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Mass spectrum of type IIB flux compactifications — comments on AdS vacua and conformal dimensions

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Abstract: In this note we study the mass spectrum of type IIB flux compactifications. We first give a general discussion of the mass matrix for F-term vacua in four-dimensional $\mathcal{N} = 1$ supergravity theories and then specialize to type IIB Calabi-Yau orientifold compactifications in the presence of geometric and non-geometric fluxes. F-term vacua in this setting are in general AdS_4 vacua for which we compute the conformal dimensions of operators dual to the scalar fields. For the mirror-dual of the DGKT construction we find that one-loop corrections to the complex-structure moduli space lead to real-valued conformal dimensions — only when ignoring these corrections we recover the integer values previously reported in the literature. For an example of a flux configurations more general than the DGKT mirror we also obtain non-integer conformal dimensions. Furthermore, we argue that stabilizing the axio-dilaton and complex-structure moduli in asymptotic regions of moduli space by fluxes implies that at least one of the corresponding mass eigenvalues diverges.

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Contents

1 Introduction

String theory is a theory of quantum gravity including gauge interactions. Its supersymmetric version is consistent in ten space-time dimensions and can be connected to four-dimensional physics through compactification. Well-understood compactification spaces are Calabi-Yau three-folds whose resulting lower-dimensional theories have a Minkowski vacuum. Deformations that preserve the Calabi-Yau condition correspond to massless scalar fields (moduli) in four dimensions, however, it is possible to deform the compact space away from being Calabi-Yau by turning on fluxes. Such fluxes generate mass terms for the moduli and typically lead to AdS_4 vacua. For gravity theories in AdS spaces one can then apply the AdS/CFT dictionary to relate, for instance, the masses of scalar fields in AdS_4 to conformal dimensions of corresponding operators in a putative three-dimensional CFT. For a certain class of type II flux compactifications, more concretely for the construction by DeWolfe, Giryavets, Kachru, and Taylor (DGKT) in type IIA string theory [1], it was observed in [2–5] that the conformal dimensions of all scalar fields in the closed-string sector are integer-valued. In [2, 3] the analysis was done for toroidal compactifications, while in [4, 5] the authors performed their computation for a general Calabi-Yau three-fold.

Obtaining integer conformal dimensions irrespective of the compactification space is a rather surprising observation which one would like to understand. In particular, one would like to know which features of the compactification are relevant for this result. This question was addressed in [5], where it was found that some non-supersymmetric type IIA vacua do not lead to integer-valued conformal dimensions. However, one can argue that such configurations are unstable [6] and therefore are not suitable for applications to the AdS/CFT correspondence [5]. In this note we approach the question of integer conformal dimensions from the type IIB side. We study orientifold compactifications of type IIB string theory with geometric and non-geometric fluxes that lead to supersymmetric AdS_4 vacua. In regard to the DGKT construction in type IIA string theory [1] we note the following differences:

1. On the type IIA side one typically considers the large-volume limit without corrections. In type IIB string theory this limit corresponds to the large-complex-structure limit, for which corrections are well-understood. We can therefore compute conformal dimensions in a setting that includes corrections to the moduli-space geometry.

2. For DGKT flux compactifications in type IIA string theory the superpotential splits into the sum of two independent terms [7]. From a type IIB perspective this is a rather special case that corresponds to turning on only a specific component of geometric and non-geometric Neveu-Schwarz–Neveu-Schwarz fluxes. For a more general flux-choice the superpotential will not split in this way.

In this work we investigate these two aspects. First, we consider the mirror-dual of the DGKT setting — including perturbative corrections to the complex-structure moduli space — and determine masses and conformal dimensions analytically. Second, we construct a flux configuration for which the superpotential does not split into two separate terms and determine the masses and conformal dimensions numerically. More concretely,

- in section 2 we give a general discussion of the mass matrix for F-term vacua of four-dimensional $\mathcal{N} = 1$ supergravity theories. In section 3 we then specialize to Calabi-Yau orientifold compactifications of type IIB string theory with O3- and O7-planes in the presence of geometric Ramond-Ramond (R-R) and geometric and non-geometric Neveu-Schwarz–Neveu-Schwarz (NS-NS) fluxes.

- In section 4 we determine masses and conformal dimensions for AdS_4 flux-vacua. For the mirror-dual of DGKT we find analytically that perturbative corrections to the prepotential lead to non-integer conformal dimensions. We also study an example with a flux choice more general than DGKT for which the superpotential does not split into two separate terms. Here we find numerically that the conformal dimensions are not integer.

- Section 5 is independent of our discussion of AdS vacua and conformal dimensions, but uses many results from sections 2 and 3. Here we study flux compactifications relevant for the KKLT and large-volume scenarios [8, 9], where only the F-terms of the complex-structure moduli and the axio-dilaton are considered. We compute the trace of the canonically-normalized mass matrix and argue that if axio-dilaton and complex-structure moduli are stabilized in an asymptotic regime of moduli space by fluxes, then at least one of the corresponding mass eigenvalues diverges.

- In section 6 we summarize our findings and in appendix A we give some technical details of the computations in the main text.

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2 Mass matrix and conformal dimensions

Let us start with a discussion of masses of chiral multiplets in four-dimensional $\mathcal{N} = 1$ supergravity theories. We consider minima of the scalar potential corresponding to vanishing F-terms and determine the general form of the mass matrix of the scalar fields. In the case of AdS_4 vacua we furthermore compute the conformal dimension of operators dual to the scalar fields.

F-term minima. Let us consider a four-dimensional $\mathcal{N} = 1$ supergravity theory with n complex scalar fields ϕ^M where $M = 1, \dots, n$. The F-term scalar potential can be written as

$$V = e^K \left[F_M G^{M\overline{N}} \overline{F}_{\overline{N}} - 3|W|^2 \right] , \tag{2.1}$$

where K denotes the real Kähler potential, W denotes the holomorphic superpotential, and $G_{M\overline{N}} = \partial_M \partial_{\overline{N}} K$ denotes the Kähler metric. The F-terms are given by $F_M = \partial_M W + K_M W$ with $K_M = \partial_M K$. In this work we are interested in F-term minima of this potential given by

$$F_M = 0 \, . \tag{2.2}$$

Mass matrix. The mass matrix for the complex scalar fields of this theory corresponds to the second derivatives of the potential (2.1). It can be arranged into the form

$$m^2 = \begin{bmatrix} m^2_{M\overline{N}} & m^2_{MN} \\ m^2_{\overline{M}\overline{N}} & m^2_{\overline{M}N} \end{bmatrix} , \tag{2.3}$$

where the blocks in the first line are related to the ones appearing in the second line by complex conjugation. For the former we find the following expressions at the minimum (2.2)

$$\begin{aligned} m^2_{M\overline{N}} &= e^K \left[\partial_M F_P \, G^{P\overline{Q}} \, \partial_{\overline{N}} \overline{F}_{\overline{Q}} - 2 G_{M\overline{N}} |W|^2 \right] , \\ m^2_{MN} &= e^K \left[-2 \, \partial_M F_N \, \overline{W} \right] . \end{aligned} \tag{2.4}$$

