Chapter 10
Mathematical Learning and Its Difficulties in Eastern European Countries

Csaba Csíkos, Szilárd András, Attila Rausch, and Anna Shvarts

Eastern European Mathematics Education as Defined by Geographical, Historical, and Political Factors

In this book, the term "Eastern Europe" is used in accordance with how other chapters in the section titled "Lessons from International System-Level Surveys" have considered their territory and field of interest. Thus, the group of countries to which we refer in this chapter is defined not strictly geographically, but we have taken them as a group of countries that previously belonged to the immediate sphere of interest of the former Soviet Union. According to a current multilingual thesaurus (Eurovoc), published by the Publications Office of the European Union, Eastern Europe consists of 21 states. Other descriptions available in the geographical or political literature may add the Baltic states (Estonia, Latvia, and Lithuania) or even Finland to this group of countries. Furthermore, the Visegrad Group (the Czech Republic, Hungary, Poland, and Slovakia) and Slovenia are often labeled as Central

C. Csíkos (✉)
Faculty of Primary and Pre-school Education, Eötvös Loránd University, Budapest, Hungary
e-mail: csikos.csaba@tok.elte.hu

S. András
Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Cluj-Napoca, Romania
e-mail: andrasz@math.ubbcluj.ro

A. Rausch
Faculty of Education and Psychology, Eötvös Loránd University, Budapest, Hungary
e-mail: rausch.attila@ppk.elte.hu

A. Shvarts
Faculty of Psychology, Lomonosov Moscow State University, Moscow, Russia

Utrecht University, Utrecht, The Netherlands
e-mail: a.y.shvarts@uu.nl

A. Fritz et al. (eds.), *International Handbook of Mathematical Learning Difficulties*, https://doi.org/10.1007/978-3-319-97148-3_10

European countries, together with Austria, Germany, and Switzerland. Having acknowledged that any such categorizations may be offensive to some or unusual to others, in this chapter we refer to Eastern Europe as a group of 24 countries: those 21 listed in the footnote and the three Baltic states.[1]

Constraints and Promises of Recent Decades in Eastern European Mathematics Education

The Eastern European countries belonged for some decades to the immediate political, economic, and/or military sphere of the former Soviet Union. In this block of countries the leading role of one (or more, but not many) Marxist political parties defined several aspects of the school system. Centralized curricula and textbooks aimed to provide the same pathway for all children. Equality has always been a central issue in the Central and Eastern European socialist states (Bankov, Mikova, & Smith, 2006); however, the Programme for International Student Assessment (PISA) studies revealed that this has not been accomplished.

Adler (1980) praised the intensive study of educational psychology in the Soviet Union and the links between school practice and the newly emerged psychological findings on how children learn. The influence of Soviet educational psychology had its effect in the region, according to Szalontai (2000). Even nowadays, the classical seminal works by Talysina, Stolyar, Davidov, Vygotsky, Leontiev, and others play important roles in Russian math educators' training. According to Goldin (2003), any kinds of ideologically set mathematics education necessarily dismiss the integrity of mathematical knowledge. Nonetheless, Eastern European mathematics and science education were seen with a kind of fear from the other side of the Atlantic Ocean from the time of the Sputnik shock (1957) until the very end of the Cold War. Stefanich and Dedrick (1985) emphasized that in Eastern Europe, 42% of Bachelor of Arts (BA) degrees were awarded in the field of engineering (while only 6% were in the USA). As Valero et al. (2015, p. 290) state, "The narrative that connects progress, economic superiority, and development to citizen's mathematical competence is made intelligible in the 20th century". Emphasizing the importance of mathematics education in the Western world was a reaction in order to maintain the supremacy of the capitalist Western world.

Was there a special kind of mathematics education that might be labeled as socialist mathematics education? In his book, Swetz (see Howson, 1980) compared seven rather different countries (all labeled as socialist countries, including Tanzania). The country profiles were provided by excellent scholars; however, some of them did not live or work in the countries they were writing about. In spite of his critical book review, Howson agrees that "mathematics education in any country

[1] Albania, Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Former Yugoslav Republic of Macedonia, Georgia, Hungary, Kosovo, Moldova, Montenegro, Poland, Romania, Russia, Serbia, Slovakia, Slovenia, Ukraine.

cannot be divorced from politics" (p. 285). In these seven "socialist" countries (among which the Soviet Union, East Germany, Yugoslavia, and Hungary were included, as part of Eastern Europe) there were three strong common features: (1) a central curriculum (not only in mathematics); (2) textbook word problems stressing industrial and societal phenomena; and (3) a strong emphasis on talent development and competitions. This third feature in itself may be the reason why the "mathematics for all" movement has not spread widely in Eastern Europe (Karp & Furinghetti, 2016). Eastern European mathematics education could rather well fulfill the role of "the gate keeper," i.e., dividing the society into two parts: those who are able to do mathematics and those who cannot (Skovsmose, 1998).

In case the reader finds a country description written by one or more (no matter how excellent) scholars incidental or nonrepresentative, a little larger sample was involved in Hawighorst's (2005) investigation; 15 parents from three different cultures were interviewed (five of them were German parents resettling from the former Soviet Union). The study focuses on some significant differences the "resettlers" expressed in their view. They seem to be particularly critical of school mathematics lessons. They expect a high level of content knowledge in mathematics; therefore, they plan to send their children to the most elitist type of secondary school—the Gymnasium.

If a kind of socialist mathematics education did exist in the bloc of Eastern European countries, it was certainly not uniform. Karp and Furinghetti (2016) analyzed how consecutive eras in the Soviet Union differed from each other with respect to aims and methods. Mathematics as a school subject was once considered a robust system to train the new bureaucracy of a new political era, while later some parts of the mathematics curricula were considered useless. What is more, the methods of instruction and evaluation changed according to what the leaders thought about the development of the so-called collective spirit.

