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Generating the Ground Truth: Synthetic Data for Label Noise Research

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Abstract

Most real-world classification tasks suffer from label noise to some extent. Such noise in the data adversely affects the generalization error of learned models and complicates the evaluation of noise-handling methods, as their performance cannot be accurately measured without clean labels. In label noise research, typically either noisy or incomplex simulated data are accepted as a baseline, into which additional noise with known properties is injected. In this paper, we propose SYNLABEL, a framework that aims to improve upon the aforementioned methodologies. It allows for creating a noiseless dataset informed by real data, by either pre-specifying or learning a function and defining it as the ground truth function from which labels are generated. Furthermore, by resampling a number of values for selected features in the function domain, evaluating the function and aggregating the resulting labels, each data point can be assigned a soft label or label distribution. Such distributions allow for direct injection and quantification of label noise. The generated datasets serve as a clean baseline of adjustable complexity into which different types of noise may be introduced. We illustrate how the framework can be applied, how it enables quantification of label noise and how it improves over existing methodologies.

1 Introduction

Classification models are of great interest to the research community and machine learning practitioners alike. When applied to real-world problems, these models are confronted with noisy data, with noise defined as anything that obscures the relationship between the dependent and independent variables (Hickey 1996). Label noise can have detrimental effects on classifier performance, model complexity, learning rates and effect size estimation (Frénay and Verleysen 2014). Therefore, a lot of research is conducted into prediction methods that are robust to such noise and into preprocessing steps for filtering label noise from data (Frénay and Verleysen 2014; Song et al. 2022).

When algorithms are evaluated with regard to their ability to handle label noise, typically, existing real-world datasets are assumed to be the ground truth, after which artificial noise is injected into the labels (Berthon et al. 2021; Cheng et al. 2020). Alternatively, data is entirely simulated, often by modelling relatively simple relationships between the dependent and independent variables (Berthon et al. 2021; Cheng et al. 2020). Lastly, a limited number of curated datasets are publicly available for which the label noise has been quantified by expert annotators (Xiao et al. 2015; Lee et al. 2018; Li et al. 2017).

Running experiments using any of the above types of datasets has drawbacks associated with it: either the relationships in the data are not sufficiently complex and therefore not realistic (simulated data), or there are no clean labels available to evaluate on and noise has to injected into already noisy data (real-world data), or the noise cannot be tailored, such that any method is only tested on a very specific noise pattern (curated data). Creating a curated dataset takes considerable effort as well. While noisy real-world data are available in abundance, the lack of a clean evaluation set cannot be easily overcome. In (Frénay and Verleysen 2014) it is stated, in the context of method comparison, that the presence of label noise in the validation data causes any estimates to be off by an unknown amount. The authors mention this as an important open research question, one which we address in this work.

In this paper we aim to improve upon the aforementioned experimental strategies. We present the SYNLABEL framework: Synthetic Labels As Baseline for Experiments with Label noise. SYNLABEL facilitates the construction of artificial tabular datasets for performance evaluation of methods dealing with label noise. We propose to define a prespecified or learned function as the ground truth relationship. Then, by applying this ground truth function to any input data contained in its domain, noiseless labels are generated. As we are interested in testing methods on known noise, rather than finding the best model for a specific real-world problem, this function is not required to exactly represent the original data. The generated ground truth set can be further transformed into a partial ground truth set for which each data point is accompanied by a soft label: first, a number of specified variables from the ground truth dataset are hidden. Then, by learning or specifying a (conditional) prior distribution, resampling values from it and combining these sampled values with fixed values for the variables that were not hidden, a posterior distribution is generated via the ground truth relationship. Although the prior distribution over the hidden variables is almost guaranteed no to match the exact underlying distribution of the real-world generative distribution over the original dataset, as before this is not necessary for our purpose. The advantage of constructing a set with soft labels compared to dealing with hard labels is that it allows for explicit quantification and direct injection of label noise. Furthermore, these sets can be used for problems from different domains that can be mapped to a label distribution problem, e.g. learning from crowds or with confidence scores. The sets with clean hard or soft labels serve as a starting point from which further transformations can be applied to the data in order to generate the specific noise of interest, allowing for any such added noise to be quantified.

