Journal of Science Education and Technology (2023) 32:759–772
https://doi.org/10.1007/s10956-023-10059-8

Designing and Evaluating Relativity Lab: A Simulation Environment for Special Relativity Education at the Secondary Level

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Accepted: 1 June 2023 / Published online: 17 June 2023

Abstract

This article describes the design and evaluation of a simulation environment for special relativity (SR) education at the secondary level. In recent years, SR has become increasingly popular in secondary school curricula worldwide. Because the key concepts in SR are very remote from everyday experience, they are difficult for students to learn. Computer simulations provide a promising approach to explore these abstract concepts in a simplified and idealized virtual environment. The currently available simulation tools for SR, however, are limited in terms of usability and flexibility. We report on the development of an online simulation environment, named Relativity Lab. In Relativity Lab, students can construct simulations themselves and freely select the inertial frame of reference from which the simulation is rendered. We performed a small-scale evaluation ($N = 16$) in which Relativity Lab was used in inquiry-learning activities. Results indicate that students found Relativity Lab a helpful tool for visualizing relative motion and relativistic light propagation. Moreover, the inquiry-learning activities helped students to recognize discrepancies between their prediction and the outcome of a simulation. We propose improvements to the current task design by providing stricter instructions with regard to constructing the simulation and switching between inertial frames.

Keywords Special relativity · Computer simulations · Secondary education

Introduction

Education in modern physics has become more popular in secondary education curricula around the globe (Choudhary et al., 2019; Kersting & Blair, 2021). The conceptual and epistemological leap between classical and modern physics illuminates scientific reasoning and attracts students' interest in physics (Villani & Arruda, 1998; Henriksen et al., 2014). Special Relativity (SR), representing a radical change in our understanding of space and time, presents a suitable topic to introduce secondary school students to the exciting world of modern physics (Dimitriadi & Halkia, 2012; Kamphorst et al., 2021).

One of the main learning objectives of SR education in secondary education is to foster understanding of Einstein's relational notion of space and time (Levrini & diSessa, 2008; Alstein et al., 2021). This relation between space and time results in relativistic effects, such as time dilation and length contraction. These phenomena are very remote from everyday experience because of two reasons. First, relativistic effects cannot be directly observed because they require relative velocities approaching the speed of light. Second, kinematic relativistic effects only become apparent when comparing measurements made in different inertial frames of reference. As a consequence, it is practically impossible for students to inquire into the consequences of SR by means of lab experiments.

Rather, the conceptual nature of SR can be explored through the imagination (Stephens & Clement, 2010; Hadzigeorgiou, 2016). By using carefully designed thought experiments (TEs), students are able to inquire into abstract concepts in simplified "what if" scenarios that relate to every day experience (Gilbert & Reiner, 2000; Galili, 2009; Asikainen & Hirvonen, 2014). More specifically, TEs provide a tool for hypothetical modeling, in which students explore the consequences of a hypothesis in an idealized setting (Reiner, 1998; Reiner & Burko, 2003; Velentzas & Halkia, 2013). This learning activity employs students'

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creative reasoning as a pedagogical tool to elicit existing conceptions (Kamphorst et al., 2019).

Within the context of SR, TEs can be used to derive kinematic relativistic effects from the theoretical basis of SR. These type of TEs require students to construct mental imagery of relative motion in different inertial frames (Monaghan & Clement, 2000). Prior research has shown that students find this very difficult, especially in secondary education (Villani & Pacca, 1987; Scherr et al., 2001; Alstein et al., 2021; Boudreaux et al., 2023). This is problematic, because an incorrectly or inconsequentially performed TE may lead to misguided conclusions about the validity of the hypothesis.

Computer simulations provide a promising solution to this problem, as they allow students to perform and evaluate TEs in a controlled virtual environment (Rutten et al., 2012; Mulder et al., 2015). Using computer simulations, students are able to perform guided inquiry activities in which they formulate a hypothesis, perform a simulation to test the hypothesis, and evaluate the hypothesis on the basis of the outcome of the simulation (Fan et al., 2018). By interacting with the simulation, students gradually infer the scientific model on which the simulation is based. This practice can serve as a basis for critical reflection upon students' existing conceptual model (de Jong & van Joolingen, 1998; Monaghan & Clement, 1999).

In this study, we investigate how simulation-based inquiry learning can be used to support students in performing and evaluating relativistic TEs. We report on the development of an online interactive simulation tool, *Relativity Lab*[1], in which students can reconstruct relativistic TEs themselves and observe the simulation from different inertial frames. We hypothesize that the iterative process of confirming or rejecting students' predicted TE outcomes leads students to recognize a discrepancy between their existing conceptual model and the scientific model of SR.

In this article, we describe the design process of *Relativity Lab* and present the results of a qualitative evaluation. Specifically, we address the following research question: "How can a simulation environment support secondary school students in performing and evaluating relativistic TEs?"

Background

This section describes the analyses that informed our design process. In the first subsection, we describe common learning difficulties in SR education. In the second subsection, we describe how these difficulties may be overcome through simulation-based inquiry learning. In the third subsection, we present the design objectives for *relativity lab*.

Learning Difficulties in SR Education

The conceptualization of inertial frames is key to describing relative motion. It is widely reported, however, that secondary school students experience a variety of difficulties in understanding inertial frames (Dimitriadi & Halkia, 2012; Kizilcik et al., 2017; Alstein et al., 2021). As these difficulties may obscure meaningful learning of SR, it is helpful to examine these difficulties more closely. In the following discussion, we focus on three main difficulties in conceptualizing inertial frames.

First, students tend to treat inertial frames as being fixed to concrete objects, in the sense that they are localized and physically extended to the dimensions of that object (Panse et al., 1994). In this view, students consider events to be either inside or outside the inertial frame's supposed physical extent (Tanel, 2014). Moreover, students tend to regard particular events as belonging to a particular inertial frame, being either the inertial frame in which the event "took place" or the inertial frame in which the event was observed (Panse et al., 1994). In the context of SR, the latter conception results in a conflict with the relativity of simultaneity.

Second, it is widely reported that students do not fully appreciate the profound consequences of the equivalence of inertial frames. In fact, many students intuitively hold on to a classical view of motion in which velocity is an intrinsic property of a body (Dimitriadi & Halkia, 2012) or even a notion of motion in which the "true motion" is determined with respect to its immediately surrounding bodies (Pietrocola & Zylbersztajn, 1999). This leads students to regard motion as real in a "stationary" inertial frame and apparent in a "moving" inertial frame (Panse et al., 1994). While students often acknowledge that measurements made in different inertial frames yield different quantities, they do not always recognize that the operational definitions of measurement are valid in all inertial frames (Boudreaux et al., 2023). Although the use of a preferred frame may be functional in everyday life, it results in the notion that relativistic effects are unilateral phenomena, appearing only in a "moving" inertial frame (Selçuk, 2011; Aslanides & Savage, 2013).

