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The age-performance relationship for a cognitive-intensive task: Empirical evidence from chess grandmasters

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ABSTRACT

To gain insight into the age-performance relationship (APR) for cognitively demanding tasks, we analyse the APR for the task of playing chess from a theoretical and empirical perspective. We set up a game-theoretic model showing that for risk-neutral players who are homogeneous in their linear payoffs, the APR can be estimated with either players' game outcomes or their Elo ratings. This result is empirically substantiated with data on outcomes of games played at an annual international chess tournament (1970–2021), and with players' Elo ratings. Further, the findings support the estimation of the APR with Elo ratings using a model that controls for player fixed effects and period random effects, which is also in accordance with the theoretical model. Next, we show that on average chess performance peaks in grandmasters' early-thirties and declines thereafter. Implications of the findings for a labour market characterised by an increase in cognitively demanding jobs and individuals working longer are discussed.

"Chess is mental torture." (Garry Kasparov, cited in Amis, 1993)

1. Introduction

During the last decades labour markets in industrialised countries have developed toward more cognitively demanding jobs, arguably through skilled-biased technological change and digitisation, and toward people working longer, arguably because of health improvements and governments' pension policies (Acemoglu & Autor, 2011; Autor et al., 2003; Coile et al., 2019; Spitz-Oener, 2006). These trends have raised questions about work-related productivity, particularly at older ages, or, more generally, questions have been raised about older individuals' functional independence in an ageing society (Bloom & Sousa-Poza, 2013; Bruine de Bruin, 2018; Kim & Lee, 2023; Krampe & Charness, 2018; Murman, 2015; Spitz-Oener, 2006; Strittmatter et al., 2020; Van Ours, 2009). Such questions are triggered by previous findings that individuals' performance on cognitive tasks can decline over their lifespans because of, e.g., normal cognitive ageing or loss of motivation as age increases (e.g., Dumas, 2015; Hertzog, 2020; Kanfer & Ackerman, 2004; Salthouse, 2009). Therefore, this study empirically analyses individuals' performances of a cognitive-intensive task over their lifespans. More specifically, the age-performance relationship (APR) for the task of playing chess is analysed.

Empirical evidence on the APR for real-world cognitive-intensive tasks is scarce either because it is difficult to measure performance, or because the task is incomparable across ages or time. Notable exceptions are studies confirming the age of peak performance (APP) to be, e.g., for major intellectual achievements in individuals' thirties (Lehman, 1953), for sports such as hockey in players' late twenties and for golf in players' early thirties (Berry et al., 1999), for the game of chess in players' mid-thirties (Elo, 1965), and for financial tasks in individuals' early fifties (Agarwal et al., 2009, pp. 51–117). Also, Shue and Luttmer (2009) show that middle-aged individuals make fewer errors than older and younger individuals performing the task of voting in the 2003 California recall election. Furthermore, a decline in performance of airline pilots due to a cognitive decline with increasing age has been shown by Causse et al. (2019). Following previous studies such as Lehman (1953), Elo (1965), Howard (2005), Vaci et al. (2015), and Strittmatter et al. (2020), we use the task of playing chess to obtain insight into the APR for a cognitive-intensive task.

The game of chess is, arguably, well-suited to investigating the APR for a cognitive-intensive task for two reasons: first, it is an individual task that is the same regardless of players' age and across time, and second,

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chess abilities are positively correlated with cognitive abilities such as fluid intelligence and short-term working memory (Burgoyne et al., 2016; Chase & Simon, 1973; De Groot, 1965; Howard, 2005; Sala et al., 2017-a; van der Maas & Wagenmakers, 2005). On these grounds, several studies have examined the APR for chess players. With data from Buttenwieser (1936), Lehman (1953) shows a hump-shaped pattern of age and tournaments' final rankings with a performance peak, on average, in the players' thirties. Rubin (1960) provides suggestive empirical evidence of a decline in chess performance with age, and Draper (1963) argues that if such a decline is real, it is caused by, e.g., increased fatigue at older ages rather than a decline in chess skills per se. Further, Strittmatter et al. (2020) found no empirical support for a decline in performance after the APP. The availability since the 1960s of players' Elo ratings as measures of their relative chess abilities has facilitated research on the APR (Elo, 1978) and Elo's (1965) hump-shaped APR estimate confirms that performance peaks, on average, in players' mid-thirties. Following Elo's work, recent studies such as Berthelot et al. (2012, 2019), Howard (2005), Roring and Charness (2007), Vaci et al. (2015) and Vaci et al. (2019) use a regression-based equivalent of Elo's method for estimating the APR, often also testing hypotheses related to explaining the APR. In contrast to Elo (1965), however, these recent empirical studies do not control for player fixed effects to account for a possible dependence between players' age and unobserved time-invariant innate chess ability, which can bias the estimated APR. For instance, if there is hump-shaped APR and players are only in the sample for the years in which they played at highly ranked tournaments, we expect that players with a relatively high innate chess ability are in the sample for a wider age range around the APP than those with a relatively low innate chess ability. Such a meritocratic sample selection can bias the results towards a flatter APR. Further, to date, no study has addressed the assumptions made about players' optimising behaviour when the Elo ratings are interpreted as measures of relative performance.

Therefore, we estimated the APR for the task of the game of chess based on an empirical model rooted in a theoretical model of optimal choice. This paper's contribution to the literature is threefold. First, we present a game-theoretical framework and show that, under certain assumptions such as risk-neutral players, the APR can be estimated using either performance measures such as outcomes of games, or (paired) players' Elo ratings. Second, we empirically substantiate the theoretical result with estimates of the APR using outcomes of games and differences in paired Elo ratings. The empirical analysis uses data on outcomes of chess games played at a prestigious international tournament, players' age, and their Elo ratings. Third, we estimate the APR based on players' Elo ratings and show that the estimation can be performed using a regression model that controls for player fixed effects and period random effects. The latter result is important for the identification of the APR, is in accordance with the theoretical model, and reflects that the Elo rating is a relative performance measure. Finally, while our empirical findings are for the cognitive task of playing chess, for the wider literature they can offer insight into issues concerning the performance of older workers. We therefore discuss our main empirical findings' implications for labour markets characterised by an increase in cognitively demanding jobs and people working longer.

