant and these constants are common to the production units, there is no bias issue. ten Raa [23] showed that the bias measures the inefficiency of the industrial organization of the production units.

## Conclusion

Activity analysis bridges the gap between input-output analysis with its fixed input proportions and neoclassical production theory with abounding substitutability, using the former as a foundation of the latter and showing that the latter encompasses the former. Substitutability of factor inputs is a manifestation of reallocations between activities with different factor intensities. Activity analysis accommodates efficiency analysis by measuring and decomposing inefficiencies.

## Cross-References

► Multiproduct Technologies
► Neoclassical Production Economics: An Overview

## References

1. Baumol WJ (1961) Economic theory and operations analysis. Prentice-Hall, New York
2. Beutel J (2017) Chapter 3. The supply and use framework of national accounts. In: ten Raa T (ed) Handbook of input–output analysis. Edward Elgar, Cheltenham
3. Blackorby C, Russell RR (1999) Aggregation of efficiency indices. J Prod Anal 12(1):5–20
4. Debreu G (1951) The coefficient of resource utilization. Econometrica 19(3):273–292
5. Diewert WE, Fox KJ (2008) On the estimation of returns to scale, technical progress and monopolistic markups. J Econ 145(1):174–193
6. Growiec J (2008) Production functions and distributions of unit factor productivities: uncovering the link. Econ Lett 101(1):87–90
7. Houthakker HS (1955) The Pareto distribution and the Cobb-Douglas production function in activity analysis. Rev Econ Stud 23(1):27–31
8. Johansen L (1972) Simple and general nonsubstitution theorems for input–output models. J Econ Theory 5(3):383–394
9. Johansen L (1972) Production functions: an integration of micro and macro, short run and long run aspects. North-Holland, Amsterdam
10. Koopmans TC (1951) Activity analysis of production and allocation. Wiley, New York
11. Kop Jansen P, ten Raa T (1990) The choice of model in the construction of input–output coefficients matrices. Int Econ Rev 31(1):213–227
12. Lagos R (2006) A model of TFP. Rev Econ Stud 73(4):983–1007
13. Leontief WW (1936) Quantitative input and output relations in the economic system of the United States. Rev Econ Stat 18(3):105–125
14. Levhari D (1968) A note on Houthakker's aggregate production function in a multifirm industry. Econometrica 36(1):151–154
15. Mirrlees JA (1969) The dynamic nonsubstitution theorem. Rev Econ Stud 36(1):67–76
16. Muysken J (1983) Transformed beta-capacity distributions of production units. Econ Lett 11(3):217–221
17. Rueda-Cantuche J (2017) Chapter 4. The construction of input–output coefficients. In: ten Raa T (ed) Handbook of input–output analysis. Edward Elgar, Cheltenham
18. Sato K (1969) Micro and macro constant-elasticity-of-substitution production functions in a multifirm industry. J Econ Theory 1(4):438–453

19. Sato K (1975) Production functions and aggregation. North-Holland, Amsterdam
20. Solow RM (1957) Technical change and the aggregate production function. Rev Econ Stat 39(3):312–320
21. ten Raa T (1995) The substitution theorem. J Econ Theory 66(2):632–636
22. ten Raa T (2008) Debreu's coefficient of resource utilization, the Solow residual, and TFP: the connection by Leontief preferences. J Prod Anal 30(3):191–199
23. ten Raa T (2011) Benchmarking and industry performance. J Prod Anal 36(3):285–292
24. ten Raa T, Shestalova V (2015) Supply-use framework for international environmental policy analysis. Econ Syst Res 27(1):77–94
25. von Neumann J (1945) A model of general economic equilibrium. Rev Econ Stud 13(1):1–9