In order to obtain the canonically-normalized mass matrix we note that the Kähler metric $G_{M\overline{N}}$ is hermitian and positive definite. We can therefore write G as the square of a positive-definite matrix Γ and we define a matrix Q as

$$G = \Gamma \Gamma^\dagger \, , \qquad\qquad Q = \Gamma^{-1} (\partial F) \Gamma^{-T} \, . \tag{2.5}$$

Here and in the following we suppress identity matrices $\delta_{M\overline{N}}$, $\delta^{M\overline{N}}$, and $\delta_M{}^N$. The canonically-normalized mass matrix is obtained by multiplying (2.3) with appropriate factors of Γ from the left and the right and we find

$$m^2_{\text{can}} = e^K \begin{bmatrix} Q Q^\dagger - 2|W|^2 & -Q\overline{W} \\ -Q^\dagger W & Q^\dagger Q - 2|W|^2 \end{bmatrix} . \tag{2.6}$$

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Mass eigenvalues. In order to determine the eigenvalues of (2.6) we first perform a singular-value decomposition of Q as

$$Q = U\Sigma V^\dagger\,, \tag{2.7}$$

where U and V are unitary matrices and Σ is a diagonal matrix that contains only real entries. We can then write the canonically-normalized mass matrix (2.6) as

$$m^2_{\text{can}} = e^K \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} \Sigma^2 - 2|W|^2 & -\Sigma\, W \\ -\Sigma\, W & \Sigma^2 - 2|W|^2 \end{bmatrix} \begin{bmatrix} U^\dagger & 0 \\ 0 & V^\dagger \end{bmatrix}, \tag{2.8}$$

and the corresponding eigenvalue equation for the mass eigenvalues m^2 is given by

$$\begin{aligned} 0 &= \det\Big[m^2_{\text{can}} - \mathsf{m}^2\Big] \\ &= \det\Big[e^{2K}\Big[(\Sigma^2 - 2|W|^2 - e^{-K}\mathsf{m}^2)^2 - \Sigma^2|W|^2\Big]\Big]. \end{aligned} \tag{2.9}$$

Denoting the entries of the diagonal matrix Σ^2 by σ_α^2, we can solve (2.9) as

$$\mathsf{m}^2_{\alpha\pm} = e^K\Big[\sigma_\alpha^2 \pm \sigma_\alpha|W| - 2|W|^2\Big]\,. \tag{2.10}$$

We finally note that $QQ^\dagger = U\Sigma^2U^\dagger$ and $Q^\dagger Q = V\Sigma^2V^\dagger$, and hence σ_α^2 are the real and positive eigenvalues of the hermitian matrices $Q^\dagger Q$ and $QQ^\dagger$.

AdS vacua and conformal dimensions. If the superpotential W is non-vanishing at the minimum, the F-term vacua of (2.1) are AdS$_4$ vacua. The corresponding AdS radius is defined as

$$R^2_{\text{AdS}} = -\frac{3}{V|_{\text{min}}} = \frac{e^{-K}}{|W|^2}\,, \tag{2.11}$$

and the mass eigenvalues (2.10) can be brought into the form

$$\mathsf{m}^2_{\alpha\pm} = e^K\left[\sigma_\alpha \pm \tfrac{1}{2}|W|\right]^2 - \frac{9}{4}\frac{1}{R^2_{\text{AdS}}}\,. \tag{2.12}$$

Hence, as expected, the Breitenlohner-Freedman bound $\mathsf{m}^2_{\text{can}} \geq -\frac{(d-1)^2}{4}R^{-2}_{\text{AdS}}$ [10] is satisfied for these vacua. Furthermore, the masses of the scalar fields are related to the conformal dimensions Δ of operators in a dual CFT as $\Delta(\Delta - d) = \mathsf{m}^2_{\text{can}}R^2_{\text{AdS}}$. For $d = 3$ we then obtain

$$\Delta = \frac{1}{2}\left[3 \pm \sqrt{9 + 4\,\mathsf{m}^2_{\text{can}}R^2_{\text{AdS}}}\right], \tag{2.13}$$

and using (2.10) for the canonically-normalized masses we have $\Delta = 1 \pm |\sigma_\alpha/W|$ and $\Delta = 2 \mp |\sigma_\alpha/W|$. In order to satisfy the unitarity bound for all values of $|\sigma_\alpha/W|$ one should choose the upper sign in the first expression and the lower sign in the second one, however, in principle the opposite sign choice is allowed as well. Here we make the choice

$$\Delta_{\alpha(1)} = 1 + \left|\frac{\sigma_\alpha}{W}\right|, \qquad\qquad \Delta_{\alpha(2)} = 2 + \left|\frac{\sigma_\alpha}{W}\right|, \tag{2.14}$$

for which the conformal dimensions of the dual operators come in pairs that differ by one.

3 Type IIB flux compactifications

In this section we consider four-dimensional $\mathcal{N} = 1$ supergravity theories that originate from type IIB flux compactifications on Calabi-Yau orientifolds. Our main result in this section is an expression for the matrix Q appearing in the canonically-normalized mass matrix (2.6).

Scalar fields. Let us consider compactifications of type IIB string theory on Calabi-Yau three-folds $\mathcal{X}$, subject to an orientifold projection leading to O3- and O7-planes [7, 11]. This projection splits the cohomology of $\mathcal{X}$ into even and odd eigenspaces $H^{p,q}_{\pm}(\mathcal{X})$ whose dimensions will be denoted by $h^{p,q}_{\pm}$. The resulting effective four-dimensional theory contains two classes of scalar fields: first, there are $h^{1,1} + 1$ Kähler-sector moduli written as (we mostly follow the conventions of [12])

$$\mathsf{T}^A = (\tau, G^\alpha, T_a) , \qquad\qquad A = 0, \dots, h^{1,1} , \tag{3.1}$$

where τ denotes the axio-dilaton, G^α with $\alpha = 1, \dots, h^{1,1}_-$ are axionic moduli, and T_a with $a = 1, \dots, h^{1,1}_+$ are the ordinary Kähler moduli (see for instance [7] for details). Our conventions are such that $\mathrm{Re}\,\mathsf{T}^A$ are axionic degrees of freedom. Second, there are $h^{2,1}_-$ complex-structure moduli z^i with $i = 1, \dots, h^{2,1}_-$. These are contained in the holomorphic three-form Ω of the Calabi-Yau three-fold. Choosing a symplectic basis $\{\alpha_I, \beta^I\} \in H^3_-(\mathcal{X})$ with $I = 0, \dots, h^{2,1}_-$, the holomorphic three-form can be expanded as

$$\Omega = X^I \alpha_I - \mathcal{F}_I \beta^I , \qquad\qquad z^i = \frac{X^i}{X^0} , \tag{3.2}$$

where the periods $\mathcal{F}_I$ depend holomorphically on the complex-structure moduli z^i. We finally note that the effective theory also contains $h^{2,1}_+$ vector fields which may give rise to a D-term potential, however, here we assume that the D-term potential vanishes.