In summary, the key contributions of this paper are:

- The SYNLABEL framework which facilitates the generation of experimental datasets for label noise research.
- A method for constructing a ground truth dataset informed by real-world data for the purpose of evaluation.
- A method for converting hard into soft labels by resampling values for features that are hidden from the model.
- An analysis showing the advantages of using soft labels for quantification and injection of label noise.

2 Related Work

Systematic experiments in label noise research require a dataset with both clean labels as a baseline to evaluate on, as well as noisy labels. Based on how these sets are obtained, the experiments in the label noise field can be placed into three categories: (1) an existing dataset for which the labels have been manually corrected is used, or artificial noise is injected into the labels of either (2) a clean dataset which has been simulated or (3) an existing real-world dataset.

The first type of experiment uses a curated real-world dataset for which noisy labels have been corrected, such as Clothing1M (Xiao et al. 2015), Food-101N (Lee et al. 2018) and WebVision (Li et al. 2017). The ground truth labels are generally decided upon by a panel of experts. Nevertheless, even among experts high inter-observer disagreement may occur (Veta et al. 2016).

In both the second and third category of experiments, first a baseline is established after which noise is injected into the labels. This injected noise is constructed based on a selected noise model. In order of decreasing commonality and increasing complexity, the injected noise can be classified as (Frénay and Verleysen 2014): Noisy Completely At Random (NCAR) (Angluin and Laird 1988), Noisy At Random (NAR) (Lawrence and Schölkopf 2001) or Noisy Not At Random (NNAR) (Jin and Ghahramani 2002; Chen et al. 2021; Xia et al. 2020; Zhang et al. 2021; Garcia et al. 2019).

When synthetic data is constructed, the true underlying function is known by design, e.g. data is sampled from Gaussian distributions or constructed using rule-based generation (Hickey 1996). Clean labels are then generated from this function, to which noise can be added. The largest downside is that these simulated sets generally lack the complex interactions that one expects between variables in real data.

In case of real-world data, when there are no resources available to curate the sets, the true labels remain unknown and thus the level of natural noise cannot be quantified (Hickey 1996). The importance of using controllable artificial data, especially in the context of noise, was already mentioned in (Langley 1988), as it enables systematic research into different aspects of a domain. While the use of real-world data is the least labour intensive experimental method, the effects of any further added label noise cannot be separated from the inherent noise present and thereby the overall noise level cannot be sufficiently controlled.

In summary, while different types of label noise experiments exists, they each suffer from shortcomings. Furthermore, when using hard labels, noise cannot be specified for any individual label, beyond the label being correct or not.

Recently, (Gu et al. 2022) proposed a framework for generating instance-dependent label noise. Our work differs from theirs in a few ways: they introduce a particular type of classifier based instance-dependent noise, whereas our framework allows for any type of noise injection. Furthermore, a dataset to be used with their work requires the labels to be clean, while we present methods for generating such data with either clean hard or soft labels.

3 The SYNLABEL Framework

We present the Synthetic Labels As Baseline for Experiments with Label noise (SYNLABEL) framework, shown in Figure 1, which facilitates generating synthetic tabular datasets for use in label noise experiments.

The SYNLABEL framework defines different types of datasets and transformations between them. Each dataset consists of input variables X, labels y and a functional relationship between the two, y = f(X). The two types of ground truth datasets depicted in the Unobservable part of Figure 1, are generally unobtainable for real-world problems, as their functional relationship is defined to be exact, i.e. noiseless. In practice, an Observable dataset is available and often the task at hand is precisely to discover a relationship between X and y that generalizes well. The datasets can be further categorized based on whether the output is a single unambiguous class, also known as a hard label, or a discrete probability distribution over the label space, commonly referred to as a soft label.

