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ORIGINAL ARTICLE

Not so simple

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Abstract
In a recent series of articles, Beall has developed the view that FDE is the formal system most deserving of the honorific "Logic". The Simple Argument for this view is a cost-benefit analysis: the view that FDE is Logic has no drawbacks and it has some benefits when compared with any of its rivals. In this paper, I argue that both premises of the Simple Argument are mistaken. I use this as an opportunity to further reflect on how such arguments can be bolstered to provide more substantial and productive support for revisionary theses about Logic.

Keywords Logical consequence · Logical monism · Logical revision · Anti-exceptionalism · FDE

1 F=L

In a recent series of articles, Beall (2013, 2015, 2017, 2018, 2019) has developed the novel view that Anderson and Belnap's system of *first-degree entailment*, FDE, is the One True Logic.[1] To spell this out more carefully, Beall triangulates on a particular role for logic in theory-building—what we might call the capital-L honorific usage of the title "Logic"—and asserts that FDE best fits this role. Throughout the rest of this paper, I use the label "F=L" to abbreviate the thesis that *FDE is Logic*. Beall's much touted *Simple Argument that F=L* runs as follows (Beall, 2018, pp.46-50).

(P1) If we accept that F=L, we lose nothing.
(P2) If we accept that F=L, we gain something.
∴ We should accept that F=L

This "optimization" strategy evinces the spirit of recent anti-exceptionalism about logic (Hjortland, 2017; Martin, 2021; Tajer, 2022). While I very much admire the

[1] For the genesis of FDE, see Anderson and Belnap (1962) and the notable semantic work of Dunn in, e.g., Anderson and Belnap (1975), Part III).

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simplicity of this strategy, I will argue that the Simple Argument fails. This invites speculation about how such an argument can be bolstered.

My commentary on the Simple Argument will not be directed at the conclusion itself, but will focus instead on subtle flaws with the premises. Briefly, I criticize each premise as follows. P1 assumes that there is just one problem that significantly detracts from F=L (a problem that can be solved), but I claim that there are other, similar, unsolved problems. Contra P1, F=L has serious drawbacks. P2 rests on a dubious conception of theory-appraisal, which reveals that there is nothing to gain from accepting that F=L. Contra P2, F=L does not offer any noteworthy benefits. I conclude that the Simple Argument rests on an incorrect analysis of the costs and benefits associated with the hypothesis that F=L.

The discussion up to this point will be "merely" destructive, but at the end of the paper I offer some constructive ideas about how to improve upon this style of argument driven by theory-building practice. My commentary throughout the paper issues from a sympathetic point of view toward Beall's overarching project. Important questions about logic are often ignored by philosophers because they are genuinely difficult to tackle. Beall's effort to address such questions is commendable and his approach is both interesting and plausible. I think his argument ultimately fails to convince, but this is for subtle reasons that can teach us something useful.

2 The role(s) of logic

In this section I will elaborate on the purpose of the Simple Argument. For Beall, what does it mean to say that a certain formal system is Logic (proper) and what virtues are meant to accrue to the hypothesis that F=L?

The term "logic" can refer to at least three different things (Caret, 2021b). While these uses are closely related, it is important not to casually conflate them when grappling with a philosophical discussion of issues in this vicinity.

(i) Logic as a tool: In this sense of the term, "logic" is a count noun used to refer to formal systems (symbolic languages coupled with a semantics or proof theory). We can speak of *a logic* and give them proper names like "FDE."
(ii) Logic as a phenomenon: In this sense of the term, "logic" collectively refers to a family of relations between propositions, viz. the *logical relations*. The most familiar examples are relations such as logical consequence and provability.
(iii) Logic as a discipline: In this sense of the term, "logic" is the name for a traditional area of academic inquiry. These days, it is most often found as part of other departments: philosophy, mathematics, linguistics, and computer science.

Logic the discipline uses logical tools to theorize about logical phenomena, but the phenomena are of primary interest. Formal systems are not an end in themselves, they just serve as the "common language" of logicians.

Beall's conception of logic lies at the second level and concerns a phenomenon in the vicinity of Aristotle's classic definition of deductive argument.

> A deduction is speech (logos) in which, certain things having been supposed, something different from those supposed results of necessity because of their being so. (*Prior Analytics* I.2, 24b18-20, trans. Smith 1989)

This notion of "following by necessity" lends itself to the familiar idea that explicit posits carry with them further implicit commitments. Beall is interested in relations of this kind. He gives the following gloss on *closure* and *consequence*.