2. Material and methods

2.1. The data

Our data are the outcomes of chess games at the Tata Steel Chess tournament, formerly known as the Hoogovens tournament (1938–1999) and the Corus chess tournament (2000–2010). The tournament is a prestigious international chess tournament held annually since 1938 in the Netherlands every January (Tata Steel Chess Tournament, 2021). At first it was restricted to Dutch players, in the second half of the 1940s it became an international chess tournament, and since the 1960s it is known as one of the strongest international chess tournaments in the world (the "Wimbledon of Chess", Barden, 2018). It has been a single round-robin tournament all along, except for the years 1993 and 1995 when a knock-out system was used. The prize money is relatively low, e.g., €10,000 for first prize in 2020, and players possibly receive appearance fees, although these are unpublished. Most importantly, it is a rated tournament which allows players to earn points for their Elo ratings. The available players' Elo ratings are valid on 1st January of each calendar year (more accurately, calculated on 31st December of the preceding year). These ratings data are available for the top 100 chess players worldwide from the online sources Chessmetrics.com (2021) and International Chess Federation (2021). We use the ratings published by Chessmetrics.com for the years until 2001 and those published by the International Chess Federation from 2001 onwards.

We restricted our sample to tournament data from 1970 onwards, i.e., from the year in which the International Chess Federation adopted the Elo system. Data on Tata Steel Challengers, introduced in 2003, are excluded (Tata Steel Chess Tournament, 2021), while the years 1993 and 1995, when a knock-out system was used, were not excluded (excluding these has no discernible influence on our findings). The data has information on 4640 games played by 233 players in the period 1970–2021. The Elo ratings are available for players who are among the top 100 worldwide. For about 32% of the games the Elo ratings for one or both players are not given because they were not among the top 100; these games were excluded from the sample. This exclusion resulted in dropping 24% of players (see Table C1 for details on the excluded number of games and players per year). Our resulting estimation sample consists of 178 chess grandmasters who played 3151 games at the Tata Steel Chess tournament in the period 1970–2021. On average, players are in the sample for 3.4 years, having played about 10 games per year at the tournament. We refer to this sample as the tournament sample. For the

Table 1
Estimates of the age-performance relationship (APR) based on games' outcomes or based on differences in paired players' Elo ratings.

	Model 1 Win-probability	Model 2 Difference in paired players' Elo ratings
	Coef.	Coef.
	(Std.Err.)	(Std.Err.)
White	0.063***	
	(0.012)	
Age/10	0.470***	382.109***
	(0.162)	(22.565)
$(Age/10)^2$	−0.103*	−85.980***
	(0.042)	(5.853)
$(Age/10)^3$	0.007	5.768***
	(0.003)	(0.479)
Age of peak performance (APP)	33.387	33.544
	(2.586)	(0.372)
H_0: No age effects[a]	0.004	<0.001
H_0: Cubic age profile (vs. age-FE)[a]	<0.001	<0.001
H_0: Player random effects[a]	<0.001	<0.001
R^2	0.119	0.848
H_0: APR Model 1 = APR Model 2 [a b]	0.965	
Number of players	178	
Number of games	3151	

Notes * $p < 0.05$; ** $p < 0.01$; *** $p < 0.005$. Model 1 is a linear probability model (Equation (7)) and Model 2 is a linear regression model (based on Equation (4)); both are estimated with least squares and clustered standard errors are reported. Age-FE: age fixed effects. Player fixed effects are included and for the identification of the APR the ones of Garry Kasparov and Magnus Carlsen are set as equal. Results with player random effects and player fixed effects without fully identifying the APR, are in Table C2. Table C3 presents the estimation results of linear probability models for a draw and a loss.

[a] Entries are p-values.

[b] The test is for the null hypothesis of the same APR for both models, accounts for the different units of measurement, and has two degrees of freedom.

years these players did not attend the tournament, their Elo ratings are also available if they were among the top 100 players worldwide. We constructed an extended sample, referred to as the top 100 sample, with data on Elo ratings for the 178 players who are also in the tournament sample, and for the years in the period 1970–2021 when they were among the top 100 players worldwide. These 178 players are in top 100 sample for, on average, 15.5 years (2753 observations).

The variables available for our empirical analysis are the outcome of the game (a win, a draw, or a loss), who plays with the white or black pieces (White or Black), the players' ages, and their Elo ratings on 1st January in the years they were among the top 100 players. Age is measured in full years, giving the average age as 30.

The flat age patterns for the percentage of losses, draws, and wins in Fig. 1 can result from meritocratic selection into the tournament sample: all players are world-class chess players at the time of their participation in the tournament. While the age pattern of Elo ratings for the top 100 sample in Fig. 2 can also have been affected by meritocratic selection, it shows a hump-shaped age pattern and suggests an age of peak performance in grandmasters' late twenties or early-thirties. Our empirical analysis provides insight on, e.g., whether these age patterns of Figs. 1 and 2 are influenced by meritocratic selection or whether there is no age pattern. (Figures C1 and C2 show the average age and average Elo rating by year).

2.2. Theoretical framework

The outcomes of the chess games at a rated chess tournament affect players' Elo ratings. The Elo system periodically updates players' Elo ratings based on their performance during the latest period with the purpose of providing measures of players' current relative chess abilities (Elo, 1978; Glickman, 1995; International Chess Federation, 2017). A player's chess ability is related to personal features such as cognitive skills, perseverance, or motivation, and for the purpose of this study the ability is the sum of an innate chess ability, which does not vary across age, and age-related chess ability. This section discusses the main components of a game-theoretical model in which both players' outcomes in individual games and their Elo rating are determined by their chess abilities. We refer to Appendix A for further details and discussions.

2.2.1. The game of chess as a rank-order tournament

We tailor the rank-order tournament model of Lazear and Rosen (1981) to the game of chess, which in our empirical analysis is part of a chess single round-robin tournament. The game of chess is modelled as a two-player simultaneous-move continuous game between two players, player 1 and player 2. Notably, this simultaneous-move game does not model the chess game *itself* in a game-theoretic way (see, e.g., Osborne & Rubinstein, 1994, p. 6).