Third, students do not always clearly delineate between Einstein's procedure of coordinate measurement, as described in his 1905 article (Einstein, 1905), and personal visual observation (Stein et al., 2023). The role of the observer may be interpreted too literally as being dependent on human visual or sensory observation (Hughes & Kersting, 2021). In particular, students often associate the time of an event with the time at which an observer receives a light signal from the event (Scherr et al., 2001; Scherr et al., 2002). This rudimentary procedure of time measurement

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[1] *Relativity Lab* will be made publicly available after completion of the research project.

results in the notion that the position of the observer determines the supposed time of an event. Students who hold this notion tend to regard two events to be simultaneous only if an observer simultaneously receives two light signals from these events (Scherr et al., 2001; de Hosson et al., 2010; Dimitriadi & Halkia, 2012).

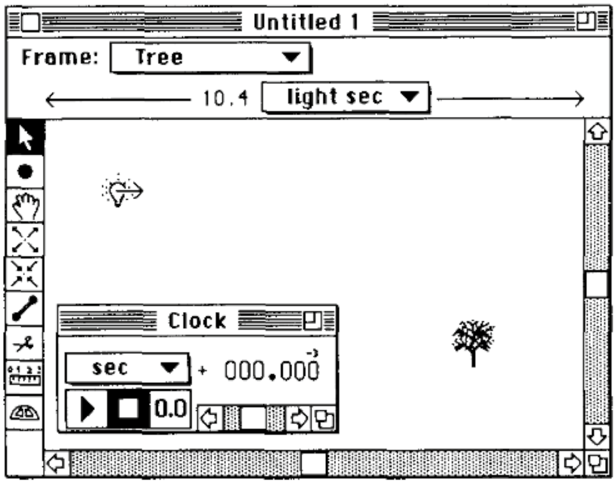
Simulation Tools in SR Education

Computer simulations provide a powerful tool to explore scientific models (de Jong & van Joolingen, 1998; Rutten et al., 2012; Banda & Nzabahimana, 2021). Typically, computer simulations are constructed in a simplified virtual environment, isolating those aspects of the simulation that require the student's attention (Adams et al., 2008). By controlling relevant parameters and observing their effect on the outcome of the simulation, students playfully inquire into the scientific model on which the simulation is based (Dalgarno et al., 2014).

By combining simulation tools with guided inquiry activities, students can be guided to test their existing conceptions in a virtual environment (Rutten et al., 2015; Mulder et al., 2015). This process requires carefully designed simulation-based inquiry cycles (Fan et al., 2018). A typical simulation-based inquiry cycle includes three phases: (1) students formulate a prediction, (2) students construct a simulation of the situation, and (3) students evaluate their prediction on the basis of the outcome of the simulation. The predictive phase of the cycle is meant to make students' existing conceptions about relative motion explicit and to promote active engagement, while the evaluate phase of the cycle is aimed at critical reflection upon students' predictions.

A number of simulation tools for SR education have already been reported on in the literature. The first of these was Horwitz and Barowy's *RelLab* (Horwitz et al., 1994; Horwitz & Barowy, 1994), which allowed students to construct simulations by placing objects on a two-dimensional grid and assigning relative velocities to each object. Students were given the option to select the inertial frame from which the simulation is rendered. A screenshot of the main interface of *RelLab* is shown in Fig. 1.

RelLab was implemented in secondary education through an instructional approach similar to the inquiry cycle described above. Students collaborated in groups to predict the result of a TE and to confirm their prediction by means of a simulation in *RelLab*. The authors conclude that this approach is helpful to make students' deeply held assumptions about relative motion more explicit and to develop a firm, qualitative intuition about SR. Moreover, the authors present evidence that the students enjoyed the freedom of designing their own simulations and that the problem-solving approach motivated them to learn about SR.



Frame: Tree

Fig. 1 A screenshot of the main interface of *RelLab*. In this simulation, a light bulb is moving relative to a tree (Horwitz & Barowy, 1994)

In subsequent studies, *RelLab* was used to study students' use of mental imagery as opposed to mathematical formalism in learning about relative motion (Monaghan & Clement, 1999; Monaghan & Clement, 2000). Based on their results, the authors hypothesize that the cognitive dissonance between students' incorrect predictions and the observed simulations presented students with new insights about relative motion. Moreover, it was shown that the memory of these visualisations provided students with a "framework for visualisation" during offline assignments.

In conclusion, the approach introduced by Horwitz and Barowy provided a powerful tool to engage students in designing and constructing TEs and to elicit and reflect on their existing conceptual model. Unfortunately, *RelLab* has run out of support and does not run on modern computers.

More recently, a number of online applets have been developed in which pre-programmed simulations are visualized (Belloni et al., 2004; Moraru et al., 2011; Kashnikov et al., 2019). While they are easily accessible on computers and mobile devices, the user's freedom in selecting inertial frames and controlling relative velocities is restricted. Moreover, as these applets lack the possibility to create simulations from scratch, their usability in simulation-based inquiry tasks is limited.

In addition to online applets, two realistic three-dimensional virtual environments have recently been developed: *Real Time Relativity* (McGrath et al., 2010) and the *OpenRelativity* game engine (Sherin et al., 2016), from which the serious game *A Slower Speed of Light* was built (Kortemeyer, 2019). While these virtual environments have been shown to enhance students' motivation and attitude toward SR (Croxton & Kortemeyer, 2018), they lack the the crucial feature to switch between inertial frames.

