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RESEARCH

Lobbying, Time Preferences and Emission Tax Policy

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Abstract

We develop a theoretical model to study the combined effect of lobbying and time preferences on emission tax policies. With a two-period model, we show that the influence of lobbying, by dirty industries and by environmental organizations, on the equilibrium tax decreases with the time horizon of the policymakers. An extension of the model to four periods shows that social welfare maximising policymakers may implement a tax higher than the marginal cost in the first period to speed up the transition to green technology. A policymaker influenced by lobby groups may, however, do the opposite, because future lobbying income will decrease if more firms invest in green technology. The results of this study indicate that countries with powerful lobby groups and a short-sighted policymaker are not likely to implement the optimal carbon tax. The influence of lobbying in combination with time preferences may explain some of the diversity in carbon taxes that we observe in practice. The results lead to the policy recommendation to combine carbon taxes with trade policies, which create an incentive for short-sighted governments to participate in carbon pricing policies.

Keywords Discounting · Carbon pricing · Emission Tax · Lobbying · Political Economy · Time preferences

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Introduction

The World Bank's High-Level Commission on Carbon Pricing (Stiglitz et al. 2017) has estimated that every country should implement a carbon price between 40 and 80 US$/tCO2e in 2020 to reach the targets of the Paris agreement. The 65 carbon pricing initiatives that are currently (December 2021) implemented or scheduled, however, only cover 21.95% of global greenhouse gas emissions (The World Bank 2021). Furthermore, the majority of those 65 initiatives have a carbon price that is far below 40US$/tCO2e (The World Bank 2021).

Among economists, there exists a strong consensus that carbon pricing is the economically efficient policy to reduce carbon emissions (Howard and Sylvan 2015). Current data show, however, that most countries are unsuccessful in implementing an efficient carbon price, which can be explained by the political reality. Climate policy is influenced by two types of lobby groups, an environmental group and an anti-environment group usually represented by industries. The weight of these lobby groups varies per country. In the United States, for example, firms in polluting sectors spend significant resources on (anti-)climate lobbying. Between 2000 and 2016, more than $2 billion was spent on climate lobbying in the United States. Most of these lobby expenditures came from the electrical utilities sector (26.4%), the fossil fuel sector (17.7%) and the transportation sector (12.0%), whereas the combined lobby expenditures of environmental organizations and the renewable energy sector only accounted for 6.1% of the total climate lobby expenditures (Brulle 2018). Research on the influence of lobbying on votes in the United States Senate on a bill that proposed a nationwide cap-and-trade system in 2010, showed that asymmetric lobbying effectiveness between gaining and losing firms has led to a 13 percentage points decrease in the likelihood that this bill would pass (Meng and Rode 2019).

In contrast to the United States, a few countries have successfully implemented a high carbon-pricing scheme. For example, Sweden has one of the highest carbon taxes in the world (137.24 US$/tCO2e—World Bank 2021) and has the political support of strong environmental lobby groups (Allen et al. 2018). Since 1991, when Sweden first implemented a carbon tax, the country has gradually increased the tax, adopting as such a long-term perspective in their climate policy (Sarasini 2009). This long-term perspective is lacking in other countries. Policymakers in countries that have implemented a carbon tax or cap-and-trade system are often averse to increasing the tax or setting a more stringent cap. The reasons are that these measures have an immediate effect on polluting industries, whereas the benefits are for future generations (Ervine 2018).

The cases of the United States and Sweden illustrate two political observations that can have an impact on carbon pricing policies: 1) the influence of lobby groups, and 2) the influence of the time preferences of the policymaker. This paper takes these observations as a motivation to set up a theoretical model that includes two types of lobby groups—an industry lobby group, and an environmental lobby group—with the aim to get insights into reasons for suboptimal climate policy, which can be used in guiding the design of more effective policies for mitigating climate change.

Out theoretical analysis employs the menu auction approach that was developed by Bernheim and Whinston (1986) and first applied to lobbying by Grossman and Helpman (1994). Fredriksson (1997) was the first to apply this method to environmental policy and used it to explain why lobbying leads to inefficient pollution taxes. Since then, multiple other studies have used this menu auction approach to analyse the influence of lobbing on environmental policy under all kinds of different conditions (e.g. Aidt 1998; Cai and Li

2020; Damania 2001; Eliste and Fredriksson 2002; Grey 2018; Habla and Winkler 2013; Hagen et al. 2021; Marchiori et al. 2017; Persson 2012). Most of these models are static one-period models and therefore do not take into account the influence of time preferences. One related study that does extend this type of lobby models to a two-stage game is the work of Lai (2008), who uses a two-stage lobby game to analyse the influence of lobbying on emission permit allocation. Lai (2008) focuses, however, on the role of lobbying on the allocation and distribution of tradable emission permits and not on the influence of time-preferences. Climate change is, however, a long-term problem and the time horizon, or discount factor that is used to evaluate a policy, has a big impact on the optimal carbon tax (Nordhaus 2017; Stern 2007; Tol 2018; van den Bergh and Botzen 2015). Empirical evidence suggests that regimes that care more about their own self-interest or interest of specific groups are likely to have a shorter time horizon and are more likely to be influenced by contributions from interest groups (Bättig and Bernauer 2009; Congleton 1992; Povitkina 2018). Taking into account time preferences is thus required to get a better understanding of the influence of lobby groups on carbon pricing. Our study combines these two key elements and examines how time preferences interact with lobbying to influence suboptimal carbon tax policies, which up to our knowledge has not been studied yet in theoretical models.

For this purpose, we specify the social welfare maximising tax in a multi-period model and analyse the impact of lobbying on this tax. Subsequently, we analyse whether the influence of lobbying changes when the time preferences of policymakers change. The paper starts with a two-period model. In the first period, the two types of lobby groups specify their lobby contributions, and the government sets the tax based on the lobby contributions and the social welfare. In the second period, the firms decide whether they invest in green technology or not. We show that an increase in lobby influence leads to a divergence of the tax to zero or to the maximum tax, depending on the relative strength of the industrial and the environmental lobby groups. This is in line with the result of previous models (e.g. Fredriksson 1997; Kalkuhl et al. 2020), but the contribution of this paper is to show that this distortion decreases with the discount factor (where the higher discount factor indicates that the government cares more about the future). A politician with a longer time horizon is likely to be influenced less by lobby contributions.

Next, we extend the model to a four-period model. Previous studies have already shown that a short-term Pigouvian tax might not lead to a long-term social optimum, because a tax changes the structure of the industry in the long-term (Carlton and Loury 1980, 1986). In line with this, we show in our four-period model that a social welfare maximizing government who cares about the future might set a different tax in the first period than the short-term optimum. Furthermore, we add the influence of lobby groups and show that future lobbying can have a negative influence on the current tax. A higher current tax will lead to a faster transition to a greener economy, which means that there is less incentive for lobbying in the future. A government that cares about future lobby contributions should, therefore, implement a lower tax than a government that aims to maximise social welfare.

The results of this paper indicate that countries with powerful lobby groups and a short-sighted government are not likely to implement the optimal carbon tax. Many countries are considering implementing a carbon-pricing scheme in the near future. The current carbon pricing initiatives are spread over 47 countries (The World Bank 2021), but 96 of the 185 countries that have ratified the Paris agreement mention carbon pricing in their Nationally Determined Contributions (NDCs) (Ramstein et al. 2019). By providing a better understanding of the combined influence of lobbying and time preferences on an emission tax, this paper raises practical policy recommendations and contributes to the decision-making process on climate policy.

The Baseline Model

This section develops the baseline two-period model. The economy consists of N firms that have to decide whether they produce with brown or green technology, and a government that is influenced by both an industry lobby group and an environmental lobby group. "Four Periods and Time Preferences" section extends the model to a four-period model.

The Economy

Consider a small open economy with N firms. The number of firms is normalized to 1. All firms produce a quantity x_i of the same homogeneous good which can be sold on the market for a fixed world price $p=1$. In the first period, all firms start with brown production technology ($f_i = B$). At the start of the second period, they can decide to invest in green production technology ($f_i = G$). The production functions and emissions for the different technologies are the same as in Grey (2018). A brown firm produces one unit of emissions per unit of output and a green firm produces zero emissions:

$$e_i(f_i) = \begin{cases} 0 \text{ if } f_i = G \\ 1 \text{ if } f_i = B \end{cases} \tag{1}$$

The government sets a tax τ at the start of each period. Each firm has to pay $\tau e_i x_i$ to the government. Green firms thus pay zero emission tax and brown firms pay τ per unit of output x_i. Production costs are strictly convex, $c = \frac{1}{4}x_i^2$, and identical for all firms.[1] Firms choose the output x_i that maximises their profit for a given tax and technology. The profit function that they maximise is:

$$\Pi_i = px_i - \frac{1}{4}x_i^2 - \tau e_i x_i \tag{2}$$

The only difference in profit between firms comes from the tax, which depends on the emission levels. Firms with the same technology thus choose the same equilibrium output and have the same equilibrium profit. We denote the equilibrium output and profit as x_B and Π_B for brown firms and as x_G and Π_G for green firms. We define the fraction of green firms as ϕ and, therefore, the fraction of brown firms as $1-\phi$. Each brown firm produces one unit of emission per unit of output, so each brown firm produces x_B units of emissions. There are $1-\phi$ brown firms (because $N=1$), thus the total emissions (E) are equal to $(1-\phi)x_B$ in each period. In the first period, $\phi = 0$ because all firms are brown. In the second period, ϕ depends on the number of firms that have invested in green technology.

[1] Strictly convex costs prevent the government from always setting a maximum tax such that all firms invest in green technology. The quadratic function $c = \frac{1}{4}x_i^2$ is the convex function with the simplest analytical results (Grey 2018).

Firms only care about financial incentives and should therefore only invest in green technology if the green profit minus investment costs is larger than the brown profit for the given emission tax. To prevent that all firms behave the same, we assume investment costs to differ per firm and to be uniformly distributed between 0 and 1. We assume that investment costs are not higher than 1 to ensure that investments costs do not exceed profits. The maximum profit that a firm can achieve in one year is also 1, so a maximum of 1 on investment costs ensures that the technology can be economically attractive in 1 year.

The Government

The government sets an emission tax $\tau \in [0, 1]$ that maximises the government's payoff Π_{GOV}. Following the standard approach of Grossman and Helpman (1994) and Fredriksson (1997), we assume that the government payoff depends both on the social welfare and the lobby contribution functions:

$$\Pi_{GOV} = W_1 + \delta W_2 + \lambda(C_E(\tau) + C_I(\tau)) \tag{3}$$

W_1 and W_2 are respectively the first- and second-period social welfare. The social welfare in period t is equal to the sum of all profits plus total tax revenue minus the total damage costs (D_t) at period t.

$$W_t = \sum \Pi_{i,t} + \tau \sum e_i x_{i,t} - D_t \tag{4}$$

In the baseline model, we use a linear damage function with a constant marginal damage of $\eta \in [0, 1]$:

$$D_t = \eta \sum e_i x_{i,t} = \eta(1 - \phi_t)x_B \tag{5}$$

The marginal damage η is a fraction of the total emissions and, therefore, has a maximum value of one. A linear damage function is a commonly applied simplification in this kind of climate policy models (e.g. Arvaniti and Habla 2021; Grey 2018; Marchiori et al. 2017). This assumption is widely accepted to be reasonable within a time-frame of 5 to 10 years, because climate damages are caused by the accumulated stock of greenhouse gas emissions and the relatively small flow of emissions in this time-frame effectively has a linear impact on the overall stock (Arvaniti and Habla 2021; Holtsmark and Weitzman 2020). As such, the linear damage function is realistic for the range of our model since we look into two to four political cycles, which stays within this time frame. Furthermore, we consider the decision-making process in only one country, which means that the total impact of emissions on the accumulated stock of greenhouse gasses is limited. Ultimately environmental damage functions are, however, convex if one considers larger amounts of emissions on a longer time-frame (Ackerman et al. 2009; Grey 2018). In appendix "Increasing marginal damage", we therefore explore the impact of a convex damage function, with increasing marginal damages, on the results of our model.