Stoilescu (2014) summarized what many Eastern European citizens felt under the socialist regimes: people's capability to make decisions and take responsibility were weakened by encouraging centralized, uniform thought through uniform and ideologically infiltrated textbooks. The freedom of research was constrained by ideological factors. A direct (and negative) intervention from governmental officers is described by Varga (1988): his teaching experiment, which was interrupted by withdrawal of authorization from the ministry of education, was able to restart later but only with modified experimental factors.

The countries in Eastern Europe have further common features beyond those originating in the decades of the Soviet sphere of interest. Eastern European states had long historical relations with the European centers (Bertomeu-Sánchez, García-Belmar, Lundgren, & Patiniotis, 2006) with respect to the circulation of scientific knowledge. From the sixteenth century it was quite common that encyclopedias written in one of the European languages were translated and used as textbooks in both central and peripheral countries in Europe. The word "textbook" itself came from English in the eighteenth century and had the meaning of a collection of texts that might be used for educative or reference purposes. However, encyclopedias were used as textbooks long before that term was coined. An example is the *Encyclopaedia* written by the Hungarian Apáczai in 1655 (Palló, 2006), which was unique not only at that time but indeed also until the end of the nineteenth century, since textbooks translated from German or French were mainly in use in Eastern Europe.

Historically, educational thoughts in the heart of Europe were formed in such a way that this tradition is now called “Didaktik,” as opposed to the Anglo-Saxon curriculum tradition (Westbury, 2000). In a Berlin–Hong Kong comparative study, Lui and Leung (2013) described several aspects of mathematics teaching common in Berlin and in Hong Kong, where the Confucianist tradition is standard. Both traditions emphasize exercises and practice (for more than one third of the time allocated to math lessons).

Even nowadays, Eastern European countries have some common characteristics that might be considered as part of their historical and cultural backgrounds. The PISA study (Mikk, Krips, Säälik, & Kalk, 2016) revealed that students’ scores in mathematics and science were, unusually, correlated with their judgment of their relations with their teachers. The five items in the background questionnaire asked them to score, on a five-point Likert scale, how much they trusted their teachers, to what extent their teachers treated them fairly, etc. From Eastern Europe, the Czech Republic, Hungary, Poland, Romania, Russia, and Slovakia were considered, and the −0.63 correlation (albeit not significant in such a small country sample) was at least alarming. No similar tendency was revealed in other country groups in the world.

Lessons from International System-Level Surveys

International educational surveys based on nationwide representative samples started in the twentieth century. The International Association for the Evaluation of Educational Achievement (IEA) focused on mathematics from the outset. The First and Second International Mathematical Studies (FIMS and SIMS) involved far fewer countries than the later TIMSS series. (Note: initially the acronym “TIMSS” was defined as “Third International Mathematics and Science Study” but later the definition was changed to “Trends in International Mathematics and Science Study”.) From Eastern Europe, only Hungary took part in SIMS, while Bulgaria, the Czech Republic, Hungary, Latvia, Lithuania, Romania, Russia, Slovakia, and Slovenia participated in TIMSS in 1995. Since 2003, TIMSS has been conducted every 4 years, making it possible to outline developmental trends in countries’ profiles. With the advent of the PISA studies, much more attention has been paid to each country’s overall educational achievement, especially in comparison with the overall state of the country’s economy, health, and culture. Each year the United Nations publishes Human Development Index (HDI) scores, which are composite scores indicating each country’s general state of development. From the Eastern European region, Slovenia, Estonia, Slovakia, and Poland had the four highest scores in 2015. There is a fairly strong connection between a country’s HDI score and its PISA score. For the 27 countries where both PISA scores and HDI scores were available, the correlation coefficients are shown in Table 10.1.

On one hand, Table 10.1 clearly indicates a strong connection between a country’s general developmental level and its PISA score. In this respect, mathematics is

Table 10.1 Correlation coefficients between Human Development Index (HDI) scores and Programme for International Student Assessment (PISA) 2000 scores ($N = 27$ countries)

	Correlation coefficients		
	HDI score	PISA reading score	PISA mathematics score
Reading	0.606		
Mathematics	0.599	0.898	
Science	0.433	0.924	0.919

Source: Modified from Csíkos (2006, p. 184)
All correlation coefficients are significant ($p < 0.05$)

not exceptional at all, implying that fostering students' performance in mathematics should by all means be embedded in the general development of the quality of education. Nonetheless, at the individual and classroom levels, mathematics does have some unique characteristics. On the other hand, Table 10.1 also demonstrates the very strong connection between PISA scores in the three fields. This further strengthens the idea that achievement will develop as a consequence and in accordance with an increase in the general quality of the educational system.

Strengths and Weaknesses as Measured by International Surveys

The Eastern European countries achieved fairly different average scores in the latest TIMSS and PISA surveys. Table 10.2 illustrates how different their positions in the two lists are.

There is a tendency for several countries to refrain from participating in the eighth graders' TIMSS survey since that age group has a large overlap with the PISA target population. Since the sample of countries participating in each survey varies and a score of 500 on the TIMSS scale refers to the actual average mean (while in PISA a score of 500 means the OECD country average in 2000), it is hard to compare the achievement results in the two studies. Of course, the large differences in the ranking lists may attract lay people's attention, but there is a large overall tendency for TIMSS and PISA to both measure the quality of education and, of course, the quality of mathematics education.

In which fields of mathematical thinking do Eastern European countries have an advantageous or disadvantageous position? In "mathematics years," i.e., when mathematics is the central field to be measured in PISA, detailed scores are available regarding three thinking processes in mathematics (Table 10.3) and four content domains within mathematics (Table 10.4).