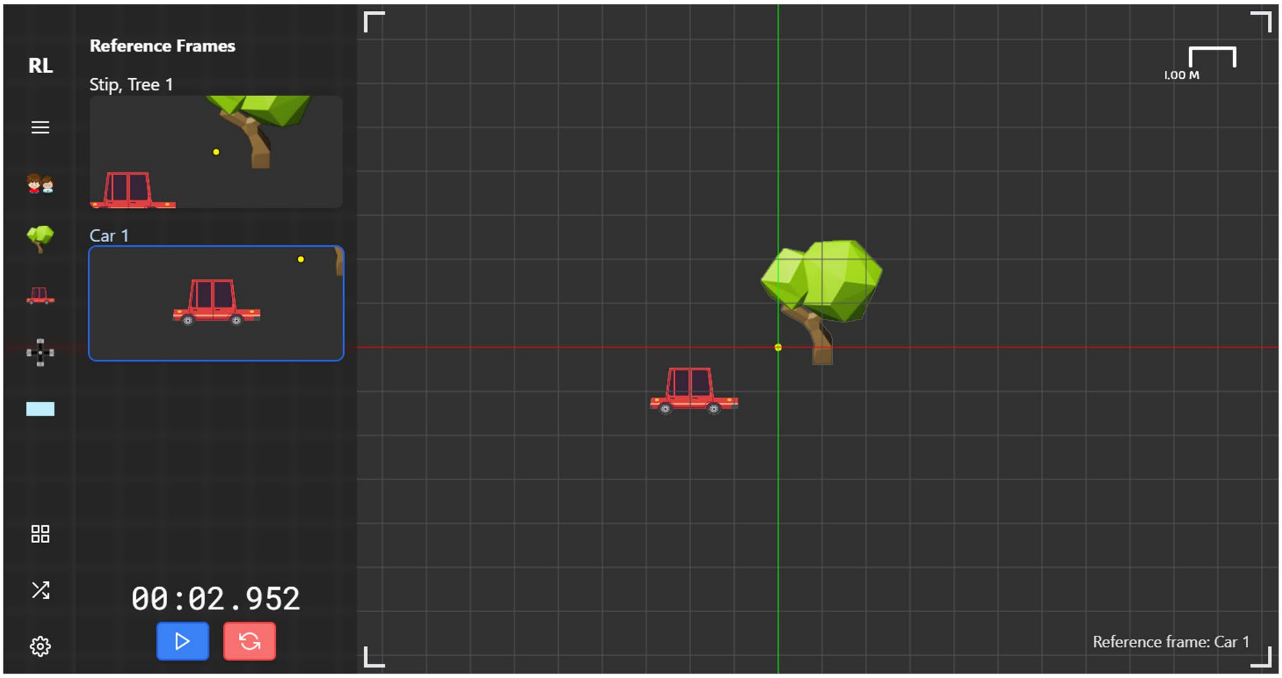


Fig. 2 A screenshot of the main interface of *Relativity Lab*. In this paused simulation, a car is moving relative to a tree

Design Objectives

From the analysis of learning difficulties, we conclude that it is difficult for secondary school students to imagine relativistic TEs, particularly when the propagation of light is to be observed in different inertial frames. While simulation tools have the potential of visualizing relativistic TEs from multiple inertial frames, the currently available simulation tools are limited in terms of flexibility and controllability. Supporting inquiry-based learning activities requires a simulation tool that allows students to create simulations themselves and to freely select inertial frames. Based on these findings, we specify three design objectives for *Relativity Lab*:

1. *Relativity Lab* provides a platform for simulating relativistic thought experiments and allows students to observe the simulation from different inertial frames.
2. *Relativity Lab* provides a user interface that allows students to construct simulations themselves, so that thought experiments can be recreated as simulation experiments.
3. *Relativity Lab* prompts students to recognize a discrepancy, if any, between their predicted outcome of a thought experiment and the outcome of the simulation experiment.

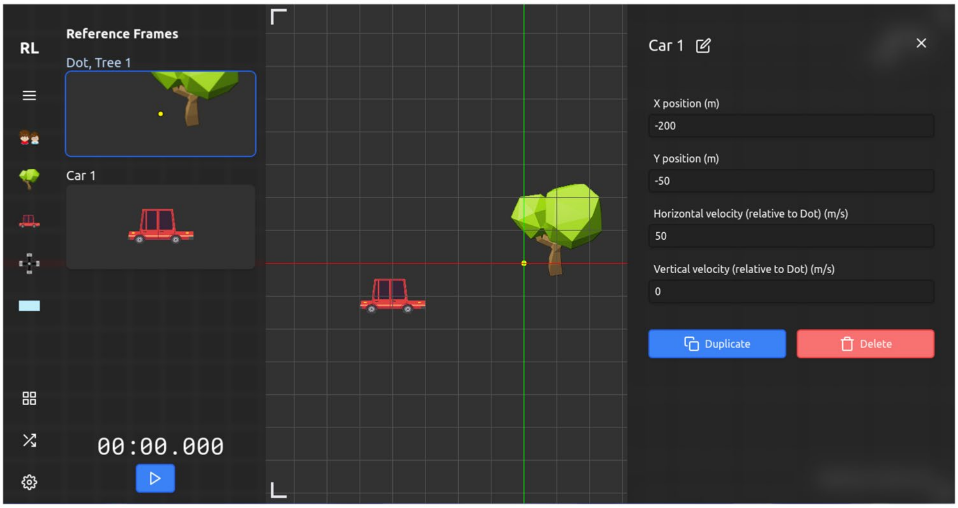
Design

This section describes *Relativity Lab*'s design. In the first subsection, we present an overview of *Relativity Lab*'s user interface. In the second subsection, we describe how the design decisions relate to the learning difficulties discussed in Sect. 2.1. In the third subsection, we present the results of a usability test and describe the resulting improvements.

User Interface

The main screen of *Relativity Lab* is a two-dimensional grid, representing the coordinate system of the selected inertial frame. The x- and y-axis are represented by a red and green line, respectively, and the origin of the coordinate system is marked by a yellow dot. The yellow dot cannot be selected as a regular object; rather, it represents the object to which the initial inertial frame is associated. The length scale of the grid is indicated in the upper right corner.

Simulations can be constructed by selecting objects, represented by icons, and specifying their positions and velocities. Objects can be placed on the grid by selecting an icon from the menu on the left side of the screen and dragging it to its desired position, as shown in Fig. 2. The currently available objects are: person, bird, tree, planet, spaceship, satellite, bus, car, airplane, tennis ball, laser, and mirror.



X position (m): -200

Y position (m): -50

Horizontal velocity (relative to Dot) (m/s): 50

Vertical velocity (relative to Dot) (m/s): 0

Fig. 3 A screenshot of the properties menu of the object "Car 1"

Once an object is placed, a pop-up menu appears in the right side of the screen, as shown in Fig. 3. In this menu, the following properties can be assigned: name, icon, *x*- and *y*-position, and *x*- and *y*-velocity. The *x*- and *y*-velocity of an object are assigned relative to the currently selected inertial frame. Hence, the velocities of the first object(s) will be assigned relative to the inertial frame of the yellow dot. The properties menu also includes options to duplicate or delete the object.