> When we form a theory of some phenomenon we throw a bunch of sentences into the theory, namely, all of those sentences that we think are true about the phenomenon. In turn, we require a theory that reflects all of the true consequences of the theory's claims. And this is the job for a closure relation: a relation that "completes" the set of truths by adding all sentences that are consequences of the theory according to the relation. (Beall, 2017, p.5)

On this picture, the theoretician records certain posits in a "seed theory" and couples them with a closure relation that "completes" the theory (Beall, 2019, p.117). This is like the axiomatic conception of mathematics extended to all theory-building.[2]

For Beall, a closure relation is a set of argument forms structured by a set of terms. These structuring terms need not be standard "logical" terms. For example, in physics, the following might be part of the relevant closure relation on theories.

that x is a quark has as a consequence *that x is a hadron*

This sort of principle is often called a lexical or material consequence because it constrains the use of terms like "hadron" and "quark." No matter. For Beall, closure relations form a family: they are relations that function to partially individuate the commitments of some theory or other. This means that principles like the one above can be part of a closure relation in this broad sense. Since relations of this kind specifically constrain the use of specialized technical terms, however, they have only *limited* applicability (e.g., in physics). Logic is different.

Tarski (1936) once argued that the relation of *logical consequence* is special and Beall agrees with this sentiment. Specifically, he holds that logical consequence is special because it is structured by a set of terms that occur in absolutely all theories whatsoever (the connectives and first-order quantifiers). At a first pass, the Beallian view is that we triangulate on the phenomenon of logical consequence by looking at what kind of closure relation is left over when we bracket out the parts of other closure relations that have strictly limited applicability. Beall (2018), p.31 refers to logical consequence as the *universal basement-level closure relation*.

There is one importat caveat to the above: the official Beallian view does not hold that logical consequence plays a role in all theories, but only in *true theories*.

> Logic is the basement-level closure/consequence relation involved in all of our true theories, where our true theories are pictured as pairs (to highlight the closure relation):

[2] Martínez-Ordaz (2022), §2.2 considers it a mistake to presume that scientific theories are, in general, logically closed. This may stem from the mistake of assuming that rational belief sets are logically closed. See Harman (1984) and Michael (2016) for more on these issues.

$$\langle T_1, \vdash_{T_1} \rangle, \langle T_2, \vdash_{T_2} \rangle, \langle T_3, \vdash_{T_3} \rangle, \dots, \langle T_n, \vdash_{T_n} \rangle$$

> Logic shows up in each such theory-specific consequence relation $\vdash_{T_i}$; it is the relation under which all true theories, so understood, are closed; it is the relation on top of which all closure relations for our true theories are built. (Beall, 2018, p.32)

Beall never explains the reason for this emphasis on true theories, but perhaps he is worried about theories that are untrue *because* they violate logical consequence. For example, a theory could reject a claim that is actually a logical consequence of other, well-known truths. If we aim to identify logical consequence by its role in theories, we certainly want to ignore theories *of this kind*. An efficient way to do that is to simply ignore all theories with untrue posits or commitments.[3]

Beall (2018), p.35 gives an "absence of counterexample" gloss on the tools of logical theory. On this understanding, a formal logic produces an account of logical consequence by defining "a space of possibilities, each of which serves as a potential counterexample to any 'argument'." If a given argument has no counterexamples, then it is judged to be valid by such an account. Any logic *cum* definition of a space of logical possibilities for first-order argument forms is a candidate for Logic (proper), i.e., the correct account of the universal closure relation.

The identity of Logic, then, is an important question because it codifies a critical threshold for theories: any theory that "violates," Logic is objectively impossible. On this view, what Logic does is to "...firmly mark the remotest boundaries of theoretical possibilities and also furnish the weakest skeletal structure of our true (closed) theories. And that's pretty much it" (Beall, 2018, p.31). The purpose of the Simple Argument is to defend a particular view about the identity of Logic.

3 Nothing to lose

As we know, Beall contends that F=L is the optimal view about Logic. In this section, I will discuss the first premise of the Simple Argument, which concerns the claim that F=L has no drawbacks when compared to any of its rivals.

3.1 Classical closure

In case it is not perfectly obvious, the claim that F=L has no costs is not a claim about utility or practical consequences, but about the way in which our view of the world hangs together. A belief can have various costs: being formed without a basis, lacking evidence or justification, introducing conflict or incoherence.

[3] This does not strike me as a particularly good reason to set things up this way, i.e., defining logical consequence by reference to true theories (only), but that is an issue for another day.

So, what costs could we be concerned about when it comes to rival hypotheses on the identity of Logic? One potential cost is a failure to explain *closure practices* in science, mathematics, and other fields of theoretical inquiry. If experts take certain (true) theories to be closed under certain principles, it is a serious drawback if closure of this kind is *inexplicable* according to our views about Logic.