In each row of Table 10.3, the first aspect of our analysis is whether there is any strikingly high or low score or whether the students' results are balanced in the fields of the three thinking processes. For a detailed analysis of what these processes mean, the reader should consult the PISA 2012 framework (OECD, 2013). Roughly speaking, the *formulating* aspect of mathematical thinking refers to the process of

Table 10.2 Eastern European countries' Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) scores, and their connections with Human Development Index (HDI) scores and gross national income (GNI) per capita

HDI ranking	Country	HDI score	2015 TIMSS score (fourth grade)	2015 TIMSS score (eighth grade)	2015 PISA score	GNI per capita (2011 PPP $)
25	Slovenia	0.880	520	516	510	27,852
28	Czech Republic	0.870	528		492	26,660
30	Estonia	0.861			520	25,214
35	Slovakia	0.844	498		475	25,845
36	Poland	0.843	535		504	23,177
37	Lithuania	0.839	535	511	478	24,500
44	Hungary	0.828	529	514	477	22,916
46	Latvia	0.819			482	22,281
47	Croatia	0.818	502		464	19,409
49	Montenegro	0.802			418	14,558
50	Belarus	0.798				16,676
50	Russian Federation	0.798	564	538	494	22,352
52	Romania	0.793			444	18,108
59	Bulgaria	0.782	524		441	15,596
66	Serbia	0.771	518			12,190
76	Georgia	0.754	463	453	404	7,164
78	Azerbaijan	0.751				16,428
81	Macedonia	0.747			371	11,780
81	Ukraine	0.747				8,178
85	Albania	0.733			413	9,943
85	Armenia	0.733				8,124
85	Bosnia and Herzegovina	0.733				9,638
107	Moldova	0.693			420	5,223
	Correlations with 2015 PISA	*0.84*	*0.72*	*0.92*		*0.88*

Even in this small sample of countries, the correlation coefficients are significant
PPP $ international dollars after conversion using purchasing power parity rates

formulating mathematical models in a given situation, usually described in a word problem. *Employing* (which has a 50% weight, while the two others each have a 25% weight) refers to mathematical concepts, facts, and procedures. *Interpreting* involves evaluation of mathematical outcomes. In several Eastern European countries, students' achievement is well balanced among the three processes. However, when there is a larger difference between columns, it might define a type of mathematical process that is relatively underdeveloped in that country. Quite often, the processes of formulating have a fairly weaker average (Slovenia, Hungary, Croatia), which may point also to the relative strength of routine processes belonging to the employing category. According to Table 10.3, Russia has a relatively weak average

Table 10.3 Eastern European countries' average scores for three different mathematical thinking processes

Country	2012 PISA score	Formulating score	Employing score	Interpreting score
Estonia	521	517	524	513
Poland	518	516	519	515
Slovenia	501	492	505	498
Czech Republic	499	495	504	494
Latvia	491	488	495	486
Slovakia	482	480	485	473
Russian Federation	482	481	487	471
Lithuania	479	477	482	471
Hungary	477	469	481	477
Croatia	471	453	478	477
Serbia	449	447	451	445
Romania	445	445	446	438
Bulgaria	439	437	439	441
Montenegro	410	404	409	413
Albania	394	398	397	379

PISA Programme for International Student Assessment

Table 10.4 Eastern European countries' average scores in four different mathematical content domains

Country	2012 PISA score	Change and relationships score	Space and shape score	Quantity score	Uncertainty and data score
Estonia	521	530	513	525	510
Poland	518	509	524	519	517
Slovenia	501	499	503	504	496
Czech Republic	499	499	499	505	488
Latvia	491	496	497	487	478
Slovakia	482	474	490	486	472
Russian Federation	482	491	496	478	463
Lithuania	479	479	472	483	474
Hungary	477	481	474	476	476
Croatia	471	468	460	480	468
Serbia	449	442	446	456	448
Romania	445	446	447	443	437
Bulgaria	439	432	442	443	432
Montenegro	410	399	412	409	415
Albania	394	388	418	386	386

PISA Programme for International Student Assessment in the interpreting cluster of mathematical thinking as compared to the other mathematical processes.

Table 10.4 indicates whether in a given country there is any field of mathematics that is relatively highly developed or underdeveloped. Often, and understandably, the country profiles are rather well balanced (e.g., Slovenia, Hungary, Romania, and Bulgaria). However, in several cases, countries have a prioritized field (at least, one may infer that the reason is massive curricular coverage or a larger body of learning material in that country). For instance, Albanian students have far better results in geometry than in other fields. Geometry, in general, is thought to have a relatively momentous role within mathematics in Eastern Europe (Aubrey & Godfrey, 2003). On the other hand, Russia (Kolmogorov's country) seems to have a weakness in uncertainty and data, which may be due to a focus on formal mathematics since Kolmogorov's reforms in 1970. These results have been seriously taken into account, and these themes are included in curricula and the national maturation exam. The aforementioned relative strengths and weaknesses usually reflect long-term curricular and instructional methodological traditions in a given country.

Socioeconomic Background and Mathematics Achievement

At the time when Hungary was the only participant from the Eastern European region in the IEA studies, and Japan and Hungary competed for the highest country achievements, the Second International Science Study (SISS) created a measure of inequalities, called the Ratio of Homogeneity (ROH) index. Although in the early 1980s there was a central curriculum in Hungary, with only one textbook, the differences between schools proved to be larger than those in other top-performing countries (see Postlethwaite & Wiley, 1992). These within-country differences reflected both parents' efforts in finding the "best available" school for their children, and traditional geographical and socioeconomic differences in the country.

The PISA studies put a special emphasis on the role of family-related background variables. Over the course of the six PISA cycles, a more and more refined measure of students' socioeconomic status (SES) has been developed. Schleicher (2014) made computations from the PISA 2012 database, and one striking illustration of how SES is related to mathematics achievement was based on comparing groups that belong to different SES deciles. For each country, ten such SES groups can be compared, and while the top decile groups usually do not differ from each other, the lowest or the lower two deciles often lag far behind. The three eye-catching examples in this respect are Slovakia, Hungary, and the Czech Republic. It is very peculiar that the average mathematics scores of the students in the lowest SES decile in these three countries are lower than those of their Mexican peers. However, at the country level, Mexico has an average score of 413.