Kähler potential. The dynamics of the scalar fields is determined by the Kähler potential. For the above setting it is given by

$$K = K_{\mathrm{K}} + K_{\mathrm{cs}} , \qquad \begin{aligned} K_{\mathrm{K}} &= -\log\big[-i(\tau - \overline{\tau})\big] - 2\log\left[\mathcal{V} + \frac{\xi}{2}\right] , \\ K_{\mathrm{cs}} &= -\log\left[+i \int_{\mathcal{X}} \Omega \wedge \overline{\Omega}\right] , \end{aligned} \tag{3.3}$$

where the Einstein-frame volume of the Calabi-Yau three-fold is denoted by $\mathcal{V}$, which depends implicitly on τ, G^α and T_a, and we included α'-corrections encoded in $\xi = -\frac{\zeta(3)\,\chi(\mathcal{X})\,(\tau-\overline{\tau})^{3/2}}{2\,(2\pi)^3\,(2i)^{3/2}}$ [13]. The Kähler potential for the Kähler-sector moduli — including the α'-corrections shown above — has some special properties. With $K_A = \partial_A K$, $G_{A\overline{B}} = \partial_A \partial_{\overline{B}} K$, and $K^A = G^{A\overline{B}} K_{\overline{B}}$ one finds

$$K^A = -(\mathsf{T} - \overline{\mathsf{T}})^A , \qquad\qquad K_A G^{A\overline{B}} K_{\overline{B}} = 4 . \tag{3.4}$$

Note, however, that the complex-structure sector does not satisfy similar relations in general. Only in certain limits one may find for instance $K_i G^{i\overline{j}} K_{\overline{j}} = 3$; we come back to this point below.

Fluxes. We furthermore consider fluxes along the compact space $\mathcal{X}$. They generate a scalar potential in the four-dimensional theory which can be described by the superpotential [14–16]

$$W = \int_{\mathcal{X}} \Omega \wedge \left(F_3 - \Xi_A \mathsf{T}^A \right). \tag{3.5}$$

Here F_3 is the R-R three-form flux and Ξ_A are geometric and non-geometric NS-NS three-form fluxes. In particular, Ξ_0 is the ordinary H_3-flux, Ξ_α correspond to geometric F-fluxes, and Ξ^a correspond to non-geometric Q-fluxes [17–19] (see [20] for a review). All fluxes are integer quantized. The Bianchi identities for the NS-NS fluxes read [12, 14, 21]

$$\int_{\mathcal{X}} \Xi_A \wedge \Xi_B = 0\,, \tag{3.6}$$

while the Bianchi identities for the R-R fields contain contributions of localized sources such as D-branes and orientifold planes. The integrated versions of the Bianchi identities are known as tadpole-cancellation conditions. The fluxes contribute as [12]

$$N_A = \int_{\mathcal{X}} F_3 \wedge \Xi_A\,, \tag{3.7}$$

where $A = 0$ corresponds to the D3-brane tadpole, $A = \alpha$ to the D5-brane tadpole, and $A = a$ to the D7-brane tadpole.

Towards canonically-normalized-mass eigenvalues. In order to compute the eigenvalues of the canonically-normalized mass matrix shown in (2.10), we need to determine the eigenvalues σ_α^2 of the matrices $QQ^\dagger$ or $Q^\dagger Q$. To do so, we first define the matrix

$$\mathsf{Q} = G^{-1} \partial F\,, \tag{3.8}$$

evaluated at the minimum. Since the matrix Γ appearing in (2.5) is invertible, we see that $\mathsf{Q}\overline{\mathsf{Q}}$ and $QQ^\dagger$ have the same eigenvalues. Let us therefore determine Q for the above setting: we denote the Kähler-covariant derivative with respect to the complex-structure moduli z^i by D_i. Its action on the $(3,0)$-form Ω leads to $(2,1)$-forms $\chi_i = D_i\Omega \in H^{2,1}_-(\mathcal{X})$ and its triple action on Ω leads to the Yukawa couplings κ_{ijk} [22, 23]. More concretely, we have (we follow the conventions of [23])

$$\chi_i = \partial_i \Omega + K_i \Omega\,, \qquad\qquad \kappa_{ijk} = -\int_{\mathcal{X}} \Omega \wedge D_i D_j D_k \Omega\,. \tag{3.9}$$

We also note that the $(1,2)$-components of the real three-forms Ξ_A are given by

$$\Xi_A^{\bar{i}} = -i\, e^{K_{\rm cs}} G^{\bar{i}j} \int_{\mathcal{X}} \chi_j \wedge \Xi_A\,, \tag{3.10}$$

which are related to the $(2,1)$-components Ξ_A^i by complex conjugation. Using then the F-term conditions $F_A = 0$ and $F_i = 0$ as well as special-geometry relations of the complex-structure moduli space [22, 23], we can determine the matrix Q shown in (3.8) as follows

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(details of this computation are shown in appendix A.1)

$$\begin{aligned}\mathsf{Q}^{\overline{A}}{}_{B} &= [-\delta^{\overline{A}}{}_{B} - K^{\overline{A}}K_{B}]\,W\,,\\ \mathsf{Q}^{\overline{A}}{}_{j} &= -i\,e^{-K_{\rm cs}}\,G^{\overline{A}B}\,\Xi^{\bar{i}}_{B}\,G_{\bar{i}j}\,,\\ \mathsf{Q}^{\bar{i}}{}_{B} &= -i\,e^{-K_{\rm cs}}\,\Xi^{\bar{i}}_{B}\,,\\ \mathsf{Q}^{\bar{i}}{}_{j} &= G^{\bar{i}m}\kappa_{jmn}\,\Xi^{n}_{\overline{B}}K^{\overline{B}}\,.\end{aligned} \tag{3.11}$$

Note that these expressions are valid for any point in complex-structure moduli space. In principle one can use them to either compute the singular-value decomposition of Q and determine the singular values σ_α or to compute the eigenvalues σ_α^2 of $\mathsf{Q}\overline{\mathsf{Q}}$. However, we were not able to obtain analytic expressions for the eigenvalues for general flux configurations.

Remarks. We close this section with two remarks. First, using (3.4) and the F-term condition of the Kähler-sector moduli, from $K^A F_A = 0$ we determine

$$W|_{\rm min} = -\frac{i}{2}\int_{\mathcal{X}}\Omega\wedge\Xi_A\,(\mathrm{Im}\,\mathsf{T}^A)\,. \tag{3.12}$$

Second, with the help of the F-term conditions we can express the tadpole charges (3.7) at the minimum as

$$N_A = \left[\int_{\mathcal{X}}\Xi_A\wedge\star\Xi_{\overline{B}} + 12\,G_{A\overline{B}}\,e^{K_{\rm cs}}|W|^2\right](\mathrm{Im}\,\mathsf{T}^B)\,. \tag{3.13}$$

At the minimum the matrix in parenthesis is semi-positive definite, which implies in particular that $N_A\,(\mathrm{Im}\,\mathsf{T}^A)\geq 0$.