The user is encouraged to utilize the framework to construct a noiseless ground truth dataset based on a known, possibly learned, functional relationship. Any further noise applied to this dataset can be quantified exactly for each individual data point. This allows for analysing method performance on a specific type of noise in isolation. We stress that SYNLABEL is not meant for optimizing models for a specific dataset, but rather for evaluating and comparing methods in the presence of label noise. In the following we describe the different components of the framework.

Notation

In the SYNLABEL framework, a dataset D is made up of objects o denoted by $D : o_i = \{X_i, y_i\}$. The framework is meant for any deterministic classification task:

Definition 1 A deterministic classification task is a task for which, given that all required information (X^G) to the outcome y^G is available, there is a true deterministic function $f^G(X^G) = y^G$. In other words, given that we know all of the relevant information to the task X_i , y_i is unambiguously assigned one true class through the function f^G for all $y_i \in y^G$. A dataset with corresponding classification task for which Definition 1 holds is defined as a Ground Truth (G) dataset D^G :

Definition 2 A Ground Truth (G) dataset D^G : $o_i = \{X_i^G, f^G(X_i^G) = y_i^G\}$ is a dataset for which any input X_i^G in the domain of f^G is mapped to its deterministic hard label y_i^G by the true function f^G .

Note that a dataset almost never satisfies this definition unless it is simulated. SYNLABEL offers the tools to generate such a dataset based on a noisy dataset.

When not all features required for deterministic classification are available, yet the true classification function is known, the dataset is referred to as a Partial Ground Truth (PG) dataset D^{PG} , defined as:

Definition 3 A Partial Ground Truth (PG) dataset D^{PG} : $o_i = \{X_i^{PG}, f^G(X_i^{PG}) = y_i^{PG}\}$ is a dataset for which any input X_i^{PG} in the domain of f^G is mapped to its soft label y_i^{PG} by the true function f^G .

Here $X^{PG} \subset X^G$ contains the available data for the task, while any unavailable features are contained in $X^{PG'} \subset X^G$ such that $X^{PG} \cup X^{PG'} = X^G$. Since some of the information $(X^{PG'})$ needed for an exact classification is missing, $f^G(X^{PG})$ produces discrete label distributions or soft labels y^{PG} , with quantifiable uncertainty, instead of hard labels.

In the (Partial) Ground Truth sets the input X is mapped to the output y by the true underlying function f^G . Such datasets with corresponding mapping are practically not obtainable in real life, i.e. they are unobservable.

Data that is observed in practice, D^O , has different characteristics: noise can be present in both the observed input data X^O , as well as in the corresponding label y^O . Often, y^O is not measured directly and is instead annotated by an expert or system, based on their own non-deterministic, noisy functional relationship f^E . Such an expert may have the same information available as is available for the classification task, i.e. $X^E \subseteq X^O$, or additional relevant information $X^{O'}$ could be available: $X^E \subseteq (X^O \cup X^{O'})$.

An annotator, either implicitly or explicitly, assigns probabilities to the different candidate labels corresponding to an object o_i , producing a Observed Soft Label (OS) set with corresponding outcome y^{OS} . This set is often discretized into a Observed Hard Label (OH) set with label y^{OH} based on some decision function f_{dec} . Depending on whether the intermediate label distribution is preserved (OS) or not (OH), the final dataset becomes either D^{OS} :

Definition 4 An Observed Soft Label (OS) dataset D^{OS} : $o_i = \{X_i^O, y_i^{OS}\}$ is a dataset for which the input X^O is associated with soft labels y^{OS} .

or more commonly D^{OH} :

Definition 5 An Observed Hard Label (OH) dataset D^{OH} : $o_i = \{X_i^O, y_i^{OH}\}$ is a dataset for which the input X^O is associated with hard labels y^{OH} .



Figure 1: A schematic overview of the SYNLABEL framework. The white boxes represent data, either input X or labels y. The gray boxes represent a type of dataset, which includes a function relating the input to the output, although this function may not be defined in case of observable data. The arrows represent the different transformations and functions as specified above. Rs: Resampling. f(): a function.

Note that there are no function-related requirements for these sets. While a function f^O may be learned from these data, it is nearly guaranteed not to match the true functional relationship for the corresponding deterministic classification task. An overview of the different datasets defined in Definition 2-5 is presented in Table 1.

