An Exploratory Study to Improve Reading and Comprehending Mathematical Expressions in Braille

Abstract Braille readers read and comprehend mathematical expressions while moving their fingertips over braille characters. The aim of this exploratory study was to investigate the effect of an intervention that teaches braille readers who use a braille display to use finger movements with a focus on the expression's mathematical structure. The finger movements involved movements where the two index fingers are about one or two braille cells apart and movements where the index fingers explore different parts of the expression. We investigated to what extent the intervention supports an interplay between finger movements and the expression's mathematical structure to make the process of calculating the value of an expression easier and to make braille readers more aware of the expression's structure. Three braille readers, respectively in grades 7, 8 and 11, received the intervention consisting of five individual lessons. During the pre-, post-, and retention test, the braille readers' finger movements were video recorded, as well as the time needed to read and process the mathematical tasks. Four expressions were selected for further analysis. The results show that during the posttest, each braille reader required at least 29% less time to read and process the expressions. The retention test results were even better. Scan paths indicated that braille readers picked up features of mathematical structures more easily after the intervention. Based on our findings, we recommend that braille readers receive lessons in tactile reading strategies that support the reading and processing of mathematical expressions and equations.

Keywords*:* braille reader ● mathematical expression ● mathematical structure ● tactile reading

1.1 Introduction

Many high school students struggle with mathematical expressions and equations, both numerical and algebraic (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). Difficulties include the order of operations (Herscovics & Linchevski, 1994; Linchevski & Herscovics, 1994), understanding of the equal sign (McNeil & Alibali, 2005), working with brackets (Hoch & Dreyfus, 2004) and working with variables (Knuth et al., 2005). Therefore, it is worthwhile to give students extra support in reading and comprehending mathematical expressions. The current study is about support for students who use braille as their primary reading medium (i.e., braille readers). In a recent study using finger-tracking technology, Van Leendert, Doorman, Drijvers, Pel and Van der Steen (2019) investigated how three braille readers read and comprehend four items involving mathematical expressions on the braille display. One of the expressions was $4 + (1 - (3 + 2))$. Using eye-tracking technology, they also investigated how five students who had typical vision (from here on print readers) read and comprehend the same expressions. They found that the two most experienced braille readers – experienced in reading in braille – needed about 3.5 times as much time as the print readers. The print readers focused directly on typical features of the expression's mathematical structure, such as brackets. In addition, they traced with their eyes the elements of the expression in an order that corresponded to the sequence of steps required to calculate the value of the expression. The braille readers, in contrast, picked up the structure's characteristics either marginally or much later than the print readers did. It was a challenge for them to match their finger movements to the sequence of steps needed to do the calculation. These findings suggest that braille readers may perform better when they use tactile reading strategies with an emphasis on the expression's structure. Therefore, the aim of the current exploratory study was to investigate the effect of an intervention that teaches braille readers who use a braille display finger movements with a focus on the expression's mathematical structure.

1.2 Theoretical Background

Mathematical expressions and equations in braille

The representation of expressions and equations in the mathematical notation, the notation that print readers use, is very different from the representation in braille on a braille display. This difference can be illustrated with the next equation:

$$\frac{(x+2)^2}{\sqrt{3}} = \frac{3(x+2)}{\sqrt{3}}$$

The symbols and numbers are arranged at different heights above, on and below the baseline to convey structure (Stöger & Miesenberger, 2015). Moreover, the symbols, numbers and operators look very different from each other. The specific features of this representation in combination with the use of sight – which has a simultaneous character (Millar, 1994; Millar, 1997), enable print readers to get an overview of this equation in a split second (Schneider, Maruyama, Dehaene, & Sigman, 2012).

The representation in braille on a braille display is very different from the representation in print. Braille is a linear output modality (Stöger & Miesenberger, 2015) and all braille cells have the same size (Karshmer & Bledsoe, 2002). The transformation from an equation in mathematical notation to an equation in braille can be executed in two steps. This is depicted in Figure 3.1. This figure consists of two blocks: Mathematical Notation and Mathematical Braille Notation. *Step 1* is a transformation from an equation into a linear-print equation. This can be done in very different ways depending on the mathematical braille notation used. In the current study, the 6-dot Dutch mathematical braille notation is used. This notation uses number signs (#). Actually, the correct linear-print equation is ((x + #b)^#b)/sqrt(#c) = (#c(x + #b))/sqrt(#c). For the sake of convenience, to make it easier to read the equation, #2 and #3 are used instead of #b and #c. *Step 2* is a conversion from the linear-print to the linear-braille equation.