The government discounts the second-period social welfare with discount factor δ. The government's openness to lobby contributions is represented by λ. A social welfare maximising government has $\lambda = 0$. An increase in λ means an increase in the influence of lobby groups. $C_E(\tau)$ and $C_I(\tau)$ are the lobby contributions for each level of tax for respectively the environmental lobby group and the industry lobby group. These lobby contribution functions are further specified in "Lobbying" section.

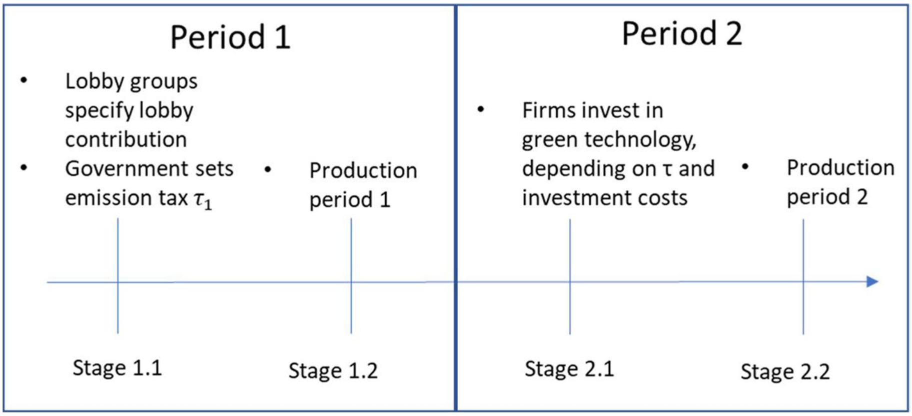

Fig. 1 Timeline of the two-period model

Timing

In the first stage of the first period, the lobby groups specify their lobby contributions, and the government sets an emission tax. Subsequently, the firms produce the output that maximises their profit for the given tax. The second period starts with an investment stage, in which firms decide whether they invest in green technology or not, followed by another production stage. The two-period model can be summarized as the four-stage game in Figure 1.

Equilibria and Simulations

To find the equilibrium tax we have to solve the model using backward induction. This section, therefore, starts with the second-period production stage.

Second Period

In the second-period production stage, all N firms will choose the output that maximises their profit for the given tax and production technology. Maximising the profit functions gives the following first order conditions:

$$x_G = 2p = 2 \tag{6}$$

$$x_B = 2p - 2\tau = 2(1 - \tau) \tag{7}$$

Plugging (6) and (7) into the profit function (2) gives the following profit for green firms:

$$\Pi_G = 2 - \frac{1}{4}2^2 = 1 \tag{8}$$

And for brown firms:

$$\Pi_B = 2(1-\tau) - \frac{1}{4}(2(1-\tau))^2 - 2(1-\tau)\tau = (1-\tau)^2 \tag{9}$$

In the investment stage, each firm invests in green technology if and only if $\Pi_G - I_i \geq \Pi_B$.

$$\begin{aligned} 1 - I_i &\geq (1-\tau)^2 \\ I_i &\leq 2\tau - \tau^2 \end{aligned}$$

The investment costs are uniformly distributed between 0 and 1, thus all firms with investment costs smaller or equal to $2\tau - \tau^2$ will invest in green technology. The fraction of firms that will invest in green technology is:

$$\phi = 2\tau - \tau^2 \tag{10}$$

The other $1 - \phi = (1-\tau)^2$ firms will stay brown. The fraction of green firms ϕ depends on τ as follows:

$$\frac{\partial\phi}{\partial\tau} = 2(1-\tau) \tag{11}$$

$\frac{\partial\phi}{\partial\tau}$ is positive and decreasing for $\tau \in [0,1)$, so an increase in the tax leads to more green firms, but this increase becomes smaller as the tax gets higher. The total profit in the economy in period 2 is[2]:

$$\begin{aligned} \Pi_2 &= \phi\Pi_G - \textstyle\sum_0^{\phi} I_i + (1-\phi)\Pi_B \\ &= 0.5\tau^4 - 2\tau^3 + 3\tau^2 - 2\tau + 1 \end{aligned} \tag{12}$$

Social welfare in period 2 is:

$$W_2 = \phi\Pi_G - \sum_0^{\phi} I_i + (1-\phi)\Pi_B + (\tau-\eta)(1-\phi)2(1-\tau) \tag{13}$$

Lemma 1: Period 2 social welfare is maximised for $\tau = \eta$, i.e. when the tax is equal to the marginal damage costs (a Pigouvian tax).

Proof. *Appendix* "Proof of Lemma 1".

First Period

All firms in period 1 have brown technology, and therefore have the same profit as brown firms in period 2. They choose the production level that maximises Π_B. From "Second Period" section, we know that Π_B is maximised for $x_B = 2(1-\tau)$. All firms have the same profit of $\Pi_B = (1-\tau)^2$. Total first period profit is thus equal to the profit of brown firms ($\Pi_1 = \Pi_B = (1-\tau)^2$). Period 1 social welfare is equal to:

[2] Proof in appendix "Equation (38), total second-period profit (Π_2)".

$$\begin{aligned} W_1 &= \Pi_B + (\tau - \eta)x_B = (1-\tau)^2 + 2(\tau - \eta)(1-\tau) \\ &= (1-\tau)(1+\tau-2\eta) \end{aligned} \tag{14}$$

$$\frac{\partial W_1}{\partial \tau} = 2(\eta - \tau) \tag{15}$$

$$\frac{\partial^2 W_1}{\partial \tau^2} = -2 \tag{16}$$

$\frac{\partial^2 W_1}{\partial \tau^2} < 0$ for all τ. Social welfare is therefore maximised when $\frac{\partial W_1}{\partial \tau} = 0$, which is for $\tau = \eta$.

Proposition 1. A social welfare maximising government sets the tax equal to the marginal damage costs (η). Both first and second period social welfare are maximised with this Pigouvian tax.

Lobbying

The government chooses a tax that maximises its payoff, which depends on both the social welfare over the two periods and the lobby contributions. This section specifies the lobby contribution functions.

Our model follows the approach of Grossman and Helpman (1994), which has become the standard approach in the political economy literature on lobbying. Similar to Marchiori et al. (2017), we assume that the industry lobby group exhibits a stake $0 \leq \alpha \leq 1$ in the profits of brown firms. The tax does not influence the profit of green firms; green firms have therefore no incentive to lobby. With brown firms, we mean here firms that have brown technology during the lobbying period. Some firms will lobby in the first period and invest in green technology in the second period. Second-period green profits of firms that were brown in the first period are thus also included in the total profit of brown firms. In the baseline case, we assume that all firms start as brown firms, so brown firms' profit is equal to total profit in the economy. The environmental lobby group exhibits a stake $0 \leq \beta \leq 1$ in the damage from pollution. The gross utilities of the lobby groups are as follows:

$$W_I = \alpha(\Pi_1 + \delta(\phi(\Pi_G) - \sum_0^{\phi} I_i + (1-\phi)\Pi_B)) = \alpha(\Pi_1 + \delta\Pi_2) = \alpha\Pi^{TOT} \tag{17}$$

$$W_E = -\beta(x_B + \delta(1-\phi)x_B)\eta = -\beta D(E) \tag{18}$$

The factors α and β express the degree of representation of the lobby group. For simplicity, we assume perfect representation, $\alpha = \beta = 1$. Each lobby group offers a contribution schedule that specifies their lobby contribution for each tax. These contribution schedules are a function of the welfare of the lobby group members:

$$C_I(\tau) = \max[0, \Pi^{TOT}(\tau) - \overline{W_I}] \tag{19}$$

$$C_E(\tau) = \max[0, -\overline{W_E} - D(E)] \tag{20}$$

$\overline{W_I}$ and $\overline{W_E}$ are the (constant) base utility levels that these lobby groups will get when they do not lobby. The marginal contributions, for strictly positive contribution levels, do not depend on $\overline{W_I}$ and $\overline{W_E}$, and are equal to $\frac{\partial \Pi^{TOT}}{\partial \tau}$ and $-\frac{\partial D(E)}{\partial \tau}$

Proposition 2: $\frac{\partial C_I(\tau)}{\partial \tau} \leq 0$ *and* $\frac{\partial C_E(\tau)}{\partial \tau} \geq 0$ *for* $\tau \in (0,1)$. *Thus, the lobby contribution of the industry lobby decreases with the tax and the lobby contribution of environmental lobby groups increases with the tax.*

Proof. *Appendix* "Proof of Proposition 2".

Following Bernheim and Whinston (1986) and Grossman and Helpman (1994), we assume that the lobby contribution functions are differentiable and truthful around the equilibrium points. We only consider the marginal lobby contributions around the equilibriums and, therefore, we do not have to define the base utility levels $\overline{W_I}$ and $\overline{W_E}$.

Government Decision

The government maximises the government payoff in Eq. (3). The equilibrium tax is the tax that maximises this government payoff function for a given value of λ and the above specified lobby contribution functions. In this section, we analyse the influence of the openness to lobby contributions (λ) on the equilibrium tax for different values of η.

The first step in the optimization of the government payoff is to define the first order condition (FOC). With both lobby groups active in lobbying,[3] the FOC becomes:

$$\frac{\partial \Pi_{GOV}}{\partial \tau} = \frac{\partial W_1}{\partial \tau} + \delta \frac{\partial W_2}{\partial \tau} + \lambda \left(\frac{\partial C_I(\tau)}{\partial \tau} + \frac{\partial C_E(\tau)}{\partial \tau} \right) = 0 \tag{21}$$

Note that the marginal contributions of the lobby groups can be replaced by the marginal utilities of the respective lobby groups, because they maximise their utility such that marginal benefits are equal to marginal costs, with costs being the lobby contributions.

The FOC in Eq. (22) implicitly defines τ as a function of the three parameters: η, δ, and λ:

$$\begin{aligned} F(\eta, \lambda, \delta, \tau(\eta, \lambda, \delta)) = \\ & 2(\eta - \tau) + \delta\left(6(\eta - \tau)(1 - \tau)^2\right) \\ & + \lambda\left(-2\delta(1 - \tau)^3 - 2(1 - \tau) + 2\eta + 6\delta\eta(1 - \tau)^2\right) = 0 \end{aligned} \tag{22}$$

The next step is to analyse the impact of openness to lobby contributions(λ) on the equilibrium tax for different values of the marginal damage (η). We start with considering the case of zero discounting ($\delta = 1$). To analyse the impact of λ on the tax we analyze two example cases for the marginal damage η, $\eta = \frac{1}{2}$ and $\eta = \frac{1}{3}$, which lead to Lemma 2:

[3] If both groups find it optimal to lobby, then the following two conditions are met:

$(1 - \tau)^2 + \delta\left(0.5\tau^4 - 2\tau^3 + 3\tau^2 - 2\tau + 1\right) - \frac{\delta}{2} > 0$
$2\eta(1 + \delta) - 2\eta(1 - \tau)\left(1 + \delta(1 - \tau)^2\right) > 0$

When $\tau = 1$, condition i. is not satisfied. Therefore $\tau < 1$. The LHS term in condition ii. is increasing in τ. To satisfy condition ii., $\tau > 0$ and $\eta > 0$. Combining these two conditions implies that if both groups lobby, $\tau \in (0, 1)$ and $\eta > 0$.