The inertial frame from which the simulation is rendered can be selected in a menu on the left side of the screen. The velocities of the objects will be transformed to match the relative velocity of the selected inertial frame, while the *x*- and *y*-axis of the grid will remain stationary relative to the screen.

The simulation can be started, paused, and reset by the buttons located in the lower left corner of the screen. The clock shows the elapsed time in milliseconds as measured in the selected inertial frame.

Light can be implemented in a simulation by placing a laser into the grid. A laser emits light flashes that propagate in a straight line in a user-specified direction or combination of directions, as shown in Fig. 4. Mirrors can be placed at a user-specified angle to change the light flash's direction of propagation. Once a laser is placed in the setup, the unit of length automatically changes from meters to light seconds. In accordance with the light postulate, light flashes propagate with a velocity of one light second per second in every inertial frame.

A number of utilities can be selected by clicking the corresponding icons in the lower left corner of the screen. The first utility is to hide the background grid. The second utility is to assign the objects' velocities by drawing vectors rather than assigning numeric values. The third utility is to pause the simulation automatically whenever two objects collide. Finally, the language can be set to Dutch or English.

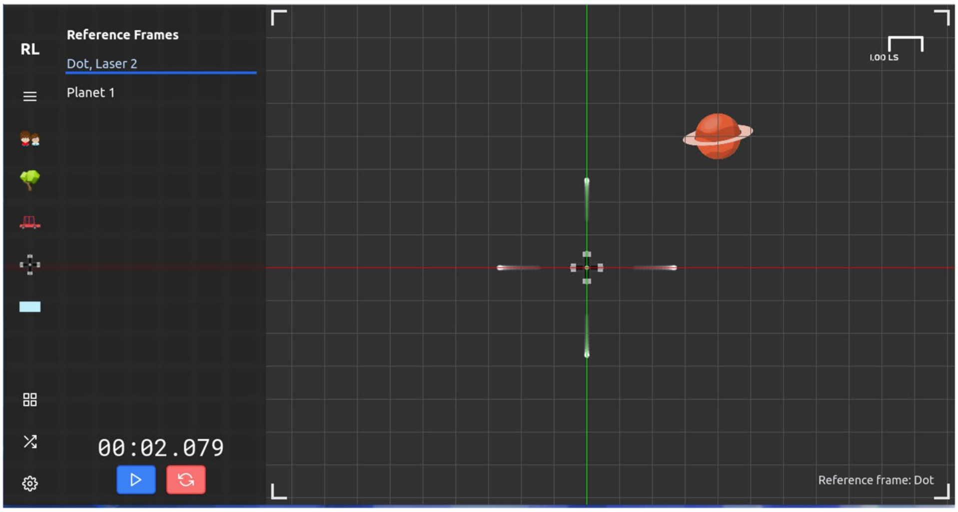


Fig. 4 A screenshot of a paused simulation in which a laser emits light flashes in four directions

Design Decisions

Relativity Lab's functionalities and limitations have been carefully designed to relate to the three main difficulties in understanding inertial frames, as described in Sect. 2.1. In the following discussion, we highlight a number of key functionalities and limitations, connected to these learning difficulties.

Students Tend to Regard Particular Events as Belonging to a Particular Inertial Frame In *Relativity Lab*, inertial frames are associated with the rest frames of the objects. Objects that are in rest relative to each other are grouped into a single inertial frame. When starting a new simulation, a single inertial frame is available, associated with a yellow dot at the origin of the coordinate system. At $t = 0$ s, the coordinate systems of all inertial frames overlap. In this regard, inertial frames behave as layers, overlapping at first and then moving away from each other.

Students Tend to Hold on to the Notion of a Preferred Inertial Frame In *Relativity Lab*, velocities of objects are assigned relative to the currently selected inertial frame. As a consequence, it is not possible to change an object's velocity while its rest frame is selected. An exception to this limitation is made when the selected inertial frame comprises multiple objects. To emphasize the equivalence of inertial frames, the origin of the coordinate system is always located at the center of the screen, and there is no background other than the grid.

Students Often Associate the Time of an Event with the Time at Which an Observer Receives a Light Signal from the Event In *Relativity Lab*, time coordinates of events are measured by a central clock that is associated with the selected inertial frame. Hence, the relativity of simultaneity cannot be interpreted as a result of signal travel time. To avoid the confusion of reading two distant clocks simultaneously, *Relativity Lab* does not allow separate clocks to be placed within the coordinate system.

Usability Test and Improvements

To investigate the usability and intuitiveness of the user interface of *Relativity Lab*, we performed a small-scale usability test. A group of Dutch upper secondary school students ($N = 10$) participated in the test. They were grouped into pairs and asked to work collaboratively and think aloud during the session. The participants were asked to construct simulations of three simple non-relativistic TEs. At the end of the session, the researcher asked the participants to reflect on their experience with *Relativity Lab* and to give suggestions for improvements. Data was collected by means of audio recordings and screen recordings. In the following, we describe the main results of the usability test and the improvements that we have made to the original design following the results.

Most students could construct simulations correctly without any difficulties. One pair of students was confused by the fact that the axes did not have a length scale, indicating that this is not intuitive to them. We addressed this confusion by implementing an indicator in the top right corner of the screen that specifies the length scale of the grid. When zooming in and out, the indicator specifies the length of a single division of the grid, while the grid itself remains stationary.

Additionally, some students found it difficult to accurately determine the time of an event, as it involves pausing the simulation exactly at the time of the event. We addressed this difficulty by implementing an optional collision detector. When the collision detector is active, the simulation will automatically pause when two objects collide, making it possible to determine the time of events with higher accuracy.

Evaluation

This section describes a qualitative evaluation of *Relativity Lab*'s design, in which we investigate whether *Relativity Lab* can be used to perform simulation-based inquiry activities. In the first subsection, we describe the participants, data collection, and data analysis. In the second subsection, we describe the structure of interview sessions and the task design. In the third subsection, we present the main results.

Method

Participants

The evaluation comprised 8 semi-structured interviews with pairs of students in the penultimate year of pre-university education at a Dutch secondary school. The students ($N = 16$) were selected from two physics classes to form a representative group in terms of age, gender, and previously obtained results. Other than selecting the participants, the teachers of the physics classes were not involved in the evaluation. The students' age ranged between 16 and 17. Ten participants were male and 6 female. The students had attended an introductory lesson on SR, in which the relativity principle and light postulate were introduced by the teacher. They were not yet familiar with time dilation or the relativity of simultaneity.