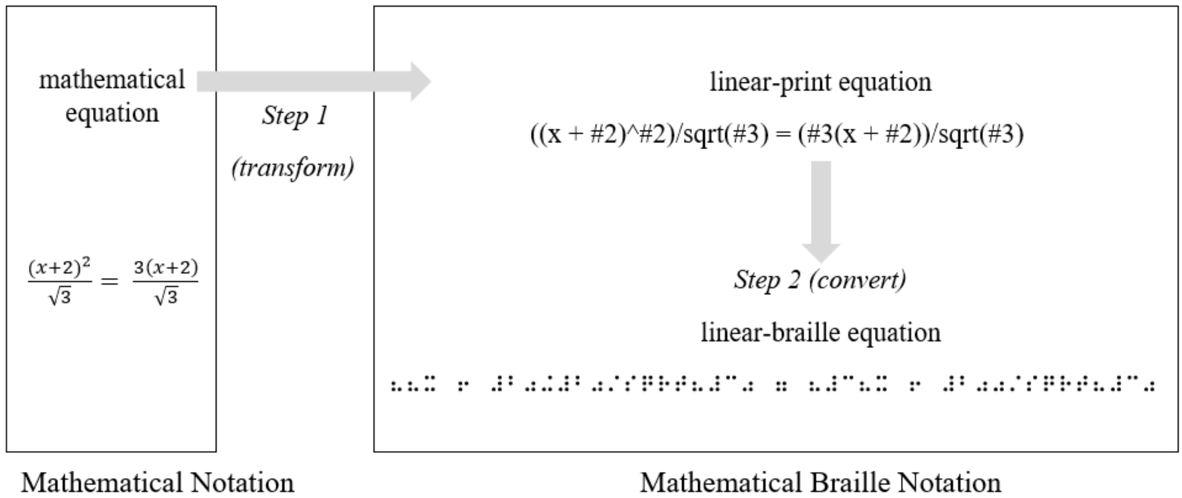

Figure 3.1 Transformation from a mathematical to a linear-braille equation
Note: # represents the number sign

Mathematical structure of expressions and equations

In the current study, the concept of mathematical structure is important. According to Hoch and Dreyfus (2004), a structure can be seen as the way in which a mathematical entity, such as an algebraic or numerical expression or equation, is composed of its parts and how these parts are connected. Awareness of the structure of an expression or equation is important because that is often needed to be able to appropriately select an operation or a solution strategy (Drijvers, Goddijn, & Kindt, 2010). The strength of recognizing the structure can be demonstrated in the following equation:

$$(x-3)(2x+1) = (x-3)(x+2)$$

First, one needs to recognize that the equal sign does not require a calculation (like in “3 + 4 = …”), but requires finding the value of x for which the left and right expression have an equal value. When one recognizes the structure of this equation, which is $\mathrm{A} * \mathrm{B} = \mathrm{A} * \mathrm{C}$, it follows rather easily that $A = 0$ or $\mathrm{B} = \mathrm{C}$. Removing brackets may also lead to the correct answer, but takes much more time and calculations. Students who have to make many calculations are more inclined to make errors; that is why using characteristics of the structure is not only faster but also more accurate (Hoch & Dreyfus, 2004).

Interplay between finger movements and comprehending structure

Mathematical structure is more difficult to grasp in braille than in print due to the linear representation in braille (e.g., Stöger & Miesenberger, 2015) and to the small perceptual view of braille readers (e.g., Millar, 1994, 1997). This does not mean that the level of detail is not present in the linear-braille expression or equation. An additional problem is that braille characters have a low redundancy, which means that braille characters are hard to distinguish (Millar, 2003; Tobin & Hill, 2015). This is particularly problematic when reading mathematical text because this type of text is very compact. For instance, in the mathematical notation for print readers, "the square root of 2x" is denoted by "$\sqrt{2x}$". Reading mathematical text in braille requires several attempts before the purpose and structure of the text becomes clear. What braille reading strategies can be recognized?

