PHYSICAL REVIEW D **80,** 046005 (2009)

Fermionic reductions of the $AdS_4 \times \mathbb{CP}^3$ superstring

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(Received 19 June 2009; published 18 August 2009)

We discuss fermionic reductions of type IIA superstrings on $AdS_4 \times \mathbb{CP}^3$ in relation to the conjectured AdS_4/CFT_3 duality. The superstring theory is described by means of a coset model construction, which is classically integrable. We discuss the global light-cone symmetries of the action and related κ-symmetry gauge choices, and also present the complete quartic action in covariant form with respect to these. Further, we study integrable (fermionic) reductions, in particular, a reduction yielding a quadratic action of two complex fermions on the string world-sheet. Interestingly, this model appears to be exactly the same as the corresponding integrable reduction found in the $AdS_5 \times S^5$ case.

DOI: 10.1103/PhysRevD.80.046005 PACS numbers: 11.25.Tq

I. INTRODUCTION

Recently Aharony, Bergman, Jafferis, and Maldacena conjectured that $\mathcal{N} = 6$ supersymmetric Chern-Simons theory in three dimensions has a holographic dual which at strong coupling can be effectively described by type IIA superstrings on $AdS_4 \times \mathbb{CP}^3$ background [1].

It appears that the Green-Schwarz superstring on the $AdS_4 \times \mathbb{CP}^3$ background admits a description in terms of a coset sigma model, quite analogous to strings on $AdS_5 \times S^5$. Normally the Green-Schwarz action contains 32 fermions, only half of which are physical due to κ-symmetry, whereas this coset model, by construction, contains 24 fermions. This apparent mismatch can be understood by considering the coset model to be a partially κ-symmetry fixed version of the Green-Schwarz action, and, in accordance, the coset model possesses local fermionic symmetry that reduces the number of physical fermions to 16. The $AdS_4 \times \mathbb{CP}^3$ background is diffeomorphic to the coset space $SO(3,2)/SO(3,1) \times SO(6)/U(3)$. The coset numerator $SO(3,2) \times SO(6)$ is the bosonic subgroup of $OSP(2,2|6)$, which suggests that fermions can be included in the theory by considering the coset space $OSP(2,2|6)/(SO(3,1) \times U(3))$.

An important advantage of the coset sigma model construction is that it exhibits manifest integrability. Analogously to the $AdS_5 \times S^5$ case [2], classical integrability of this model can be concluded immediately from the existence of the corresponding Lax pair [3,4]; the corresponding algebraic curve encoding solutions of classical equations of motion has been obtained in [5]. We also note that recently the full Green-Schwarz action for strings on $AdS_4 \times \mathbb{CP}^3$ has been constructed, but the integrable properties of this formulation still remain an open issue [6,7]. The pure spinor formulation for this background has been explored in [8].

The question of quantum integrability is rather subtle, especially, taking into account that classical integrability of the $\mathbb{CP}^3$ bosonic model is known to be spoiled by quantum corrections. Following an analogous construction for the $AdS_5 \times S^5$, and the leading perturbative result found in field theory [9] (see also [10–13]), an all-loop asymptotic Bethe ansatz has been proposed in [14]. It would be important to find further support for its validity from direct field-theoretic computations on the string world-sheet, following the lines, e.g., in [15–23,26,27].

In order to shed some more light on the integrability of superstrings on the $AdS_4 \times \mathbb{CP}^3$ background, in this note we will investigate possible consistent truncations of the string sigma model. This problem has already received some attention in the bosonic sector of the theory [26], but since fermions are expected to play an important role in maintaining quantum integrability, we will consider consistent *fermionic* reductions of the superstring Lagrangian.