There can easily be a *prima facie* asymmetry between the explanatory resources of different logical systems. Suppose that $L1$ and $L2$ are candidate accounts of Logic that stand in the following relation to our closure practices. First, there are theories T closed under the principle $\phi \vdash_T \psi$.[4] Second, logic $L1$ is properly stronger than $L2$.[5] Third, the two systems differ over the judgment that $\phi \vdash_{L1} \psi$ but $\phi \nvdash_{L2} \psi$. This makes $L1$ more attractive than $L2$ because it offers a ready explanation of the salient closure practice. If we accept that $L1$ is Logic, this already implies that *all (true) theories* T are closed under the principle $\phi \vdash_T \psi$. True theories have to be closed under Logic, that is its "job description" after all. The advocate of $L2$, on the other hand, has an up-hill battle to explain how the local "connective and quantifier" laws of a theory can *exceed* what they (the advocate of $L2$) consider to be Logic.

Beall highlights such a problem and offers a solution, overcoming what he takes to be the most significant hurdle in the way of accepting that F=L. This problem arises when we think about the standard textbook system of classical quantified logic, CQL, understood as a rival to FDE in the present debate. For now, all you need to know is that CQL and FDE are analogous to $L1$ and $L2$ above. I use the label "C=L" to abbreviate the thesis that *CQL is Logic*. Beall (2019), p.119 describes the potential cost of F=L as follows: "Perhaps the main worry that confronts the FDE account of logic (qua universal closure) is the ubiquity of true classically closed theories." If this problem can be solved, then nothing should stand in the way of accepting that F=L. But what does that take and how is it to be accomplished?

To say that a theory T is classically closed means this: if a certain "connective and quantifier" principle is part of classical logic $\phi_1,...,\phi_n \vdash_{CQL} \psi$, it is also part of the closure principles of theory T, i.e., $\phi_1,...,\phi_n \vdash_T \psi$. This does not ascribe any special status of CQL as the genuine, correct account of Logic, it is just describing a kind of closure that may or may not hold in certain theories.

As a matter of fact, however, some (true) theories are classically closed. The experts attest as much. An important example of this kind of theory is first-order Peano Arithmetic (PA) with the induction schema. This well-trod axiomatic theory codifies a classical conception of the natural numbers and it is quite explicitly intended to be closed under CQL. To explain this closure practice, the advocate of F=L apparently has to do a lot more work to do than the advocate of C=L.

This is a hard problem because of the inherent weakness of FDE as a formal system. Without going into all the details, predicates in FDE are interpreted with a positive and negative extension. A given object may fall into one side or the other, it may

[4] We do not have to only consider "single-input-single-output" principles, but adding more details on either side does little to clarify the current point.

[5] Everything that $L1$ judges to be valid, $L2$ also judges to be valid, but there are things that $L2$ also judges to be valid and $L1$ does not.

occur in neither side (making a "gap"), or it may occur in both (making a "glut"). The formal semantics allows all four options. As a result, for example, $\forall x(Px \vee \neg Px)$ is not a logical truth according to FDE because there are interpretations on which an object does not occur in either the positive or negative extension of P. Even more importantly, *modus ponens* for the material conditional "→" is not valid in FDE, so for example there are theories in which $\forall x(Qx \rightarrow Rx)$ and Qc hold but Rc does not hold and yet the theory is fully FDE-closed. Such theories obviously fall short of classical closure, so how can there be classically closed theories if F=L?

Beall's answer to this question relies on the following facts about what he calls "shrieked" and "shrugged" theories, which are formal theories in the language of FDE.

> To *shriek* a predicate P in the language of theory T one imposes the following condition on T's closure relation $\vdash_T$, where $\bot$ is true in no models of T:
>
> $$\exists x(Px \wedge \neg Px) \vdash_T \bot$$
>
> Imposing this condition on a theory's closure/consequence relation has the effect of reducing the space of logical possibilities with respect to predicate P to only non-glutty ones… To *shrug* a predicate P in the language of theory T one imposes the following condition on T's closure relation $\vdash_T$, where $\top$ is true in all models of T:
>
> $$\top \vdash_T \forall x(Px \vee \neg Px)$$
>
> Imposing this condition on a theory's closure/consequence relation has the effect of reducing the space of logical possibilities with respect to predicate P to only non-gappy ones. (Beall, 2018, pp.47-48)

The upshot is that even if F=L, it is possible to achieve classical closure for a given theory T by enriching its internal closure relation $\vdash_T$ in specific ways.

If every predicate of T is shrieked and shrugged, it reduces the admissible models to those that "behave classically" and eliminates all others as "T-impossibilities." Beall (2018), p.47 considers the best reason to accept C=L as "the mistaken thought that logic precludes a vast portion of the space of...possibilities," whereas from the point of view of F=L, this is not work done by Logic, "we—qua theorists of the target phenomenon—rule them out as theoretical *im*possibilities." All we need to explain classical closure is to shift where that explanation is located.