In reading non-mathematical text, several types of finger cooperation can be recognized: conjoint and disjoint (Bertelson, Mousty, & D'Alimonte, 1985). In conjoint reading, the fingertips of the two index fingers are at a distance of one or two braille cells from each other, often touching each other. In disjoint reading, the two index fingers explore different parts of the line. Another type of finger movement is when a reading finger slows down and rests on a braille cell. This movement is referred to as lingering on a braille cell (Perea, Jiménez, Martín-Suesta, & Gómez, 2015). Conjoint reading is most suitable for efficient and deep reading and disjoint reading for global scanning of text (Breidegard, Jönsson, Fellenius, & Strömqvist, 2006). Our assumption is that braille readers can be supported in comprehending the expression's mathematical structure and in the calculation or solving process by using specific finger movement strategies. In this study, we only focus on the movements of the index fingers. We assume that the first step in comprehending the structure is to decode every element of the expression or equation accurately. For this, a conjoint reading style seems to be suitable (Breidegard, Jönsson, Fellenius, & Strömqvist, 2006). During this activity, the braille readers are expected to try to get a global overview of the expression. The next step, which can be skipped if the structure is very clear, is to look for relations between different parts of the expression or equation. For this, a disjoint reading style is appropriate, because that enables the braille reader to physically relate parts of the expression or equation to each other and compare these parts almost simultaneously. The following step, inevitable in mathematical activity, is to calculate the value of the expression or solve the equation. For this, one can use different reading styles, depending on the required process and the skills and knowledge of the braille reader. Figure 3.2 gives an illustration of the proposed method. This figure illustrates the finger movements over a braille display while reading and processing $24 + 15 =$. The expression and the time are shown on the horizontal and vertical axis, respectively. The movements of the left and right index finger are shown in light and dark grey, respectively. The diagram shows that the braille reader starts with conjointly reading the entire expression, continues with disjoint reading and ends with conjoint reading (Figure 3.2c). Using a disjoint reading style helps to perform the calculation by first focusing on tens and next on ones in both numbers (20 + 10 = 30 and 4 + 5 = 9).

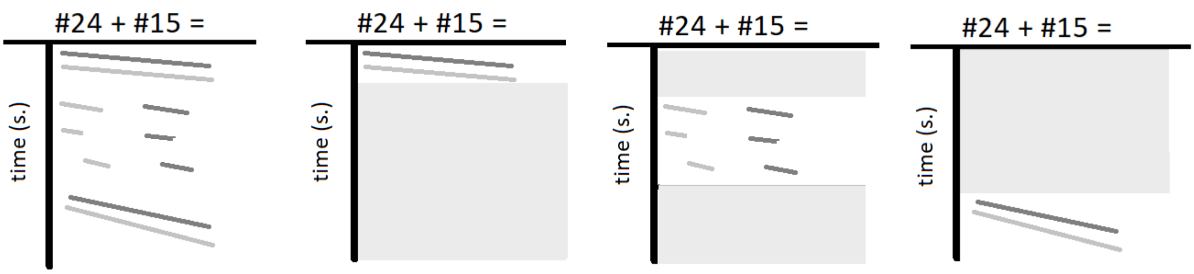

Figure 3.2a | Figure 3.2b conjoint reading | Figure 3.2c disjoint reading | Figure 3.2d conjoint reading

Figure 3.2 Schematic representation of the finger movements while reading the expression $24 + 15 =$ *in braille. Figure 3.2a illustrates the whole reading process. Figure 3.2b, 3.2c and 3.2d show the successive stages of this process*

Note: Only the movements of the index fingers in the left-to-right direction are depicted. Light grey indicates the movements of the left index finger, dark grey the movements of the right one.

Research questions

The current study investigated whether an intervention on tactile reading supports braille readers to read and comprehend mathematical expressions. In this intervention, we made braille readers aware of their finger movements and reading strategies. Moreover, we suggested and trained ways for improving these finger movements over the braille display to better understand the structure of an expression and to support the calculation process. The following questions were addressed:

> *Research question 3.1: To what extent does an intervention that gives support on tactile reading with a focus on structure improves the braille readers' performances in reading and comprehending mathematical expressions?*

> *Research question 3.2: Does the intervention make the braille readers more aware and make more use of the mathematical structure of expressions?*

The first question uses the term performance. In this study, the operationalization of performance is the time needed to read and calculate the value of an expression and the number of correct answers. The attention for structure is expected to lead to better performance in reading and comprehending mathematical expressions (Drijvers et al., 2010; Hoch & Dreyfus, 2004). The second research question addresses awareness and use of the mathematical structure while reading and solving mathematical expressions and tasks. In this study, awareness and use of the structure is operationalized in tactile scan path patterns. It is expected that the intervention changes these patterns towards finger movements that reflect structure or mathematical problem solving. For example, they will use a disjoint reading style when they need to compare two parts of the expression with each other.

1.3 Methods

Design of the study

An intervention consisting of five individual lessons was developed to be able to answer the research questions. The focus was on tactile reading strategies for processing mathematical expressions and equations. A mixed-method approach was taken for investigating the effect of the intervention on the braille readers, using a pre-, post- and retention test. The quantitative data (test results and time needed) and qualitative data (video recordings of finger movements) are used to answer the first and second research question respectively.

This study was approved by the medical ethical committee of the Erasmus Medical Centre (MEC-2012-097 and MEC-2012-524) and adheres to the tenets of the Declaration of Helsinki (2013) for research involving human subjects. Informed consent was obtained from the subjects.