Our treatment starts with describing a particular parametrization of the coset element $SO(3,2)/SO(3,1) \times SO(6)/U(3)$ suitable for the light-cone gauge fixing [27]. We then identify the manifest bosonic symmetry group of the light-cone Lagrangian, which appears to be in agreement with earlier findings in [18,21,22]. Further we determine the specific form of the κ-symmetry parameter for our current coset parametrization based on the analysis of κ-symmetry presented in [3]. We then proceed to fix a κ-symmetry gauge compatible with these bosonic symmetries. Next we present the complete sigma model Lagrangian in the expansion up to quartic order in fields, given in a manifestly covariant form with respect to the manifest bosonic symmetries. This Lagrangian can be used to compute the full tree level world-sheet S-matrix. Finally, we discuss a consistent truncation of the full Lagrangian down to a model containing two complex fermions which, being a consistent truncation, maintains integrability inherited from the full model. Interestingly, the emerging reduced model appears to be completely equivalent to an integrable system arising under the corresponding consistent truncation of the $AdS_5 \times S^5$ superstring [28,29].

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II. SIGMA MODEL LAGRANGIAN

The coset sigma model is based on a coset element that parametrizes a space, such that the fields take values in a nonlinear way on a chosen manifold, and does not require the knowledge of the Green-Schwarz action. In particular the $AdS_4 \times \mathbb{CP}^3$ coset sigma model is constructed on the coset space

$$\frac{OSP(2,2|6)}{SO(3,1) \times U(3)},$$

where we note that USP(2, 2), a bosonic subgroup of OSP(2, 2|6), is locally isomorphic to SO(3, 2). The complete construction of this model is discussed in [3].

A. Superalgebra $\mathfrak{osp}(2,2|6)$ and $\mathbb{Z}_4$-grading

An arbitrary element belonging to the $\mathfrak{osp}(2,2|6)$ superalgebra is given by a 10×10 matrix

$$A = \begin{pmatrix} X & \theta \\ \eta & Y \end{pmatrix},$$

where X and Y are 4×4 and 6×6 matrices containing bosonic fields, and η and θ are 6×4 and 4×6 blocks containing fermionic fields, respectively, where A has to obey the following two conditions

$$A^{st}\begin{pmatrix} C_4 & 0 \\ 0 & \mathbb{I}_{6\times6} \end{pmatrix} + \begin{pmatrix} C_4 & 0 \\ 0 & \mathbb{I}_{6\times6} \end{pmatrix} A = 0, \tag{2.1}$$

$$A^{\dagger}\begin{pmatrix} \Gamma^0 & 0 \\ 0 & -\mathbb{I}_{6\times6} \end{pmatrix} + \begin{pmatrix} \Gamma^0 & 0 \\ 0 & -\mathbb{I}_{6\times6} \end{pmatrix} A = 0. \tag{2.2}$$

Here C_4 denotes an arbitrary real, skew symmetric, charge conjugation matrix satisfying $C_4^2 = -\mathbb{I}$. For purpose of this paper we have chosen $C_4 = i\Gamma_0\Gamma_3$. Additionally we define the supertranspose of a matrix to be

$$A^{st} = \begin{pmatrix} X^t & -\eta^t \\ \theta^t & Y^t \end{pmatrix}.$$

For the odd blocks of A, Eqs. (2.1) and (2.2) imply the following fermionic transposition and reality conditions,

$$\eta = -\theta^t C_4, \qquad \theta^* = i\Gamma^3\theta.$$

These reduce the number of fermionic degrees of freedom from 48 complex, down to 24 real. This algebra possesses a fourth order outer automorphism

$$\Omega(A) = \Upsilon A \Upsilon^{-1},$$

where the matrix Υ is given in the Appendix A. This gives the algebra a $\mathbb{Z}_4$ grading

$$\mathcal{A} = \mathcal{A}^{(0)} \oplus \mathcal{A}^{(1)} \oplus \mathcal{A}^{(2)} \oplus \mathcal{A}^{(3)},$$

such that $[\mathcal{A}^{(k)}, \mathcal{A}^{(m)}] \subseteq \mathcal{A}^{(k+m)}$ modulo $\mathbb{Z}_4$, and where the subspace $\mathcal{A}^{(k)}$ is an eigenspace of the map Ω

$$\Omega(\mathcal{A}^{(k)}) = i^k \mathcal{A}^{(k)}.$$

In particular, the stationary subalgebra of Ω is determined by the conditions

$$[\Gamma^5, X] = 0, \qquad [K_6, Y] = 0,$$

where $\Gamma^5 = -i\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = K_4C_4$ is a matrix defined in terms of anticommuting Γ^i matrices used to parametrize the AdS_4 space. The space $A^{(2)}$ is spanned by matrices satisfying