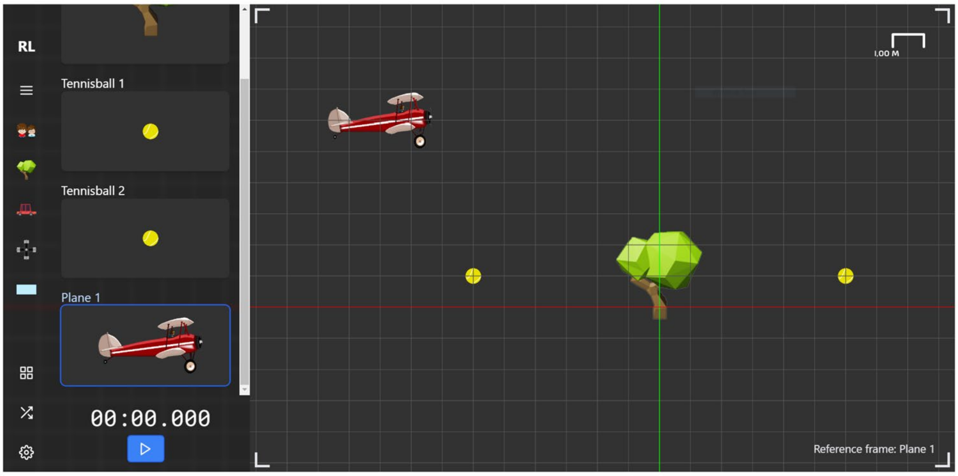


Fig. 5 Screenshot of a simulation of TE 1 (tennis balls). The inertial frame of the airplane is selected

Data Collection and Analysis

Each student pair participated in a 45 min semi-structured interview session with the researcher. Students were asked to think aloud during the session and to discuss similarities and differences in their reasoning (Whitelock et al., 1995). Data were collected by means of audio recordings and screen recordings. By the end of the interview, students were asked to individually fill in an exit card containing three evaluative questions.

The audio recordings and screen recordings were coded in terms of the correctness of students' responses and the activities performed in *Relativity Lab*. Relevant excerpts of the interview were transcribed and translated manually.

Task Design

During the sessions, the student pairs performed two tasks with *Relativity Lab*, as well as an introductory task. In the introductory task, students performed a number of activities guiding them to explore *Relativity Lab*'s user interface and main features. In the two main tasks, students followed the three phases of the simulation-based inquiry cycle, as described in Sect. 2.2.

In the first phase, students were presented with a written description of a TE. After individually reading the TE, students were asked to formulate a joint answer to the TE's central question. Students were stimulated to think aloud in determining their individual predictions and to discuss any differences between each other's predictions. In this phase, students were not allowed to use any means of visualization, such as drawings or animations. In the second phase, students were asked to reconstruct the TE in *Relativity Lab*. Students were allowed to construct the simulation however they believed it would match the TE. In the third phase, students observed the simulation and compared its outcome to their prediction. Students were free to observe the simulation from different inertial frames; however, this was not a mandatory part of the task. If the observation did not match their prediction, the students were asked to identify any discrepancies.

Thought Experiment 1 (Tennis Balls)

In the first task, students were presented with a TE that expresses the concept of events in an inertial frame. The TE's description is as follows:

> Two tennis balls are being thrown toward a tree, one coming from the left and one coming from the right. The tennis balls hit the tree simultaneously in the inertial frame of the tree. There is also an airplane flying over the tree.
> Question: do the tennis balls hit the tree simultaneously in the inertial frame of the airplane?

The correct answer is that the tennis balls will hit the tree simultaneously in every inertial frame, because it is a single event. A screenshot of a simulation of TE 1 in *Relativity Lab* is shown in Fig. 5.

Thought Experiment 2 (Einstein's Train)

In the second task, students were presented with a classic relativistic TE, commonly referred to as "Einstein's train":

> A laser is placed in the middle of a train carrier that is travelling at a velocity near the speed of light. The laser emits flashes of light toward the left and the right ends of the carrier, where observers are located. In the inertial frame of the carrier, the observers detect the light beams simultaneously.
> Question: is the light detected simultaneously in an inertial frame that is moving with respect to the carrier?

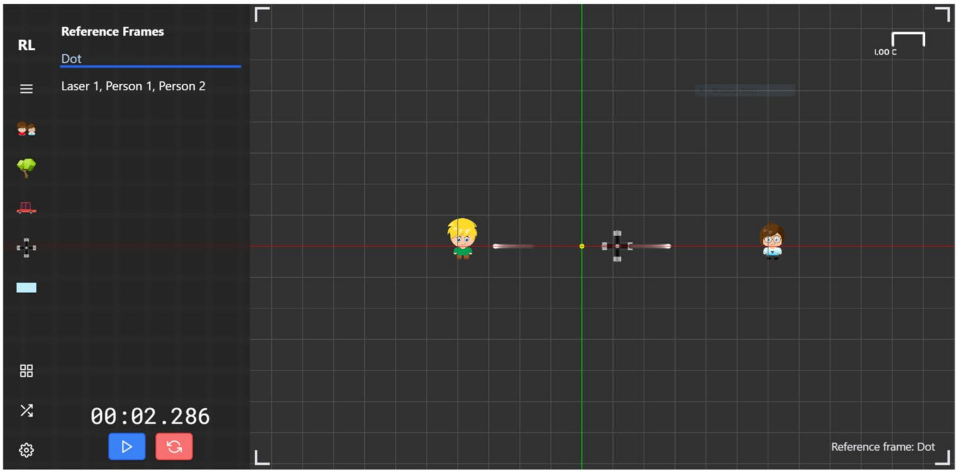


Fig. 6 Screenshot of a simulation of TE 2 (Einstein's train). The inertial frame of the yellow dot is selected

The correct answer is that the events of light detection are not simultaneous in any inertial frame that is not the carrier's rest frame. Due to the invariant speed of light, the light flashes require less time to reach one end of the carrier and more time to reach the other end of the carrier in an inertial frame that is moving with respect to the carrier. A screenshot of a simulation of this TE in *Relativity Lab* is shown in Fig. 6. Note that the simulation does not strictly require a train carrier.

Results

This subsection presents the main results of the interviews, categorized according to the three phases of the task described above. An overview of the results is given in Table 1.