3.2 Tallying the costs

When Beall says that we have "nothing to lose" by accepting that F=L, he means that "we lose no true theories by accepting the FDE account" (Beall 2019, p.119). Many theories are classically closed, but since this can be adequately explained under the hypothesis that F=L, the view has no significant costs.

In response, I offer two comments on P1. The first concerns how shrieking and shrugging occur in *theory-building* understood as a dialectical activity instigated by "flesh and blood" human beings. The second concerns how far shrieking and shrugging can take us in the landscape of scientific and mathematical theories; I will argue that there are *additional hurdles* they do not overcome.

On the going picture, theory-building has two ingredients: a "seed theory" and a theory-specific closure relation. Together, these uniquely individuate theory T by precisely demarcating what is and is not a commitment of T. For the advocate of F=L, the explanation of classical closure appeals to aspects of theory-specific closure relations (shrieking and shrugging predicates). The division of theory in two parts is important to this explanation, but it raises some pressing questions.

When speaking of theories, I take it that we are not limiting our attention to formal theories like PA. Although we previously used PA as a motivating example, we are broadly interested in theories of all kinds. Theories are most often articulated through informal, dialectical activities of speaking, and writing. How do the two parts of theory-building manifest in these activities? One part of this story is as easy to understand as it is familiar: a seed theory is simply a set of explicit posits and a theoretician can make it clear that ϕ is in their seed theory simply by *asserting that* ϕ.

What about the theory-specific closure relation? How is that determined? Unfortunately, Beall says nothing about this. In fairness, it is a difficult question for the advocate of F=L. This is where the inherent weakness of FDE comes home to roost. As mentioned above, *modus ponens* for the material conditional "$\rightarrow$" is not valid in FDE. This means that a theoretician who uses only the FDE connectives and quantifiers *cannot* shriek predicate P in their theory simply by asserting that $\exists x(Px \wedge \neg Px) \rightarrow \bot$ (i.e., putting this sentence into their seed theory). Adding this sentence to theory T is insufficient to ensure that $\exists x(Px \wedge \neg Px) \vdash_T \bot$, hence it does not achieve the constraint on models required for shrieking. Similar remarks apply to shrugging.

The point is that there is a lacuna in the explanation we have been offered and it is quite unclear how to fill it in. Beall has pointed out that certain features of a closure relation suffice to constrain the commitments of a theory in desirable ways. Granted, but what kind of dialectical activity could determine that a given closure relation has these features? Asserting sentences of the underlying language will not do the trick. One wants to go dispositional, e.g., perhaps what manifests that $\phi \vdash_T \psi$ is that the T-theorist *sees to it that* whenever ϕ is in T, this guarantees that ψ is in T. Yet this is at best a start of a story and a problematic one at that.

None of this amounts to a knock-down argument against premise P1. It merely shows that premise to be dubious. To explain classical closure, it is not enough to point out features of a closure relation that would have the right effect, we also need to know how these features are manifested in theory-building practice.

For the sake of argument, suppose that a satisfying story can be told to fill the lacuna above. Would premise P1 then be warranted? I think not. The reason is that there is more to explain in heaven and earth than are dreamt of in the realms of classical closure. Consider any of the theories developed in intuitionistic mathematics such as first-order Heyting Arithmetic (HA) with the induction schema. This axiomatic theory codifies a constructive conception of the natural numbers and it is explicitly intended to be closed under intuitionistic quantified logic or IQL.

Here, we find another candidate for Logic. I use the label "I=L" to abbreviate the thesis that *IQL is Logic*. Say that a theory T is intuitionistically closed if the following holds: if a "connective and quantifier" principle is part of intuitionistic logic $\phi_1,...,\phi_n \vdash_{IQL} \psi$, it is part of the closure relation of T, i.e., $\phi_1,...,\phi_n \vdash_T \psi$. This is meant to be descriptive and neutral on the question of whether IQL is genuine Logic. Since there are intuitionistically closed theories, we should explain how such theories can successfully be defined if it is the case that F=L.

As a first step, it would be nice to at least have something like Beall's shrieking and shrugging story, i.e., one which identifies features of a closure relation that reduce the admissible models of the theory to those that "behave intuitionistically." This, however, is impossible for the advocate of F=L.

The reason is that $\neg\neg\phi \vdash_{FDE} \phi$ and $\neg(\phi \wedge \psi) \vdash_{FDE} \neg\phi \vee \neg\psi$, whereas intuitionistic theories like HA are not closed under such principles. If we start with FDE models, there is absolutely no way of restricting the nonconstructive principles of that system by isolating "just some" of its models. So, the claim that *there are (true) theories* that are not closed under double-negation and DeMorgan equivalences is incompatible with F=L. The greatest cost of F=L is that it implies that there are no (true) intuitionistic theories.