$$\Omega(A) = \Upsilon A \Upsilon^{-1} = -A.$$

This implies the following conditions

$$\{X, \Gamma^5\} = 0 \qquad \{Y, K_6\} = 0,$$

which are solved by

$$X = x_\mu \Gamma^\mu, \qquad Y = y_i T_i,$$

providing a parametrization of the coset space of $AdS_4 = \frac{SO(3,2)}{SO(3,1)}$ and $\mathbb{CP}^3 = \frac{SO(6)}{U(3)}$ respectively. The two sets of matrices Γ^μ and T_i are given in Appendix A.

B. The Lagrangian

To further progress in the construction of the Lagrangian, let us use an element of the coset g to build the following current (one-form)

$$A = -g^{-1}dg = A^{(0)} + A^{(2)} + A^{(1)} + A^{(3)},$$

where on the right hand side we exhibited its $\mathbb{Z}_4$-decomposition. By construction A has vanishing curvature

$$\partial_\alpha A_\beta - \partial_\beta A_\alpha - [A_\alpha, A_\beta] = 0.$$

The sigma model is given by the following action

$$S = -\frac{\sqrt{\lambda}}{4\pi}\int d\tau d\sigma \mathcal{L},$$

where λ is the 't Hooft coupling constant, related to the AdS radius as $\sqrt{\lambda} = R^2/\alpha'$. The Lagrangian density $\mathcal{L}$ is the sum of the kinetic and the Wess-Zumino terms

$$\mathcal{L} = \gamma^{\alpha\beta}\mathrm{str}(A_\alpha^{(2)}A_\beta^{(2)}) + \kappa\epsilon^{\alpha\beta}\mathrm{str}(A_\alpha^{(1)}A_\beta^{(3)}). \tag{2.3}$$

where str denotes the supertrace, $\gamma^{\alpha\beta}$ is the Weyl-invariant world-sheet metric with $\det\gamma = -1$, and $\epsilon^{\tau\sigma} = 1$. The parameter κ in front of the Wess-Zumino term is kept arbitrary, however the requirement of κ-symmetry and integrability will fix it to $\kappa = \pm 1$.

The equations of motion derived from (2.3) are

$$\partial_\alpha \Lambda^\alpha - [A_\alpha, \Lambda^\alpha] = 0, \tag{2.4}$$

where we have defined,

$$\Lambda^\alpha = \gamma^{\alpha\beta}A_\beta^{(2)} - \frac{1}{2}\kappa\epsilon^{\alpha\beta}(A_\beta^{(1)} - A_\beta^{(3)}).$$

For further details see [3].

C. Kappa symmetry

Kappa symmetry is a well known local fermionic symmetry in string theory. In the case of type IIA Green-Schwarz string theory it typically allows one to gauge away 16 of the 32 fermionic degrees of freedom. As was argued in [3], our coset model is to be understood as a Green-Schwarz string theory with κ-symmetry *partially* fixed. In accordance with this picture, there is a local fermionic symmetry in this coset model that allows one to gauge away precisely the eight fermionic degrees of freedom needed.

Contrary to the global bosonic symmetries which are realized by multiplication from the left, κ-symmetry is here understood as the right local action of a fermionic element $G = \exp\epsilon$ from OSP(2, 2|6) on our coset representative g [3]:

$$gG(\epsilon) = g' g_c$$

where $\epsilon \equiv \epsilon(\tau, \sigma)$ is a local fermionic parameter, and g_c is a compensating element from $SO(3,1) \times U(3)$. The action is not invariant under arbitrary variations of this form. In order for this to be the case, the fermionic parameter has to be of a form presented below, and the whole transformation will have to be accompanied by a transformation of the metric.