4 Conformal dimensions

Since it is difficult to determine the eigenvalues of the mass matrix for general flux choices analytically, in this section we consider two specific settings. First, we study the mirror-dual of the type IIA DGKT construction [1]. Second, we analyze numerically an example with geometric and non-geometric NS-NS fluxes for $h^{1,1} = h^{2,1}_{-} = 1$.

4.1 The type IIA mirror

We start with a setting that is mirror-dual to the type IIA DGKT construction [1]. This configuration is special since the superpotential splits into a sum of two terms that only depend on the complex-structure and only on the Kähler-sector moduli, respectively.

Fluxes. The setting that we consider on the type IIB side is characterized by the following choice of NS-NS fluxes

$$\Xi_A = -\,(\Xi_A)_0\,\beta^0\,, \tag{4.1}$$

where we expanded Ξ_A into the symplectic basis $\{\alpha_I,\beta^I\}\in H^3_-(\mathcal{X})$. On the type IIA side $(\Xi_A)_0$ are the components of the H_3-flux, and we remark that the R-R three-form flux F_3 is not restricted besides the tadpole-cancellation condition. From the expansion of the holomorphic three-form shown in (3.2) we see that the superpotential (3.5) indeed splits into two terms depending only on the complex-structure and only on the Kähler-sector moduli

$$W = \int_{\mathcal{X}} \Omega \wedge F_3 + \left[X^0 \left(\Xi_A\right)_0 \mathsf{T}^A \right] . \tag{4.2}$$

Furthermore, from (3.12) we see that in this case W/X^0 at the minimum is purely imaginary which matches the discussion in [1]. Using then $\int_{\mathcal{X}} \partial_i \Omega \wedge \Xi_A = 0$ and the F-term condition $F_A = 0$, we find for the $(1,2)$-components of Ξ_A

$$\Xi_A^{\bar{i}} = -i\, e^{K_{\rm cs}} K_A K^{\bar{i}} W . \tag{4.3}$$

Large-complex-structure limit. For the type IIA setting one considers the large-volume limit in which corrections are typically neglected. On the type IIB side this limit corresponds to the large-complex-structure limit, where subleading corrections are however well-understood. In particular, the holomorphic three-form is determined by the following prepotential at the perturbative level

$$\mathcal{F} = -\frac{1}{3!} \frac{\kappa_{ijk} X^i X^j X^k}{X^0} + \frac{1}{2!} a_{ij} X^i X^j + b_i X^i X^0 + \frac{1}{2!} c (X^0)^2 , \tag{4.4}$$

where a_{ij} and b_i are real while c is purely imaginary. In this work we ignore instanton corrections to the prepotential. The periods $\mathcal{F}_I$ appearing in (3.2) are given by $\mathcal{F}_I = \partial_{X^I} \mathcal{F}$, from which one can determine the Kähler potential, the first derivatives K_i, and the Kähler metric $G_{i\bar{j}}$. Let us define

$$z^i = u^i + i v^i , \qquad\qquad \gamma = \frac{3\, \mathrm{Im}\, c}{\kappa_{ijk} v^i v^j v^k} \ll 1 , \tag{4.5}$$

and note that in the large-complex-structure limit we have $\gamma \to 0$. We then compute the following expressions

$$K^i = -i\, \frac{2-\gamma}{1+\gamma} v^i , \tag{4.6a}$$

$$K_i G^{i\bar{j}} K_{\bar{j}} = \frac{3}{1+\gamma} , \tag{4.6b}$$

$$\kappa^{\bar{i}}{}_{jk} K^k = \frac{i}{2} \frac{X^0}{\overline{X}^0} e^{-K_{\rm cs}} \frac{2-\gamma}{1+\gamma} \left[\delta^{\bar{i}}{}_j + K^{\bar{i}} K_j \right] . \tag{4.6c}$$

The eigenvalues of $QQ^\dagger$. With the help of (4.3) and (4.6) we determine the matrix Q and subsequently $\mathsf{Q}\overline{\mathsf{Q}}$. Noting that (3.12) together with (4.1) implies that W/X^0 is purely imaginary at the minimum, we obtain the following four sub-blocks

$$\begin{aligned}
(\mathsf{Q}\overline{\mathsf{Q}})^{\overline{A}}{}_{\overline{B}} &= |W|^2 \left[\delta^{\overline{A}}{}_{\overline{B}} + \frac{5+2\gamma}{1+\gamma} K^{\overline{A}} K_{\overline{B}} \right] , \\
(\mathsf{Q}\overline{\mathsf{Q}})^{\overline{A}}{}_{\bar{j}} &= |W|^2 K^{\overline{A}} K_{\bar{j}} \left[3 - 2 \left(\frac{2-\gamma}{1+\gamma} \right)^2 \right] , \\
(\mathsf{Q}\overline{\mathsf{Q}})^{\bar{i}}{}_{\overline{B}} &= |W|^2 K^{\bar{i}} K_{\overline{B}} \left[3 - 2 \left(\frac{2-\gamma}{1+\gamma} \right)^2 \right] , \\
(\mathsf{Q}\overline{\mathsf{Q}})^{\bar{i}}{}_{\bar{j}} &= |W|^2 \left[4 \left(\frac{2-\gamma}{1+\gamma} \right)^2 \delta^{\bar{i}}{}_{\bar{j}} + \left(4 + 4 \left(\frac{2-\gamma}{1+\gamma} \right)^2 \frac{1-2\gamma}{1+\gamma} \right) K^{\bar{i}} K_{\bar{j}} \right] .
\end{aligned} \tag{4.7}$$

eigenvalues $\sigma_\alpha^2/\lvert W\rvert^2$	eigenvectors	multiplicity
1	$(L^{\overline{A}},0)^T$	$h^{1,1}$
$\left(\frac{4-2\gamma}{1+\gamma}\right)^2$	$(0,L^{\bar{i}})^T$	$h^{2,1}_{-}-1$
$16+\frac{96}{13}\gamma+\mathcal{O}(\gamma^2)$	$(K^{\overline{A}},\eta_{(1)}K^{\bar{i}})^T$	1
$81-\frac{5400}{13}\gamma+\mathcal{O}(\gamma^2)$	$(K^{\overline{A}},\eta_{(2)}K^{\bar{i}})^T$	1