In stage 1, nature determines each player i's innate ability α_i, age-related ability $\Omega_i = \Omega(A_i)$ (where Ω_i equals a function $\Omega(.)$ of the player's age A_i), handicap h_i (playing with White vs. playing with Black), and random component ε_i (e.g., luck, or idiosyncratic variation in performance). We assume that each player observes both his own and the other player's innate ability, and his own and the other players' age-related ability. Player i, however, observes neither his own random component nor the other player's. Each player i's performance q_i is now assumed to be equal to the sum of the player's innate ability, his age-related ability, his effort level μ_i, his handicap, and his random component:

$$q_i = \alpha_i + \Omega_i + \mu_i + h_i + \varepsilon_i \text{for } i \in \{1, 2\}. \quad (1)$$

Without loss of generality, other factors or interactions between factors that influence performance can be included if they enter the performance function in an additively separable way with effort.

In stage 2 of the game, each player i sets his effort level μ_i. We assume that player i is risk neutral, and that his payoff in playing against player j is additively separable and equals his expected Elo rating after the game, plus the expected contribution of the game's outcome to the player's ranking in the chess tournament which includes the game, minus his cost of effort. While we assume homogeneous cost of effort, Appendix A shows that the results extend to heterogeneous costs, modelled by costs that are a function of effort plus an idiosyncratic cost parameter. Note that Elo ratings are updated based on players' actual performance (a win, a draw, or a loss) relative to their expected performance (the predicted probability of a win, determined by the Elo rating system before the game), which is the essence of using an Elo rating system (Elo, 1978). To accommodate the possibility of a draw, we assume that the game ends in a draw if $\left|q_i - q_j\right| < \gamma$, that is if the difference between the players' performances does not exceed a certain performance threshold $\gamma > 0$ (Nalebuff & Stiglitz, 1983). Player i then wins when $q_i - q_j \geq \gamma$ and loses when $q_j - q_i \geq \gamma$. With a strictly convex function for the cost of effort, a Nash equilibrium is obtained when each player i maximises his expected payoff with respect to effort μ_i, given the effort μ_j of the other player

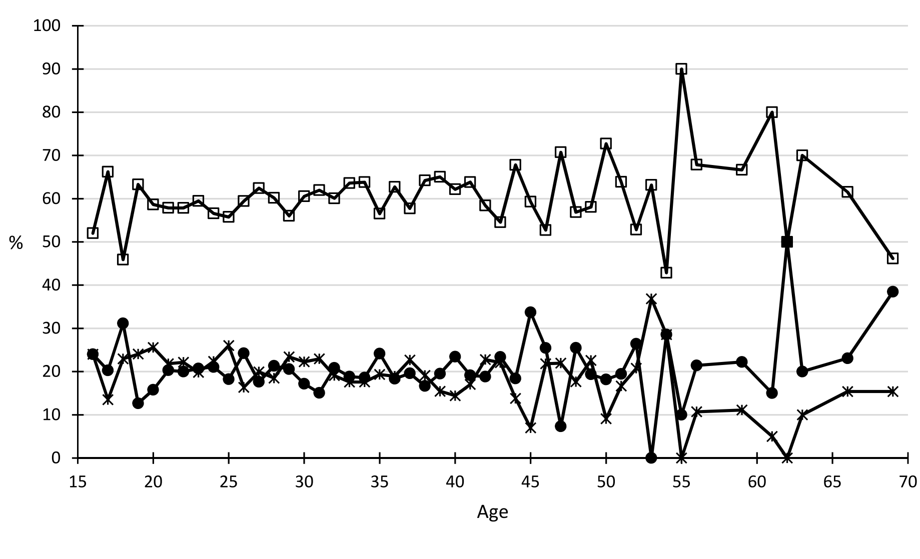

Fig. 1. Percentages of game outcomes by age.
Note. Based on 3151 games of 178 chess grandmasters for the period 1970–2021.

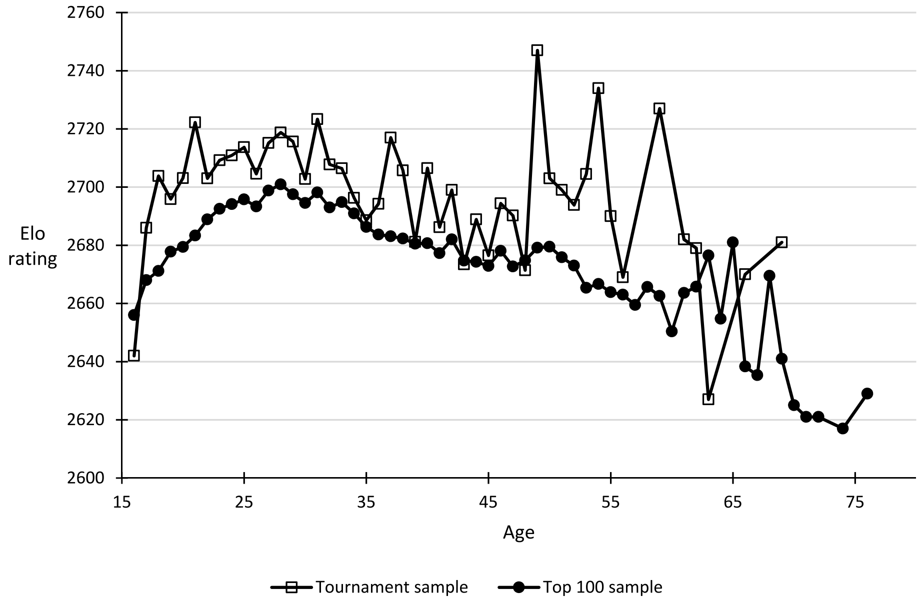

Fig. 2. Average Elo ratings by age for the tournament sample and for the top 100 sample.
Notes. Both samples: 178 chess grandmasters for the period 1970–2021. Players are, on average, 3.4 years in the tournament sample and 15.5 years in the top 100 sample.

(Lazear & Rosen, 1981). In equilibrium, efforts are equal ($\mu_i = \mu_j$ and $\Delta\mu_{i,j} = 0$, cf. Lazear & Rosen, 1981).