For his own part, Beall is fairly clear about the fact that he is not trying to solve such problems. He only intends the Simple Argument to mediate the choice between two rival hypotheses: F=L and C=L. No other logical traditions are being considered, no other candidates for Logic matter to the cost-benefit analysis. This is why the explanation of classical closure is "the main worry" because it represents the only rival conception of logical standards. It is, of course, perfectly fine to limit one's attention in this way, but when we realize this only this narrow comparison is intended it seriously diminishes the value of the Simple Argument.

For this reason, I find it instructive to stretch the imagination a bit and consider a broader range of candidates for Logic, including the hypothesis that I=L. Then, we need to think harder about the phenomenon of intuitionistic closure. One way that the advocate of F=L could resolve this problem is by arguing that intuitionistic theories like HA are irrelevant to the debate. We said that Logic is involved in the closure of all true theories. One could try to argue that HA and its ilk are *systemically untrue*. Presumably, this demands something like a non-constructive account of the very nature of mathematical truth, which paves the way to dismissing the explanatory challenge. If there are no (true) intuitionistically closed theories, there is nothing to explain and the problem simply evaporates.

I will not fully assess this idea, but I will say that it does not seem promising. The reason is that it requires us to draw pretty arbitrary lines between true and untrue pockets of mature mathematics. By analogy, consider Euclidean geometry and Hyperbolic geometry. It is weird to ask which one is true and which one is untrue because these mathematical theories simply describe different kinds of spatial structures. Each theory seems to be true about its own kind of structure. The same thought extends to different arithmetics like PA and HA. [6]

These considerations show that P1 of the Simple Argument is likely wrong. For Beall, the claim that F=L has no drawbacks amounts to the claim that the problem of classical closure—the only problem that counts against F=L—can be solved. As we have seen, however, the (shrieking and shrugging) explanation of classical closure is incomplete. Furthermore, the strategy of this solution is fragile. There are similar problems (intuitionistic closure) lurking in the wings that seemingly cannot be solved on this view. F=L has some heavy costs after all.

4 Something to gain

Moving on to our next topic, in this section I will discuss the second premise of the Simple Argument, which concerns the claim that F=L has notable advantages when compared to rival views on the identity of Logic.

4.1 Live options

Again, this is not a claim about practical utility. It is a claim about how F=L hangs together with and augments our broader view of the world. The central benefit advertised for the hypothesis that F=L pertains to the way in which Logic demarcates a space of objective possibilities from impossibilities.

We said previously that a formal logic produces an account of logical consequence by defining a set of models to serve as counterexamples. If an argument has no counterexamples, it is classified as valid. Each model represents a *logical possibility*, which is meant to be a species of objective possibility, thus closer in spirit to so-called "metaphysical" rather than "epistemic" possibility. Accordingly, if a theory cannot be satisfied by any model, it is classified as being objectively impossible. Beall does not use the terminology of "objective vs. subject" himself, but it seems fairly clear that this is integral to his thought on F=L. Let me expand on that point.

On the picture of theories under consideration, a theory-specific closure relation restricts the logical possibilities. The set of *theoretical possibilities* relative to a given theory T are the logical possibilities consonant with its closure relation $\vdash_T$. For example, we gave an example in §2 of a closure principle of physics that rules out the possibility of quarks that fail to be hadrons. This is theory-specific because it is only plays a role in theories in which physical concepts occur and no role in other theories. In other words, "quarks that fail to be hadrons" describes a situation that

[6] Some internally coherent mathematical theories are particularly dramatic. HA can be extended with the additional axiom known as Intuitionistic Church's Thesis to make HAC. There is also an intuitionistic approach to analysis with infinitesimal quantities known as Smooth Infinitesimal Analysis (SIA). Both the extended HAC and SIA prove *counter-classical* theorems of the form $\neg\forall x(\phi\vee\neg\phi)$. Thanks to an anonymous referee for reminding me that these theories pose yet another distinctive challenge to advocates of F=L and, especially, to advocates of C=L.

is theoretically impossible relative to physics, but theoretically possible relative to other theories (they *do not care* to rule it out). Theoretical possibilities vary from one theory to the next, but theorists are also aware that other theories might be true. This assumes an objective notion of possibility in the background, one which transcends the possibilities associated with any one specialized theory.

According to Beall (2018), p.31, Logic accounts for this by marking "the remotest boundaries of theoretical possibilities," i.e., *what could be true* in a theory-independent sense. This is why I call it objective in nature. At least, on my understanding, this topic is only interesting to the extent that it represents an impartial benchmark for theories rather than a classification of what is *acceptable* or *conceivable* or merely subjectively possible from the perspective of some agent or community.