Table 1. Eigenvectors and eigenvalues of the matrix $\mathsf{Q}\overline{\mathsf{Q}}$ shown in (4.7). The vectors $L^{\overline{A}}$ and $L^{\bar{i}}$ satisfy $K_{\overline{A}}L^{\overline{A}}=0$ and $K_{\bar{i}}L^{\bar{i}}=0$, and the parameters $\eta_{(1)}$ and $\eta_{(2)}$ take the form $\eta_{(1)}=\frac{1}{3}+\frac{25}{39}\gamma+\mathcal{O}(\gamma^2)$ and $\eta_{(2)}=-4+\frac{48}{13}\gamma+\mathcal{O}(\gamma^2)$. The precise expressions for the eigenvalues and eigenvectors in the last two lines are shown in appendix A.2.

mass $\mathsf{m}^2/R_{\rm AdS}^{-2}$	conformal dimension Δ	multiplicity
0	2	$h^{1,1}$
-2	3	$h^{1,1}$
$\frac{18(1-\gamma)}{(1+\gamma)^2}$	$\frac{5-\gamma}{1+\gamma}$	$h^{2,1}_{-}-1$
$\frac{2(5-\gamma)(1-2\gamma)}{(1+\gamma)^2}$	$\frac{6}{1+\gamma}$	$h^{2,1}_{-}-1$
$18+\frac{108}{13}\gamma+\mathcal{O}(\gamma^2)$	$5+\frac{12}{13}\gamma+\mathcal{O}(\gamma^2)$	1
$10+\frac{84}{13}\gamma+\mathcal{O}(\gamma^2)$	$6+\frac{12}{13}\gamma+\mathcal{O}(\gamma^2)$	1
$88-\frac{5700}{13}\gamma+\mathcal{O}(\gamma^2)$	$10-\frac{300}{13}\gamma+\mathcal{O}(\gamma^2)$	1
$70-\frac{5100}{13}\gamma+\mathcal{O}(\gamma^2)$	$11-\frac{300}{13}\gamma+\mathcal{O}(\gamma^2)$	1

Table 2. Masses and conformal dimensions for the mirror-dual of the type IIA DGKT setting. In the strict large-complex-structure limit $\gamma=0$ and one obtains integer-valued conformal dimensions.

These expressions are in line with corresponding type IIA formulas found in [4, 24, 25]. In order to determine the eigenvalues σ_α^2 of $\mathsf{Q}\overline{\mathsf{Q}}$, we first note that K_A is a $(h^{1,1}+1)$-dimensional vector and that there are $h^{1,1}$ vectors $L_{\overline{A}}$ perpendicular to K_A with respect to the Kähler metric, i.e. they satisfy $K_A L^A=0$. Similarly, K_i is a $h^{2,1}_{-}$-dimensional vector and there are $h^{2,1}_{-}-1$ vectors $L_{\bar{i}}$ that satisfy $K_i L^i=0$. Using then (3.4) and (4.6b), we can compute the eigenvalues and eigenvectors of the matrix $\mathsf{Q}\overline{\mathsf{Q}}$. They are summarized in table 1.

Masses and conformal dimensions. Using the results shown in table 1, we can now determine the canonically-normalized masses and corresponding conformal dimensions using (2.10) and (2.14). The resulting expressions are summarized in table 2, where we assumed that $W\neq 0$ at the minimum. Our main observation is that when taking into account the corrections to the large-complex-structure limit encoded in γ, the conformal dimensions of the dual operators are not integer-valued. Only in the strict large-complex-structure limit $\gamma=0$ we reproduce the integer conformal dimensions found in [2–4].

4.2 General fluxes for $h^{1,1} = h^{2,1}_- = 1$

In this section we analyze a compactification with a minimal set of moduli but with a more general choice of fluxes. We stabilize moduli in an AdS_4 vacuum at large complex structure, large volume, and weak coupling without taking into account perturbative corrections to the prepotential. We then study the masses and corresponding conformal dimensions numerically.

Setting. Let us consider a setting with two Kähler-sector moduli and one complex-structure modulus. This corresponds to a compactification manifold and orientifold projection with Hodge numbers

$$h^{1,1} = h^{1,1}_+ = 1\,, \qquad h^{2,1}_- = 1\,. \tag{4.8}$$

We furthermore assume that the complex-structure modulus is stabilized at large complex structure. Ignoring the perturbative corrections and choosing $\kappa_{111} = 1$ for simplicity, the corresponding prepotential (4.4) simplifies to

$$\mathcal{F} = -\frac{1}{3!}\frac{(X^1)^3}{X^0}\,. \tag{4.9}$$

In the Kähler sector we ignore α'-corrections and we express the Einstein-frame volume of the Calabi-Yau manifold in terms of the Kähler modulus T_1 as

$$\mathcal{V} = \frac{1}{6}\left[-i(T_1 - \overline{T}_1)\right]^{3/2}\,. \tag{4.10}$$

Fluxes. Turning to the fluxes, we expand the R-R and NS-NS three-form fluxes in the symplectic basis $\{\alpha_I, \beta^I\}$ as $F_3 = f^I\alpha_I - f_I\beta^I$ and $\Xi_A = (\Xi_A)^I\alpha_I - (\Xi_A)_I\beta^I$. Arranging the components f^I, f_I and $(\Xi_A)^I$, $(\Xi_A)_I$ into a vector and matrix, respectively, we make the following choice

$$F_3 = \begin{pmatrix} 60 \\ 0 \\ 0 \\ -2\mathsf{a}^2(9+2\mathsf{b}) \end{pmatrix}, \qquad \Xi_A = \begin{pmatrix} 0 & 0 \\ 0 & 10\mathsf{b} \\ -2\mathsf{a}^2(2+\mathsf{b}) & -\mathsf{a}^2(12+\mathsf{b}) \\ 0 & 0 \end{pmatrix}, \tag{4.11}$$

where $\mathsf{a}^2 \in \mathbb{Z}$ and $\mathsf{b} \in \mathbb{Z}$. The corresponding tadpole charges (3.7) are determined as

$$N_A = \Big(120\,\mathsf{a}^2(2+\mathsf{b})\,,\ 40\,\mathsf{a}^2(18 - 3\mathsf{b} - \mathsf{b}^2)\Big)\,. \tag{4.12}$$

Note that $\mathsf{b} = 0$ corresponds to a setting that is mirror dual to a type IIA DGKT construction. In the absence of corrections to the prepotential, as we are considering here, for $\mathsf{b} = 0$ we therefore expect integer conformal dimensions.