4 Data Transformations

At the core of the SYNLABEL framework lie the different operations that enable a user to transform the datasets from one type to another. These allow a user to obtain both a (Partial) Ground Truth dataset for validation as well as realistic datasets that contain the specific type of label noise for which an experiment is to be conducted.

The different types of datasets can be transformed in two directions. The direction in which data is most often transformed, to obtain observed sets containing varying label noise, is down the chain: Ground Truth $(G) \rightarrow$ Partial Ground Truth $(PG) \rightarrow$ Observed Soft Label $(OS) \rightarrow$ Observed Hard Label (OH). Transformations in this direction ensure that the objects that are contained in each set remain coupled, i.e. two objects o_i in different sets following a transformation down the chain still represent the same entity and remain linked to the original relationship that governs the ground truth data from which they originate. The reverse direction, up the chain, is made possible by using learned functions. In this case, however, a distinct dataset is generated, that is, the objects in the original set are not the same as those in the transformed set, since a new ground truth relationship is defined. The objects become decoupled.

Objects remain coupled following an arbitrary number of transformations both up and down the chain only when all sets are identical, thus $D^G = D^{PG} = D^{OS} = D^{OH}$. This implies that the soft labels, y_i^{PG} and y_i^{OS} , have probability 1 for the class in y_i^{OH} and y_i^G and probability 0 for the other classes and that the true functional relationship f^G required for the (Partial) Ground Truth is known.

The transformations and functions and their corresponding types are shown in Figure 1. A transformation can either be an identity transformation, through which the data are not altered, or a non-identity transformation which adds noise to the data. A function can either be a true function or some other function, e.g. a learned function or specified decision function, denoted by any function.

Down the chain

In the following we describe the supported transformations down the chain. These transitions allow the objects *o* to remain coupled between the different datasets.

From Ground Truth to Partial Ground Truth An identity transformation exists between X in D^G and in D^{PG} . The variables contained in X^G have identical values in D^{PG} , but they may be split into two sets, X^{PG} and $X^{PG'}$. Additionally, the function f^G describes the true relationship between X and y: $f^G(X_i^G)$ is equal to the true class of object o_i with absolute certainty. $f^G(X_i^{PG})$ is equal to the true soft label of o_i , given that some information contained in X_i^G is now contained in $X_i^{PG'}$ and thus missing from X_i^{PG} , whereby the classification task becomes ambiguous.

To obtain D^{PG} , $X^{PG'}$ cannot simply be ignored and a function f^{PG} learned on $\{X^{PG}, y^G\}$, as the resulting function would not equal the original relationship f^G . This follows from the fact that the variables in X^{PG} would have to be irrelevant to the task and such variables are not contained in X^G by definition, and thus $X^{PG'}$ by extension. The exception is when $X^{PG'}$ is empty, resulting in the special case $X^{PG} = X^G$ and $D^G = D^{PG}$. Otherwise, the missing information from $X^{PG'}$ will cause a different function to be learned. The labels produced by f^{PG} are then guaranteed to differ from y^G for some objects in the domain of f^G . Two such labels must both be the true label for such an object, which is contradictory, hence the sets become decoupled.

In addition to the identity transformation, $X^{PG} = X^G$,

Dataset type	Label	Function
Ground Truth (G)	Hard	True
Partial Ground Truth (PG)	Soft	True
Observed Soft Label (OS)	Soft	Any
Observed Hard Label (OH)	Hard	Any

Table 1: The different dataset types defined in SYNLABEL.