Denoting $\Delta\alpha_{i,j} = \alpha_i - \alpha_j$ $\Delta\Omega_{i,j} = \Omega_i - \Omega_j$, $\Delta h_{i,j} = h_i - h_j$, and given the result that $\Delta\mu_{i,j} = 0$, the probability $P^w_{i,j}$ of a win for player i can now be expressed as a function of the cumulative distribution function $F(.)$ of $(\varepsilon_j - \varepsilon_i)$:

$$P^w_{i,j} = F\left(\Delta\alpha_{i,j} + \Delta\Omega_{i,j} + \Delta h_{i,j} - \gamma\right). \tag{2}$$

Likewise, the probability $P^l_{i,j}$ of a loss for player i is $1-F(\Delta\alpha_{i,j} + \Delta\Omega_{i,j} + \Delta h_{i,j} + \gamma)$ and his probability $P^d_{i,j}$ of a draw is $1 - P^w_{i,j} - P^l_{i,j}$. The larger the performance difference $\Delta\alpha_{i,j} + \Delta\Omega_{i,j} + \Delta h_{i,j}$, the higher the probability that player i wins, which provides the basis for our empirical analysis which relates age to performance in individual games. The result that $\Delta\mu_{i,j} = 0$ and, therefore, that the difference in players' performances in a game is solely a function of their ability difference $\Delta\alpha_{i,j} + \Delta\Omega_{i,j} + \Delta h_{i,j}$ requires the assumptions that players are risk neutral and homogeneous in their linear payoffs.

2.2.2. Elo ratings and the age-performance relationship

Next, we show that the Elo ratings reflect players' abilities in the same manner as the individual game outcomes do. For initial Elo ratings r^0_i and r^0_j at the start of the chess game, assume that the probability of winning is reflected by the players' ratings, so that player i wins when $r^0_i + \eta_i \geq r^0_j + \eta_j$ iff $r^0_i - r^0_j \geq \eta_j - \eta_i$, where η_i and η_j are error terms. Denoting by $H(.)$ the normalised cumulative distribution function of $\eta_j - \eta_i$, the probability of winning is then

$$\pi_{i,j} = H\left(\frac{r^0_i - r^0_j}{\sigma}\right), \tag{3}$$

where $H(.)$ is, e.g., a cumulative standard normal or logistic distribution function (Elo, 1978), and where σ is the standard deviation of the difference between paired players' Elo ratings. The probability $\pi_{i,j}$ does not account for who is playing with White or Black, nor for the possibility of a draw, and determines the payoffs in terms of Elo ratings (Elo, 1978). Concerning the latter, the Elo rating system determines the probability $\pi_{i,j}$ before the game and it is known to the players.

Consider now in the same way $P^w_{i,j}$ in Equation (2) for $\Delta h_{i,j} = 0$, $\gamma = 0$, so that $P^w_{i,j} = F(\Delta\alpha_{i,j} + \Delta\Omega_{i,j})$. If the distribution of $\eta_j - \eta_i$ is identical to the distribution of $\varepsilon_j - \varepsilon_i$, so that $F(.) = H(.)$ and $\pi_{i,j} = F(\Delta\alpha_{i,j} + \Delta\Omega_{i,j})$, then

$$r^0_i - r^0_j = \sigma\Delta\alpha_{i,j} + \sigma\Delta\Omega_{i,j}. \tag{4}$$

Because a common performance factor included in Equation (1) would be differenced out in Equation (2) and not influence the payoffs in terms of Elo ratings, there is no common factor in Elo ratings, and

$$r^0_i = \sigma\alpha_i + \sigma\Omega_i. \tag{5}$$

This also reflects that Elo ratings are relative measures of performance.

Yet, since the Elo rating system is a heuristic algorithm, players' Elo ratings can be measured with error. For instance, discrete bins of rating differences are used in FIDE's $\pi_{i,j}$ calculations (International Chess Federation, 2017), which causes the distribution functions $F(.)$ and $H(.)$ to differ and leads to discrepancies between Elo ratings and relative chess abilities. Rating deflation can be another source of measurement error (Elo, 1978). Measurement error can be accommodated by adding it to Equations (4) and (5), as we also do in our empirical work. While such measurement error in a player's initial rating affects the expected payoffs for both players through the probability $\pi_{i,j}$, for risk-neutral players it does not affect their optimal efforts nor the outcome of a game. The APR based on the win-probability (Equation (2)) is, therefore, unaffected by measurement error in Elo ratings. The APR based on Elo ratings (Equation (4) or (5)) is unaffected by measurement error if it is independent of age.

2.2.3. Financial incentives

Previous studies such as Ehrenberg and Bognanno (1990a; 1990b) have for the task of playing golf shown the incentive effects of tournaments' prize distribution on players' efforts. For this they examined the level of performance which includes effort (e.g., Equation (1)). Though the effect of financial incentives on effort is beyond the scope of our paper because it would require different data, one may still wonder to what extent financial incentives affect our estimates of the APR. Yet, these estimates are insensitive to changes in, e.g., total prize money if the distribution of prizes remains the same for both players. This is because a change in the total prize money affects efforts of both players equally (an increase in v^w in Equation (A.5)) and it is only the difference in efforts ($\Delta\mu$) that matters for the outcomes of games. Under the assumption that players are risk neutral and homogeneous in their linear payoffs, $\Delta\mu = 0$ and the APR can be estimated based on outcomes of games or Elo ratings (Equation (2), (4), or (5)).

Further, appearance fees are a way for tournament organisers to attract top players. Under the assumption of risk neutrality such fees do not affect players' efforts, nor the APR. The argument for this is the same as the one for why optimal efforts are insensitive to players' Elo ratings before the game: it does not affect players' decision making on how much effort to provide (Equations (A.9)-(A.12), Appendix A).

In short, under the assumptions that players are risk neutral and homogeneous in their linear payoffs, financial incentives such as a rise in total prize money or appearance fees do not affect the APR.

2.3. Empirical framework

Three empirical models are outlined below based on, respectively, Equations (2), (4) and (5). The models were estimated with data on outcomes of chess games, on players' Elo ratings and their age, and the estimated APRs are presented.

The Tata Steel Chess tournament aims to draw the best players worldwide to participate; therefore, players are not randomly selected into our raw sample (see also Bertoni et al., 2015; Linnemer & Visser, 2016). In addition, we selected players who were in the top 100 worldwide. Player fixed effects are included in the three models to control for meritocratic sample selection. They also control for possible cohort effects (e.g., generational differences in the use of chess computers). Further, the three empirical models outlined below are estimated using a cubic age function:

$$\Omega(A;\boldsymbol{\beta}) = \beta_0 + \beta_1 A + \beta_2 A^2 + \beta_3 A^3, \tag{6}$$

where the parameter vector $\boldsymbol{\beta}$ determines the APR. A is the age of a player in full years.