Moduli stabilization. For the above Kähler potential, superpotential, and choice of fluxes we can now solve the F-term conditions. These fix the moduli as

$$z^1 = i\,\mathsf{a}\,, \qquad \mathsf{T}^0 = \tau = i\,\mathsf{a}\,, \qquad \mathsf{T}^1 = T_1 = i\,\mathsf{a}\,, \tag{4.13}$$

and the AdS radius takes the value

$$\frac{1}{R_{\rm AdS}^2} = \frac{27(2+\mathsf{b})^2}{\mathsf{a}}\,. \tag{4.14}$$

Let us discuss two particular choices for the parameter b. As mentioned above, for the mirror-dual of the DGKT setting we expect integer conformal dimensions. And indeed, for $\mathsf{b}=0$ the eigenvalues of $QQ^\dagger$ are $\sigma_\alpha^2/|W|^2 = (1,16,81)$ which lead to the canonically-normalized masses and conformal dimensions

$$\begin{aligned} \frac{\mathsf{m}^2}{R_{\rm Ads}^{-2}}\bigg|_{\mathsf{b}=0} &= (\ 0\ ,\ -2\ ,\ 18\ ,\ 10\ ,\ 88\ ,\ 70\ )\,, \\ \Delta\big|_{\mathsf{b}=0} &= (\ 2\ ,\quad 3\ ,\quad 5\ ,\quad 6\ ,\ 10\ ,\ 11\ )\,. \end{aligned} \tag{4.15}$$

On the other hand, the choice $\mathsf{b}=1$ corresponds to a more general setting that is different from the DGKT mirror. Here we obtain $\sigma_\alpha^2/|W|^2 = (0.91, 13.45, 42.16)$ which leads to

$$\begin{aligned} \frac{\mathsf{m}^2}{R_{\rm Ads}^{-2}}\bigg|_{\mathsf{b}=1} &= (\ -0.14\ ,\ -2.05\ ,\ 15.12\ ,\quad 7.78\ ,\ 46.66\ ,\ 33.67\ )\,, \\ \Delta\big|_{\mathsf{b}=1} &= (\quad 1.91\ ,\quad 2.91\ ,\ 14.45\ ,\ 15.45\ ,\ 43.16\ ,\ 44.16\ )\,. \end{aligned} \tag{4.16}$$

In particular, for a choice of fluxes that is slightly more general than the mirror-dual of the type IIA DGKT setting, the conformal dimensions of the operators dual to the scalar fields are not integer. A similar observation was made in [3].

5 Asymptotic regions

Our discussion in this section is independent from our analysis of AdS vacua and conformal dimensions, but it utilizes many results from sections 2 and 3. We study the mass matrix for type IIB flux compactifications that are relevant for the KKLT and Large-Volume scenarios [8, 9]. Using the trace of this matrix, we argue that stabilizing the axio-dilaton and complex-structure moduli in an asymptotic region of moduli space by fluxes implies that at least one the corresponding mass eigenvalues diverges. This computation was part of the master thesis [26] and has been verified numerically (in a slightly different setting) in the master thesis [27].

Scalar potential. We consider type IIB flux compactifications with R-R and NS-NS three-form fluxes F_3 and H_3. In this case only the dilaton τ and the complex-structure moduli z^i appear in the superpotential and hence W is independent of the remaining Kähler-sector moduli. When ignoring the α'-corrections to the Kähler potential $K_{\rm K}$ shown in (3.3), the Kähler-sector moduli (without the axio-dilaton) satisfy the no-scale condition

$$K_A G^{A\overline{B}} K_{\overline{B}} = 3 \qquad\qquad \text{for} \quad A,B = 1,\ldots,h^{1,1}\,. \tag{5.1}$$

In this case the scalar F-term potential can be brought into the form

$$V = e^K\, F_M G^{M\overline{N}}\, \overline{F}_{\overline{N}}\,, \tag{5.2}$$

where M, N label the axio-dilaton τ and the complex-structure moduli z^i but not the remaining Kähler-sector moduli G^α and T_a. In the following we are interested in F-term minima given by

$$F_M = 0\,, \tag{5.3}$$

but we ignore the F-terms corresponding to the moduli G^α and T_a. In the KKLT and large-volume scenarios these are stabilized in a second step using non-perturbative effects.

Mass matrix. Following a discussion similar to the one in section 2, we find that the canonically-normalized mass matrix can be expressed as

$$m^2_{\rm can} = e^K \begin{bmatrix} QQ^\dagger + |W|^2 & 2Q\overline{W} \\ 2Q^\dagger W & Q^\dagger Q + |W|^2 \end{bmatrix}. \tag{5.4}$$

Denoting the eigenvalues of $QQ^\dagger$ again by σ^2_α, we determine the eigenvalues of (5.4) as

$$\mathsf{m}^2_{\alpha\pm} = e^K \left(\sigma_\alpha \pm |W|\right)^2. \tag{5.5}$$

The matrix $\mathbf{Q\overline{Q}}$. We note that the eigenvalues of $QQ^\dagger$ are the same as the eigenvalues of $\mathsf{Q}\overline{\mathsf{Q}}$, where Q was defined in (3.8). From the superpotential

$$W = \int_{\mathcal{X}} \Omega \wedge (F_3 - H_3\tau)\,, \tag{5.6}$$

we then compute

$$\begin{aligned}
\mathsf{Q}^{\overline{\tau}}{}_\tau &= 0\,, \\
\mathsf{Q}^{\overline{i}}{}_\tau &= -i\, e^{-K_{\rm cs}}\, h^{\overline{i}}\,, \\
\mathsf{Q}^{\overline{\tau}}{}_j &= +i(\tau-\overline{\tau})^2\, e^{-K_{\rm cs}}\, g_{j\overline{i}}\, h^{\overline{i}}\,, \\
\mathsf{Q}^{\overline{i}}{}_j &= (\tau-\overline{\tau})\,\kappa^{\overline{i}}{}_{jk} h^k\,,
\end{aligned} \tag{5.7}$$

where $h^{\overline{i}}$ with $i = 1, \ldots, h^{2,1}_-$ are the $(1,2)$-components of H_3 (cf. equation (3.10)). Denoting by $R_{i\overline{j}m\overline{n}}$ the Riemann tensor of the complex-structure moduli-space metric (we follow the conventions of [23]) and by $K_\tau = -\log[-i(\tau-\overline{\tau})]$ the Kähler potential of the axio-dilaton, we determine

$$\begin{aligned}
\left[\mathsf{Q}\overline{\mathsf{Q}}\right]^{\overline{\tau}}{}_{\overline{\tau}} &= e^{-2(K_\tau + K_{\rm cs})}\, h^i g_{i\overline{j}} h^{\overline{j}}\,, \\
\left[\mathsf{Q}\overline{\mathsf{Q}}\right]^{\overline{\tau}}{}_{\overline{j}} &= e^{-K_\tau - K_{\rm cs}} (\tau-\overline{\tau})^2\, \kappa_{\overline{j}\overline{m}\overline{n}} h^{\overline{m}} h^{\overline{n}}\,, \\
\left[\mathsf{Q}\overline{\mathsf{Q}}\right]^{\overline{i}}{}_{\overline{\tau}} &= -e^{-K_\tau - K_{\rm cs}}\, \kappa^{\overline{i}}{}_{mn} h^m h^n\,, \\
\left[\mathsf{Q}\overline{\mathsf{Q}}\right]^{\overline{i}}{}_{\overline{j}} &= e^{-2(K_\tau + K_{\rm cs})} \left(-R^{\overline{i}}{}_{\overline{j}m\overline{n}} h^m h^{\overline{n}} + \delta^{\overline{i}}{}_{\overline{j}} h^m g_{m\overline{n}} h^{\overline{n}} + 2h^{\overline{i}} h_{\overline{j}}\right).
\end{aligned} \tag{5.8}$$