The PISA studies developed an index to measure students' economic, social, and cultural status (ESCS). In general, in top-performing countries, ESCS tends to have a relatively weak correlation with students' performance. Conceptually, ESCS can be connected to the idea of social inclusion, and the within-school and between-school differences in ESCS in a given country indicate the level of social inclusion. Although social justice and equity have long been catchphrases in Eastern Europe, according to the aforementioned ROH index, these countries may still suffer from lack of inclusion and lack of equity in their mathematics classrooms. Two interconnected phenomena should be investigated here: the question of low performers and whether the school system provides a chance for them to succeed (Table 10.5).

There is a clear connection between a country's overall average achievement and the percentage of low-performing students in that country. Estonia's 10.5% is the lowest value in Europe, a little lower than those of Finland (12.3%) and Switzerland (12.4%). In the majority of Eastern European countries, it is not only the high percentage of low performers that hinders their future development, but also the relatively low level of inclusion, as measured by means of ESCS. In general, ESCS explains around 15% of the PISA score variance in the overall country pool, but in certain countries, a higher percentage (i.e., a more expressed role of) explained variance appears. The OECD (2016) provided a statistical analysis (https://doi.org/10.1787/9789264250246-en) to compare the percentages of low-performing students in the top and bottom quartiles of ESCS. Ranking the countries in ascending order of the difference between the two rates of low-performing students, some Eastern European countries are at the far end of this; Bulgaria, Romania, Hungary, and Slovakia have more than a 40% difference in the rate of low performers in the two groups. It means that in these school systems the socioeconomic differences are deepened or at least not decreased. Conversely, in Estonia there is only a 12.6% difference in the rate of low-performing students in the most advantaged and most disadvantaged ESCS quartiles.

Table 10.5 Eastern European countries' percentages of low-performing students in mathematics in the 2012 Programme for International Student Assessment (PISA)

Country	Low-performing students (%)
Estonia	10.5
Poland	14.4
Slovenia	20.1
Czech Republic	21.0
Latvia	19.9
Slovakia	27.5
Russian Federation	24.0
Lithuania	26.0
Hungary	28.1
Croatia	29.9
Serbia	28.9
Romania	40.8
Bulgaria	43.8
Montenegro	56.6
Albania	60.7

The Organization for Economic Co-operation and Development (OECD) overall average rate is 23.0%

Member states of the European Union should decrease their percentage of low performers (not only in mathematics but also in the other key fields) below 15% by 2020. This aim seems to be unattainable, and in order to approach a 15% or at least 20% rate, school reforms in some Eastern European countries can be exemplary.

In Poland and Estonia, educational reforms have been aimed at decreasing the within-country differences by means of letting prospective vocational school students stay for one more year in the general schooling system (World Bank, 2010). As a philosophical basis for this reform movement, it is worth highlighting that Poland has a relatively fortunate situation within Eastern Europe. According to Turnau (1993), Polish scholars had the freedom to build international relations. Nevertheless, he is critical of the level of scientific achievement (which should have been much stronger in this state of research freedom). As an explanation, the still-existing complicating factor of language difficulties has been mentioned, along with the lack of strong theoretical embeddedness in math educators' scientific works.

The Estonian reforms (Lees, 2016) have contained elements that decreased inequality: individual psychological support, consultancy offered in the case of learning difficulties, free lunch, etc. Some of these elements were started even before the country's independence was regained. These seem to have little to do with mathematics achievement, but—as revealed from system-level data—it is the overall quality of education that will increase mathematical performance.

Russian reforms in mathematics education, according to a "Conception of Mathematics Education Development in the Russian Federation" government document, deal with individualization, where each student is supposed to receive education in accordance with his or her abilities, including talent recognition and support. At the same time, much more attention is paid to the development of gifted children and improving scientific achievements than to the support of children with learning difficulties. The need to establish a system of additional "leisure-time groups in mathematics" and to popularize mathematics is stressed throughout the conception.

There is a long list of educational reforms in Romania with a lot of positive effects in general (see UNESCO, 2015) but, as Nicu (2016) stated, there is a lack of consistency in introducing elements of reform and pursuing their effects in the Romanian education system. One of the reasons could be the fact that there have been too many and too rapid changes at the policy level. In the last 17 years there have been 12 different prime ministers and 17 different persons as minister of education, while the Romanian educational system is still highly centralized with almost no professional autonomy for teachers or teacher organizations.

Some Current Features and Tendencies in Eastern European Mathematics Education

Looking into Classrooms: Methodological Challenges

Blömeke, Suhl, and Döhrmann (2013) conducted an international comparative study on teachers' knowledge. An important aspect of their research is that they conducted an item-level analysis of different aspects of teachers' knowledge: strengths and weaknesses in mathematical pedagogical content knowledge and (pure) mathematical content knowledge. Here, Russia and Poland represented the Eastern European region, and the results pointed to a shared culture of mathematics teacher education in these two countries. Furthermore, Polish and Russian prospective teachers' advantages have been revealed (as compared to other countries in the sample: Taiwan, Hong Kong, Norway, the USA, and Germany) when solving difficult mathematical tasks.

In another study, Kaiser and Blömeke (2013) provided an example of how Eastern–Western dichotomies can be handled in large sample investigations. In this analysis, Poland and Russia, of course, belong to the Western culture countries, but when comparing future mathematics teachers' mathematical content knowledge and mathematical pedagogical content knowledge, these two countries proved to be similar to some traditional Eastern culture countries. In Poland, Russia, Taiwan, Thailand, Germany, Switzerland, and Georgia, students had greater mathematical content knowledge. Conversely, in the USA, Norway, the Philippines, Malaysia, Chile, Spain, and Botswana, prospective teachers' mathematical pedagogical content knowledge proved to be greater. Consequently, in the two representative Eastern European countries, preservice mathematics teachers are relatively well trained in mathematics and relatively poorly trained in pedagogy.