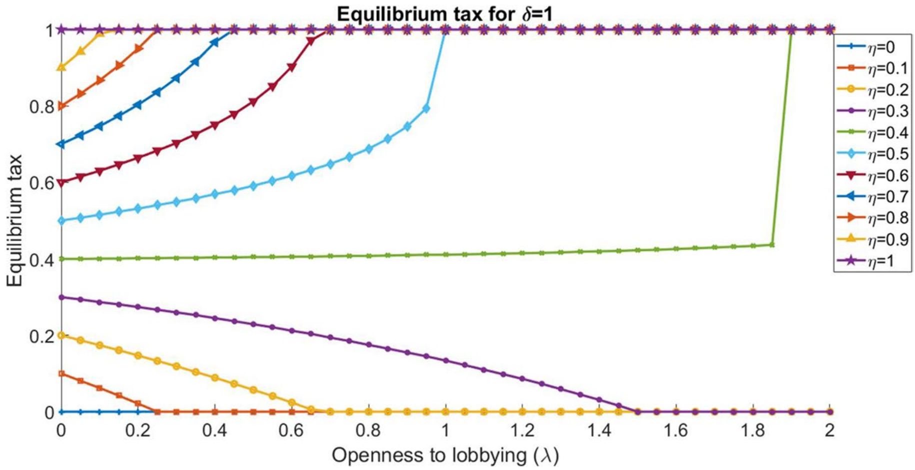

Fig. 2 Equilibrium tax as a function of openness to lobby contributions λ, for δ = 1

Lemma 2: When $\delta = 1$, *the equilibrium carbon tax rate is increasing in the degree of openness to lobby contributions* $\left(\frac{\partial \tau}{\partial \lambda} > 0\right)$ *for* $\eta = \frac{1}{2}$ *and decreasing in the degree of openness to lobby contributions* $\left(\frac{\partial \tau}{\partial \lambda} < 0\right)$ *for* $\eta = \frac{1}{3}$ *if* $\lambda < 2$.

Proof. Appendix "Proof of Lemma 2".

Our analytical derivations show that an increase in the openness to lobby contributions increases the equilibrium tax for marginal damage equal to $\frac{1}{2}$ and decreases the equilibrium tax for marginal damage equal to $\frac{1}{3}$. We use MATLAB simulations to analyse the impact of openness to lobby contributions for other values of marginal damage. Figure 2 shows the equilibrium tax as a function of the openness to lobbying contributions (λ) for different values of the marginal damage η when $\delta = 1$. The equilibrium tax is equal to the marginal damage ($\tau = \eta$) for all η if $\lambda = 0$. A government with $\lambda = 0$ is not influenced by lobby groups, thus maximises social welfare. As shown in "Second period" and "First Period" sections, social welfare is maximised with a Pigouvian tax.

An increase in the openness to lobby contributions leads to a deviation from this Pigouvian tax for almost all values of η. Lobbying thus leads to a decrease in social welfare in this simple two-period model. The deviation from the Pigouvian tax increases with λ. The direction and speed of this deviation depends however on the marginal damage costs (η). For a marginal damage cost of 0.4, the equilibrium tax stays relatively constant for values of λ below 1.9. Only after this high threshold, the equilibrium tax increases to the maximum of $\tau = 1$. For marginal damage costs lower than 0.4, the equilibrium tax decreases with λ and eventually goes to zero. For larger marginal damage costs, the equilibrium tax increases with λ. An increase in lobby influence thus leads to a divergence of the tax to one of the extremes ($\tau = 0$ or $\tau = 1$).

The simulations and analytical derivations show that a strong lobby by brown industry can prevent an emission tax when the marginal damage of emission is relatively small. This means that the transition to green technology does not take place. This result suggests that lobbying is especially problematic in countries where the perceived climate damage is low. Lobbying could worsen the problem of low climate risk awareness in those countries.

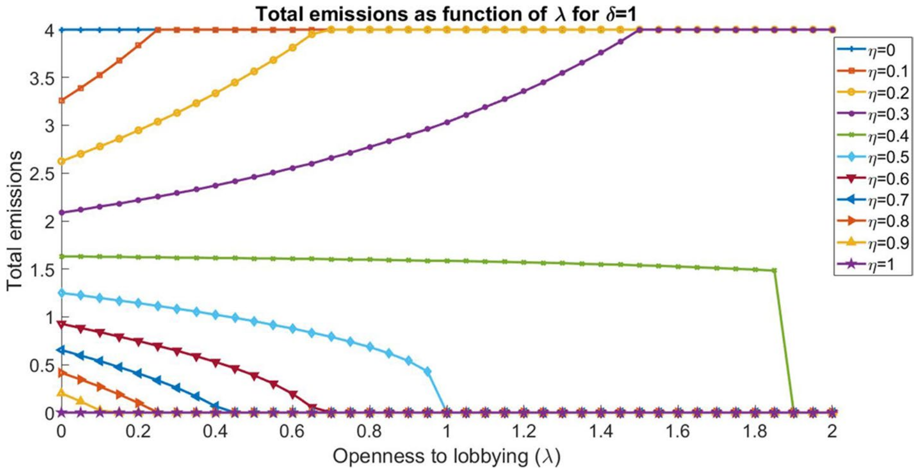

Fig. 3 Total emissions as a function of openness to lobby contributions λ

On the other hand, the model shows that the environmental lobby will be stronger than the brown industry lobby for large marginal damage costs. This results in a tax higher than the social optimum, which leads to lower profits and a faster transition to a green economy. The World Bank Carbon Pricing Dashboard shows that there are many countries with a very low tax and a few countries with a high tax (The World Bank 2021). This observation could be consistent with the influence of lobbying found here.

Since lobbying influences the transition to green technology, it also influences the total emissions over the two periods. Figure 3 plots total emissions as a function of openness to lobby contributions. Total emissions are the sum of the emissions in both periods as a result of the equilibrium tax of Fig. 2. A Pigouvian tax ($\lambda = 0$) leads to positive emissions for all marginal damage cost levels except for $\eta = 1$ for which the Pigouvian tax is 1. When the openness to lobby contributions increases, the emissions move in the opposite direction as the tax. For high marginal damage costs, the tax increases with λ and therefore the emissions decrease with λ. For smaller marginal damage an increase in λ leads to an increase in emissions. A high openness to lobby contributions thus either reduces the emissions to zero or increases the emissions to the maximum quantity of emissions.

The switching point from increasing emissions to decreasing emission is just below $\lambda = 0.4$ in these simulations. This point depends on how well the lobby groups are organized. Both lobby groups perfectly represent their interest groups in the simulations ($\alpha = \beta = 1$ in Eq. (17) and (18)), but in reality, one lobby group can be less organized than the other can. Industry lobby groups are often better organized and have larger lobby budgets than environmental groups (Yu 2005). Moreover, Cai and Li (2020) show that clean firms find it more difficult to reach objectives through lobbying than dirty firms. Introducing this asymmetry in lobby groups in our model would lead to lower taxes and more emissions. Environmental organizations that want to decrease emissions should thus make sure that they are well organized and collect enough money for lobbying to offset the industry lobby.

Discount Factor

In the previous section, we set the discount factor at $\delta = 1$ (no discounting), to analyze how marginal damage and openness to lobby contributions influence the government's decision on the tax. This section analyses the influence of the discount factor on the tax decision.

First, consider the case without lobbying ($\lambda = 0$). The government maximises social welfare.

$$\Pi_{GOV}(\lambda = 0, \delta, \tau, \eta) = W_1(\tau, \eta) + \delta W_2(\tau, \eta) \tag{23}$$

From Proposition 1, we know that both W_1 and W_2 are maximised for $\tau = \eta$ and that therefore the social welfare is also maximised for $\tau = \eta$ irrespective of the discount factor.

Second, consider the case with a relatively small openness to lobby contributions of $\lambda = \frac{1}{2}$. If we plug $\lambda = \frac{1}{2}$ in the FOC of Eq. (22), we get:

$$F(\eta, \delta, \tau) = 3\eta - 2 + (1 - \tau) + (9\delta\eta - 6\delta)(1 - \tau)^2 + 5\delta(1 - \tau)^3 = 0 \tag{24}$$

Let $y = 1 - \tau$, then the above equation can be rewritten as:

$$F(\eta, \delta, y) = 5\delta y^3 + 3\delta(3\eta - 2)y^2 + y + 3\eta - 2 = 0 \tag{25}$$

This FOC is well defined only if the value of η is in a reasonable range. The left-hand side of Eq. (25) is zero for $\eta = \frac{2}{3}$ and always positive for $\eta > \frac{2}{3}$. It is therefore optimal to set the carbon tax at one for $\eta \geq \frac{2}{3}$. We only need to check $\frac{\partial\tau}{\partial\delta}$ when $\eta < \frac{2}{3}$, therefore we calculate the derivative for $\eta = \frac{1}{2}$ and $\eta = \frac{1}{3}$, which leads to Lemma 3:

Lemma 3: For $\lambda = \frac{1}{2}$, the equilibrium tax is increasing with the discount factor $\left(\frac{\partial\tau}{\partial\delta} > 0\right)$ for both $\eta = \frac{1}{2}$ and $\eta = \frac{1}{3}$. The equilibrium tax is at its maximum ($\tau = 1$) for $\eta \geq \frac{2}{3}$.

Proof. *Appendix "Proof of Lemma 3".*

Third, consider the case with an openness to lobby contributions of $\lambda = 1$, which leads to the following FOC:

$$F(\eta, \lambda = 1, \delta, \tau) = 2\eta - 1 + 2\delta(1 - \tau)^3 - 3\delta(1 - \tau)^2 + 6\delta\eta(1 - \tau)^2 = 0 \tag{26}$$

The left-hand side of Eq. (26) is positive for any value of $\tau \in [0, 1]$ if $\eta > \frac{1}{2}$. The marginal damage is large and combined with sufficient lobbying ($\lambda = 1$) this leads to the maximum tax of $\tau = 1$ irrespective of the discount factor.

For $\eta = \frac{1}{2}$ this FOC becomes $F(\eta, \delta, \tau) = 2\,\delta(1 - \tau)^3 = 0$. This equation holds for $\tau = 1$ or $\delta = 0$. For $\eta = \frac{1}{2}$, we can thus also conclude that $\tau = 1$ for any positive discount factor. After simplifying the FOC in Eq. (27) and considering the case with $\eta = \frac{1}{3}$, we derive Lemma 4. Similarly, we consider the case with $\eta = \frac{1}{4}$ to derive Lemma 4.

Lemma4: For $\lambda = 1$ and $\eta = \frac{1}{3}$ the carbon tax is increasing in $\delta\left(\frac{\partial\tau}{\partial\delta} > 0\right)$ for *$\delta > \frac{1}{3}$ and zero for $\delta \leq \frac{1}{3}$. The carbon tax is zero for $\eta = \frac{1}{4}$ and $\lambda = 1$, irrespective of the discount factor.*

Proof. *Appendix "Proof of Lemma 4".*

We use MATLAB simulations to plot the equilibrium tax as a function of the discount factor for other values of marginal damage: for $\lambda = \frac{1}{2}$ in Fig. 4 and $\lambda = 1$ in Fig. 5. Based on these figures and the analytical derivations we can conclude that with lobbying 1) the tax goes to one for high values of marginal damage, 2) the tax is increasing in the discount factor for intermediate values of marginal damage, and *3)* the tax is zero for low values of marginal damage. The results confirm the conclusion of "Government Decision" section that lobbying

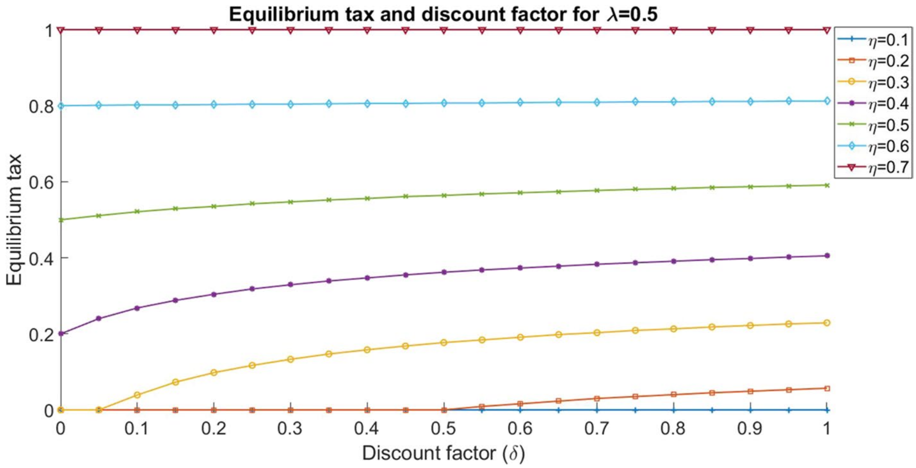

Fig. 4 Equilibrium tax as a function of the discount factor δ, for λ = 0.5

leads to a tax that differs from the optimal Pigouvian tax. How much the tax deviates from the Pigouvian tax depends on the discount factor (δ), the openness to lobby contributions (λ), and the marginal damage cost (η). Comparing Fig. 4 with Fig. 5 shows that an increase in λ leads to a decrease in the range of values for η and δ that lead to an intermediate tax. The larger the government's openness to lobby contributions the more likely that the tax will be at one of the extremes ($\tau = 0$ or $\tau = 1$).