While our discussion has mostly focused on the "completing" role of Logic in theories—perhaps its essential role—Logic just as much plays a "boundary policing" role with respect to theories. Rival accounts of Logic go hand-in-hand with rival accounts of where this boundary lies and, in particular, on how stringent they are about the range of objectively (logically) impossible theories.

One important contrast between the semantic models of FDE and CQL is how they handle "gaps" and "gluts" in theories.[7] A theory is gappy when it is silent on some pair of sentences ϕ and $\neg\phi$, i.e., the theory does not include either of these sentences in its closure. Call this a ϕ gap in the theory. A theory is glutty when it accepts both members of some pair of sentences ϕ and $\neg\phi$, i.e., the theory includes both of these sentences in its closure. Call that a ϕ glut in the theory.

If C=L, it is not logically possible to have (true, closed) theories with gaps or gluts, whereas both options are possible on the hypothesis that F=L. This is crucial to the Simple Argument. Beall (2019), p.122 says that there are "natural—and currently live-option—candidates for true theories of various extraordinary phenomena that appear, prima facie, to be 'gluts' or 'gaps'." He has in mind the kind of logic-centric solutions to truth-theoretic and property-theoretic paradoxes advanced by the likes of Priest (2006) and Field (2008). This looks very much like evidence in favor of F=L and, more damningly, it looks like decisive evidence against C=L.

To drive this home, Beall (2018), p.49 goes so far as to suggest that the theories of truth and paradox above "can be at best treated as mere doodles in conceptual space" by the advocate of C=L. They cannot even assert that those theories *might be true*! Frankly, that is a very bad look. If we were to accept that C=L, we would have to view certain influential philosophical theories as falling below a minimum threshold of objective possibility, hence "automatically out of bounds" as far as theories go. The advantage of F=L is that it does not impose such limitations.

4.2 Checking the benefits

When Beall says that we have "something to gain" by accepting that F=L, he means that "we gain live options for true theories" (Beall 2019, p.119). Many of the

[7] See also §3.1 on gappy and glutty semantic interpretations in FDE.

theories he has in mind fall short of classical closure. The hypothesis that F=L has a great advantage insofar as it classifies these theories as logical possibilities.

I offer a three stage reply to P2 starting with "live options." If this status entails that a theory is objectively possible, then this premise simply *begs the question* against C=L. On the other hand, if a live option is a kind of epistemic possibility, then such considerations seemingly have *no bearing* on our view of Logic. I then drill down on these themes. I argue that P2 requires the implausibly strong assumption that there cannot be an epistemically possible yet logically impossible theory.

When we say that *T* is a live option, we seem to be saying something like this: *T* engages with ongoing discussions in its discipline, it makes productive contributions to those discussions, and it is taken seriously by discussants in that domain. Many theories fall below that threshold. The interesting question is how this relates to the other threshold we discussed in this section, viz. our background conception of logical possibility or "the remotest boundaries of theoretical possibility."

To digress briefly, one glaring problem with the strategy of P2 is that it generalizes to support conclusions incompatible with Beall's. Many theories are live options in the sense above, but when they have very different internal structure from FDE they suggest conflicting views on Logic. We mentioned the example of HA already, which fits the bill. This is certainly a live option in its own domain of inquiry, so it challenges the thesis that C=L as much as any other theory. We can then apply Beall's strategy: assert that HA is just "a doodle in conceptual space" to the advocates of C=L, and infer that we have much to gain from accepting that I=L. The reason this works is because the strategy revolves around a binary comparison between one sub-classical logic and classical logic. The problem is that *any* sub-classical logic *L* will have the same advantage in such a comparison, so long as there is *any* live option theory explicitly formulated with *L* as its intended, internal logic. Even if P2 is true, it is true for reasons that would generalize to lots of other logical systems, so we learn very little from this about the broader question of the real identity of Logic.

Putting that aside, however, I now want to return to the topic of live options and consider two interpretations of how this relates to logical possibility. On the first interpretation, when we *take theory T seriously* we regard *T* as an objective possibility, hence at the very least a logical possibility.[8] Call this the objective reading. If we interpret claims about live options in this way, then premise P2 is very obviously true, but only because it begs the question. After all, the regimented theories of truth and paradox marshaled in support of P2 are explicitly formulated with an intended, internal logic in the vicinity of FDE. On an objective reading, the claim that such a theory is a live option *just is* the claim that the space of logical possibilities is more expansive than the space of CQL models. In other words, the alleged evidence against C=L *is indistinguishable from* the brute denial of C=L.