there is a transformation that we call feature hiding, which preserves the coupling between the objects in both sets when $X^{PG} \neq X^G$ and which preserves the truth relationship f^G . Instead of trying to learn a function from the known features X^{PG} , as before, we use f^G and the information we have about the missing features in $X^{PG'}$ to construct y^{PG} as follows: first we resample a number of values j for the features contained in $X^{PG'}$ for each object i in accordance with a probability density function which was constructed based on X^G . This can be either a conditional density function, $P(X^{PG'}|X^{PG})$ or even $P(X^{PG'}|X^{PG} \cup y^G)$, or a marginal density function, $P(X^{PG'})$. These multiple sampled values $X_{i,j}^{PG'}$ are then joined with the known values X_i^{PG} for each object o_i after which f^G is applied to all combinations of resampled and known values to obtain a corresponding label. We then aggregate the obtained labels into a soft label:

$$y_{i,c}^{PG} = \sum_{j=1}^{n} \frac{\mathbb{1}_{c} (f^{G}(X_{i,j}^{PG'} \cup X_{i}^{PG}))}{n},$$
(1)

which returns the probability of class c for object i, with $\mathbb{1}_c$ the indicator function. As $n \to \infty$, an exact soft label for y_i^{PG} is obtained given the density function.

The values in X^G could be sampled from an infinite number of distributions. Therefore, it is impossible to determine the exact prior distribution for $X^{PG'}$ from which we should resample. On the other hand, any distribution from which X^G could possibly be sampled is a valid choice and allows for generating soft labels that reflect the ground truth given that specific distribution. Resampling from any valid distribution thus allows for the creation of a new D^{PG} from a D^G by which the coupling between objects remains intact.

From Partial Ground Truth to Observed Soft Label The following non-identity transformations from D^{PG} to D^{OS} serve to produce experimental datasets tailored to a variety of different classification tasks, with corresponding (Partial) Ground Truth labels being available to evaluate on:

- X^{PG} to X^O . By using any other relationship than the identity transformation, X^O may be altered directly by for instance applying Gaussian noise to (some of) the variables in X^{PG} . This can result in label noise when X^O is used to generate y^{OS} further on via X^E and f^E .
- y^{PG} to y^{OS} . If y^{OS} is not established based on the variables in X^O and/or $X^{O'}$, but is measured directly, noise can be introduced directly to y^{PG} . This facilitates the NCAR and NAR noise models.
- X^{PG} and/or $X^{PG'}$ and possibly y^{PG} to y^{OS} . If y^{OS} is

determined based on X^{PG} and/or $X^{PG'}$ and y^{PG} , noise can be generated according to the NNAR model.

- X^O to X^E . If, in contrast to the previous transformation, y^{OS} is decided upon through f^E , for instance by an expert panel or system, using information X^E which is based on X^O , the labels y^{OS} can be manipulated by adding noise to X^O .
- $X^{PG'}$ to $X^{O'}$ to X^E . If as in the previous transformation y^{OS} is decided upon by an expert through f^E using X^E , and this expert has more relevant information $X^{O'}$ available than is contained in X^O alone, noise can be added to y^{OS} by using a non-identity transformation between $X^{PG'}$ and $X^{O'}$. An example of $X^{O'}$ would be textual descriptions that can be used by a physician when labelling for the presence of some disease which are not readily available to be used by a classification model.
- X^E to y^{OS} . If, as per the previous two transformations, y^{OS} is obtained trough f^E based on X^E , the labels may be transformed by adjusting the annotation function f^E .

The specific transformation used to add label noise to the data is decided upon by the user. The transformation should be such that the resulting dataset is suited toward the classification task for which a method is to be validated. In case the user is interested in soft label research instead of label noise research, the identity transformations may also be applied such that $X^O = X^{PG}$ and $y^{OS} = y^{PG}$.

From Observed Soft Label to Observed Hard Label Given X^O and y^{OS} , transformation to D^{OH} is straightforward. A decision function f_{dec} needs to be defined, which converts the soft labels into hard labels. Examples of such a function would be sampling the soft label distribution or simply selecting the class with the highest probability, although many more decision functions are possible. Note that this function may be stochastic, allowing for random tie breaks in case of equal probabilities, as there exists no truth requirements for this functional relationship.