The fermionic parameter ϵ can of course be decomposed into an element of degree one, $\epsilon^{(1)}$, and one of degree three, $\epsilon^{(3)}$. The transformation above is a symmetry of the action, provided the following form of the κ-symmetry parameter $\epsilon^{(1)}$

$$\epsilon^{(1)} = A^{(2)}_{\alpha,-} A^{(2)}_{\beta,-} \kappa^{\alpha\beta}_{++} + \kappa^{\alpha\beta}_{++} A^{(2)}_{\alpha,-} A^{(2)}_{\beta,-} + A^{(2)}_{\alpha,-} \kappa^{\alpha\beta}_{++} A^{(2)}_{\beta,-} - \frac{1}{8} \mathrm{str}(\Sigma A^{(2)}_{\alpha,-} A^{(2)}_{\beta,-}) \kappa^{\alpha\beta}_{++}, \tag{2.5}$$

where $\kappa^{\alpha\beta}_{++}$ is the κ-symmetry parameter which is assumed to be independent of the dynamical fields of the model. Obviously, $\kappa^{\alpha\beta}_{++}$ must be an element of $\mathfrak{osp}(2,2|6)$, and $\epsilon^{(1)} \in \mathcal{A}^{(1)}$ provided $\kappa^{\alpha\beta}_{++}$ is. The form of the κ-symmetry parameter $\epsilon^{(3)}$ is analogous. The accompanying variation of the metric is then given by

$$\delta\gamma^{\alpha\beta} = \frac{1}{2} \mathrm{str}(\Sigma A^{(2)}_{\delta,-} [\kappa^{\alpha\beta}_{++}, A^{(1),\delta}_{+}]) + \frac{1}{2} \mathrm{str}(\Sigma A^{(2)}_{\delta,+} [\varkappa^{\alpha\beta}_{--}, A^{(3),\delta}_{-}]),$$

where $\varkappa^{\alpha\beta}_{--} \subset \mathcal{A}^{(3)}$ is another independent κ-symmetry parameter that comes in with $\epsilon^{(3)}$. Finally, along with the above transformations, the parameter κ in the Lagrangian is required to be equal to plus or minus one to guarantee invariance of the action. For details on the derivation of κ-symmetry and the explicit form of the κ-symmetry parameter ϵ, see [3].

This symmetry can be used to gauge away eight fermionic degrees of freedom. The question then is what the best suited gauge choice is, or equivalently, what the most convenient form of the gauge fixed fermionic coset element is. In the next sections we will discuss the manifest bosonic symmetry of the light-cone gauge fixed action for our choice of coset parametrization,[1] and its action on the bosonic and fermionic fields. Ideally one would like to choose a κ-symmetry gauge that is preserved under the action of the complete manifest bosonic symmetry group. We will present one such choice, directly after discussing the bosonic symmetries of course.

III. κ-GAUGE FIXED LAGRANGIAN

A. Coset paramtrization, symmetries and κ-gauge choice

In anticipation of imposing a uniform light-cone gauge, with the light-cone coordinates taken to be combinations of the AdS time variable t and the $\mathbb{CP}^3$ spherical coordinate angle ϕ, we take the following choice of coset parametrization

$$g = \Lambda(t, \phi) g_\chi g_B, \tag{3.1}$$

where

$$\Lambda(t,\phi) = \begin{pmatrix} e^{(i/2)t\Gamma^0} & 0 \\ 0 & e^{-(\phi/2)(T_{34}+T_{56})} \end{pmatrix}, \quad g_B = \begin{pmatrix} e^{ix_i\Gamma^i} & 0 \\ 0 & e^{iy_jT_j} \end{pmatrix},$$

and

$$g(\chi) = \exp(\chi), \quad \text{for } \chi = \begin{pmatrix} 0 & \kappa \\ -\kappa^t C_4 & 0 \end{pmatrix}.$$

With this choice of parametrization shifts in t and ϕ are realized linearly. Therefore this choice makes sure that all fermions and the other bosons remain unchanged under the action of group elements corresponding to shifts in t and ϕ, which is a desirable feature when planning to impose a uniform light-cone gauge [16].

The choice of the matrix $T_{34} + T_{56}$ might be unexpected, but it is of course exactly this matrix that generates shifts in ϕ on the full $\mathbb{CP}^3$ coset element. Further details on this and the parametrization of $\mathbb{CP}^3$ employed here are discussed in Appendix A.