We know that there are plenty of theories in metaphysics and philosophy of language that deploy non-standard logic to solve problems surrounding, e.g., vagueness

[8] This is not redundant. There are other kinds of objective possibilities: metaphysical possibilities are narrower than logical ones, physical possibilities are even narrower.

and composition. Despite the fact that such theories are taken seriously, there are skeptics who attest to having very low credence that such theories could be true. On the objective reading of live options, these people are confused, because as soon as one takes such theories seriously, one thereby rejects the standard account of logical possibilities. This is not very charitable. It suggests that these skeptics accept the view that C=L in one frame of mind and reject it in another.

For this reason, we may want to finesse our interpretation of live options. On the second interpretation, when we *take theory T seriously* we merely regard *T* as an epistemic possibility. Call this the subjective reading. Independently of any other considerations, this just seems like an overwhelming natural way to understand live options in a debate. The problem is that this retreat to a weaker interpretation opens a gulf between live options and logical possibilities. One lesson of logical omniscience is that the *epistemic content* of sentences is more fine-grained than their *truth-conditions* across objective possibilities. The range of epistemic possibilities for the average "bounded" agent is not reducible to a set of logical possibilities.[9]

There are several ways to flesh this out. First, consider an impossible world's semantics for individual, agent-relative epistemic possibilities (Jago 2014). On this approach, an agent can at once implicitly accept $\phi \vdash \psi$ as a logical consequence, yet at the same time entertain a *subtly* impossible world where ϕ holds and ψ as an epistemic possibility. Second, consider an expressivist account of epistemic modals on which "communal" epistemic possibilities arise from (weak) assertions of the form "perhaps..." (Incurvati and Schlöder 2021). On this approach, a group can at once implicitly accept $\phi \vdash \psi$ as a logical consequence, yet at the same time "block" the conversational common ground from being closed under this inference for the sake of *leaving open* certain questions. Both of these approaches say that rational agents sometimes countenance logically impossible theories as live options. This happens when a rationally supererogatory effort is required to bring the inference $\phi \vdash \psi$ to bear on the current epistemic context.

In some sense, the preceding discussion reinforces a gulf between live options and logical possibilities, but an ardent defender of the Simple Argument might be unmoved by this. The reason is that the discussion focused on *oversights*, i.e., contexts of inquiry in which Logic-violating theories are "permitted" due to the epistemic limitations of agents. But does that really pose a problem for P2 of the Simple Argument? Is the idea that *we, active participants in the philosophical discourse, are such agents who unwittingly admit live options that are logically impossible*?

No, that cannot be the end of the story. Interlocutors in the present debate are well aware that certain theories violate their favored views on Logic. Yet they still consider such theories live options. But how can this be? I offer a speculative account of theory-appraisal that accommodates this practice. In Caret (2021b), I floated an idea about deduction inspired by Field (2015). The idea is that deduction is sometimes a simulacrum of "real" inference, one which allows for a kind of pretense detached from the aim of truth. I envision that this kind of "off-line" deduction functions by slavish conformity to the internal "rules of the game" of a given theory. This grasp of structure

[9] See Jago (2014) and Caret (2022) on the indeterminacy of logical closure of knowledge states.

may underpin an important epistemic achievement, viz. structural understanding of a theory (Kvanvig, 2003). The stronger claim I want to make is that theory-appraisal can sometimes emanate from nothing more than such understanding.

The question is how agents could remain genuinely open-minded toward the possibility of accepting a theory that is logically impossible by their own lights. Can structural understanding facilitate *rational change of view* when the structure itself violates what we deem to be Logic? I think this is possible in part because logical concepts are not aspects of reality that we find "fully formed" prior to theorizing about them. With Glanzberg (2015), I take individual or communal views on Logic to actively involve abstraction from natural language. Our views on Logic should, thus, evolve by a holistic calculus, somewhat as Carnap (1937) believed. Coming to understand a theory that deploys "not my logic" can be transformative—providing insight into a different way of seeing the world—which may *appeal* to me as fruitful or simply interesting in some way. The more this theory appeals to me, the more it erodes credence in my original view of Logic. As such understanding accrues, I may feel the bounds of logical possibility shift beneath my feet.

Change of view about Logic is not inevitable when we engage with theories that use non-standard logics. Far from it. The point is simply that *it can occur*. Even if a theory is logically impossible by our lights, we can still fairly appraise it and take it just as seriously as anything else. Beall says that FDE-based theories are "mere doodles in conceptual space" to agents who accept that C=L. He is partly right and partly wrong. Right because such agents believe that such theories cannot be true. At first glance, such a theory looks like an aimless "doodle." But he is wrong to suggest that an agent could never progress to a state of better understanding or even acceptance of the rival theory (with implicit rival view on Logic). So, the mere fact that FDE-based theories are live options is not a damning accusation against C=L after all.