2.3.1. Empirical Model 1

The first empirical model (Model 1) is based on the win-probability, i.e., Equation (2). A player's innate chess ability (α in the theoretical model) is unobserved and controlled for by including a player fixed effect. In general, however, a nonlinear model cannot be consistently estimated when controlling for player fixed effects. Therefore, the marginal effects on the probability of a win are modelled with a linear probability model (Wooldridge, 2010). This model allows for player fixed effects and is specified as follows:

$$W_{i,j} = \theta_0 + \Delta\alpha_{i,j} + \Delta\Omega_{i,j} + \theta_1 \Delta h_{i,j} + \zeta_{i,j}, \tag{7}$$

where $W_{i,j}$ is equal to 1 if player i wins the game against player j and equal to 0 if player i draws or loses, $\Delta\Omega_{i,j} = \Omega(A_i;\boldsymbol{\beta}) - \Omega(A_j;\boldsymbol{\beta})$ with $\Omega(.)$ defined in equation (6), $\Delta h_{i,j} = h_i - h_j$, $\Delta\alpha_{i,j}$ is the difference in players' fixed effects ($\alpha_i - \alpha_j$), θ_1 is the effect of playing White on the probability of a win, and ζ is an error term. Period effects play no role in this model because they are eliminated by taking differences in the performances of the two players in each game. To identify the age effect β_1 a model restriction is required because player fixed effects also control for birth cohort effects, hence the age difference between two players is equal to the difference in their years of birth. To identify β_1 we assume that for the, arguably, two best chess players in our sample, namely (former) world chess champions Garry Kasparov and Magnus Carlsen, meritocratic selection is not an issue. Both players are, or were, in the top 100 for their entire active careers and when they did not participate in the tournament, we assume it was for reasons unrelated to their chess abilities. The identifying assumption is implemented by restricting their player fixed effects to be equal. This approach still allows for random differences between the two players, which are assumed unrelated to their age in the sample.

2.3.2. Empirical Model 2

The second empirical model (Model 2) is based on Equation (4) and the APR is estimated using the differences in paired Elo ratings of players in the tournament sample. We add an idiosyncratic error to Equation (4), e.g., because of measurement error (see discussion below Equation (5)). Just as in the Model 1, the APR is identified with the restriction that the player fixed effects of Garry Kasparov and Magnus Carlsen are equal. We test the null hypothesis of the same APRs in Models 1 and 2 and thereby account for the different units of measurement. The null hypothesis can be rejected if the premise that Elo ratings measure relative performance does not hold. The null hypothesis can, however, also be rejected for different reasons, hence the alternative hypothesis is undefined. For instance, it can be rejected if measurement error is related to age because of rating deflation for relatively young players (Elo, 1978).

2.3.3. Empirical Model 3

The third empirical model (Model 3) is based on Equation (5) and the APR is estimated using players' Elo ratings from the tournament sample or from the top 100 sample. An idiosyncratic error term and a random period effect are added to the model. While there is no need to add a random period effect from a theoretical point of view, it is added to account for a possible dependency between observations within a period, e.g., because of rating deflation (Elo, 1978).

The main advantage of estimating Model 3 instead of Model 1 or Model 2 is that the APR is identified without the restriction that the player fixed effects of Garry Kasparov and Magnus Carlsen are equal. Further, our theoretical model implies the absence of period fixed effects, which reflects that the Elo rating is a relative performance measure and is in accordance with Elo's (1965) approach to estimating the APR. Random period effects can, therefore, be assumed. Nevertheless, the period random effects assumption is violated if, e.g., adopting a chess innovation such as a new chess computer is related to both age and chess performance. The age parameter β_1 in Equation (6) is not identified for a specification with period fixed effects; therefore, we test the period random effects assumption by testing the null hypothesis that the parameters β_2 and β_3 of, respectively, the age squared and age cubic terms, are the same for a period random effects specification and a period fixed effects specification.

2.3.4. Empirical models 1-3

All three models are estimated with least squares, standard errors are adjusted for two-way clustering at the levels of the two players for Models 1 and 2 and at the levels of the player and period for Model 3 (Gu & Yoo, 2019). The importance of controlling for player fixed effects when estimating the APR is for all models tested with the null hypothesis of player random effects (Wooldridge, 2010).

3. Results

For plausibly replicable results and following Benjamin et al.'s (2018) recommendation, we use 0.5% levels of significance for assessing the null hypotheses of statistical tests.

The theoretical finding of the same APR based on games' outcomes (Model 1) or based on paired differences in Elo ratings (Model 2) is empirically substantiated in Table 1 (bottom panel): the empirical evidence is in favour of the null hypothesis of the same APR for these two models (p-value = 0.965). This finding is reflected in an estimated APP of 33.4 for Model 1 and 33.5 for Model 2. Further, the test results do not support the null hypothesis of player random effects, opposed to player fixed effects (p-values <0.001).

The predicted probability of winning a chess game, holding the age of the opponent constant, is more than two times larger at the APP than in