Participants

Three braille readers from a school for visually impaired students participated in the study. The braille readers, called T., R., and S., were in grade 7, 8 and 11 of havo, which is the senior general secondary track, respectively. They all developed blindness at a very young age. They had no comorbidities. The braille readers started at the age of six with reading in braille on paper. In grade 6, they switched gradually to using the braille display in mathematics lessons. They did not learn how to read and explore mathematical text in braille, not on paper and not on the braille display. Consequently, they developed a very personal way of reading and comprehending mathematical expressions and equations. Over time, they had learned to only use their index fingers when reading in braille, regardless of the type of text. Their mathematics teachers classified the braille readers as above average in mathematics.

Design of the intervention

The intervention consisted of five 45-minute lessons. The braille readers were individually taught by an instructor (this paper’s second author) who had a good knowledge of mathematics and braille. They did not get exactly the same intervention; adjustments were made according to their grade level. For example, the grade 7 braille reader was given equations with only a variable on one side of the equal sign. The other two braille readers were given equations that are more complicated. Table 3.1 provides an overview of the lessons. In the first lesson, the focus was on the decoding of braille characters and on the mathematical structure of expressions. The braille readers were taught which characteristics of the structure are important – and why – and how to recognize them by touch. For example, they were asked to point out the operator in an expression or the equal sign in an equation. In the second lesson, the emphasis was on using finger movements that support reading and processing mathematical expressions. In the third lesson, the focus was on finger movements that support reading and solving equations. An important strategy was to relate the two sides of an equation in disjoint reading. In the fourth and fifth lessons, the knowledge and skills acquired in the first three lessons were further developed with different expressions and equations. In addition, much attention was paid to mathematical language.

Table 3.1 Design of the intervention

Lesson	Topic of the Lesson
1	Global structure of mathematical expressions
2	Finger movements that support reading and processing mathematical expressions
3	Finger movements that support reading and solving equations
4	Mixed exercises
5	Mixed exercises

Test design

The pre- post- and retention tests consisted of 22 items. Each test took 45 minutes. Examples of the tasks are: point out operators in expressions, read the expression or equation, simplify an expression, calculate the value of an expression, and solve an equation. The tasks were the same for each braille reader, but dependent on the grade level, there were small differences in the expressions and equations involved. For each individual braille reader, the items on the pre-, post- and retention test were identical (see Appendix for the test taken by braille reader R.).

Procedure

The instructor provided the lessons over a period of three weeks. The lessons and tests were scheduled during regular mathematics lessons and were administered in a room separated from other students. The pre-test was administered one week before the start of the intervention, the posttest one week after the end of the intervention and the retention test was conducted six weeks after the posttest. The expressions were typed – represented as linear-print expressions – in Word and converted into braille with a screen reader. The instructor started with explaining the task. Then the braille readers were asked to "think aloud" while reading the expressions and equations on the braille display and to answer orally.

The screen reader software NVDA – NonVisual Desktop Access – and the so-called NL literature table, the 6-dot Dutch braille table, were installed on the laptop used for the lessons and the tests. The braille readers were used to using this braille table.

Data collection and analysis

During the tests, a video camera assessed the movements of the readers' index fingers over the braille display. The finger movements were analyzed with the help of video analysis software (Kinovea). Every 200 milliseconds, the position of both index fingers was noted in Excel. The positions were matched with the spatial locations of the braille cells that represent separate elements in the expressions. The successive positions of each index finger on the braille display formed a tactile scan path.

To answer the first research question, we first selected the test items whose task was "calculate the value of the expression." From these five test items, we then selected four items for further analysis. These items are depicted in Table 3.2. This table illustrates that every braille reader has their own test. For example, item *d* involves the expression 3/4 * 2/5 (braille reader T.) and 5/7 * 2/9 (braille readers R. and S.).

Table 3.2 Selected test items

	T. grade 7	R. grade 8	S. grade 11
Item a	4 + 2 * (10 - 3)	4 + 2 * (10 - 3)	4 + 2(10 - 3)
Item b	2 + 1 - (2^2 + 1)	2 + 1 - (2^2 + 1)	2 + 1 - (5 - 3)^2
Item c	45.7 + 13.4	45.7 - 13.4	45.7 - 13.4
Item d	3/4 * 2/5	5/7 * 2/9	5/7 * 2/9

Note: In the Netherlands, we use a decimal comma as a decimal separator.