As an example, in Romania a regular teacher training program consists of 180 credits in the scientific field (bachelor's level) and 30 credits in the pedagogical module (offered by the teacher training institute, which has a fixed national curriculum that is 80% the same for all specializations). For those completing a master's program (2 years, 120 credits) there is a second pedagogical module with 30 additional credits. During these studies there is only one course of subject didactics and one discipline (during one semester) of practical training in schools. Thus, the main knowledge of how to be a mathematics teacher is not sourced in the worldwide recognized knowledge base (books and research papers) but in mathematical problem books. This viewpoint and structure (scientific specialization and pedagogical module) is historically rooted in the Romanian educational system; it was used from 1918 onward (when the modern and unified Romanian state was proclaimed) and at the beginning it was determined by the necessity for a large number of teachers in a relatively short time period (at the outset for unifying the four different educational systems that existed in the different regions before the unification, and subsequently for elimination of illiteracy, till 1959). This system practically allows the possibility for each student from higher education to become a teacher with minimal practical training, without any preliminary selection. In this context it is not surprising that according to the TIMSS 2007 teacher reports, rote-learning strategies were used in more than half of the lessons for at least 60% of students in the eighth grade in Bulgaria, Cyprus, Lithuania, Romania, and Turkey.

In Russia, there is also a traditional emphasis on mathematical content knowledge, with a 50–70% rate of preservice math teachers' university courses being dedicated to pure mathematics. According to the aforementioned "Conception of Mathematics Education Development in the Russian Federation" document, the curriculum for teachers needs to be changed in order to add extra tasks in elementary mathematics, including creative tasks and tasks at an advanced level, which teachers need to be able to solve by themselves. The conception also stresses the role of practices at schools, which would motivate teachers to acquire deeper pedagogical and psychological knowledge, but it does not point toward reformation of pedagogical or psychological courses that could be based on contemporary findings in mathematics education research.

There are many anecdotal cases where mathematics teachers from different cultures have observed each other's lessons. Such an experience is described by Woodrow (1997): Hungarian colleagues in a British school observed that instead of forcing students to achieve well, the British colleagues considered it more important not to hurt their students' self-image.

A current trend in Europe is the widespread dissemination of inquiry-based learning (IBL; often called problem-based learning in mathematics education). According to Maaß and Dorier's (2010) analysis, three Eastern European countries participating in the Promoting Inquiry in Mathematics and Science Education across Europe (PRIMAS) project—Hungary, Romania, and Slovakia—can be characterized by late introduction of IBL into their curricula. In this way, these countries proved to be similar to Malta, Spain, and Cyprus. Remarkably, problem-based learning was introduced in Russia back in 1832 by P. S. Gur'yev in his "Arithmetic leaflets" ["Arifmeticheskie listki"] (Polyakova, 2011) and then was spread in curricula during the 1940s and 1950s (Karp, 2011); the current programs still honor this tradition.

Moreover, the analysis of the Mathematics and Science Across Europe (MASCIL) project (see Maaß and Engeln (2016)) revealed that teachers have a positive attitude about IBL and about connecting IBL with the "World of Work" (WoW) in some Eastern European countries (Romania and Bulgaria), but neither IBL nor the WoW context is frequently used in daily teaching practice. Teachers from Romania feel less supported than teachers from other European countries in implementing IBL or using WoW contexts. Romania puts an emphasis on active participatory methods and active learning using cooperative strategies (in pairs or in groups). In other words, it recommends a shift from teaching from the front to cooperative teaching and learning in order to improve motivation and engagement in mathematics.

Fostering Students' Mathematics Learning Talent Development, Remedial Education, School Readiness, and Attitudes

Hungary is said to be first country where a nationwide high school mathematical competition was organized, at the end of the nineteenth century (Kontorovich, 2011). Frank (2012) cited George Pólya's reason as to why mathematics was so important and highly developed in the first decades of the twentieth century in Hungary: it is the least expensive science. The "competition cult" that was so greatly expressed in the Eastern European countries during the Soviet regime had strong antecedents in Hungarian mathematics and physics competitions. Also, the existence of specialized high schools aimed at developing mathematical and science talent originates in Budapest, e.g., the Lutheran High School, where several Nobel Prize winners studied high-level mathematics (see Marx, 1996).

Currently, Russia maintains a strong tradition of specialized mathematical education in schools; in many schools, children are divided into mathematics and humanities classes after the eight to ninth grades. Educational standards and the approximate curriculum for schools (which is officially provided by the ministry of education) assume two levels of competence and corresponding programs: ordinary and advanced. For example, the advanced level for primary school (first to fourth grades) includes the ability to solve logical tasks; to read simple pie charts; to plan, conduct, and analyze simple empirical investigations; and other tasks. There is also a number of special mathematics schools for the most talented children. These schools have a unique system to teach mathematics, which is called the "system of leaflets" (e.g., Shen, 2000). In this two-level system, all curricula for advanced mathematics are presented as sequences of problems, and a student needs to solve them on his own and then explain his solutions to a teacher assistant. This system develops the skill to think, to achieve a new mathematical result on one's own, and to experience a mathematical discovery together with the team of teacher assistants and schoolmates, as they not only go through mathematics problems but also experience out-of-school activities together (Yurkevich & Davidovich, 2008). As a result, each student is able to enter the mathematical departments of the best universities; many students successfully participate in all-Russian and other mathematical competitions, although successful participation in competitions is mostly considered an incidental result of education.