The discount factor only influences the equilibrium tax for intermediate values of the marginal damage. Figure 5 clearly shows that the equilibrium tax increases with the discount factor (a higher discount factor means that the government cares more about the future) for these

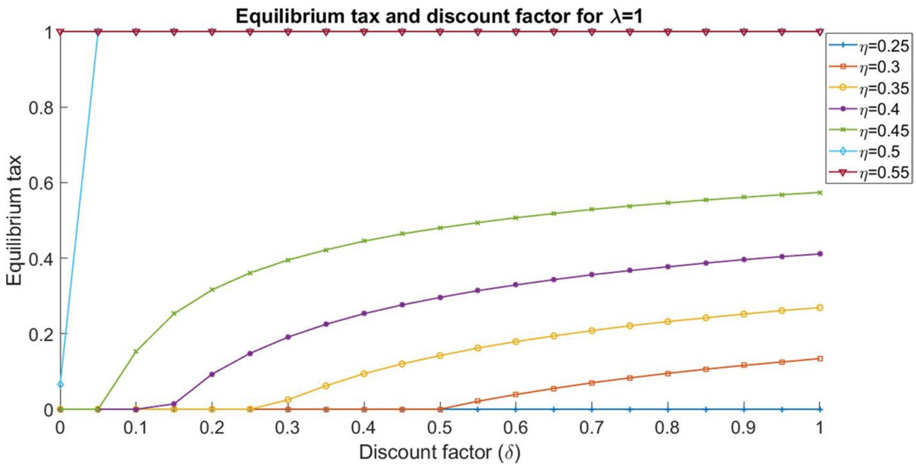

Fig. 5 Equilibrium tax as a function of the discount factor δ, for λ = 1

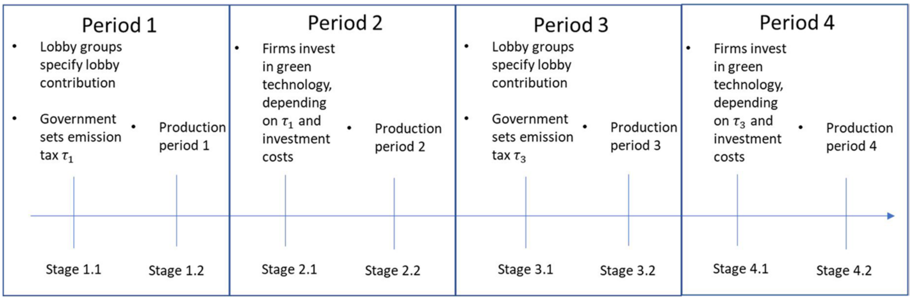

Fig. 6 Timeline four-period model

intermediate situations. These results indicate that placing a low weight on the future reinforces lobbying impacts from polluting industries, resulting in a lower tax, whereas a high weight on the future reinforces impacts of environmental lobby groups, resulting in a higher tax.

Four Periods and Time Preferences

In this section, we add two more periods to the model. The aim of this section is to analyse the influence of the discount factor on the tax decision over a longer period and to analyse the lobby dynamics over multiple decision-making periods. The stages in periods 3 and 4 are the same as in periods 1 and 2. Figure 6 gives and overview of all stages in the model.

The main difference between period 1 and period 3 is that period 1 start with only brown firms, while in period 3 part of the firms are green, depending on the investment decision in period 2. The government in period 1 should therefore take into account how the tax decision influences the emissions in period 3 and period 4. We assume that green firms will never go back to brown production and that the investment in green technology is a one-time investment. The number of green firms can therefore only increase in period 4 when the tax is higher than the first-period tax. A lower tax does not change the fractions of green and brown firms. First, "Social Welfare Maximising Policymaker" section analyses how the discount factor and time preferences of policymakers influence the tax decisions of a social welfare maximising policymaker who does not get influenced by lobby contributions. Subsequently, "Lobbying in the Third Period" section provides an analysis of the interaction between time preferences and lobbying.

Social Welfare Maximising Policymaker

Firms in stage 4.2, the production stage in period 4, will maximise their profit, so they behave the same as in "Second Period" section. In the investment stage (stage 4.1), brown firms will again only invest if that leads to a higher profit ($\Pi_G(\tau_3) - I_i > \Pi_B(\tau_3)$). Profit in this stage depends on τ_3, which is the tax set by the government in period 3. Brown firms invest if $I_i \leq 2\tau_3 - \tau_3^2$, but all firms with investment costs smaller or equal to ϕ already have invested in period 2. Only the firms with $\phi < I_i \leq 2\tau_3 - \tau_3^2$ will invest in period 4. Total period 4 investment costs are therefore equal to $I_4 = \int_{\phi}^{2\tau_3-\tau_3^2} x dx = \frac{1}{2}((2\tau_3 - \tau_3^2)^2 - \phi^2)$ if $2\tau_3 - \tau_3^2 > \phi$ and $I_4 = 0$ if $2\tau_3 - \tau_3^2 \leq \phi$. The fraction of green firms in period 4 is $\theta = \max[\phi, 2\tau_3 - \tau_3^2]$ and the fraction of brown firms is $1 - \theta = \min[1 - \phi, (1 - \tau_3)^2]$. Period 4 social welfare is equal to:

$$W_4 = \theta\Pi_G + (1-\theta)\Pi_B(\tau_3) - I_4 + (1-\theta)(\tau_3 - \eta)x_B(\tau_3) \tag{27}$$

The fraction of green firms in period 3 is the same as the fraction of green firms in period 2, which is ϕ. Social welfare in period 3 is therefore equal to:

$$W_3 = \phi\Pi_G + (1-\phi)\Pi_B(\tau_3) + (1-\phi)(\tau_3 - \eta)x_B(\tau_3) \tag{28}$$

A social welfare maximising policymaker in period 3 chooses a tax that maximises $SW_3 = W_3 + \delta W_4$.

For $2\tau_3 - \tau_3^2 \leq \phi$, this function is equal to:

$$SW_3 = (1+\delta)(\phi\Pi_G + (1-\phi)\Pi_B(\tau_3) + (1-\phi)(\tau_3 - \eta)x_B(\tau_3)) \tag{29}$$

For $2\tau_3 - \tau_3^2 > \phi$, we get:

$$SW_3 = W_3 + \delta(\theta\Pi_G - I_4 + (1-\theta)\Pi_B(\tau_3) + (1-\theta)(\tau_3 - \eta)x_B(\tau_3)) \tag{30}$$

With $\theta = 2\tau_3 - \tau_3^2$ and $I_4 = \frac{1}{2}(\theta^2 - \phi^2)$.

Both of these social welfare functions are maximised for $\tau = \eta$.[4] A social welfare maximising policymaker in period 3 will thus always implement the Pigouvian tax. This leads to a social welfare of:

$$SW_3 = \begin{cases} (1+\delta)\left[\phi + (1-\phi)(1-\eta)^2\right] & \text{if } 2\eta - \eta^2 \leq \phi \\ \phi + (1-\phi)(1-\eta)^2 + \delta\left[(2\eta - \eta^2) - \frac{1}{2}((2\eta-\eta^2)^2 - \phi^2) + (1-\eta)^4\right] & \text{if } 2\eta - \eta^2 > \phi \end{cases} \tag{31}$$

Social welfare in period 3 and period 4 depends on τ_1 because ϕ depends on τ_1. A period 1 social welfare maximising policymaker should therefore not only maximise period 1 and period 2 welfare but also take into account the impact of the tax on period 3 and 4.

$$\frac{\partial SW_3}{\partial \phi} = \begin{cases} (1+\delta)(1-(1-\eta)^2) > 0 & \text{if } 2\eta - \eta^2 \leq \phi \\ 1 - (1-\eta)^2 - \delta\phi \geq 0 & \text{if } 2\eta - \eta^2 > \phi \end{cases} \tag{32}$$

SW_3 is increasing in ϕ for every value of $\phi \in (0,1)$ and ϕ is increasing in τ_1. A policymaker who considers the influence of the first-period tax on the third and fourth period should implement a higher first-period tax than a policymaker who does not consider those.

The optimal first-period tax can thus differ from the optimal first-period tax in the two-period model for two reasons. First, because the policymakers consider the influence of the policy they set in the first period on the third and fourth period. Second, because the firms will also consider the consequences of their investment decision on their profit in the third and fourth period, which will lead to a different ϕ than the one in "Social Welfare Maximising Policymaker" section.

To analyse the second-period investment decision, we need to take into account whether the firms will invest in the fourth period if they did not already invest in the second period. Firms with $I_i > 2\tau_3 - \tau_3^2$ will not invest in the fourth period and firms with $I_i \leq 2\tau_3 - \tau_3^2$ will invest in the fourth period. With a social welfare maximising policymaker, the firms

[4] Proof in appendix "Social welfare maximising third-period tax".

know that the third-period tax will be $\tau_3 = \eta$, so they know what their decision will be if they wait until the fourth period. The firms with high investment costs ($I_i > 2\eta - \eta^2$), know that they will have three periods of brown technology if they do not invest in the second period. These firms will therefore only invest in the second period if the discounted profit over three periods for green technology minus the investment costs is larger than the discounted profit over three periods for brown technology:

$$(1 + \delta + \delta^2)\Pi_G - I_i \geq \Pi_B(\tau_1) + (\delta + \delta^2)\Pi_B(\tau_3) \tag{33}$$

Plugging in $\Pi_G = 1$, $\Pi_B(\tau_1) = (1 - \tau_1)^2$ and $\Pi_B(\tau_3) = (1 - \eta)^2$ and solving the inequality leads to the following threshold investment costs:

$$I_h^* = 2\tau_1 - \tau_1^2 + (\delta + \delta^2)(2\eta - \eta^2) \tag{34}$$

Firms with lower investment costs ($I_i \leq 2\eta - \eta^2$) will always invest in the fourth period if they did not invest already in the second period. They will therefore invest in the second period if and only if:

$$(1 + \delta + \delta^2)\Pi_G - I_i \geq \Pi_B(\tau_1) + \delta\Pi_B(\tau_3) + \delta^2(\Pi_G - I_i) \tag{35}$$

Plugging in $\Pi_G = 1$, $\Pi_B(\tau_1) = (1 - \tau_1)^2$ and $\Pi_B(\tau_3) = (1 - \eta)^2$ and solving the inequality leads to the following threshold investment costs for firms with low investment costs[5]:

$$I_l^* = \frac{2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2)}{1 - \delta^2} \quad \text{for} \quad \delta < 1 \tag{36}$$

The firms only invest in green technology in the second period if their investment costs are below their respective threshold (I_l^* or I_h^*).

The fraction of firms that invests in green technology in the second period is[6]:

$$\phi = \begin{cases} \min\left[1, \max[2\tau_1 - \tau_1^2 + \left(\delta + \delta^2\right)\left(2\eta - \eta^2\right), 2\eta - \eta^2]\right] & \text{if } 2\tau_1 - \tau_1^2 \geq (1 - \delta - \delta^2)(2\eta - \eta^2) \\ \min\left[\frac{2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2)}{1 - \delta^2}, 2\eta - \eta^2\right] & \text{if } 2\tau_1 - \tau_1^2 < (1 - \delta - \delta^2)(2\eta - \eta^2) \end{cases} \tag{37}$$

For a given tax, the fraction of firms that will invest in the second period is in this four-period model larger or equal to that fraction in the two-period model ($\phi \geq 2\tau_1 - \tau^2$). The fraction of green firms increases with the discount factor, so firms that put more value on the future are more likely to invest early.