Thought Experiment 1 (Tennis Balls)

Phase 1: Prediction All of the student pairs succeeded in formulating a joint prediction to TE 1. Seven of the eight pairs formulated a correct prediction of the TE's outcome. For example:

> *Student 1 (pair 2): [The tennis balls] are coming from different directions but they're going at the same speed, so I think that they arrive simultaneously (...) The velocities of the tennis balls do not suddenly change. It is just that they are now different compared to the airplane.*

Pair 4 formulated an incorrect prediction, stating that the tennis balls would not hit the tree simultaneously due to the relative velocities of the tennis balls:

Table 1 Overview of student activities during the interview. *x* indicates that the corresponding activity was performed by the student pair while *(x)* indicates that the activity was performed by one student of the pair

TE 1 (tennis balls)	Pair 1	Pair 2	Pair 3	Pair 4	Pair 5	Pair 6	Pair 7	Pair 8
Agreed on a prediction	x	x	x	x	x	x	x	x
Formulated a correct prediction	x	x	x		x	x	x	x
Constructed a correct simulation	x	x	x	x	x	x	x	x
Switched inertial frames				x	x		x	
Confirmed correct prediction	x	x	x		x	x	x	x
Rejected incorrect prediction				x				
TE 2 (Einstein's train)	**Pair 1**	**Pair 2**	**Pair 3**	**Pair 4**	**Pair 5**	**Pair 6**	**Pair 7**	**Pair 8**
Agreed on a prediction	x	x		x	x		x	
Formulated a correct prediction	x		(x)			(x)		(x)
Constructed a correct simulation	x	x	x	x		x	x	
Switched inertial frames	x	x		x				
Confirmed correct prediction	x		(x)			x		(x)
Rejected incorrect prediction		x	(x)	x			x	(x)

> *Student 1 (pair 4): From the perspective of the airplane, one [tennis] ball goes faster than the other, because they are going in opposite directions.*

Interestingly, one student brought up the difference between measurement and observation:

> *Student 2 (pair 6): If [the TE] is about actually hitting the tree, then the inertial frame of the airplane does not influence it. Then it is just about one moment. But suppose that it seems from the airplane that the tennis ball has hit the tree already, while it may not yet be so, then it is [dependent on] the inertial frame of the airplane.*

As she was unsure of her interpretation of an inertial frame, she struggled to make a clear prediction.

Phase 2: Simulation All of the student pairs succeeded in reconstructing TE 1 in *Relativity Lab* accordingly. Moreover, all pairs succeeded in selecting only the objects that are essential to the TE: a tree, two tennis balls, and an airplane. None of the students considered that the position of the airplane would influence the simulation's simultaneity of events, as was indicated by Scherr et al. (2001).

Phase 3: Evaluation The seven student pairs who had predicted TE 1 correctly claimed that the simulation had confirmed their prediction. For example:

> *Student 1 (pair 1): This is what we thought would happen, so that is nice.*

Pair 4, who had made an incorrect prediction, immediately recognized that the simulation did not match their prediction after viewing the simulation in the inertial frame of the airplane:

> *Student 1 (pair 4): [The tennis balls] do arrive simultaneously (...) Ah yes, the tree also moves along.*
> *Student 2 (pair 4): Ah yes, that's it. The tree also moves in that direction.*
> *Student 1 (pair 4): I forgot that the tree also moves.*

This excerpt indicates that they had not regarded the motion of the tree relative to the inertial frame of the airplane. After reading the velocities of the tree and the tennis balls in the inertial frame of the airplane, they realized that the velocities of all objects transformed equally.

> *Student 2 (pair 4): Ah yes, these two [velocities] compensate for each other.*

Finally, it is noteworthy that five student pairs observed the simulation only from the inertial frame of the airplane. They did not verify that they had constructed the TE correctly by confirming that the tennis balls hit the tree simultaneously in the inertial frame of the tree.

Thought Experiment 2 (Einstein's Train)

Phase 1: Prediction It was clearly more difficult for students to predict the outcome of TE 2 compared to TE 1. Student pair 1 was the only pair to agree upon a correct prediction of the TE that respects the invariant speed of light:

> *Student 1 (pair 1): Not simultaneous (...) I think light always goes at the speed of light, no matter how you look at it (...) The light particles do not go faster in one direction than in other direction (...) Then [the light] should travel the same distance at the same time in both directions.*

Student pairs 2, 4, 5 and 7 settled on a prediction of the TE that is based on a classical notion of light propagation. For example:

> *Student 1 (pair 7): Now everything is moving, because we are not in the inertial frame of the train.*
> *Student 2 (pair 7): [The scientists] also move along, so it should be so that they [detect the light] at the same moment. Because everything is just going toward the right.*
> *Student 1 (pair 7): The laser does not move, right? It is just that the whole thing is moving.*

Pairs 3, 6, and 8 could not reach agreement on the prediction, with one student predicting the correct outcome and the other predicting the outcome incorrectly. The following excerpt illustrates a discussion between students who use two different notions of light propagation:

> *Student 1 (pair 6): Not [simultaneous], because the light is moving along with the speed of the train, so the one who is standing behind will get [the light] earlier than the one who is standing at the front.*
> *Student 2 (pair 6): In principle the wagon is just something by itself, and the scientists are standing inside of it, so the fact that it moves should not matter, right? The scientists and the light source are moving at the same speed so then it does not matter that it moves, right?*
> *Student 1 (pair 6): Yes it does because [the train] is moving toward the right and one [scientist] is standing right and the other is standing left, so the light has to go further to reach [the right scientist] than [the left scientist].*

Phase 2: Simulation Six student pairs succeeded in reconstructing TE 2 in *Relativity Lab* accordingly. Pairs 5 and 8 did not assign velocities to the observers; hence, they remained at rest with respect to the "stationary" inertial frame. Pair 5 also did not assign velocities to the laser. After the researcher