Back up

Transformations down the chain, i.e. $D^G \rightarrow D^{PG} \rightarrow D^{OS} \rightarrow D^{OH}$, can be used to generate arbitrarily many observed datasets from a single ground truth set by adding some specified noise. Before this can be done, however, a ground truth dataset has to be constructed. This can be achieved using a real-world observed dataset by utilizing a transformation up the chain. Such transformations have to respect the constraints posed on the different datasets per Definition 2-5, summarized in Table 1, which generally forces the objects in the different sets to become decoupled:

• When transforming from D^{PG} to D^G , a decision function f_{dec} must be applied to y^{PG} to transform the soft labels into hard labels. Unless $X^{PG'}$ is empty and thus $X^{PG} = X^G$, $X^{PG'}$ contains information that is taken into account by f^G by definition. This information then has to result in different labels for $f_{dec}(f^G(X^{PG} \cup X^{PG'}))$ compared to $f_{dec}(f^G(X^{PG}))$ for some values

in the domain of f^G . Due to the missing information in $X^{PG'}$, this transformation then decouples the objects.

• When transforming from D^{OS} or D^{OH} to D^G or D^{PG} , the true function f^G for the task would need to be discovered. This is generally impossible, as observed data is always finite and theoretically infinite functions can be found that describe it perfectly, with no further information being available that enables the selection of the true governing function. Furthermore, duplicate instances in X^O can have different hard or soft labels, whereas y^G is constrained to contain deterministic hard labels.

As stated before, the exception is when sets along the chain are identical and thus noiselessly observed with a known ground truth function, which is generally not the case.

It is possible, however, to construct a different ground truth dataset based upon the observed dataset, such that its requirements are fulfilled. This will cause the objects in the new dataset to become decoupled from those in the original dataset. While this is an issue when the task of interest is to find the best model for a specific dataset, when the aim is to create a suitable dataset for validation of a method for a certain type of label noise this is of no concern.

To construct a new D^{PG} , we have to meet Definition 3, i.e. we must obtain both soft labels and the true relationship between X and y. Since the latter is generally not possible given X^O and y^{OS} or y^{OH} , we propose a different approach: first a function is learned based on X^O and y^{OS} or more commonly y^{OH} , i.e. f^O . We then set this function equal to f^G , X^{PG} to be X^O and y^{PG} to be $f^G(X^{PG})$. In effect, we disregard the original labels and obtain new soft labels that are generated by the application of the selected ground truth function to the input data.

Furthermore, by imposing a deterministic decision function f_{dec} upon f^G , a new D^G can be constructed in a similar manner from X^{PG} by setting $y^G = f_{dec}(f^G(X^{PG}))$. Note that f_{dec} has to be deterministic, so if for a binary problem $f_{dec}(f^G(X^{PG}))$ returns 0.5, the decision may not be randomly taken. Again, the transformation results in a new dataset consisting of different, decoupled objects: the items in the newly constructed D^G are not necessarily the same as those in D^{PG} and the truth function f^G is altered as well: $f_{new}^G = f_{dec}(f^G)$. From the newly constructed D^G , any of the transformations described in Section 4 can be applied to construct any number of new datasets for experimentation.

The SYNLABEL framework together with all of the transformations previously described has been implemented and made available publicly on GitHub to encourage its use: https://github.com/sjoerd-de-vries/SYNLABEL.

5 Application of the Framework

In the following we demonstrate how SYNLABEL might be used in practice by the developer of a noise-robust algorithm and highlight differences with existing methods for label noise experimentation. We believe this to be more valuable than a comparison of different algorithms on generated datasets - one of the main uses of the framework - would be, as the results would speak to the algorithmic performance instead of the quality of the framework. The dataset used here



Figure 2: The level of label noise generated by feature hiding as measured by the mean entropy of the resulting soft labels for different probability density estimation methods and different numbers of features hidden. Average over 50 runs. KDE: Kernel Density Estimation. MICE: Multivariate Imputation by Chained Equations.

and in Section 6 is the Keel Vehicle Silhouette set (Triguero et al. 2017), which consists of 18 features and 4 classes.

Constructing a Ground Truth To thoroughly evaluate the performance of an algorithm, we need a number of datasets for which the noise has been quantified. Some curated datasets are available, for which this has been done by experts. Not all noisy instances may have been identified, however, and the type of noise these sets contain is fixed and might not be the of the type that interests us.