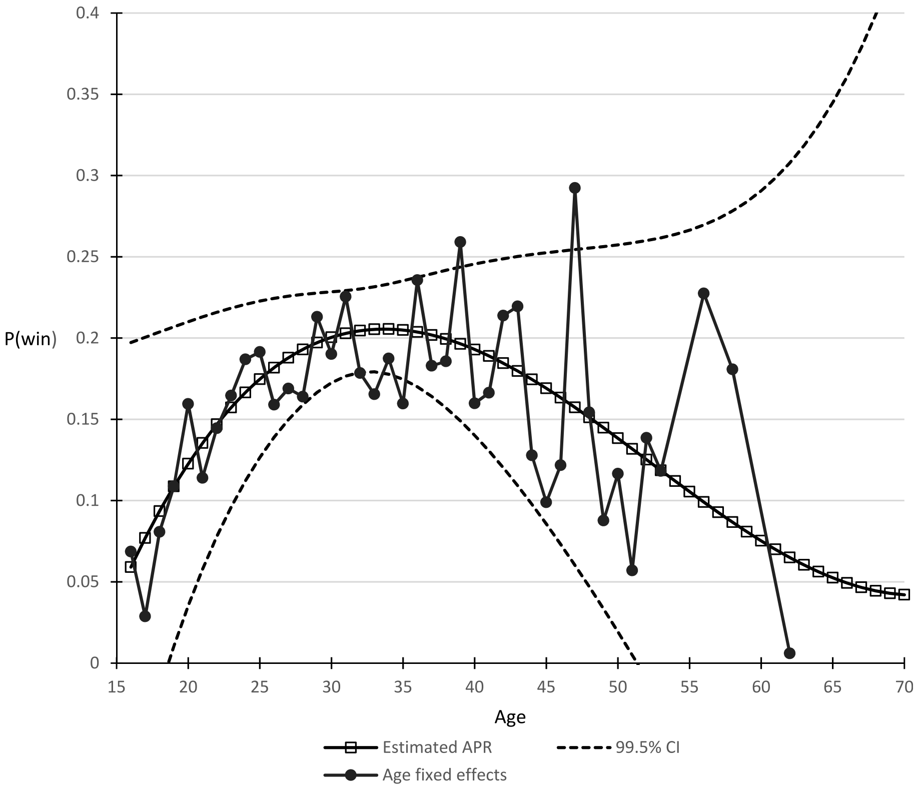

Fig. 3. The estimated age-performance relationship (APR) in terms of the probability of a win.
Notes. Based on the results of Table 1 (Model 1). The APP is about 33 years of age. The profiles are for a reference player with average innate chess ability, and an opponent with average innate chess ability and 32 years of age. Age fixed effects: APR based on included dummy variables for each age up to age 53 and age-groups for higher ages.

the two extremes at ages 18 and 60 (Fig. 3). There is a strong increase in performance at younger ages and a performance decline at older ages, albeit that the predictions are rather imprecise in the older age group. The latter imprecision limits the interpretation of the APR. The predicted APR based on paired Elo ratings is more precise than the one based on the win-probability; it shows a clear performance decline at older ages (Fig. 4).

The assumption of a cubic age function is rejected for both models (Table 1; 'H_0: Cubic age profile (vs. age-FE)'). Therefore, Figs. 3 and 4 also show the predicted APRs based on model specifications with dummy variables for each age ('Age fixed effects'). The figures show that the main conclusions remain when we allow for such a flexible age profile, confirming a rise in performance until, on average, players' early to mid-thirties and a decline in their performance after that age.

Comparing the results for Model 3 that uses Elo ratings (Table 2, specification 4) to the results for Model 2 that uses the paired Elo ratings (Table 1, second column) we find that their estimated APRs are about the same. This finding reflects that the APR can be estimated based on the levels of players' Elo ratings, which is in accordance with our theoretical model. Further, the empirical test results of specification 3 in Table 2 support a model for estimating the APR with period random effects and player fixed effects (i.e., specification 2).

Increasing the sample size and using the top 100 sample to estimate Model 3 with the same specifications as for the tournament sample

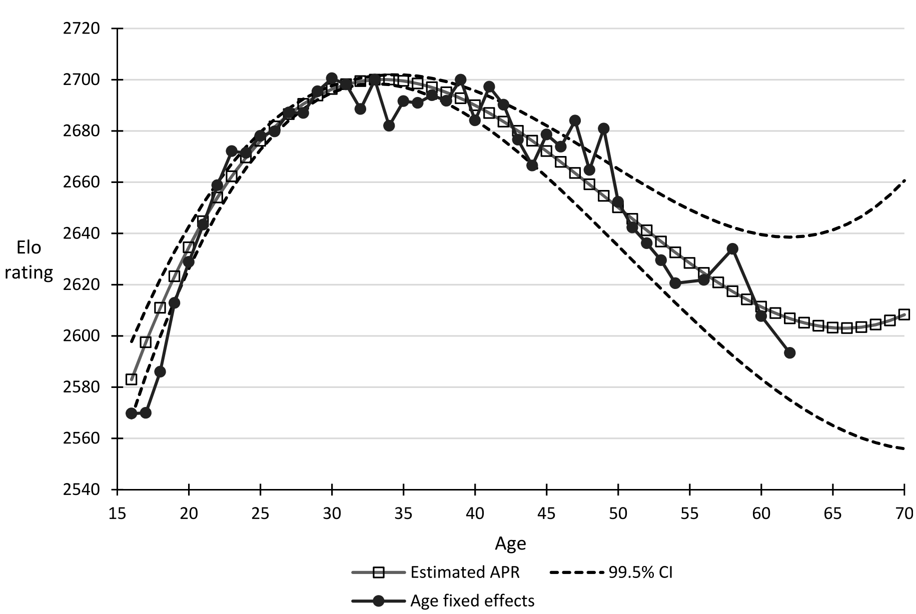

Fig. 4. The estimated age-performance relationship (APR) in terms of the Elo rating.
Notes. Based on the results of Table 1 (Model 2). The APP is about 34 years. The profiles are for a reference player with average innate chess ability, and an opponent with average innate chess ability and 32 years of age. Age fixed effects: APR based on included dummy variables for each age up to age 62 with the dummy for 62 including all ages over 61. For this graph, the Elo rating is normalised to 2700 at the APP.

Table 2
Estimates of the age-performance relationship (APR) based on players' Elo ratings using the tournament sample.

Model 3, based on Equation (5) Specification:	1	2	3	4
Period effects	Fixed	Random	Fixed	Fixed
Player effects	Random	Fixed	Fixed	Fixed, FE-restricted
	Coef.	Coef.	Coef.	Coef.
	(Std.Err.)	(Std.Err.)	(Std.Err.)	(Std.Err.)
Age/10	189.152*	369.743***		378.650***
	(84.308)	(49.121)		(39.059)
$(Age/10)^2$	−47.090*	−82.210***	−84.899***	−84.899***
	(23.362)	(12.694)	(9.868)	(9.868)
$(Age/10)^3$	3.652	5.406***	5.658***	5.658***
	(1.973)	(1.036)	(0.786)	(0.786)
Age of peak performance (APP)	31.983	33.673		33.559
	(2.674)	(0.816)		(0.496)
H_0: Player random effects[a]			<0.001	
H_0: Period random effects[a]			0.763	
H_0: Cubic age profile (vs. age-FE)[a]	<0.001	<0.001	<0.001	<0.001
H_0: No age effects[a]	0.011	<0.001	<0.001	<0.001
R^2	0.429	0.882	0.907	0.907

Notes. Tournament sample: 178 players produce 607 observations. Only for specification 4: FE-restricted refers to the identification of the APR by assuming the same player fixed effects for Garry Kasparov and Magnus Carlsen. Age-FE: age fixed effects. Linear regression models are estimated with least squares and clustered standard errors are reported.
$*p < 0.05$; $**p < 0.01$; $***p < 0.005$.
[a] Entries are p-values.