In Eastern European mathematics education, much less attention has been paid to remedial education. Currently the system is undergoing serious reconstruction in Russia: all children are supposed to be taught in inclusive classes, thus the system of specialized schools is going to be renewed and new educational standards are being elaborated in order to adjust school curricula to specify what needs to be taught at each of three disability levels of each disorder (Malofeev, Nikol'skaya, Kukushkina, & Goncharova, 2009).

Talent Development and Participation in the International Mathematics Olympiad

Since the beginning of mathematics competitions, the main aim has been not to win prizes or praise good students, but to find future creative mathematicians (Kontorovich, 2011). The International Mathematics Olympiad (IMO) was initiated in the socialist countries with the aim of promoting excellence in mathematics (Adler, 1980).

The first IMO was held in Romania (and still Romania has hosted the most IMOs—five times). The Romanian team has participated in all 57 contests over the years. Several times, Romania has been first in the unofficial country rankings (the last time was in 1996), and there is a very strong tradition in Romania for preparing children for mathematical contests. If we focus only on the countries of the European Union, we can see that the results for the Romanian team are in the forefront of the rankings: in first position in 2011 and 2012, and in second position in 2014 and 2015. Obviously, in the worldwide ranking, the results of the Romanian team show a declining trend (see Table 10.6).

Of course, the results of the Romanian IMO team can be analyzed from several viewpoints. If we see the numbers of medals won over the years (75 gold, 138 silver, and 98 bronze), they are impressive. But if we relate this number to the number of Romanian students participating in the IMO competition (380), we see that the efficiency is around 82%, which is less than the efficiency obtained by the Hungarian teams (91%) or by the Russian teams (100%).

The way in which talented students are selected in Eastern European countries has a long tradition. In Romania, there is a multistep testing procedure for high-achieving students (they are tested at the local level in schools, at the county level, and nationally), a huge collection of background materials (the *Gazeta Matematica* journal and publications from several specialized publishing houses like GIL), and an excellent study program for those included in the enlarged national teams. It is also worth mentioning that from a professional point of view the educational system is controlled by the ministry of education (through local inspectorates), while the contests are supervised more or less by members of the Romanian Mathematical Society. This duality seems to be persistent in the Romanian educational system, and there are no concrete signs that there is a (common or political) will to change it at the national level. Despite the good Romanian results in international competitions, the talent recognition, talent development, and talent support programs are

Table 10.6 Ranking of the Romanian International Mathematics Olympiad (IMO) team in the unofficial country rankings

	2010	2011	2012	2013	2014	2015	2016
Romania's overall ranking	16	8	10	22	11	13	22
Romania's ranking among European Union countries	6	1	1	5	2	2	6

not visible at a systemic level in schools. Extra classes for remedial education are allowed, but for talent development these are transferred to centers of excellence (mostly created in cities), which are unavailable for most children. There are several civil initiatives (sponsored by foundations), but most of them are not embedded in the regular (not private) school system.

Our second example here is Russia. As the cessionary of the former Soviet Union's several first rankings, Russia traditionally has had quite good results in IMOs. It achieved second to fourth places in the worldwide ranking during the last 20 years, though in the last 2 years the results got worse; Russia won no gold medals and took eighth place in 2015, but it came seventh and won four gold medals in 2016. The mass media stressed that the results in 2016 were noticeably better than those in 2015, but they need to be improved further since Russia is expected by politicians and mathematicians to come first or second every year, as it did many times during the Soviet years. The 11th place it achieved in 2017 signified a failure of the current efforts. As has been mentioned, education for gifted students receives special attention; for example, a special center for gifted children was opened in Sochi under the personal control of the president. The preparations of the Russian team for IMOs, together with other conferences and summer and winter camps, are conducted at this center.

There are a few levels in the selection process of the IMO participants. Results from two all-Russian mathematical competitions, an open Chinese competition, and a "Romanian Masters" competition are taken into account. Around 50 selected participants are invited for 2 weeks of preparations a few times during the last 2 years of school. At the end the final team is formed, and a lot of attention is paid to individual preparation during the last 2 weeks of preparation before the IMO: each pupil solves the tasks that are chosen for him or her in accordance with his or her difficulties during the previous competitions and preparations. Tasks in international and all-Russian competitions are different: tasks in all-Russian competitions need more creative thinking, while tasks in the IMO are more technical, and it is exactly creative thinking in which Russian participants are so strong, while they lose in comparison with their Asian colleagues in technics and stability of calculation skills. This is what needs to be approached during the preparations.

School Readiness in Mathematics

The importance and topicality of school readiness investigations in the Eastern European region is illustrated here by the cases of Hungary and Poland. The first kindergarten in Central and Eastern Europe was established in 1828 in Buda by Terézia Brunszvik. Since that time, kindergarten education has been a central topic in Hungary. The current national kindergarten curriculum considers mathematics "a tool for observing and learning in the world through activities" (Government decree 363/2012). In Poland the development of mathematics skills is an important part of the core curriculum in kindergarten. Children learn about counting, numeracy, classification, addition, and subtraction through playful activities (Smoczyńska et al., 2014).

Children start school at age 6 in both countries. School readiness assessments are not compulsory in Hungary. The decision as to whether a child is ready for primary school is generally made by kindergarten teachers based on mostly social and physical characteristics. Cognitive development is also an important indicator, but language and vocabulary are usually more relevant than early numerical skills. The test battery most commonly used to assess the key cognitive and social skills for school readiness in Hungary is called the Diagnostic System for Assessing Development for Four- to Eight-Year-Old Children (DIFER), which includes a basic counting and numeracy test (see Csapó, Molnár, & Nagy, 2014). However, kindergarten teachers barely use these tools, considering that face-to-face measures are time consuming for them. To overcome these problems there are new research projects to extend technology-based assessments to early childhood as well (Rausch & Pásztor, 2017). A newly developed online test is used at school entry to assess early numerical skills from age 5–7, including five subtests: basic counting, the number word sequence, numeral recognition, magnitudes, and numerals and relations. The results of the first nationwide measurements are promising (Rausch, 2016).