These items have been selected because the structure of the expressions provokes braille readers to use lingering or disjoint reading. This enables braille readers to show that that they use a reading strategy with a focus on the structure. In the first two items, the braille readers can use lingering to keep track of the locations of the brackets and solve the sub-expressions between the brackets before solving the whole expression. In the last two items, the braille readers can use a disjoint reading style while comparing the tens, ones and decimals (item *c*) or the denominators (item *d*).

To answer the first research question, we did a descriptive analysis of the quantitative data. For each test item and for each braille reader, the number of correct answers and the reduction in time needed to give a correct answer were calculated. In addition, for all selected test items in the pre-, post-, and retention test, and for each braille reader the total time necessary to give a correct answer and the reduction in time needed to give a correct answer were calculated.

A qualitative approach was used to answer the second research question. We visualized the scan path patterns of the items *a* and *c* by tracing finger locations on the braille display. These items were selected because the expressions' mathematical structures are very different. We analyzed the resulting scan path patterns and the changes in these patterns for each braille reader and for both items. We tried to recognize the finger movements that braille readers used and investigated whether these can be related to the expression's mathematical structure and the sequence of steps necessary to perform the required calculations.

1.4 Results

Research question 3.1: Performances in reading and comprehending expressions

The braille readers calculated the values of the selected expressions correctly. For each braille reader and for each test, the time needed to read and process the selected expression is represented in Table 3.3. The braille readers performed better on the post- and retention test than on the pre-test, except in four cases. T., for example, needed 47 s. in the pre-test and 49 s. in the posttest for processing item *b*. However, he performed much better on this item in the retention test, in which he only needed 27 s. For item c., T. and R. performed worse in the retention test than in the pre-test. In contrast, S. performed much better on this item in the post- and retention test than in the pre-test. The total time needed to read and process the four items is depicted in Table 3.4. In the posttest, the reduction in time was 29%, 54% and 49% for T., R. and S., respectively. The results on the retention tests were even better.

Table 3.3 Time (s.) for reading and processing Item a (Table 3.3a), Item b (Table 3.3b), Item c (Table 3.3c) and Item d (Table 3.3d)

Table 3.3a Time (s.) for reading and processing Item a

Braille reader	Expression	Pre-test	Posttest (reduction %)	Retention test (reduction %)
T. (grade 7)	4 + 2 * (10 - 3)	71	42 (41%)	55 (23%)
R. (grade 8)	4 + 2 * (10 - 3)	87	27 (69%)	15 (83%)
S. (grade 11)	4 + 2(10 - 3)	57	14 (75%)	16 (72%)

Table 3.3b Time (s.) for reading and processing Item b

Braille reader	Expression	Pre-test	Posttest (reduction %)	Retention test (reduction %)
T. (grade 7)	2 + 1 - (2^2 + 1)	47	49 (-4%)	27 (42%)
R. (grade 8)	2 + 1 - (2^2 + 1)	86	33 (62%)	21 (76%)
S. (grade 11)	2 + 1 - (5 - 3)^2	53	35 (34%)	13 (75%)

Table 3.3c Time (s.) for reading and processing Item c

Braille reader	Expression	Pre-test	Posttest (reduction %)	Retention test (reduction %)
T. (grade 7)	45.7 + 13.4	28	10 (64%)	10 (64%)
R. (grade 8)	45.7 - 13.4	25	28 (-12%)	18 (28%)
S. (grade 11)	45.7 - 13.4	32	26 (19%)	30 (6%)

Table 3.3d Time (s.) for reading and processing Item d

Braille reader	Expression	Pre-test	Posttest (reduction %)	Retention test (reduction %)
T. (grade 7)	3/4 * 2/5	8	8 (0%)	12 (-50%)
R. (grade 8)	5/7 * 2/9	22	14 (36%)	24 (-9%)
S. (grade 11)	5/7 * 2/9	30	12 (60%)	8 (73%)

Table 3.4 Total Time (s.) for reading and processing all four Items

Braille reader	Pre-test	Posttest (reduction %)	Retention test (reduction %)
T. (grade 7)	154	109 (29%)	104 (32%)
R. (grade 8)	220	102 (54%)	78 (65%)
S. (grade 11)	172	87 (49%)	67 (61%)