Alternatively we could generate, i.e. simulate, clean datasets from scratch and add the exact noise we are interested in later, e.g. by sampling from different normal distributions or constructing concentric circles. Once more this approach is less than optimal, as the resulting datasets typically do not capture the complexity of real-world data.

By using SYNLABEL and the transformations up the chain defined in Section 4, we can construct a D^G based on observed, noisy data D^O . Most commonly, real datasets contain hard labels and as such we take $D^O = D^{OH} = \{X^O, y^{OH}\}$. We then transform this dataset as follows: based on D^{OH} a function f^O is learned. We set X^G to be X^O , f^G to be f^O with a deterministic decision function f_{dec} applied to it: $f^G = f_{dec}(f^O)$. Then we simply set $y^G = f^G(X^O)$, as for the simulated data, to construct a real-world data inspired Ground Truth dataset.

The properties of this dataset depend on the f^G used. If a simple linear model is used, the resulting dataset will likely not contain the complexities expected to be present in a real world system. On the other hand, if an overfit neural network is used, the relationships may well be overly complicated. As model expressivity is varied, baselines of corresponding complexity are constructed, approximating real-world problem difficulty to different extents.

Partial Ground Truth via Feature Hiding Having generated D^G as a baseline set for evaluation, we then generate

an additional set with soft labels. These allow for alternative, more direct ways of quantifying and injecting label noise compared to hard labels, as shown in Section 6.

 D^{PG} can be constructed by applying feature hiding, as specified by Equation 1. First, we define which features to hide, i.e. add to $X^{PG'}$, and thereby which features remain in X^{PG} . Next, we specify the method for constructing the prior distribution from which the data in $X^{PG'}$ is resampled. Several methods are implemented in SYNLABEL by default and these can easily be extended to include custom methods. Then we specify the number of samples drawn and apply the transformation to obtain soft labels y^{PG} .

In Figure 2 we show how resampling from prior distributions for $X^{PG'}$ constructed via different methods results in different levels of uncertainty in the obtained posterior distributions via Equation 1, as measured by the Shannon entropy, for different numbers of features hidden. As expected, sampling according to a conditional density function $P(X^{PG'}|X^{PG})$ or even $P(X^{PG'}|X^{PG} \cup y^G)$, in this case constructed using MICE (Volker and Vink 2021), produces soft labels with lower entropy than using a marginal density function $P(X^{PG'})$ does.

Introducing Noise Now that we have obtained both a baseline dataset with hard labels D^G and with soft labels D^{PG} , we need to introduce the specific type of label noise we are interested in so that we can compare methods on the resulting sets. This noise may be added via any of the transformations that have been described in detail in Section 4. Note that such noise injection can also be applied to a noisy real-world dataset. In this case, however, we would add noise upon pre-existing, unspecified noise, which makes it impossible to study the effect of the added noise in isolation, as we show in Section 6.

6 Quantifying Label Noise

An inherent advantage of having soft labels available is that noise can be quantified by measures such as the Shannon entropy of the resulting distribution, or when two distributions P and Q are to be compared, the total variation distance:

$$D_{TV}(P,Q) = \frac{1}{2} ||P - Q||_1.$$
(2)

The latter enables us to apply any noise generation method directly on D^{PG} to generate D^{OS} and quantify the strength of the label noise by evaluating $D_{TV}(D^{PG}, D^{OS})$. As most classification tasks are concerned with hard labels rather than soft labels, a decision function f_{dec} can be utilized to transform y^{OS} into y^{OH} and thereby generate D^{OH} . Such a decision function could be for instance taking a sample in proportion to the label distribution or selecting the class with the highest probability. When a classifier has to be evaluated against some introduced noise, either its probabilistic output can be compared directly to y^{PG} , or its hard label to y^{G} or y^{PG} , depending on whether any noise due to partly unobserved data is of interest.