The period 1 social welfare maximising policymaker chooses a tax τ_1 that maximises $SW_1 = \sum_{j=1}^{4} \delta^{j-1} W_j$. Figure 7 shows MATLAB simulations of the social welfare maximising first-period tax as a function of the discount factor for values of η between 0 and 0.8. Based on the above analysis we set the third-period tax equal to the marginal damage ($\tau_3 = \eta$). For $\delta = 0$, the policymaker only focuses on first-period welfare, which is maximised for a tax equal to the marginal damage costs. An increase in the discount factor means that the policymaker attaches more value to the effect of the tax on later periods. The effect of an increase in the discount factor on the optimal tax differs with the marginal cost (η). For most values of η we first observe an increase in the optimal tax when δ increases. This is caused by the positive effect of τ_1 on W_3 and W_4 through the increases in ϕ. At some

[5] See appendix "Proof of equation (36), threshold investment costs I_l^*" for proof of Eq. (37)

[6] See appendix "Proof of equation (37), first-period investment in four-period model" for proof of Eq. (39)

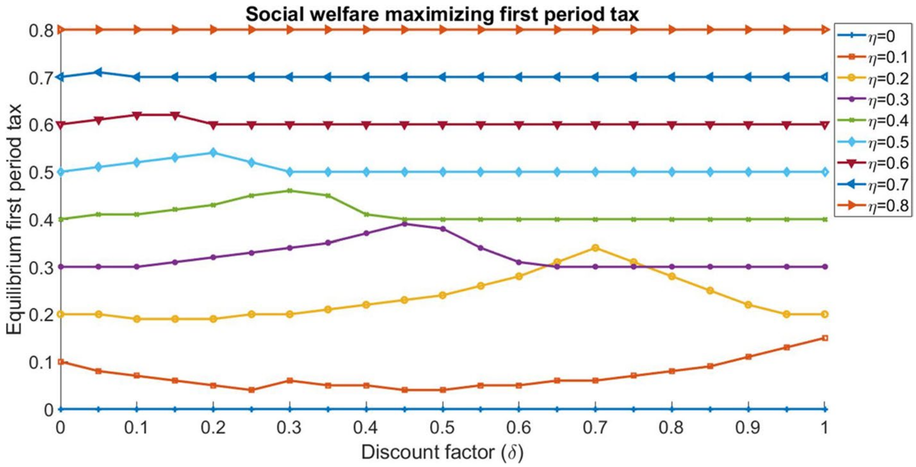

Fig. 7 Equilibrium first period tax as a function of the discount factor δ

point, the tax decreases again and goes back to $\tau_1 = \eta$ (for most values of η). The reason for this is that ϕ becomes one if η and δ are large enough, which means that all firms invest in green technology. After this point, a further increase in τ_1 does not have a positive effect on the third- and fourth-period welfare. The government should therefore implement the tax that maximises first- and second-period welfare, which is the Pigouvian tax. For a marginal damage of 0.8 and higher, ϕ already becomes one for a very small discount factor, which results in a tax that stays equal to the marginal damage.

In general, an increase in the discount factor increases the tax until $\phi = 1$. There is however an exception for $\eta < 0.2$. For these low marginal damage cases, the optimal tax first decreases with the discount factor before it increases. The reason for this is that a higher discount factor increases the investment in the second period, which has a small negative effect on second-period profits. A higher tax can thus have a negative effect on the second-period welfare. The positive effect in the third and fourth period is larger for most combinations of η and δ, but the negative effect in the second period can be larger when both η and δ are small. A social welfare maximising government will thus only set an inefficient low first-period tax if (the perceptions of) the marginal damage costs are low and the time horizon is very short.

Lobbying in the Third Period

When we include lobby contributions, the analysis of the first-period tax becomes more difficult because the third-period tax cannot be fixed at $\tau_3 = \eta$. The third-period tax will be influenced by lobby contributions and by the fraction of firms that have invested in green technology in the second period. Both the first-period tax, the first-period lobby contribution, and the second-period investment decision depend on the third-period tax. Firms should base their second-period investment decision on the expected third-period tax, but this third-period tax also depends on the investment of other firms.

We start with analysing how the investment decision influences the government payoff in the third period. Based on this influence we can reason how third-period lobbying might influence the first-period tax. The third period lobby contribution schedules are again a function of the welfare of the lobby group members. Green firms do not have an incentive to lobby because the tax does not influence their profit. We, therefore, assume that the industry lobby group only consists of the firms that are still brown at the start of period 3. The industry lobby contribution is thus a function of the discounted third and fourth period profit of the firms that have brown technology at the start of period 3. As in "Lobbying" section, we assume that the lobby contribution functions are differentiable and truthful around the equilibrium points. The marginal lobby contributions of the lobby groups are again equal to the marginal utilities of the two groups, which is marginal utility of period 3 brown firms for the industry lobby and marginal damage for the environmental lobby group. An important assumption of this model is that lobby groups do not behave strategically, meaning that in each period they provide the lobby contribution that maximises their own utility and they do not try to manipulate future lobby contributions of the other lobby group.

To analyse the influence of lobbying on government payoff we first plot the equilibrium government payoff as a function of the fraction of green firms without lobby influence ($\lambda = 0$) in Fig. 8, which we compare with the same plot for a government that is open for lobby contributions ($\lambda = 1$) in Fig. 9. The plots are the result of MATLAB simulations, where the equilibrium government payoff is the government payoff that corresponds to the equilibrium tax. We set $\delta = 1$ (no discounting) because we are focusing on the influence of lobby contributions.

The government in Fig. 8 is not influenced by lobby contributions and therefore maximises social welfare. The figure confirms the conclusion of "Social Welfare Maximising Policymaker" section that period 3 and 4 social welfare increases with ϕ. In Fig. 9 we observe that ϕ has the opposite effect on the equilibrium government payoff when the government is open for lobby influences. The more firms invest in period 2 the less the period 3 government payoff will be. The reason behind this is that total lobby contributions will decrease when there are fewer brown firms. A decrease in brown firms results in a smaller lobby contribution of the industry lobby groups because their contribution depends on the

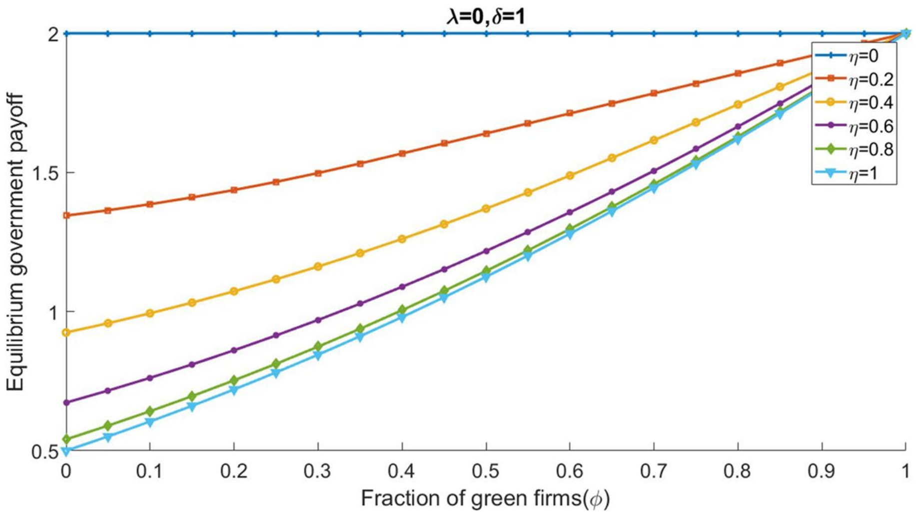

Fig. 8 Equilibrium government payoff as a function of the fraction of green firms ϕ, $\lambda = 0$ and $\delta = 1$

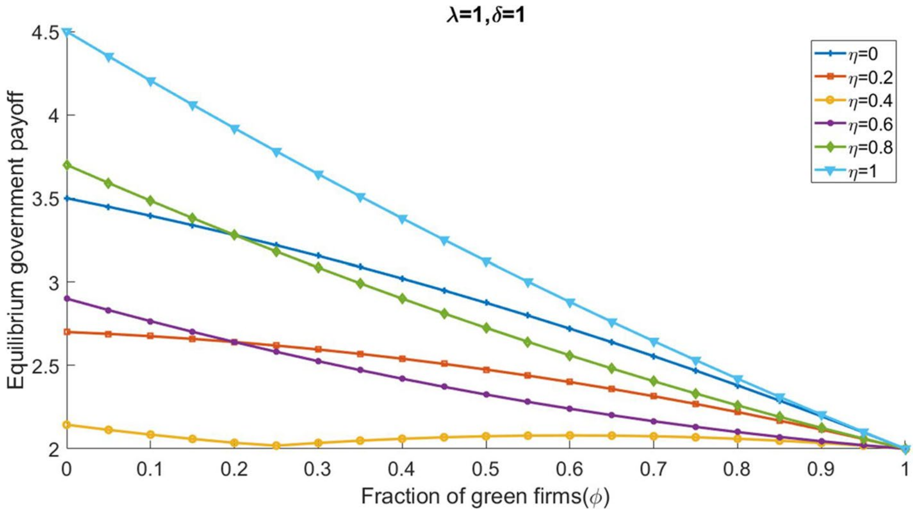

Fig. 9 Equilibrium government payoff as a function of the fraction of green firms ϕ, λ = 1 and δ = 1

total brown firms' profit. It, however, also decreases the lobby contribution of the environmental lobby groups, because their contribution depends on the damage. Less brown firms means fewer emissions which results in lower damage costs. A policymaker who cares about lobby contributions has an incentive to limit the investment in green technology to extract more lobby contributions in later periods. The government can influence the investment decision with the period 1 tax, so period 3 lobbying can have a negative effect on the period 1 tax. This effect will be larger when the government cares more about the future (uses a higher discount factor).

Conclusion

Although carbon pricing is becoming more widely implemented around the world, a large variation in the prevailing levels of carbon prices can be observed. Most prices are too low for meeting the emission reductions that countries agreed upon in the Paris Agreement, while some countries managed to introduce sufficiently high carbon prices. Theoretical models can give insights into the causes behind suboptimal climate policy decisions, based on which one can draw lessons for more effective policies.

The aim of this paper is to analyse the combined influence of lobbying, from industry and environmental groups, and time preferences on carbon emission tax policies to get insights into reasons for suboptimal climate policy. Based on these insights, we provide recommendations to design more effective policies for mitigating climate change. To achieve this, we extend a one-period models to two- and four-period models, to allow for the inclusion of time preferences of politicians, which previous theoretical studies on carbon emission tax policies do not account for.

Our model confirms the conclusion of previous literature that lobbying leads to inefficient tax policies. Furthermore, we learn from the two-period model that the inefficiency decreases with the discount factor. The influence of lobby groups is larger on politicians with a short-term focus (using a low discount factor) than on politicians with a long-term focus. The direction of the lobby influence depends on the marginal damage costs and the relative strength of both lobby groups. Lobbying especially results in inefficiently low carbon taxes when (perceptions of) the marginal damage costs of climate change are low, which suggests that lobbying could worsen problems of insufficient climate action in countries with low awareness about climate change. The reason is that in such countries lobbying from dirty industry dominates the environmental lobby.

The four-period model indicates that a social welfare maximising government might implement a tax higher than the marginal cost in the first period to speed up the transition to green technology which will lead to higher welfare in later periods. This effect increases when the government uses a larger discount factor and thus has a longer-term focus. A government might however do the opposite, when influenced by lobby groups. This government wants to extract as many lobby contributions as possible, but future lobby contribution will decrease if more firms invest in green technology. Which of those two effects is stronger depends on the trade-off that the government makes between lobby contributions and the social welfare.