Research question 3.2: Awareness and use of the mathematical structure

To answer the second research question, the scan paths of the expressions that belong to item *a* and item *c* were analyzed. The scan paths during the pre-test (left), the posttest (middle) and the retention test are shown for item *a* (Figure 3.3) and *c* (Figure 3.4). The scan paths of braille reader T., R. and S. are shown in the top, middle and bottom panel, respectively. Because braille readers can only properly decode braille characters if they move their fingers from left to right on the braille display, those are the only finger movements that are displayed. We start with the analysis of the scan paths for item *a*. Overall, for this item, the scan path patterns in the retention test are very similar to the scan path patterns in the post-test. In contrast, the scan path patterns in the pre-test are different and much longer. Let us look at the patterns of each braille readers in more detail. In the pre-test, T. started to read the expression with his left index finger and continued with his right index finger. He stopped just before the open bracket. He spent some time on '#2' and on '*'. After about 30 seconds from the beginning, he continued in a more conjoint reading style. He read the entire expression a few times. The sub-expression between brackets was read more often than the other parts of the expression. In the posttest, he first explored the first part of the expression – using a conjoint and disjoint reading style. Then he read almost 10 seconds only with his left index finger. Finally, he reread the entire expression in conjoint reading. In the retention test, he also used a conjoint and disjoint reading style. After about 10 seconds from the beginning, he spent more than 5 seconds on '('. R. started in the pre-test with a conjoint reading style. Then she continued with a more disjoint reading style. She also read part of the expression with only one index finger. She read the first elements of the expression more than 10 times. In the posttest, she started with conjointly reading the entire expression. Then she made little jumps while moving her fingers from right to left in the expression. In the retention test, she started with a similar strategy as in the posttest, but did not reread so much in the end. S. started in the pre-test with conjointly reading the entire expression. She conjointly read parts of the expression again and switched, after about 25 seconds, to a more disjoint reading style. In the posttest, she started with conjointly reading the entire expression. Then she switched to a more disjoint reading style. In the retention test, she used similar reading strategies as in the posttest.

Figure 3.4 shows the scan paths for item *c*. In the pre-test, T. started with conjoint reading and switched, after about 13 seconds, to disjoint reading. After about 20 seconds from the beginning, he read #45.7 with his left index finger and, almost simultaneously, #13.4 with his right index finger. In the posttest as well as in the retention test, he started with reading the entire expression conjointly and continued in disjoint reading. R. read the entire expression once using different reading styles and continued with disjoint reading. In the post- and retention-test, she started with conjointly reading the entire expression. Then she continued in disjoint reading. For R., the three patterns are not very different from each other. S. started in the pre-test with reading the entire expression. During the whole process of reading, she alternated between different reading strategies: conjoint reading, disjoint reading and reading with just one index finger. In the post- and retention test, she started with conjointly reading the entire expression and continued in disjoint reading. For S., the scan path patterns in the post and retention test are very similar. The pattern in the pre-test, on the other hand, is very different.

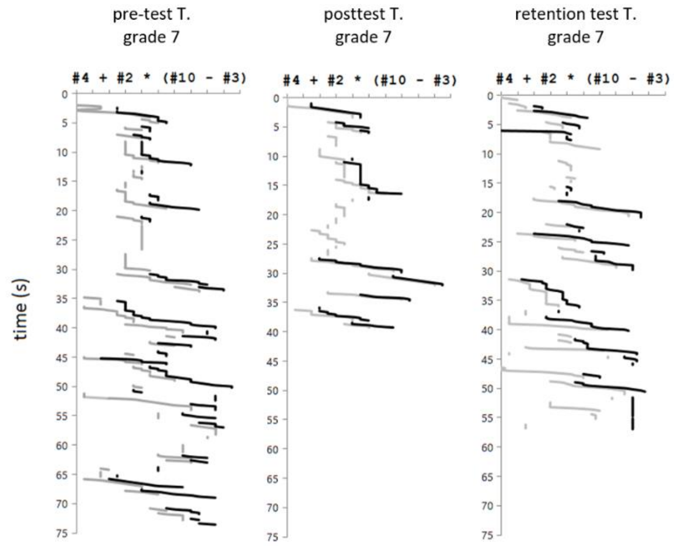

Figure 3.3a The tactile scan paths of braille reader T.

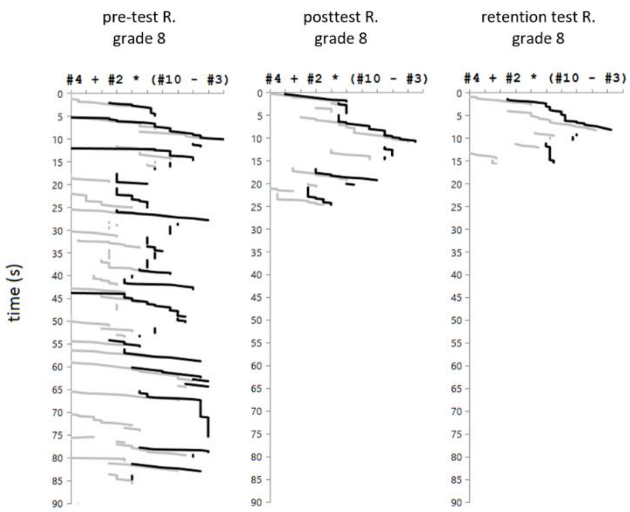

Figure 3.3b The tactile scan paths of braille reader R.