In Poland, kindergarten teachers are requested to make school readiness assessments, which is called preschool diagnosis, based on the instructions of the core curriculum (Smoczyńska et al., 2014). Assessing basic counting skills is usually part of these measurements. Integrating information and communication technologies (ICT) into early mathematics education and assessments is a rapidly developing research area in Poland as well. The Test of Abilities at the Start of School (TUNSS), done using tablet computers, is used to assess school achievements in mathematics, reading, and writing from preschool up to second grade students at primary school. The mathematics subtest has items related to numbers, measurements, space and shape, relations, and dependencies (Szram, 2016).

The worldwide growing importance of mathematics education and the unquestioned importance of the early years of schooling define a research field that has brought some important findings from Eastern European colleagues. A cross-cultural investigation into early arithmetic by Rodic et al. (2015) found similar knowledge structures in 5- to 7-year-old students in the participating countries: the UK, Russia, China, and Kyrgyzstan.

Conclusion

Eastern European countries have a rather famous (sometimes labeled as infamous) heritage of school mathematics education. Having built on both the European didactical tradition and the Soviet ideas of psychology, Eastern Europe's mathematics education has produced impressive results in talent recognition and talent development, as indicated by the outstanding participation at International Mathematics Olympiads. From the 1980s, however, the average results of students in mathematics have tended to decrease, as measured by large-scale international surveys. At the root of the problems is an increasing difference between students with advantaged and those with disadvantaged socioeconomic status, and (not independently of that) the increasing proportion of low-performing students may lead to the conclusion that many countries in the region may and should follow some elements of the Polish and Estonian school reforms.

Acknowledgements This research was supported by the Content Pedagogy Research Program of the Hungarian Academy of Sciences.

References

Adler, I. (1980). Mathematics education under socialism. *Science & Society, 44*, 357–359.

Aubrey, C., & Godfrey, R. (2003). The development of children's early numeracy through key stage 1. *British Educational Research Journal, 29*, 821–840.

Bankov, K., Mikova, D., & Smith, T. M. (2006). Assessing between-school variation in educational resources and mathematics and science achievement in Bulgaria. *Prospects, 36*, 447–473.

Bertomeu-Sánchez, J. R., García-Belmar, A., Lundgren, A., & Patiniotis, M. (2006). Introduction: scientific and technological textbooks in the European periphery. *Science & Education, 15*, 647–665.

Blömeke, S., Suhl, U., & Döhrmann, M. (2013). Assessing strengths and weaknesses of teacher knowledge in Asia, Eastern Europe and Western countries: differential item functioning in TEDS-M. *International Journal of Science and Mathematics Education, 11*, 795–817.

Csapó, B., Molnár, G., & Nagy, J. (2014). Computer-based assessment of school readiness and early reasoning. *Journal of Educational Psychology, 106*(2), 639–650.

Csíkos, C. (2006). Nemzetközi rendszerszintű felmérések tanulságai az olvasástanítás számára [Lessons for reading education from international system-level surveys]. In K. Józsa (Ed.), *Az olvasási képesség fejlődése és fejlesztése* (pp. 175–186). Budapest, Hungary: Dinasztia.

Frank, T. (2012). Teaching and learning science in Hungary, 1867–1945: schools, personalities, influences. *Science & Education, 21*, 355–380.

Goldin, G. A. (2003). Developing complex understanding: on the relation of mathematics education research to mathematics. *Educational Studies in Mathematics, 54*, 171–202.

Government decree 363/2012. (XII. 17.) on the Core Programme for Kindergartens (effective from 1 September 2013). Budapest, Hungary.

Hawighorst, B. (2005). Parents' views on mathematics and the learning of mathematics—an intercultural comparative study. *ZDM Mathematics Education, 37*, 90–100.

Howson, A. G. (1980). Socialist mathematics education: does it exist? *Educational Studies in Mathematics, 11*, 285–299.

Kaiser, G., & Blömeke, S. (2013). Learning from the Eastern and the Western debate: the case of mathematics teacher education. *ZDM Mathematics Education, 45*, 7–19.

Karp, A. (2011). Reforms and counter-reforms: schools between 1917 and the 1950s. In A. Karp & B. R. Vogeli (Eds.), *Russian mathematics education: history and world significance* (pp. 43–87). Singapore: World Scientific.

Karp, A., & Furinghetti, F. (2016). Survey of the state of the art. In *History of mathematics teaching and learning* (pp. 3–27). Cham, Switzerland: Springer.

Kontorovich, I. (2011). What makes an interesting mathematical problem? A perception analysis of 22 adult participants of the competition movement. In B. Rösken & M. Casper (Eds.), *Current state of research on mathematical beliefs XVII. Proceedings of the MAVI-17 conference* (pp. 129–139). Bochum, Germany: Professional School of Education, Ruhr-Universität Bochum.

Lees, M. (2016). *Estonian education system 1990–2016: reforms and their impact*. Available online: http://4liberty.eu/wp-content/uploads/2016/08/Estonian-Education-System_1990-2016.pdf

Lui, K. W., & Leung, F. K. S. (2013). Curriculum traditions in Berlin and Hong Kong: a comparative case study of the implemented mathematics curriculum. *ZDM Mathematics Education, 45*, 35–46.