To illustrate the importance of a clean baseline and how the use of label distributions allows both quantification of noise and direct noise injection, we conducted experiments



Figure 3: Different noise measures for varying noise rates. Left: the mean D_{TV} . Feature hiding was done by sampling from a marginal distribution constructed via Kernel Density Estimation (KDE). Uniform noise (NCAR) was added through T_r . Right: the mean entropy. Feature hiding was done by sampling from a conditional distribution constructed using MICE. Random class-conditional noise (NAR) was introduced by a randomly generated T_r , with equal probabilities on the main diagonal. T_r : transition matrix. ID: instance-dependent (NNAR). FH: feature hiding. Δ_1 : noise introduced by FH. Δ_2 : noise introduced by applying T_r to D^{PG} .

with the Keel Vehicle Silhouette dataset, of which the results are shown in Figure 3. To obtain D^G from this observed dataset, we trained a Random Forest classifier on the original labels and set it equal to f^G , a transformation up the chain.

On the left side the Mean Total Variation Distance is shown, either with respect to the labels in D^G or in case of Δ_3 with respect to D^{PG} , which measures how often a label is expected to change due to the noise injection. We observe that there is a difference between the level of noise introduced when applying a uniform noise matrix T_r to a D^{PG} that has been constructed via feature hiding (FH)from D^G , i.e. $T_r(FH(D^G))$, and the separately added noise of $\Delta_1 = D^{PG} = FH(D^G)$ and $\Delta_3 = T_r(D^{PG})$, $\Delta_1 + \Delta_3$. The same applies when this noise is added to D^G directly $\Delta_2 = T_r(D^G)$, and then added to the noise introduced by $FH: \Delta_1 + \Delta_2$. This illustrates how label noise applied to a set for which the baseline noise is unknown cannot be retrospectively isolated and properly quantified.

 $T_r(D^{OH})$ is the result of sampling hard labels from D^{PG} (100 times) and then applying the uniform flipping probability via T^r to each sample (100 times as well), while for $T_r(FH(D^G)$ the noise matrix is simply applied to the y^{PG} directly. As desired, $T_r(D^{OH}) = T_r(FH(D^G))$, which shows how the direct injection of noise into the label distribution makes repeated sampling from D^{PG} , followed by repeated application of random noise functions to individual objects, is redundant, in this case saving 10.000 repeated actions. Furthermore, the noise level is exact instead of an estimation. In addition, some types of noise are more naturally applied directly to a label distribution.

Finally, $ID(T_r(D^{OH}))$, where ID stands for instancedependent (NNAR), is added to illustrate that similar results are obtained for a more complex type of noise. The uniform T_r from before is still used, but we applied it twice as often to the objects with the largest ratio of distance to their nearest neighbour of the same label to distance to a neighbour of the other label, as in (Garcia et al. 2019).

On the right hand side the mean entropy of the label distributions is shown and class-conditional noise is added. As entropy is not a measure between distributions, Δ_3 equals $T_r(FH(D^G))$, and is omitted. The same patterns can be observed as for D_{TV} , demonstrating that noise added to preexisting, unknown noise cannot be studied in isolation.

7 Conclusion

In this work, we present the SYNLABEL framework which facilitates the generation of synthetic data for label noise experiments. Standard procedure would have the user utilize the framework to generate a ground truth dataset inspired by a real-world dataset by learning a classification function from the real-world data, setting it to be the ground truth and applying it to the input data to obtain new hard labels. This dataset can be transformed into a set with soft labels by hiding a number of the input features contained in the domain of the selected function and resampling values from learned or pre-specified distributions for these hidden features, evaluating the ground truth function on the resulting data and aggregating the resulting labels. This method called feature hiding adds measurable uncertainty into the labels, which we show can be useful for direct injection and more thorough quantification of label noise. These ground truth sets provide a clean baseline to evaluate method performance on, to which any noise of interest may be added. Conducting experiments using datasets generated by the framework offers advantages over the three types of datasets typically used in label noise research: the generated data are more complex than data simulated from scratch, provide a clean baseline for evaluation which is lacking from real-world data and allow for the noise to be controlled, in contrast to curated data, in addition to being constructed at a low cost.

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