Climate change is a long-term problem and emission tax policy is therefore characterized by long-term decision-making. That time preferences can have a large influence on optimal climate policy outcomes has already been shown by the literature on economic integrated assessment models of optimal climate policy, which found that the optimal carbon tax is higher if a higher weight is given to the future (van den Bergh and Botzen 2015). An addition we make with our study to the existing literature is that time preferences of the government in an interaction with lobbying influence the emergence of suboptimal carbon tax policies in a complex manner. If policymakers place less weight on the future, then this reinforces lobbying effects from dirty industries, while placing a higher weight on the future reinforces impacts on the carbon tax of lobbying from environmental groups. Future research on the political processes behind environmental policy should therefore consider the combined effect of time preferences and lobbying.

This study indicates that countries with powerful lobby groups and a short-sighted government are not likely to implement an optimal carbon tax. Lessons for more effective climate policy can be drawn from this result. International climate policy should include short-term incentives for higher carbon taxes if it wants to achieve higher carbon pricing in countries that are lacking behind in this respect. We suggest that one way to achieve efficient carbon pricing is to combine carbon taxes with trade policies. Trade policies, such as carbon tax border adjustments, would stimulate countries with inefficient low carbon taxes to adopt higher taxes, because trade policies have an immediate economic implication (van den Bergh et al. 2020).

Our model only considers a maximum of four periods. One could argue that a realistic evaluation of climate policy requires many more periods, since current climate policy has an impact on climate damages for multiple decades in the future. Nevertheless, the four-period model gives relevant first insights into the influence of lobbying on policymakers that take into account future periods in their emission tax policy decisions. Future research could include more periods for a more realistic comparison between a long-term and short-term perspective. Including more periods would especially be interesting in a model with increasing damage costs over time, which would enlarge the difference in policy decisions between politicians with a short- and long-term focus.

Further research in this direction could also include inconsistencies in time preferences and allow for differences in time preferences between policymakers and lobby groups.

Appendix

Equation (38), total Second-Period Profit (Π_2)

Total second-period profit in the two-period model "Second Period" section is:

$$\Pi_2 = \phi \Pi_G - \sum_0^{\phi} I_i + (1 - \phi)\Pi_B \tag{38}$$

The sum of investment cost is:

$$\sum_0^{\phi} I_i = 0.5\phi^2 = \frac{(2\tau - \tau^2)^2}{2} \tag{39}$$

Plugging (8), (9), (10) and (39) into (38) gives:

$$\begin{gathered} \Pi_2 = (2\tau - \tau^2) - \frac{(2\tau - \tau^2)^2}{2} + (1 - \tau)^4 \\ = 2\tau - \tau^2 - (2\tau^2 - 2\tau^3 + 0.5\tau^4) + (1 - 4\tau + 6\tau^2 - 4\tau^3 + \tau^4) \\ = 0.5\tau^4 - 2\tau^3 + 3\tau^2 - 2\tau + 1 \end{gathered}$$

Proof of Lemma 1

Lemma 1: Period 2 social welfare is maximised for $\tau = \eta$, i.e. when the tax is equal to the marginal damage costs (a Pigouvian tax).

To maximise the second period social welfare function in Eq. (13), we first take the derivative with respect to τ. The second period social welfare function is equal to the second-period profit plus tax income minus damage costs. The derivative of W_2 is the sum of the derivatives of those three parts:

$$\frac{\partial W_2}{\partial \tau} = \frac{\partial \Pi_2}{\partial \tau} + \frac{\partial T_2}{\partial \tau} - \frac{\partial D_2}{\partial \tau} \tag{40}$$

Equation (38) specifies the second-period profit. The derivative of Eq. (38) with respect to τ is:

$$\frac{\partial \Pi_2}{\partial \tau} = 2\tau^3 - 6\tau^2 + 6\tau - 2 = 2(\tau^3 - 3\tau^2 + 3\tau - 2) = -2(1 - \tau)^3 \tag{41}$$

The second-period tax income is τ times the second-period emission, which leads to the derivative in Eq. (43):

$$T_2 = \tau(1 - \phi)x_B = \tau(1 - \tau)^2 * 2(1 - \tau) = 2\tau(1 - \tau)^3 \tag{42}$$

$$\frac{\partial T_2}{\partial \tau} = 2(1-\tau)^3 - 6\tau(1-\tau)^2 \tag{43}$$

For the damage function, we insert Eq. (7) and (10) into (5), leading to the derivative in Eq. (45):

$$D_2 = 2\eta(1-\tau)^3 \tag{44}$$

$$\frac{\partial D_2}{\partial \tau} = -6\eta(1-\tau)^2 \tag{45}$$

Combining the above derivatives gives:

$$\begin{aligned}\tfrac{\partial W_2}{\partial \tau} &= -2(1-\tau)^3 + 2(1-\tau)^3 - 6\tau(1-\tau)^2 + 6\eta(1-\tau)^2 \\ &= 6(\eta-\tau)(1-\tau)^2\end{aligned} \tag{46}$$

This first order derivative is equal to zero for $\eta = \tau$ and/or $\tau = 1$. Analysing whether these points are a maximum requires taking the second order derivative:

$$\frac{\partial^2 W_2}{\partial \tau^2} = -6(1-\tau)(1-3\tau+2\eta) \tag{47}$$

The second order derivative of W_2 is negative for $\tau = \eta$ and $\tau \in [0,1)$, so there is a local maximum at $\tau = \eta$. The second order derivative is zero for $\tau = 1$. The first order derivative is negative just above and just below $\tau = 1$ for $\eta < 1$, which means that $\tau = 1$ is a saddle point. Period 2 social welfare is thus maximised when the tax is equal to the marginal damage costs (a Pigouvian tax).

Proof of Proposition 2

Proposition 2 states that $\frac{\partial C_I(\tau)}{\partial \tau} \le 0$ and $\frac{\partial C_E(\tau)}{\partial \tau} \ge 0$ for $\tau \in (0,1)$. We start with proofing that $\frac{\partial C_I(\tau)}{\partial \tau} \le 0$. In "Lobbying" section, we demonstrate that the derivative of the industry contribution function is the same as the derivative of the total brown firm profit. Since all firms are brown in the first period, total brown firm profit is the total profit in the economy (Π^{TOT}). The total profit is the discounted sum of the first- and second-period profits of all firms:

$$\frac{\partial C_I(\tau)}{\partial \tau} = \frac{\partial \Pi^{TOT}}{\partial \tau} = \frac{\partial \Pi_1}{\partial \tau} + \delta \frac{\partial \Pi_2}{\partial \tau} \tag{48}$$

Using $\frac{\partial \Pi_1}{\partial \tau} = \frac{\partial \Pi_B}{\partial \tau} = -2(1-\tau)$ and Eq. (41), $\frac{\partial \Pi_2}{\partial \tau} = -2(1-\tau)^3$, gives:

$$\frac{\partial C_I(\tau)}{\partial \tau} = -2(1-\tau) + \delta(-2(1-\tau)^3) = -2(1-\tau)(1+\delta(1-\tau)^2) \tag{49}$$

$-2(1-\tau) \le 0$ for $\tau \in (0,1)$ and $(1+\delta\left(1-\tau)^2\right) \ge 0$, therefore $\frac{\partial C_I(\tau)}{\partial \tau} \le 0$

The derivative of C_E with respect to τ is the derivative of the total damage function:

$$\frac{\partial C_E(\tau)}{\partial \tau} = -\frac{\partial D(x_B^1, x_B^2)}{\partial \tau} = -\frac{\partial(\eta x_B)}{\partial \tau} - \delta \frac{\partial(\eta(1-\phi)x_B)}{\partial \tau}$$

Plugging in Eqs. (7) and (10) gives:

$$\frac{\partial C_E(\tau)}{\partial \tau} = -\frac{\partial(2\eta(1-\tau)}{\partial \tau} - \delta\frac{\partial(2\eta(1-\tau)^3)}{\partial \tau}$$
$$= 2\eta - \delta(-6\eta(1-\tau)^2) = 2\eta(1 + 3\delta(1-\tau)^2) \geq 0$$

Proof of Lemma 2

Lemma 2: When $\delta = 1$, the equilibrium carbon tax rate is increasing in the degree of openness to lobby contributions $\left(\frac{\partial \tau}{\partial \lambda} > 0\right)$ for $\eta = \frac{1}{2}$ and decreasing in the degree of openness to lobby contributions $\left(\frac{\partial \tau}{\partial \lambda} < 0\right)$ for $\eta = \frac{1}{3}$ if $\lambda < 2$.

Proof:

To proof the first part of Lemma 2 ($\eta = \frac{1}{2}$), we start with the FOC in Eq. (22). We plug in $\delta = 1$ and for simplification we take $y = 1 - \tau$, which leads to the following rewritten FOC:

$$F(\eta, \lambda, \delta = 1, y) = \eta - 1 + y + \left(3(\eta - 1)y^2 + 3y^3\right) + \lambda\left(-y^3 - y + \eta + 3\eta y^2\right) = 0 \quad (50)$$

Using Eq. (50) we can express λ as a function of y and η :

$$\lambda = \frac{3y^3 + 3(\eta - 1)y^2 + y + \eta - 1}{y^3 - 3\eta y^2 + y - \eta} \quad (51)$$

If we take $\eta = \frac{1}{2}$ and let $z = y^{-1} = \frac{1}{1-\tau}$ then the FOC in Eq. (50) can be simplified to:

$$F(\lambda, y) = 1 - z^2 + (1 - z)^3 - \frac{4}{\lambda - 1} = 0 \quad (52)$$

To analyse the influence of the lobby group on the tax we want to calculate $\frac{\partial \tau}{\partial \lambda}$. Since $z = y^{-1} = \frac{1}{1-\tau}$, we can calculate $\frac{\partial \tau}{\partial \lambda}$ by first calculating $\frac{\partial z}{\partial \lambda}$. Using the implicit function theorem and the FOC in Eq. (52), we get:

$$\frac{\partial z}{\partial \lambda} = -\frac{\frac{\partial F(\lambda,z)}{\partial \lambda}}{\frac{\partial F(\lambda,z)}{\partial z}} = -\frac{F_\lambda}{F_z} = \frac{4(\lambda - 1)^{-2}}{2z + 3(1 - z)^2} > 0 \quad (53)$$

Recall: $z = y^{-1} = \frac{1}{1-\tau}$, therefore:

$$\frac{\partial z}{\partial \lambda} = \frac{\partial(y^{-1})}{\partial \lambda} = -y^{-2}\frac{\partial y}{\partial \lambda} = -\frac{1}{(1-\tau)^2}\frac{\partial(1-\tau)}{\partial \lambda} = \frac{1}{(1-\tau)^2}\frac{\partial \tau}{\partial \lambda} = \frac{4(\lambda-1)^{-2}}{2y+3(1-y)^2} > 0$$
$$\Rightarrow \frac{\partial \tau}{\partial \lambda} = 4\frac{(1-\tau)^2(\lambda-1)^{-2}}{2z+3(1-z)^2} > 0$$

To proof the second part of Lemma 2, we plug $\eta = \frac{1}{3}$ in Eq. (51), rewriting gives:

$$\lambda = 2 + \frac{y^3 - y}{y^3 + y - y^2 - \frac{1}{3}} \quad (54)$$

$$\lambda - 2 = \frac{y^3 - y}{y^3 + y - y^2 - \frac{1}{3}} \quad (55)$$

If $\lambda = 2$, and $\eta = \frac{1}{3}$ then the left hand-side of the above equation is equal to zero. The right hand-side is equal to zero only if $y = 0$ or $y = 1$. Which leads to the conclusion that $\lambda \neq 2$.