Maaß, K., & Dorier, J.-L. (2010). PRIMAS WP2—analysis of national contexts: international synthesis report comparing national contexts, pointing out differences, commonalities, and interesting resources and initiatives proper to be adapted to an international use, EUProject PRIMAS, Deliverable No 2.1. Retrieved from http://www.primas-project.eu/servlet/supportBinaryFiles?referenceId=3&supportId=1297

Maaß, K., & Engeln, K. (2016). Report on the large-scale survey about inquiry based learning and teaching in the European partner countries, EUproject mascil, Deliverable No 10.2. Retrieved from http://www.mascil-project.eu/images/pdf/mascilD102FinalVersion.pdf

Malofeev, N. N., Nikol'skaya, O. S., Kukushkina, O. I., & Goncharova, E. L. (2009). Edinaya kontseptsiya spetsial'nogo federal'nogo gosudarstvennogo standarta dlya detei s ogranichennymi vozmozhnostyami zdorov'ya: osnovnye polozheniya [Unified conception of a special federal state standards for children with disabilities: main statements]. Al'manakh Instituta Korrektsionnoi Pedagogiki [Almanac of the Institute of Correctional Pedagogy], 13. Retrieved from http://alldef.ru/ru/articles/almanah-13/edinaja-koncepcija-specialnogo-federalnogo-gosudarstvennogo

Marx, G. (1996). The myth of the Martian and the golden age of Hungarian science. *Science & Education, 5*, 225–234.

Mikk, J., Krips, H., Säälik, Ü., & Kalk, K. (2016). Relationships between student perception of teacher–student relations and PISA results in mathematics and science. *International Journal of Science and Mathematics Education, 14*, 1437–1454.

Nicu, A. (2016). Romanian education reform between policy and action. *Journal Plus Education, 14*, 21–33.

OECD. (2013). *PISA 2012 assessment and analytical framework: mathematics, reading, science, problem solving and financial literacy*. Paris: OECD Publishing. https://doi.org/10.1787/9789264190511-en

OECD. (2016). *Low-performing students: why they fall behind and how to help them succeed*. Paris: OECD Publishing. https://doi.org/10.1787/9789264250246-en

Palló, G. (2006). Encyclopedia as textbook. *Science & Education, 15*, 779–799.

Polyakova, T. (2011). Mathematics education in Russian before the 1917 revolution. In A. Karp & B. R. Vogeli (Eds.), *Russian mathematics education: history and world significance* (pp. 1–42). Singapore: World Scientific.

Postlethwaite, T. N., & Wiley, D. E. (1992). *The IEA study of science II: science achievement in twenty-three countries* (Vol. 2). Oxford, UK/Elmsford, NY: Pergamon.

Rausch, A. (2016). Online assessment of early numeracy at school entry. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 226). Szeged, Hungary: PME.

Rausch, A., & Pásztor, A. (2017). Exploring the possibilities of online assessment of early numeracy in kindergarten. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (pp. 89–96). Singapore: PME.

Rodic, M., Zhou, X., Tikhomirova, T., Wei, W., Malykh, S., Ismatulina, V., et al. (2015). Cross-cultural investigation into cognitive underpinnings of individual differences in early arithmetic. *Developmental Science, 18*(1), 165–174.

Schleicher, A. (2014). Developing educational policies in a holistic skills framework. In S. K. Lee, W. O. Lee, & E. L. Low (Eds.), *Educational policy innovations* (pp. 29–48). Singapore: Springer.

Shen, A. (Ed.). (2000). *Zadachi po matematike [Mathematics problems]*. Moscow: MCCME. Retrieved from http://www.mccme.ru/free-books/57/shen.pdf

Skovsmose, O. (1998). Linking mathematics education and democracy: citizenship, mathematical archaeology, mathemacy and deliberative interaction. *ZDM Mathematics Education, 30*, 195–203.

Smoczyńska, A., Górowska-Fells, M., Maluchnik, B., Płatos, B., Chojnacki, M., & Smolik, M. (2014). *The system of education in Poland*. Warsaw: Publisher Foundation for the Development of the Education System.

Stefanich, G., & Dedrick, C. (1985). Addressing concerns in science and mathematics education: an alternative view. *The Clearing House, 58*, 274–277.

Stoilescu, D. (2014). A critical multicultural analysis of a Romanian textbook taught in elementary international language program. *Acta Didactica Napocensia, 7*, 33–48.

Szalontai, T. (2000). Some facts and tendencies in Hungarian mathematics teaching. *International Journal of Mathematics Teaching and Learning*. www.cimt.org.uk/journal/tshungmt.pdf

Szram, N. J. (2016). Children schooling achievements in the course of early education and their cognitive development. *International Journal of Information and Education Technology, 6*(12), 927.

Turnau, S. (1993). Mathematics education research in Poland. *Educational Studies in Mathematics, 24*, 235–238.

UNESCO. (2015). *Education for all national review report: Romania*. Bucharest, Romania: Ministerul Educaţiei Naţionale.

Valero, P., Graven, M., Jurdak, M., Martin, D., Meaney, T., & Penteado, M. (2015). Socioeconomic influence of mathematics achievement: what is visible and what is neglected. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematics Education* (pp. 285–301). Cham/Heidelberg/New York/Dordrecht/London: Springer.

Varga, T. (1988). Mathematics education in Hungary today. *Educational Studies in Mathematics, 19*, 291–298.

Westbury, I. (2000). Teaching as a reflective practice: what might Didaktik teach curriculum? In I. Westbury, S. Hopmann, & K. Riquarts (Eds.), *Teaching as a reflective practice: the German Didaktik tradition* (pp. 15–39). Mahwah, NJ: Erlbaum.

Woodrow, D. (1997). Democratic education: does it exist: especially for mathematics education? *For the Learning of Mathematics, 17*, 11–16.

World Bank. (2010, November). Successful education reform: lessons from Poland. *Europe & Central Asia Knowledge Brief: Vol. 34*. Washington, DC: World Bank.

Yurkevich, V. S., & Davidovich, B. M. (2008). Russian strategies for talent development. In T. Balchin, B. Hymer, & D. J. Matthews (Eds.), *The Routledge international companion to gifted education* (pp. 101–105). Abingdon, UK: Routledge.