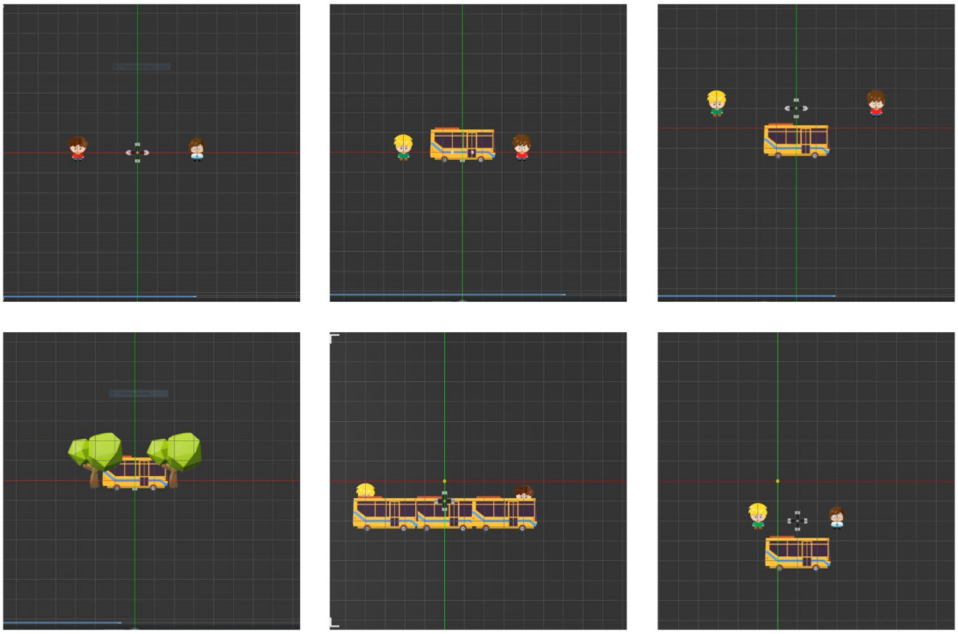


Fig. 7 A comparison of various ways in which student pairs constructed TE 2

had asked them whether the simulation matched the given TE, both pairs responded positively. This made it clear that they believed that the observers are located “outside" the train carrier.

Compared to TE 1, students were more inclined to add additional objects to the simulation that are not essential to the TE, such as the train carrier itself. To investigate whether students would recognize that the train carrier as a non-essential object, we deliberately chose not to include a train carrier as an available icon. Pairs 1 and 2 immediately recognized that the train carrier is unnecessary:

> *Student 2 (pair 1): But we don’t have a carrier.*
> *Student 1 (pair 1): No man, I don’t think that matters.*

Pairs 3 to 8 decided to use an icon of a bus instead:

> *Student 1 (pair 4): There isn’t really a train carrier.*
> *Student 2 (pair 4): Ah, then we will use a bus.*

There are many more differences in the way in which students chose to construct TE 2. A comparison of various constructions of TE 2 is shown in Fig. 7.

Phase 3: Evaluation Those students who had predicted TE 2 according to the invariant speed of light claimed that the simulation matched their prediction. For example:

> *Student 1 (pair 6): It is correct that the light has to traverse a shorter distance toward [the left scientist] than toward [the right scientist], so it arrives earlier at [the left scientist].*

Most students who had predicted TE 2 incorrectly recognized that the simulation did not match their prediction. The following excerpt describes students’ reaction to observing the simulation from the inertial frame of the train carrier:

> *Student 1 (pair 7): This is not what we expected at all.*
> *Student 2 (pair 7): The [light flashes] are both equally far from where they had started, but this [light flash] goes to the left so it hits the [left] scientist much earlier (...) When we thought about [the TE], we viewed it from the laser, where everything is standing still, but now we see that everything moves to the right.*

Interestingly, one student did not recognize that the simulation did not match his prediction. In phase 1, this student had predicted the light flashes would reach the scientists simultaneously. While the simulation contradicted his prediction, he responded positive to the questions whether his prediction was confirmed:

> *Student 2 (pair 4): Yes, this is what I had in mind.*

It is noteworthy that only three student pairs spontaneously switched between multiple inertial frames. These student pairs selected the “stationary" inertial frame to verify that their simulation matched the TE description. By switching back and forward between the “stationary" inertial frame and the inertial frame of the train carrier, they were able to recognize the frame-dependence of the relativity of simultaneity:

> *Student 1 (pair 2): Why do [the light flashes] reach the two persons simultaneously in this inertial frame and not in the other? (...) Here [the observer] is moving toward the light burst. I expected that it would look the same from the wagon as in the [stationary inertial frame].*

The other five student pairs selected a different inertial frame after a suggestion by the researcher.

General Evaluation

The participants reflected on the use of *Relativity Lab* in the exit cards that were handed out at the end of the interview, as well as an informal evaluation afterwards. Answering the first question: "What have you learned from working with *Relativity Lab*?", multiple students noted that *Relativity Lab* had helped them to visualize TEs more clearly. In addition, students noted that they had learned about using inertial frames to describe relative motion.

Answering the second question: "What did you think was difficult in working with *Relativity Lab*?", students gave various critical comments. One student expressed that it was difficult for her to make the simulation look as realistic as possible. Another student noted that he was confused by the tasks because he was not able to explain the outcome of the simulation:

> *Student 1 (pair 8): I found that it was sometimes unclear because I could not really give a good explanation of the events.*

Finally, the researcher asked the students how they would use *Relativity Lab* if it were available to them. Multiple students suggested that they would use *Relativity Lab* to verify their answers to offline assignments.

> *Student 1 (pair 1): First, you work out [the TE] on paper. Then you run the [simulation] experiment and from that you check whether your hypothesis was correct. Then you draw your conclusion from that.*

One student noted that she would be able to transfer the visualization that she had observed in *Relativity Lab* to offline assignments:

> *Student 1 (pair 5): On a test, for example, I've got this picture in my head, and then perhaps I could understand it better (...) Actually I am going to project this into my mind now to see what happens.*

Conclusion and Discussion

Conclusion

This study aims to answer the research question: "How can a computer simulation environment support secondary school students in performing and evaluating relativistic TEs?" To ensure that *Relativity Lab* meets this demand, three design objectives were specified, as described in Sect. 2.3. In the following, we evaluate each design objective.

The first design objective is: "*Relativity Lab* provides a platform for simulating relativistic thought experiments and allowing students to observe the simulation from different inertial frames." The evaluation shows that students are able to use *Relativity Lab* to construct simulation experiments without requesting any additional instruction. However, some of *Relativity Lab*'s features, in particular the feature to switch between inertial frames, were only used sporadically. While it can be concluded that the first design objective is met, improvements could be made to the task design to stimulate students to view the simulation from multiple inertial frames.