Table 3
Estimates of the age-performance relationship (APR) based on players' Elo ratings using the top 100 sample.

Model 3, based on Equation (5) Specification:	1	2	3	4
Period effects	Fixed	Random	Fixed	Fixed
Player effects	Random	Fixed	Fixed	Fixed, FE-restricted
	Coef.	Coef.	Coef.	Coef.
	(Std.Err.)	(Std.Err.)	(Std.Err.)	(Std.Err.)
Age/10	99.365*	335.487***		337.904***
	(39.047)	(33.049)		(24.392)
$(Age/10)^2$	−24.722*	−74.584***	−74.345***	−74.345***
	(10.447)	(8.435)	(6.143)	(6.143)
$(Age/10)^3$	1.788*	4.778***	4.736***	4.736***
	(0.850)	(0.683)	(0.480)	(0.480)
Age of peak performance	29.602	32.879		33.360
	(1.402)	(0.457)		(0.364)
H_0: Player random effects[a]			<0.001	
H_0: Period random effects[a]			0.903	
H_0: Cubic age profile (vs. age-FE)[a]	<0.001	<0.001	<0.001	<0.001
H_0: No age effects [a]	0.003	<0.001	<0.001	<0.001
R^2	0.185	0.782	0.811	0.811

Notes. Top 100 sample: 178 players produce 2753 observations. Only for specification 4: Only for specification 4: FE-restricted refers to the identification of the APR by assuming the same player fixed effects for Garry Kasparov and Magnus Carlsen. Age-FE: age fixed effects. Linear regression models are estimated with least squares and clustered standard errors are reported.
$*p < 0.05$; $**p < 0.01$; $***p < 0.005$.
[a] Entries are p-values.

improves the precision of the estimated APR (Table 3). Just as in Table 2, the empirical test results in Table 3 support using period random effects and player fixed effects. Specification 4 controls for period and player fixed effects and, in the same way as in Table 1, identifies the APR by assuming the same fixed effects for Garry Kasparov and Magnus Carlsen. The results for specification 4 in Table 3 are in line with the ones for specification 2. Nevertheless, some caution is warranted when using specification 4 because it is, after all, based on an untestable assumption. Fig. 5 shows the predicted APRs for different model specifications. The confidence intervals in this figure are based on estimates of specification 2 with age fixed effects ('Player fixed effects (specification 2) & Age fixed effects'). The latter results are not presented in a table. Specification 1 with player random effects results in a flatter APR than does the specifications with player fixed effects (specifications 2 or 4). This shows the importance of controlling for, arguably, meritocratic sample selection through the inclusion of player fixed effects. Also, when we use period random effects, player fixed effects, and a cubic age profile (specification 2), the APR falls in the reported confidence intervals.

Finally, Model 3 with specification 2 is a regression-based equivalent of the estimation methodology used by Elo (1965). We have estimated this model on the dataset used by Elo (1965) and replicated his findings (see Table D1 and Figure D1). The estimated shape of the APR for Elo's sample is in line with the one of Table 3 or Fig. 5 (specification 2; player fixed effects and period random effects), and with the one of Elo (1965). The estimated APP with Elo's sample is, however, on average about three years higher (99.5% CI: 34.2–38.5; Appendix D) than the estimate reported in Table 3 (99.5% CI: 31.6–34.2, specification 2).

4. Discussion

Based on an empirical model firmly rooted in a theoretical model of optimal choice, our empirical findings show that the estimated age-performance relationship (APR) for the cognitive-intensive task of the game of chess is hump-shaped with, on average, an age of peak performance (APP) in chess grandmasters' early thirties. Arguments for a hump-shaped APR are a combination of, e.g., increased learnt knowledge during adulthood that dominates until the APP and a cognitive decline that is associated with biological ageing of the human brain (Elliott et al., 2019; Jastrzembski et al., 2006; Kaufman & Horn, 1996; Salthouse, 2009; van der Maas & Wagenmakers, 2005). Other arguments are related to, e.g., motivation and fatigue (Draper, 1963; Hertzog, 2020; Kanfer & Ackerman, 2004).

4.1. *This study's limitations and an assessment of previous studies' methods and findings*

While the hump-shaped APR estimated by Arpad Elo (Elo, 1965) stands the test of time, our estimated APR differs in finding an APP of about three years lower. A possible explanation for this difference is that the younger cohorts started with competitive chess earlier than the older cohorts (Strittmatter et al., 2020). The results in Appendix E confirm the finding of Strittmatter et al. (2020) that the APP decreased with an increase in players' year of birth and, in contrast to the findings of Strittmatter et al. (2020), show that for all birth cohort groups there is a decline in performance after APP. A limitation of our study implied by the results in Appendix E is that our estimated APR is the average APR across birth cohorts.