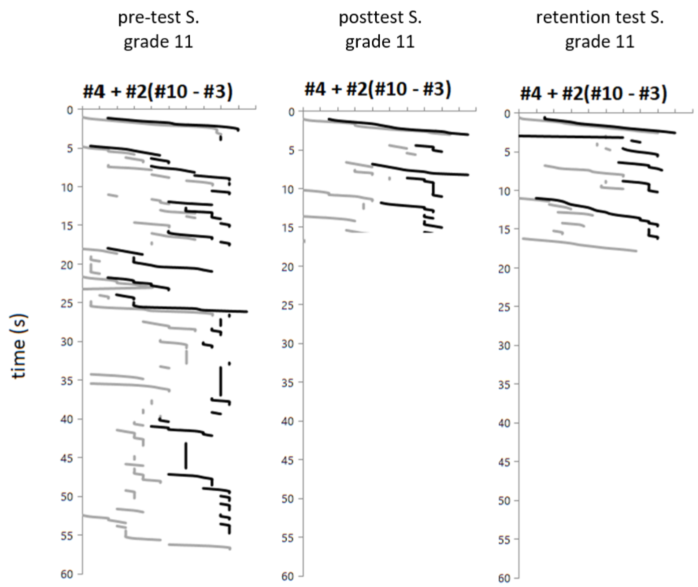

Figure 3.3c The tactile scan paths of braille reader S.

Figure 3.3 The tactile scan paths for item a during the pre-test (left), the posttest (middle) and the retention test (right). # stands for the number sign. Only the movements of the index fingers in the left-to-right direction are depicted. The movements of the left and right index finger are shown in light and dark grey, respectively

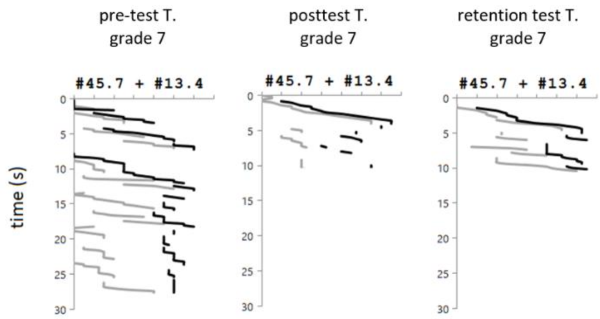

Figure 3.4a The tactile scan paths of braille reader T.

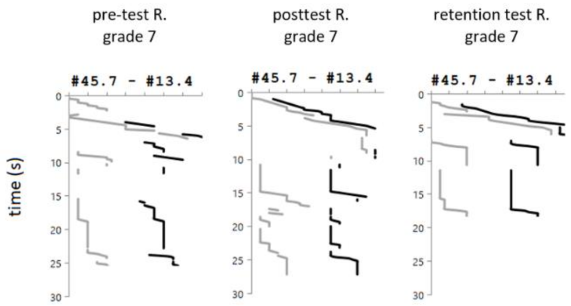

Figure 3.4b The tactile scan paths of braille reader R.

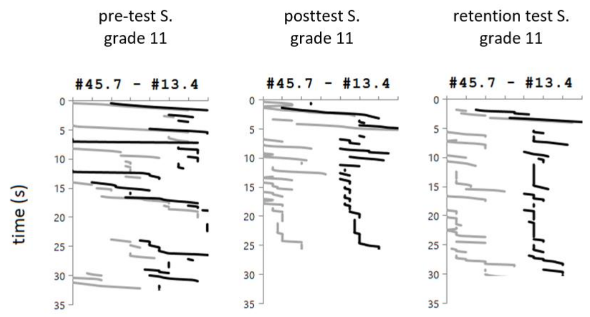

Figure 3.4c The tactile scan paths of braille reader S.

Figure 3.4 The tactile scan paths for item c during the pre-test (left), the posttest (middle) and the retention test (right). # stands for the number sign. Only the movements of the index fingers in the left-to-right direction are depicted. The movements of the left and right index finger are shown in light and dark grey, respectively

1.5 Conclusions and Discussion

RQ 1: Performances in reading and comprehending expressions

The first research question concerned the performances in reading and comprehending mathematical expressions. In all tests, the braille readers calculated the values of the expressions correctly. The results show that the braille readers improved their performance, because each braille reader needed at least 29% less time to read and process the expressions. A reduction in time was expected, because attention for the mathematical structure should lead to improved results (Drijvers et al., 2010; Hoch & Dreyfus, 2004). The results also show that the performances of the braille readers differ from each other. Overall, it appears that braille reader T. – the youngest braille reader – had benefitted less from the intervention than the other braille readers. In some cases, the braille readers performed much better on the retention test than on the posttest. This may indicate that other aspects related to experiences in mathematical learning were also relevant for improving performance. Moreover, for item *d*, there were large differences between the performance and performance change of the braille readers. This is also an indication that other aspects may be relevant.