With $\lambda \neq 2$, we can take the inverse of Eq. (55):

$$(\lambda - 2)^{-1} = \frac{y^3 + y - y^2 - \frac{1}{3}}{y^3 - y} \tag{56}$$

Which leads to the following FOC:

$$F(y, \lambda) = \left(y^3 - y\right)(\lambda - 2)^{-1} - \left(y^3 + y - y^2 - \frac{1}{3}\right) \tag{57}$$

Using the implicit function theorem, we can calculate $\frac{\partial y}{\partial \lambda}$:

$$\frac{\partial F}{\partial \lambda} = -(\lambda - 2)^{-2}\left(y^3 - y\right) = -(\lambda - 2)^{-2}y\left(y^2 - 1\right) \tag{58}$$

$$\begin{aligned}\frac{\partial F}{\partial y} &= (\lambda - 2)^{-1}\left(3y^2 - 1\right) - \left(3y^2 + 1 - 2y\right) \\ &= \left((\lambda - 2)^{-1} - 1\right)\left(3y^2 - 1\right) + 2(y - 1)\end{aligned} \tag{59}$$

$$\frac{\partial y}{\partial \lambda} = -\frac{\frac{\partial F}{\partial \lambda}}{\frac{\partial F}{\partial y}} = \frac{(\lambda - 2)^{-2}y\left(y^2 - 1\right)}{\left((\lambda - 2)^{-1} - 1\right)\left(3y^2 - 1\right) + 2(y - 1)} \tag{60}$$

To proof that $\frac{\partial \tau}{\partial \lambda} < 0$, we need to proof that $\frac{\partial y}{\partial \lambda} > 0$, because $\frac{\partial \tau}{\partial \lambda} = \frac{\partial (1-y)}{\partial \lambda} = -\frac{\partial y}{\partial \lambda}$. We have $y \in (0, 1)$ therefore, $(\lambda - 2)^{-2}y\left(y^2 - 1\right) \leq 0$. Since the numerator in Eq. (60) is negative, the denominator, Eq. (59), should also be negative to make $\frac{\partial y}{\partial \lambda} > 0$. We consider the case for $\lambda \leq 2$, $\left((\lambda - 2)^{-1} - 1\right) < 0$ and $2(y - 1) < 0$. We thus need $\left(3y^2 - 1\right) > 0$, which is for $y > \sqrt{1/3} \approx 0.57735$ or $\tau < 1 - \sqrt{1/3} \approx 0.42265$

$$\frac{\partial F}{\partial y} = \underbrace{((\lambda - 2)^{-1} - 1)}_{<0} \underbrace{(3y^2 - 1)}_{>0 \text{ if } y>0.57735} + \underbrace{2(y - 1)}_{<0}$$

We have proved that $\frac{\partial \tau}{\partial \lambda} = -\frac{\partial y}{\partial \lambda} \leq 0$ if $y > 0.57735$ or $\tau < 0.42265$ and $\lambda \leq 2$.

Proof of Lemma 3

Lemma 3: For $\lambda = \frac{1}{2}$, the equilibrium tax is increasing with the discount factor $\left(\frac{\partial \tau}{\partial \delta} > 0\right)$ for both $\eta = \frac{1}{2}$ and $\eta = \frac{1}{3}$. The equilibrium tax is at its maximum ($\tau = 1$) for $\eta \geq \frac{2}{3}$.

Proof:

First, we plug in $\eta = \frac{1}{2}$ in the FOC in Eq. (25):

$$F\left(\eta = \frac{1}{2}, \delta, y\right) = 5\delta y^3 - \frac{3}{2}\delta y^2 + y - \frac{1}{2} = 0 \tag{61}$$

When $\delta = 0$, $y = \frac{1}{2}$ and $\tau = \frac{1}{2}$. When $\delta = 1$, $5y^3 - \frac{3}{2}y^2 + y - \frac{1}{2} = 0$, solving this nonlinear equation gives $y \approx 0,409$ and $\tau = 1 - y = 0.591$. This gives us the range for $\tau \in [0.5, 0.591]$ or $y \in [0.409, 0.5]$.

Based on the range of y obtained above, we get:

$$\frac{\partial F}{\partial y} = 15\delta y^2 - 3\delta y + 1 = 3\delta y(5y - 1) + 1 > 0 \tag{62}$$

$$\frac{\partial F}{\partial \delta} = 5y^3 - \frac{3}{2}y^2 = y^2\left(5y - \frac{3}{2}\right) = \delta^{-1}\left(\frac{1}{2} - y\right) > 0 \tag{63}$$

Using the implicit function theory, we get:

$$\frac{\partial y}{\partial \delta} = -\frac{\frac{\partial F}{\partial \delta}}{\frac{\partial F}{\partial y}} < 0 \tag{64}$$

Since $\frac{\partial y}{\partial \delta} = -\frac{\partial \tau}{\partial \delta}$, we can conclude that $\frac{\partial \tau}{\partial \delta} > 0$.
Second, we plug in $\eta = \frac{1}{3}$ in the FOC in Eq. (25):

$$F\left(\eta = \frac{1}{3}, \delta, y\right) = 5\delta y^3 - 3\delta y^2 + y - 1 = 0 \tag{65}$$

When $\delta = 0, y = 1, \tau = 0$. When $\delta = 1$, solving $5y^3 - 3y^2 + y - 1 = 0$ gives $y \approx 0.713$, and $\tau = 1 - y = 0.287$. This gives us the range for $\tau \in [0, 0.287]$ or $y \in [0.713, 1]$.
We can again check the sign of $\frac{\partial \tau}{\partial \delta}$ by using the implicit function theorem:

$$\frac{\partial F}{\partial y} = 15\delta y^2 - 6\delta y + 1 = 3\delta y(5y - 2) + 1 > 0 \tag{66}$$

$$\frac{\partial F}{\partial \delta} = 5y^3 - 3y^2 = \frac{1 - y}{\delta} > 0 \tag{67}$$

$$\frac{\partial y}{\partial \delta} = -\frac{\frac{\partial F}{\partial \delta}}{\frac{\partial F}{\partial y}} < 0 \Rightarrow \frac{\partial \tau}{\partial \delta} > 0 \tag{68}$$

$$\frac{\partial \tau}{\partial \delta} > 0 \tag{69}$$

Proof of Lemma 4

Lemma 4: For $\lambda = 1$ and $\eta = \frac{1}{3}$ the carbon tax is increasing in $\delta\left(\frac{\partial \tau}{\partial \delta} > 0\right)$ for *$\delta > \frac{1}{3}$ and zero for $\delta \leq \frac{1}{3}$. The carbon tax is zero for $\eta = \frac{1}{4}$ and $\lambda = 1$, irrespective of the discount factor.*
Plugging $\eta = \frac{1}{3}$ and $y = 1 - \tau$ into the FOC in Eq. (26) gives:

$$F\left(\eta = \frac{1}{3}, \delta, y\right) = -\frac{1}{3} + 2\delta y^3 - \delta y^2 = 0 \tag{70}$$

When $\delta = 1$, solving $-\frac{1}{3} + 2y^3 - y^2 = 0$ gives $y = 0.77645$ and $\tau = 1 - y = 0.22355$, which is the upper bound for carbon tax when $\eta = \frac{1}{3}$. With $\delta = 0$, the above equation does not hold ($= -\frac{1}{3} + 2\delta y^3 - \delta y^2 \neq 0$). So, no carbon tax will be charged with $\eta = \frac{1}{3}$ and $\delta = 0$ because future does not matter.

We know that that $y = 0.77645$ is the lower bound of y. The term $2y^3 - y^2$ is positive with $y > 0.5$ and increases in y. Therefore, the maximum of $2y^3 - y^2$ occurs when $y = 1$. Then $2y^3 - y^2 = 1$ and $\delta\left(2y^3 - y^2\right) = \delta$. Plugging in $y = 1$ in Eq. (70) gives:

$$F\left(\eta = \frac{1}{3}, \delta, y = 1\right) = -\frac{1}{3} + \delta = 0 \tag{71}$$

Solving Eq. (71) gives $\delta = \frac{1}{3}$. At $y = 1$, $\tau = 0$, thus we can conclude that when $\eta = \frac{1}{3}$ and $\delta = \frac{1}{3}$ there will be no carbon tax charged as $y = 1$ and $\tau = 1 - y = 0$. The range the carbon tax can take is $[0, 0.2235]$ for $\eta = \frac{1}{3}$. Using the implicit function theorem, we get:

$$\frac{\partial y}{\partial \delta} = -\frac{\frac{\partial F}{\partial \delta}}{\frac{\partial F}{\partial y}} = -\frac{2y^3 - y^2}{6\delta y^2 - 2\delta y} = -\frac{y}{2\delta}\frac{2y - 1}{3y - 1} < 0 \tag{72}$$

$$\frac{\partial \tau}{\partial \delta} > 0 with \tau \leq 0.2235 \tag{73}$$

Plugging η = $\frac{1}{4}$ and $y = 1 - \tau$ into the FOC in Eq. (26) gives:

$$F\left(\eta = \frac{1}{4}, \delta, y\right) = -\frac{1}{2} + 2\delta y^3 - \frac{3}{2}\delta y^2 = 0 \tag{74}$$

This equation only holds when $\delta = 1$ and $y = 1$, which implies that $\tau = 0$ for $\delta = 1$ and $\eta = \frac{1}{4}$. For $\delta < 1$, the equation is not well defined. To conclude, with $\lambda = 1$ and $\eta = \frac{1}{4}$, the carbon tax will always be zero because the marginal damage is small, and the lobby power is not strong enough to make the carbon tax happen.

Social welfare maximising third-period tax

In "Social Welfare Maximising Policymaker" section, we claim that the social welfare maximising third-period tax is $\tau = \eta$. To get this tax we have to find the tax τ that maximises $SW_3 = W_3 + \delta W_4$. For $2\tau_3 - \tau_3^2 \geq \phi$, $W_3 = W_4$ which is specified in (29). The derivative of (29) with respect to τ is:

$$\frac{\partial SW_3}{\partial \tau} = (1 + \delta)\frac{\partial W_3}{\partial \tau} \tag{75}$$

Combining (26) with (7), (8) and (9) gives:

$$W_3 = \phi + (1 - \phi)(1 - \tau_3)^2 + (1 - \phi)(\tau_3 - \eta)2(1 - \tau_3) \tag{76}$$

$$\begin{aligned} \frac{\partial W_3}{\partial \tau_3} &= -2(1 - \phi)\left(1 - \tau_3\right) + 2(1 - \phi)\left(1 - \tau_3\right) - 2(1 - \phi)\left(\tau_3 - \eta\right) \\ &= -2(1 - \phi)(\tau_3 - \eta) \end{aligned} \tag{77}$$

$$\frac{\partial SW_3}{\partial \tau_3} = (1 + \delta)(-2(1 - \phi)(\tau_3 - \eta)) \tag{78}$$

$$\frac{\partial^2 SW_3}{\partial {\tau_3}^2} = -2(1+\delta)(1-\phi) \tag{79}$$

The first order derivative of SW_3 is zero for $\tau = \eta$ and the second order derivative is negative for $\phi > 0$, thus we have a maximum at $\tau = \eta$.

For $2\tau_3 - \tau_3^2 > \phi$, SW_3 is specified in Eq. (30), the derivative of (30) is:

$$\frac{\partial SW_3}{\partial \tau_3} = \frac{\partial W_3}{\partial \tau_3} + \delta \frac{\partial W_4}{\partial \tau_3}$$

For this, we need to have the derivative of W_4 with respect to τ. Combining (27) with (7), (8) and (9), and plugging in $\theta = 2\tau_3 - \tau_3^2$ and $I_4 = \frac{1}{2}((2\tau_3 - \tau_3^2)^2 - \phi^2)$ gives:

$$W_4 = 2\tau_3 - \tau_3^2 + (1-\tau_3)^4 - \frac{1}{2}((2\tau_3 - \tau_3^2)^2 - \phi^2) + 2(1-\tau_3)^3(\tau_3 - \eta) \tag{80}$$

$$\begin{aligned} \tfrac{\partial W_4}{\partial \tau_3} &= 2 - 2\tau_3 - 4(1-\tau_3)^3 - (2-2\tau_3)(2\tau_3 - \tau_3^2) \\ &\quad -6(1-\tau_3)^2(\tau_3 - \eta) + 2(1-\tau_3)^3 \\ &= -6(1-\tau_3)^2(\tau_3 - \eta) \end{aligned} \tag{81}$$

$$\frac{\partial SW_3}{\partial \tau_3} = \frac{\partial W_3}{\partial \tau_3} + \delta \frac{\partial W_4}{\partial \tau_3} = -2(1-\phi)(\tau_3 - \eta) - \delta 6(1-\tau_3)^2(\tau_3 - \eta) \tag{82}$$

$$\frac{\partial^2 SW_3}{\partial {\tau_3}^2} = -2(1-\phi) + \delta(12(1-\tau_3)(\tau_3 - \eta) - 6(1-\tau_3)^2) \tag{83}$$

$\frac{\partial SW_3}{\partial \tau_3} = 0$ for $\eta = \tau$ and $\frac{\partial^2 SW_3}{\partial {\tau_3}^2}(\tau = \eta) = -2(1-\phi) - 6(1-\eta)^2 < 0$, thus this function is also maximised for $\tau = \eta$.