These considerations show that P2 of the Simple Argument is likely wrong. For Beall, the claim that F=L has benefits amounts to the claim that it draws the boundary for live options in the right place. As we have seen, this claim either begs the question or it involves an unwarranted jump from epistemic possibility to logical possibility. Even for logically impossible theory T, we can assign content to claims that T might be true and make sense of rational appraisal of T. So, the grounds for treating a theory as a live option are completely unrelated to our current views on logical possibility. In particular, accepting that F=L is unnecessary to treating FDE-based theories as live options. F=L seemingly has nothing to offer after all.

5 Reflections

The weakness with the Simple Argument stems from the fact that it plays things too safe and it often leaves too little conceptual space between its starting point and its end goals. The claim that F=L is only compared to C=L; as a result, some of the salient costs of the view are hidden from sight. The observation that there are live option theories using non-standard logics almost looks like a veiled assertion that the standard account of logical possibilities is wrong. Since this is only meant to be a head-to-head comparison in the first place, this assertion is the same as the conclusion it is meant to support. We cannot make meaningful progress this way.

It reminds me of what Lewis (2004) once said about engaging in debate over the law of non-contradiction: "My feeling is that since this debate instantly reaches deadlock, there's really nothing much to say about it...[T]he principles *not* in dispute are so very much less certain than noncontradiction itself that it matters little whether or not a successful defence of non-contradiction could be based on them." Does that mean there is no way to progress on debate about Logic? Are appeals to theory-building practice useless? Was the Simple Argument hopeless from the start?

I do not think so. In fact, I sincerely think the argument has the right outlook. Some of my objections to P1 and P2 are quibbles about the specific formulation of these claims. If, however, we set aside the choice of FDE or even set aside the assumption of *logical monism*, there is a productive strategy here.

I am optimistic about a *practice-based* philosophy of logic which proceeds by steps roughly as follows. First, we identify a normative role of theory closure. This would be something like a bridge principle that says: if one reason about theory T internally closed under $\phi \vdash_T \psi$, then it is incorrect to suppose that ϕ while resisting that ψ.[10] What is needed at this first step is a normative story, but it does not have to be the specific one above. That is just an example. Second, we need to operationalize "logical expertise" by deciding who or what counts as paradigmatically conforming to Logic. Nothing guarantees that we are getting things right here. This is conjectural. But, for example, it is reasonable to believe that mathematical proofs are logically correct, hence the mathematical community count as experts for our purposes. Once we put these together, we have the following sort of "test" for logical theory: if a logical system judges that $\phi \vdash \psi$ is a logical consequence, this predicates that when members of the "expert community" reason about their theories, they never suppose that ϕ while resisting that ψ. We can devise a number of ways to check this. How do the experts actually argue? What is the logical structure of their best theories? The point is that this makes clear what we are looking for: do they ever accept ϕ and reject ψ? If so, this would count as evidence against a system that judges that $\phi \vdash \psi$ is a logical consequence. I have alluded to this method in Caret (2021), b, although I think my explanation of the method is clearer now than it was back then.

This could lead us to F=L just as Beall says, but it would take substantially more work to make the case in the way and that is only a *good thing*. It is not helpful to appeal to regimented, formal philosophical theories to support philosophical conclusions about Logic because the two are so inextricably intertwined. Someone who works extensively on FDE-based theories is already disposed to have a higher credence in F=L than others. That makes it hard to reach common ground. What is wanted is a source of evidence that is independent of one's own logical agenda, and that is what a practice-based approach offers. I think this approach can be read into the explanatory arguments for *logical pluralism* offered by Shapiro (2014) (more to the point, for present purposes, reading his arguments against classical monism which is very much the top priority of the Simple Argument as well).

10 For related discussion on the normativity of logic, see Steinberger (2019).

I take no issue with Beall's contention that F=L. He may be right about that. I think that some sort of argument drawing on extra-philosophical data could eventually make a strong case for this view or another sub-classical view of Logic. The practice-based approach offers the best way to make such debates substantive and productive.

6 Conclusion

The Simple Argument that F=L offers an attractive picture: that FDE affords the resources to explain everything we want Logic to explain and it provides methodological benefits we cannot get anywhere else. The cost-benefit analysis is clear as day. The problem is that both premises of this argument are mistaken.

The advocate of F=L cannot explain intuitionistic closure, which forces them to say that there are no (true) intuitionistic theories. That is a heavy cost. The hypothesis that F=L also seems unnecessary simply for the sake of acknowledging that FDE-based theories are live options. The alleged benefit is missing.

Despite all of this, the simplicity and clarity of the argument are admirable. It is only because of this that we can learn so much by dissecting the argument and thinking through its components. For that, we can only be grateful.

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Declarations

Competing interest The authors declare no competing interests.

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