RQ 2: Awareness and use of the mathematical structure

The second question concerned whether, after the intervention, the braille readers were more aware of and made more use of the expressions' mathematical structure. The scan paths patterns for item *a* show that after the intervention R. used reading strategies that were more in line with the sequence of steps needed to process the expression. After reading the entire expression, she almost immediately started to reread the sub-expression between brackets. To a lesser extent, this was also the case for S. For item *a*, the results also show that all braille readers reread less after the intervention. The scan paths for item *c* illustrate that T. and R. used a disjoint reading style in all tests. This enabled them to relate 45.7 to 13.4. After the intervention, braille reader S. also used this strategy. The results also illustrate that T. reread less after the intervention. Overall, the results indicate that the braille readers picked up characteristics of the structure more easily. They used finger movements that were more in accordance with the steps needed to do the calculations. This is in line with what we expected.

We recognized in almost all tactile scan paths patterns that are depicted in this article that the changes in scan strategies – that indicated that braille readers were more aware of the expression's structure and use the structure more – improve the braille readers' reading and comprehension time. This is in accordance with what we expected (Drijvers et al., 2010; Hoch & Dreyfus, 2004). However, for braille reader S., the changes in the scan path patterns for item c were large but did not result in great reductions in the reading and comprehension time. It is also possible that another calculation strategy affected reading and processing time.

This study had some limitations. The first is that the small number of participants offered little basis to generalize findings. However, the braille readers differed in their knowledge of mathematical concepts and in their braille reading skills, but they all benefited from the intervention. Secondly, the braille readers were all classified as good students. It is possible that braille readers who perform on or below average level in mathematics will respond differently on the intervention. Thirdly, in all tests, the braille readers made no errors in calculating the values of the expressions. These results do not make clear whether and to what extent the braille readers have improved their comprehension. The use of more complicated test items may provide insight into the development of the braille readers' comprehension. The fourth limitation concerns threats to the internal validity of the study design, in this case a history and a testing threat. A history threat refers to the possibility that other aspects related to the braille readers' experiences in mathematics learning may have been important for their performances and their awareness and use of the expressions' structure. A testing threat refers to the possibility that taking the pre-test affects how participants do on the posttest. It is possible that there was an interaction between the pre-test and instruction. It is also possible that the braille readers remembered the answers from the pre-test. Adding a comparable control group and using equivalent – and not identical – items, would counter these threats to internal validity.

The current study has provided a new way of supporting braille readers in reading and comprehending mathematical expressions and equations. The results indicate that the presented approach stimulates braille readers to keep improving their tactile reading strategies, since they performed better not only after the intervention but also after a retention period. Follow-up research should investigate whether the change in strategy use is also relevant for braille readers who use more than one or two fingers while reading. This is important, because it is known that some braille readers do (Radojichikj, 2015; Wanja, Murugami, & Bunyasi, 2021).

Based on our findings, we recommend that braille readers receive individual instruction on how to read and comprehend expressions and equations in braille. This instruction should be given by a mathematics teacher who is able to connect finger movements to the mathematical structure of an expression or equation.

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Appendix A: Test for Braille Reader R. (Grade 8)

I Point out the +, - and * on the braille display

9641 + 83153 - 4732

728.3 - 481.2 * 3.1

II Point out the variable on the braille display

x + 1 + x = 5

4x - x + 3 = 6

III Read the expression

45.86 + 11.34 - 3.4

IV Read the equation

3 + sqrt(9) = 3 * 2

V Enter the number two on the dots and calculate the value of the expression

4 * .. + 3

2 * .. + 1 + ..

VI What number should be on the dots?

4 + .. = 6

3 * .. + 1 = 7

VII Simplify the expression

x + 4 + 2x

3x - 2x + 1

VIII Calculate the value of the expression

45.7 - 13.4

5/7 * 2/9

45.8 + 10.4 - 2.4

4 + 2*(10 - 3)

2 + 1 - (2^2 + 1)

IX Is the equation correct?

4 + 2 * 3 = 10

3 + 1/2 = 2

(5 + 1)/2 = 3

X Solve the equation

4 + x = 3

4x = 1 + 2