Trace of the canonically-normalized mass matrix. The trace of the canonically-normalized mass matrix (5.4) can be computed using (5.8) as

$$\operatorname{tr} m^2_{\rm can} = \frac{2}{\mathcal{V}^2}\Big[e^{-K_{\rm cs}-K_\tau}h^m H_{m\overline{n}}h^{\overline{n}} + e^{+K_{\rm cs}+K_\tau}(h^{2,1}+1)|W|^2\Big]\,, \tag{5.9}$$

where all expressions are evaluated at the minimum. The Hodge metric $H_{i\bar{j}}$ is defined in terms of the Ricci tensor $R_{i\bar{j}}$ on the complex-structure moduli space and satisfies (see e.g. [28])

$$H_{i\bar{j}} = R_{i\bar{j}} + (h^{2,1}_- + 3)\,g_{i\bar{j}}\,, \qquad\qquad H_{i\bar{j}} \geq 2\,g_{i\bar{j}}\,. \tag{5.10}$$

We furthermore note that with the help of the F-term conditions the flux number appearing in the D3-brane tadpole-cancellation condition can be written as

$$N_{\rm flux} = e^{-K_{\rm cs}-K_\tau}h^m g_{m\overline{n}}h^{\overline{n}} + e^{+K_{\rm cs}+K_\tau}|W|^2\,, \tag{5.11}$$

and using the second relation in (5.10) we find from (5.9) the bound

$$\operatorname{tr} m^2_{\rm can} \geq \frac{2}{\mathcal{V}^2}\Big[2\,N_{\rm flux} + e^{K_{\rm cs}+K_\tau}(h^{2,1}-1)|W|^2\Big]. \tag{5.12}$$

Requiring $h^{2,1}_- \geq 1$ the bound above implies

$$\operatorname{tr} m^2_{\rm can} \geq \frac{4}{\mathcal{V}^2}N_{\rm flux} \quad\Rightarrow\quad \mathsf{m}^2_{\rm max} \geq \frac{2}{\mathcal{V}^2}\frac{N_{\rm flux}}{h^{2,1}_- + 1}\,, \tag{5.13}$$

where $\mathsf{m}^2_{\rm max}$ is the largest eigenvalue of the canonically-normalized mass matrix. (For $h^{2,1}_- = 0$ we obtain $\operatorname{tr} m^2_{\rm can} = 2\,N_{\rm flux}/\mathcal{V}^2$.) For the expression on the right-hand side in (5.13) we used that $\operatorname{tr} m^2_{\rm can}$ is the sum of all mass eigenvalues and that $\operatorname{tr} m^2_{\rm can}/2(h^{2,1}_- + 1)$ is the average mass eigenvalue. Note that in [29] a similar relation for the average mass eigenvalue has been estimated, whereas here we give a precise derivation.

Moduli stabilization in asymptotic regions. In the paper [30] we argued that when stabilizing the axio-dilaton and complex-structure moduli in the weak-string-coupling or large-complex-structure limit by fluxes, the flux number $N_{\rm flux}$ is expected to diverge. In [31] this argument has been extended to arbitrary boundary limits using asymptotic Hodge theory. If this expectation is true, then (5.13) implies that stabilizing these moduli in an asymptotic region of moduli space means that at least one of the axio-dilaton and complex-structure mass-eigenvalues will diverge — provided that the overall volume $\mathcal{V}$ remains the same

$$N_{\rm flux} \xrightarrow{\text{asymptotic region}} \infty \quad\Rightarrow\quad \mathsf{m}^2_{\rm max} \xrightarrow{\text{asymptotic region}} \infty\,. \tag{5.14}$$

In a consistent string-theory compactification the flux number $N_{\rm flux}$ is bounded by the tadpole-cancellation condition, however, it has been argued that this bound can be rather large [32]. Therefore, also $\mathsf{m}^2_{\rm max}$ can be large. In order to have a separation of scales between the moduli masses and the Kaluza-Klein masses $\mathsf{m}^2_{\rm max} \ll m^2_{\rm KK}$, one has to ensure

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a sufficiently large volume $\mathcal{V}$ and sufficiently small string coupling. In particular, we have to require

$$\mathsf{m}^2_{\mathrm{max}} \ll m^2_{\mathrm{KK}} \quad \Rightarrow \quad \frac{N_{\mathrm{flux}}}{h^{2,1}_{-}+1} \ll 2\pi^2\,\mathrm{Im}(\tau)\,\mathcal{V}^{2/3}\,. \tag{5.15}$$

For the large-volume scenario this is only a mild restriction, but for KKLT it may become relevant.

6 Summary and conclusions

In this note we studied the mass spectrum of flux compactifications of type IIB string theory. Let us summarize and discuss our findings.

AdS vacua of type IIB flux compactifications. In section 2 we considered F-term vacua of general four-dimensional $\mathcal{N}=1$ supergravity theories and determined in equation (2.10) the eigenvalues $\mathsf{m}^2_{\alpha\pm}$ of the canonically-normalized mass matrix for the scalar fields. In general these vacua are AdS$_4$ vacua, for which we compute the conformal dimensions of dual operators in a putative three-dimensional CFT. They are shown in (2.14). We find that both quantities are determined in a simple way by the value of the superpotential at the minimum and by the singular values σ_α of the matrix

$$\mathsf{Q}^{\overline{M}}{}_{N} = G^{\overline{M}P}\partial_P F_N\,. \tag{6.1}$$

In section 3 we specialized our discussion to four-dimensional $\mathcal{N}=1$ theories coming from compactifications of type IIB string theory on general Calabi-Yau orientifolds with geometric and non-geometric fluxes. Our main result in this section is an explicit expression for the matrix Q mentioned above, however, we were not able to determine analytic expressions for its singular values for general flux configurations.

(Non-)integer conformal dimensions. In section 4 we therefore consider two particular cases for which we can determine the singular values σ_α analytically and numerically, respectively. Here we are interested in the question of what features of the compactification lead to the integer-valued conformal dimensions observed in [2–5].

- In section 4.1 we study the mirror-dual of the type IIA DGKT construction [1]. On the type IIB side we have good control over perturbative corrections in the large-complex-structure limit and were able to take them into account for the computation of the masses and conformal dimensions. As summarized in table 2, we find that mass eigenvalues (in units of the AdS radius) and conformal dimensions are in general not integer-valued — only in the strict large-complex-structure limit we obtain integer conformal dimensions.