Further, we find that also Elo's methodology for estimating the APR based on Elo ratings stands the test of time. Our findings support the assumption of period random effects when we control for player fixed effects. These findings, therefore, do not support the empirical approach recent studies such as Roring and Charness (2007) and Vaci et al. (2015) took of controlling for player random effects (and period fixed effects). The importance of controlling for meritocratic sample selection by

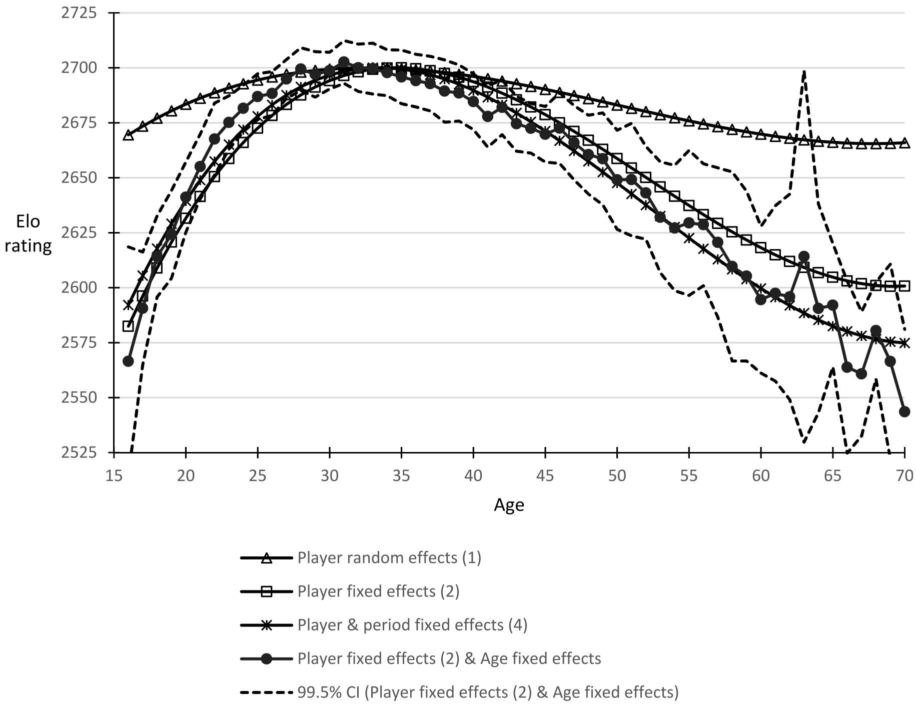

Fig. 5. Age-performance relationships (APRs) based on individuals' Elo ratings for different model specifications.
Notes. Based on the estimates of Table 3 (top 100 sample). In the Figure's legend, the empirical specifications of Table 3 are in parentheses. 'Age fixed effects' includes a full set of age dummies up to age 70 instead of a cubic age profile. The confidence intervals (CI) are based on the latter specification. For this graph, the predicted Elo ratings are normalised to 2700 at the APP.

including player fixed effects points to a further limitation of our study: the estimation sample is conditional on tournament participation or being in the top 100. Hence, for two relatively old players with different declines in performance after the APP, the player with the strongest decline is less likely to be in our estimation sample (ceteris paribus). Such heterogeneity in the decline in performance after the APP suggests that our estimated average decline in performance after the APP is a lower bound of the true average decline. Appendix F provides empirical insight into the effect of positive sample selection on the APR.

From a methodological point of view, our study unites the two streams of empirical studies that use either Elo ratings (e.g., Berthelot et al., 2019; Elo, 1965; Roring & Charness, 2007) or performance measures (e.g., Rubin, 1960; Strittmatter et al., 2020) for estimating the APR for the game of chess. These two approaches are linked on the premise that the heuristic Elo system measures relative performance. There are, under the stipulated assumptions of our theoretical model, no theoretical advantages to using either of these approaches since both can be rooted in a version of Lazear and Rosen's (1981) rank-ordered tournament model tailored to chess. This model assumes risk-neutral players who are homogeneous in their linear payoffs and choose their levels of effort in a way that maximises their expected Elo rating and tournament ranking. Further, both approaches are facilitated by the increased data availability on players' Elo ratings and their game performances (e.g., Strittmatter et al., 2020; Vaci & Bilalić, 2017).

There is, however, an important empirical advantage to using Elo ratings instead of performance measures such as games' outcomes or levels of performance, for estimating the APR. Based on levels of performance of players, which requires controlling for period fixed effects, or on outcomes of games, the APR is not fully identified and cannot be estimated when player fixed effects are controlled for unless an additional modelling assumption is made, e.g., grouping of periods and birth cohorts (Strittmatter et al., 2020) or a restriction on player fixed effects, which is a further limitation of our study when using outcomes of games (Table 1). Based on Elo ratings, however, the APR is identified and can be estimated when player fixed effects are controlled for because there is no need to control for period fixed effects (Table 3). The latter finding reflects that Elo ratings measure relative performance and common period effects are eliminated.

4.2. Generalisation of the methods and findings

Our estimate of the APR is for the specific cognitive-intensive task of the game of chess and for a specific population (chess grandmasters). The APR will differ for tasks with different cognitive or physical demands (Berry et al., 1999; Börsch-Supan & Weiss, 2016; Hertzog, 2020; Lehman, 1953). Nevertheless, for the literature on APRs, the theoretical and empirical insights our study gives when using Elo ratings to estimate the APR for the task of playing chess, can facilitate future research on estimating the APR for competitive tasks and for which relative performance measures such as Elo ratings are available. Other examples of such tasks in the domain of (e−)sports are the games of table tennis, Go, tennis, or League of Legends. Also, in the domains of the labour market and education, tasks such as software development (Boudreau et al., 2016) or e-learning (Mangaroska et al., 2019; Pelánek, 2016) can be illustrative.

4.3. Labour market implications

Insofar as cognitive-intensive tasks have an APP during working life, our empirical finding of a decline in performances after the APP illustrates the concern that in a labour market characterised by an increase in cognitively demanding jobs and people working longer, aggregate productivity can be at risk.

From a career perspective, workers who start their careers with a cognitively demanding job (e.g., software developers), experience after their APP a performance decline for their cognitively demanding task and can increase their job performance by taking up tasks that, e.g., require relatively more skills related to crystallised intelligence obtained from work experience (Bruine de Bruin, 2018), or more social skills (Weinberger, 2014). This suggests that an increase in cognitively demanding jobs (Acemoglu & Autor, 2011) can increase employers' demand for, e.g., workers who can combine cognitive and social skills to stay productive throughout their careers. Such a view is supported with empirical evidence on a stronger complementarity over time between social and cognitive skills (Deming, 2017; Weinberger, 2014). Working longer because of, e.g., physical health improvements and pension policies (Coile et al., 2019) would strengthen such a complementarity that, arguably, becomes more relevant at older ages. Such trends also underline the importance of social skills, or soft skills, in educational programs (Heckman & Kautz, 2012).

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Appendix A. Supplementary data

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