Proof of Eq. (36), Threshold Investment Costs I_l^*

Equation (36) follows from the inequality in Eq. (84):

$$(1+\delta+\delta^2)\Pi_G - I_i \geq \Pi_B(\tau_1) + \delta \Pi_B(\tau_3) + \delta^2(\Pi_G - I_i) \tag{84}$$

Plugging in $\Pi_G = 1$, $\Pi_B(\tau_1) = (1-\tau_1)^2$ and $\Pi_B(\tau_3) = (1-\eta)^2$ and solving the inequality gives the following:

$$\begin{aligned} 1 + \delta + \delta^2 - I_i &\geq (1-\tau_1)^2 + \delta(1-\eta)^2 + \delta^2(1 - I_i) \\ 1 - \delta + \delta^2 - I_i &\geq 1 - 2\tau_1 + \tau_1^2 + \delta(1 - 2\eta + \eta^2) + \delta^2(1 - I_i) \\ I_i &\leq 2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2) + \delta^2 I_i \\ I_i &\leq \tfrac{2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2)}{1-\delta^2} \quad \text{for} \quad \delta < 1 \end{aligned} \tag{85}$$

Which gives the threshold in Eq. (36):

$$I_l^* = \frac{2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2)}{1-\delta^2} \text{ for } \delta < 1$$

Proof of Eq. (37), First-Period Investment in Four-Period Model

Equation (37), states that the fraction of firms that invest in green technology in the first period is:

$$\begin{gathered}\phi = \min\left[1, \max\left[2\tau_1 - \tau_1^2 + \left(\delta + \delta^2\right)\left(2\eta - \eta^2\right), 2\eta - \eta^2\right]\right] \\ \text{if } \ 2\tau_1 - \tau_1^2 \geq (1 - \delta - \delta^2)(2\eta - \eta^2)\end{gathered} \tag{86}$$

And

$$\begin{gathered}\phi = \min[\tfrac{2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2)}{1-\delta^2}, 2\eta - \eta^2] \\ \text{if } \ 2\tau_1 - \tau_1^2 < (1 - \delta - \delta^2)(2\eta - \eta^2)\end{gathered} \tag{87}$$

We show in the paragraphs before Eq. (37) that firms who will invest in the fourth period ($I_G \leq 2\eta - \eta^2$) will also invest in the second period if:

$$I_G \leq \frac{2\tau_1 - \tau_1^2 + \delta\left(2\eta - \eta^2\right)}{1 - \delta^2} \text{ if } \ \delta < 1 \tag{88}$$

All firms in this group will invest in the second period if:

$$\begin{gathered}2\eta - \eta^2 \leq \tfrac{2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2)}{1-\delta^2} \\ (1 - \delta^2)(2\eta - \eta^2) \leq 2\tau_1 - \tau_1^2 + \delta(2\eta - \eta^2) \\ (1 - \delta - \delta^2)(2\eta - \eta^2) \leq 2\tau_1 - \tau_1^2\end{gathered} \tag{89}$$

Firms with higher investment costs ($I_G > 2\eta - \eta^2$) will invest in the second period if:

$$I_G \leq 2\tau_1 - \tau_1^2 + (\delta + \delta^2)(2\eta - \eta^2) \tag{90}$$

None of the firms in this group will invest if:

$$\begin{gathered}2\eta - \eta^2 > 2\tau_1 - \tau_1^2 + (\delta + \delta^2)(2\eta - \eta^2) \\ (1 - \delta - \delta^2)(2\eta - \eta^2) > 2\tau_1 - \tau_1^2\end{gathered} \tag{91}$$

The indifferent firm is thus in the groups with lower investment costs if $2\tau_1 - \tau_1^2 < (1 - \delta - \delta^2)(2\eta - \eta^2)$ and in the group with higher investment costs if $2\tau - \tau^2 \geq (1 - \delta - \delta^2)(2\eta - \eta^2)$.

Increasing Marginal Damage

The social welfare maximising equilibrium tax in the two-period model stays constant when the discount factor increases. The literature on the social cost of carbon however shows that the social welfare maximising tax increases with the discount factor (e.g. Tol 2018). The reason that the optimal tax stays constant in our model is that we use a damage function with constant marginal damage. Constant marginal damage simplifies the analysis, but in reality, marginal damages are increasing with carbon emissions. Integrated Assessment Models of climate and the economy, such as Nordhaus' DICE model, therefore use damage functions with increasing marginal damage (Nordhaus 2017). In this section, we analyse the impact of a damage function with increasing marginal damage and cumulative emissions on the equilibrium tax in our two-period model. The damage function we use is:

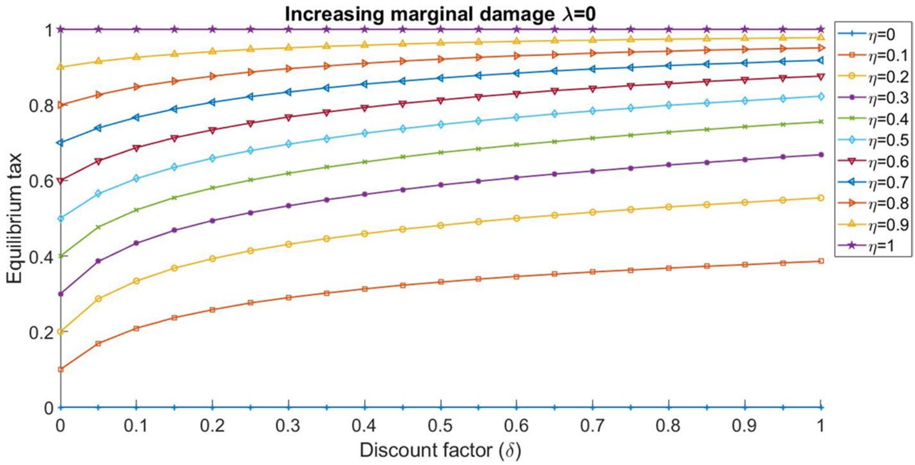

Fig. 10 Equilibrium tax as a function of the discount factor δ, λ = 0

$$D_t = \eta(\sum_{j=1}^{t} e_j)^2 \tag{92}$$

The damage in each period is a quadratic function of the current emission and the emission in previous periods. First-period damage is thus equal to $D_1 = \eta e_1^2 = \eta x_B^2$ and second-period damage is $D_2 = \eta(e_1 + e_2)^2 = \eta(x_B + (1-\phi)x_B)^2$. The total discounted sum of the damage is:

$$D = \sum_{j=1}^{2} \delta^{j-1} D_j = \eta x_{B1}^2 + \delta\eta(x_{B1} + (1-\phi)x_{B2})^2 \tag{93}$$

$$\frac{\partial D}{\partial x_{B1}} = 2\eta x_{B1} + 2\delta\eta(x_{B1} + (1-\phi)x_{B2}) > 0 \tag{94}$$

The discounted marginal damage increases with δ, so the optimal social welfare maximising tax should also increase with δ. Figure 10 indeed shows that the optimal social welfare maximising tax increases with the discount factor.

The changing damage function has an influence on the lobby contribution. The contribution functions are still specified as follows:

$$C_I(\tau) = \max[0, \Pi^{TOT}(\tau) - \overline{W_I}] \tag{95}$$

$$C_E(\tau) = \max[0, \overline{W_E} - D(e_1, e_2)] \tag{96}$$

The environmental lobby contribution depends on the damage. Another damage function thus influences the environmental lobby contribution. In "Lobbying" section, we assume that the industry lobby group decreases the tax to zero when there is no environmental lobbying. This assumption is less realistic with an increasing damage function, because the damage becomes very large when $\tau = 0$ and the loss of social welfare can only

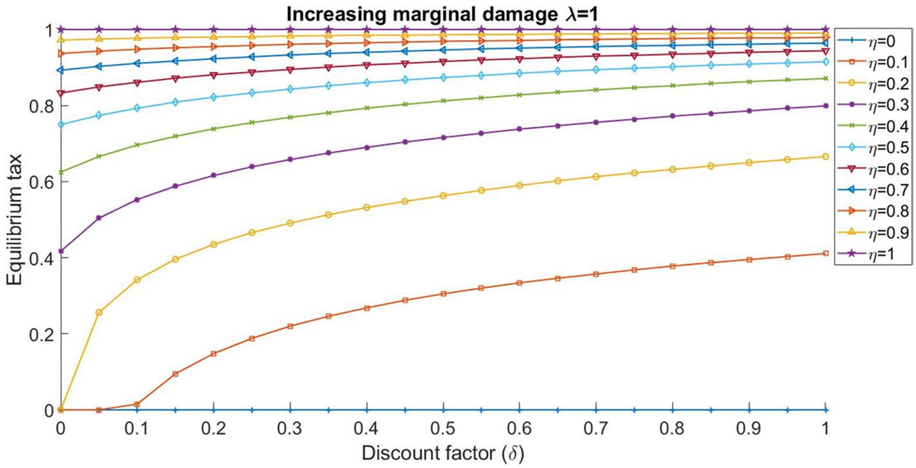

Fig. 11 Equilibrium tax as a function of the discount factor δ, λ = 1

be compensated by industry lobby contribution when λ is very large. The equilibrium outcome with only industry lobby groups depends on η, δ and λ, and will be larger than zero in most cases. For simplicity, we can however assume that the environmental lobby group is naive and takes the worst-case scenario of ($\tau = 0$) as their baseline contribution $\overline{W_E}$:

$$\overline{W_E} = D(\tau = 0) = \eta 2^2 + \delta\eta 4^2 = 4\eta(1 + 4\delta) \tag{97}$$

For the industry lobby, we assume that they take the profit at $\tau = 1$ as the baseline contribution, which is the same as in "Lobbying" section. Figure 11 plots the equilibrium tax as a function of δ with $\lambda = 1$ using these lobby contributions. Compared to Fig. 10, this figure shows that, with lobbying, the equilibrium tax decreases for small η and increases for larger η. Furthermore, the figure shows that the deviation from the optimal tax is larger when δ is smaller. These conclusions are the same as with the constant marginal damage in "Discount Factor" section. The impact of lobbying on the equilibrium tax is, however, less extreme. The figure also shows that lobbying already increases the equilibrium tax for relatively small values of η. This is both influenced by the fact that the chosen environmental lobby contribution schedule gives relatively big environmental lobby contributions, and the damage function increases relatively fast with η. This exploratory analysis, however, indicates that the direction of the lobby influence stays the same when increasing marginal damage functions are used. It can therefore be justified to focus on constant marginal damage costs for the purpose of this paper. Future research could try to analyse the influence of lobbying and time preferences on emission tax policies using different damage functions and baseline lobby contributions. The impact of damage functions becomes especially interesting when studying the impact of lobbying on long-term policy, using more time periods. Such an analysis, however, exceeds the scope of this paper.

Author Contribution Teun Schrieks took the lead in the study and conducted the analyses. The first draft of the manuscript was written by Teun Schrieks, and all authors commented on previous versions of the manuscript. Julia Swart and Wouter Botzen contributed to the writing and Fujin Zhou contributed with mathematical derivations of the model. All authors read and approved the final manuscript.

Declarations

Competing Interests The authors have no conflicts of interest to declare that are relevant to the content of this article.

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