The second design objective is: "*Relativity Lab* provides a user interface that allows students to construct simulations themselves, so that thought experiments can be recreated as simulation experiments." The evaluation shows that all students succeeded in constructing a simulation of the non-relativistic TE (Tennis balls), while most students succeeded in constructing a simulation of the relativistic TE (Einstein's train). Those students who constructed "Einstein's train" incorrectly did not assign velocities to the observers. A possible explanation for this result is that these students may have misread or misinterpreted the TE, believing that the observers are located "outside" of the train carrier. This is problematic, because an incorrectly constructed simulation obscures meaningful evaluation of its outcome. It is noteworthy that students often added more objects to the simulation than strictly necessary. In particular, students often unnecessarily added an icon to represent a train carrier in "Einstein's train." This indicates that it is important to students to construct simulations as close to the written description as possible. From these results, it can be concluded that *Relativity Lab* has the potential to meet the second design objective. To ensure that students construct simulations correctly, the task design could be improved to include instructions to verify the construction of the simulation.

The third design objective is: "*Relativity Lab* prompts students to recognize a discrepancy, if any, between their predicted outcome of a thought experiment and the outcome of the simulation experiment." The evaluation shows that the task to formulate a joint prediction of the TEs triggered students to use their imaginative skills to create mental imagery of the TE. In some cases, lively discussion evolved between the student pairs in which differences in predictions were debated, often using hand gestures as a means to express the imagined trajectories of the objects. In the non-relativistic task (tennis balls), all students succeeded in comparing the outcome of the simulation with their prediction and drawing a consequent conclusion from the comparison. One student pair predicted the TE incorrectly, and they immediately identified the flaw in their prediction after observing the simulation. In the relativistic task (Einstein's train), the majority of participants predicted the TE based on a classical notion of light propagation. Almost all students who had predicted the TE incorrectly recognized the discrepancy between the simulation and their prediction. While some students spontaneously realized that they had based their prediction on a different notion of light propagation, other students could not provide an explanation of what they had observed. From this, it can be concluded that the third design objective is met; however, improvements could be made to the task design to help students understand and explain the observed simulation.

Overall, it can be concluded that *Relativity Lab*'s design, in combination with the task design, provides a promising approach to address the conceptual difficulties that students experience when performing relativistic TEs.

Discussion

While we have provided evidence that *Relativity Lab* provides a promising learning approach to SR, a number of difficulties in performing the tasks have been found. These difficulties may be overcome by an improved task design featuring stricter instructions to constructing and verifying the simulations. First, the task design could be improved as to ensure that students construct the simulation according to the given TE. In the case of simultaneity, for example, this could be achieved by asking students to verify that two events are simultaneous in a particular inertial frame before observing the simulation in a different inertial frame. Second, it is key for students to switch between different inertial frames to observe changes in time intervals and the time order of events. The evaluation shows that most student pairs did not spontaneously switch between multiple inertial frames. The task design could be improved by including suggestions to observe the simulation from multiple inertial frames and to observe relevant changes.

There are similarities between Horwitz and Barowy's study on *RelLab* (Horwitz & Barowy, 1994) and the present study. For example, *RelLab* was evaluated by analysing predict-observe-explain activities similar to our inquiry-based task design. In line with our findings, Horwitz and Barowy found that the discrepancy between students' prediction and evaluation of the simulation served as a basis to overcome conceptual obstacles. Moreover, our study replicates Horwitz and Barowy's finding that students enjoyed the freedom of designing and evaluating their own TEs and that it motivated them to learn more about SR. In contrast to Horwitz and Barowy's study, our findings are based on qualitative analysis of semi-structured interview sessions, where students were asked to discuss their reasoning in pairs. This method has provided the insight that students' collaboration plays a pivotal role in explicating assumptions that students may hold implicitly. For example, we have found in multiple interviews that one of the students made the other student realize that their prediction was based on a false assumption. Some of *Relativity Lab*'s features that were not featured in *RelLab*, such as the option to reflect light using mirrors, will investigated in our next study.

To the authors knowledge, there are currently no simulation tools available with similar functionalities to *Relativity Lab*. There are, however, a number of applications available that visualize relativistic effects in a realistic virtual environment, such as Real Time Relativity (McGrath et al., 2010) and the serious game A Slower Speed of Light (Kortemeyer, 2019). While these virtual realities have proven to enhance students' motivation and attitude toward learning SR (Croxton & Kortemeyer, 2018), they do not allow students to view a simulation in different inertial frames. Rather, these virtual realities aim at visualising a realistic three-dimensional representation of the world at velocities near the speed of light. In order to thoroughly compare *Relativity Lab*'s learning gains to existing virtual realities, further investigation on *Relativity Lab's*'s effects on students' understanding of SR is needed.

This study focuses on evaluation of *Relativity Lab*'s design on the basis of its design objectives, rather than an investigation of its potential effects on students' understanding of SR. We recognize that the insights provided in the present study on students' interaction with *Relativity Lab* are prerequisite to an investigation of the effects on conceptual understanding. In the present study, we have shown that *Relativity Lab* can create a cognitive dissonance that leads students to reconsider their existing conceptions about relative motion. However, it was also found that students sometimes struggled to understand and explain the outcome of a simulation. For example, students found it difficult to accept that two events are simultaneous in one inertial frame and not simultaneous in another. It can hardly be expected that students would understand this deeply abstract relativistic phenomenon after a single intervention with *Relativity Lab*.

In order to gain meaningful understanding of SR, the approach described in the present study should be extended to multiple interventions with *Relativity Lab* embedded in a whole-class introductory lesson series. Ideally, the first intervention would focus on describing relative motion in the non-relativistic limit, while subsequent interventions focus on the relativity of simultaneity and time dilation. Implementation of *Relativity Lab* in a lesson series paves the way for transfer of learning gains during sessions with *Relativity Lab* to offline assignments, as was shown by Clement and Mongahan (1999). Whole-class implementation of *Relativity Lab* and its effects on students' understanding of SR will be central to our next studies.

Author Contributions All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Paul Alstein, Kim Krijtenburg-Lewerissa, and Wouter van Joolingen. The first draft of the manuscript was written by Paul Alstein, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Funding This work was financially supported by The Dutch Research Council (NWO) under Grant No. 023.013.003. The authors have no relevant financial or non-financial interests to disclose.

Data Availability The datasets generated during and analysed during the current study are not publicly available due to EU privacy policy (EU-GDPR) but are available under embargo from the corresponding author on reasonable request.

Declarations

Informed Consent This study was performed in line with the principles of the Declaration of Helsinki. Informed consent was obtained from all individual participants included in the study to participate in the study and to publish the anonymized data.

Ethical Approval Ethical approval was granted by the Science-Geosciences Ethics Review Board (SG ERB) of Utrecht University (Date 24 June 2022 / No. S-22767).

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