- In section 4.2 we consider a concrete type IIB example with one complex-structure modulus, one Kähler modulus, and the axio-dilaton. We ignore corrections to the large-complex-structure limit and stabilize moduli using geometric and non-geometric fluxes. For a choice of fluxes mirror-dual to the DGKT setting we find integer conformal dimensions — as expected — however, when considering a slightly more general flux choice the masses (in units of R^2_{AdS}) and conformal dimensions are no longer integer-valued.

Our findings therefore suggests that integer conformal dimensions occur a) for a specific choice of fluxes for which the superpotential splits into two separate terms, i.e. the DGKT setting, and b) when ignoring perturbative corrections to the large-complex-structure limit. When deviating from either of those properties the conformal dimensions are in general no longer integer-valued. However, in our analysis we focussed only on the closed-string sector. It would be interesting to take into account the open-string sector and repeat the computation of masses and conformal dimensions.

Moduli stabilization in asymptotic regions. Our discussion in section 5 is independent of our analysis of conformal dimensions, but uses many results from sections 2 and 3. We consider type IIB flux compactifications with only geometric F_3- and H_3-fluxes in the large-volume limit. These configurations are relevant for the KKLT and large-volume scenarios. We compute the trace of the canonically-normalized mass matrix of the axio-dilaton and complex-structure moduli and argue that when stabilizing these moduli in an asymptotic region of moduli space by fluxes at least one of the corresponding mass eigenvalues will diverge. This observations highlights that when stabilizing moduli one does not only need to ensure that the lightest modes are sufficiently heavy — but also that the heaviest modes are separated from the Kaluza-Klein scale. This point has recently been emphasized also in [33].

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A Computational details

In this appendix we summarize some details relevant for the computation of masses and conformal dimensions in section 4.1.

A.1 The computation of Q

Let us explain the computation of the matrix Q shown in equation (3.11). We first note that the F-term conditions $F_i = 0$ for the superpotential (3.5) take the form

$$0 = \int_{\mathcal{X}} \chi_i \wedge \left(F_3 - \Xi_A \, \mathsf{T}^A \right). \tag{A.1}$$

Next, we recall that the Kähler potential K_{K} shown in (3.3) only depends on the imaginary parts of the moduli T^A and therefore

$$K_{AB} = \partial_A \partial_B K = -\partial_{\overline{A}} \partial_B K = -G_{\overline{A}B} \,. \tag{A.2}$$

Using this relation and the fact that the superpotential (3.5) is linear in T^A we obtain

$$\mathsf{Q}^{\overline{A}}{}_j = G^{\overline{A}C}\partial_C F_B = G^{\overline{A}C}\Big[K_{CB} - K_B K_C\Big]W = \Big[-\delta^{\overline{A}}{}_B - K^{\overline{A}}K_B\Big]W\,. \tag{A.3}$$

With the help of the relation (3.10) we also determine

$$\mathsf{Q}^{\overline{A}}{}_j = G^{\overline{A}B}\partial_B F_j = G^{\overline{A}B}\int_{\mathcal{X}}\chi_j\wedge(-\Xi_B) = -i\,e^{-K_{\rm cs}}G^{\overline{A}B}\,\Xi^{\bar{i}}_B G_{\bar{i}j}\,, \tag{A.4}$$

and along similar lines the expression for $\mathsf{Q}^{\bar{i}}{}_B$ is found. Finally, with $\overline{\chi}_{\bar{i}}$ the complex conjugate of χ_i, we recall from [23] that

$$D_i\chi_j = -i\,e^{K_{\rm cs}}\,\kappa_{ij}{}^{\overline{m}}\overline{\chi}_{\overline{m}}\,, \tag{A.5}$$

where indices are raised by the inverse Kähler metric $G^{i\bar{j}}$. (This result was used recently also in [34] to determine the mass matrix.) We then compute using (3.4) and the complex-conjugates of (A.1) and (3.10)

$$\begin{aligned}
\mathsf{Q}^{\bar{i}}{}_j = G^{\bar{i}k}\partial_k F_j &= G^{\bar{i}k}\int_{\mathcal{X}}D_k\chi_j\wedge\Big(F_3 - \Xi_A\mathsf{T}^A\Big)\\
&= -i\,e^{K_{\rm cs}}\,\kappa^{\bar{i}}{}_j{}^{\overline{m}}\int_{\mathcal{X}}\overline{\chi}_{\overline{m}}\wedge\Big(F_3 - \Xi_A\mathsf{T}^A\Big)\\
&= -i\,e^{K_{\rm cs}}\,\kappa^{\bar{i}}{}_j{}^{\overline{m}}\int_{\mathcal{X}}\overline{\chi}_{\overline{m}}\wedge\Big(F_3 - \Xi_A\overline{\mathsf{T}}^A - \Xi_A\big(\mathsf{T}^A - \overline{\mathsf{T}}^A\big)\Big)\\
&= -i\,e^{K_{\rm cs}}\,\kappa^{\bar{i}}{}_j{}^{\overline{m}}\int_{\mathcal{X}}\overline{\chi}_{\overline{m}}\wedge\Xi_A K^A\\
&= \kappa^{\bar{i}}{}_{jn}\,\Xi^n_{\overline{A}}K^{\overline{A}}\,.
\end{aligned} \tag{A.6}$$

A.2 Eigenvalues and eigenvectors of $\mathbf{Q}\overline{\mathbf{Q}}$

In this appendix we summarize the exact expressions for the eigenvalues and eigenvectors shown in table 1. In particular, the eigenvalues in the last two lines are given by

$$\begin{aligned}
\frac{\sigma^2_{(1)}}{|W|^2} &= \frac{\begin{array}{r}-5\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}+\gamma\Big(14\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}\\ +\gamma\Big(\gamma(13\gamma+28)+\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}+222\Big)-20\Big)+97\end{array}}{2(\gamma+1)^4}\\
&= 16+\frac{96}{13}\gamma+\mathcal{O}(\gamma^2)\,,\\
\frac{\sigma^2_{(2)}}{|W|^2} &= \frac{\begin{array}{r}5\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}+\gamma\Big(\gamma\Big(\gamma(13\gamma+28)-\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}+222\Big)\\ -2\Big(7\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}+10\Big)\Big)+97\end{array}}{2(\gamma+1)^4}\\
&= 81-\frac{5400}{13}\gamma+\mathcal{O}(\gamma^2)\,.
\end{aligned} \tag{A.7}$$

The corresponding eigenvectors shown in the last two lines of table 1 are characterized by two parameters of the form

$$\begin{aligned}\eta_{(1)} &= \frac{\gamma(2-5\gamma)+\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}-11}{6(\gamma+1)}\\ &= \frac{1}{3}+\frac{25}{39}\gamma+\mathcal{O}(\gamma^2)\,,\\ \eta_{(2)} &= \frac{\gamma(2-5\gamma)-\sqrt{\gamma(\gamma(\gamma(25\gamma+28)+258)+100)+169}-11}{6(\gamma+1)}\\ &= -4+\frac{48}{13}\gamma+\mathcal{O}(\gamma^2)\,.\end{aligned} \tag{A.8}$$

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