B. Linearly realized bosonic symmetries

The global bosonic symmetry group $USP(2,2) \times SO(6)$ acts on the coset element by multiplication from the left. For $G \in USP(2,2) \times SO(6)$ we have

$$Gg = g' g_c,$$

where g_c is a compensating transformation from

[1]The precise form of the κ-symmetry parameter is affected by this choice of parametrization, as discussed in the appendix.

$SO(3,1) \times U(3)$. Different coset parametrizations have consequences as to which symmetries are linearly realized, and on which fields they are linearly realized. For example for the coset parametrization $g = g_\chi g_B$, the fermions undergo an adjoint linear action by G.

While our choice of coset parametrization is well suited for imposition of a uniform light-cone gauge, it does not allow for a linear realization of all bosonic symmetries. Here we will determine the symmetries that are linearly realized, which will then form the manifest bosonic symmetry group of the light-cone gauge fixed Lagrangian.

The linearly realized symmetries will be given by the subgroup of $USP(2,2) \times SO(6)$ forming the centralizer of group elements corresponding to the U(1) isometries of shifts in t and ϕ. The Lie algebra centralizer of these $\mathfrak{u}(1)$-isometries in $\mathfrak{usp}(2,2) \oplus \mathfrak{so}(6)$ is easily found to be the subalgebra

$$\mathfrak{C} = \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1), \tag{3.2}$$

in agreement with [18,21,22], where this was found for different (coset) parametrizations. Here the first factor corresponds to $\mathfrak{su}(2) \subset \mathfrak{usp}(2,2)$, while the second and third correspond to $\mathfrak{su}(2) \oplus \mathfrak{u}(1) \subset \mathfrak{so}(6)$. This stems from the fact that in the AdS sector the centralizer is spanned by the Γ^{ij} for i, $j = 1, 2, 3$, since these all commute with Γ^0, forming a four dimensional reducible representation of the Lie algebra $\mathfrak{su}(2)$, while in the $\mathbb{CP}$ sector it is spanned by the set $\{T_{12}, (T_{45} - T_{36}), (T_{34} - T_{56}), (T_{35} + T_{46})\}$, commuting with $T_{34} + T_{56}$, forming a reducible representation of the Lie algebra $\mathfrak{su}(2) \oplus \mathfrak{u}(1)$.

Note that a group element, G, generated by elements of the algebra, (3.2), has the following action on the coset element

$$Gg = \Lambda G g_\chi G^{-1} G g_B G^{-1} G,$$

since $G\Lambda G^{-1} = \Lambda$. As both the centralizer subalgebras $\mathfrak{su}(2)$ and $\mathfrak{su}(2) \oplus \mathfrak{u}(1)$ are in fact subalgebras of the coset denominator Lie algebras, we see that we can recognize the last G above as nothing else than a compensating transformation from $SO(3,1) \times U(3)$. Thus we see that the element G has a linear adjoint action on the fermionic coset element g_χ and the bosonic element g_B.

While the four dimensional reducible representation of SU(2) in the AdS sector is, by our convention for the gamma matrices, in canonical direct sum form (see below), the set of T-matrices above does not present a reducible representation in such canonical form. By doing a change of basis that diagonalizes T_{12}, T_{34}, and T_{56}, we obtain the more pleasant canonical direct sum representation of our symmetry group. Labelling the set of T-matrices introduced above as $\{a_0, a_1, a_2, a_3\}$, we have

$$a_0 \to \tilde{a}_0 = S a_0 S^{-1} = \mathrm{diag}(i\sigma_3, 0, 0),$$
$$a_i \to \tilde{a}_i = \mathrm{diag}\left(0, \frac{i\sigma_i}{2}, \frac{i\sigma_i}{2}\right),$$

where the σ_i are the Pauli matrices, and for completeness

$$S = \frac{1}{2}\begin{pmatrix} i\sqrt{2} & \sqrt{2} & 0 & 0 & 0 & 0 \\ -i\sqrt{2} & \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & i & i & 1 \\ 0 & 0 & -i & -1 & 1 & -i \\ 0 & 0 & 1 & i & -i & 1 \\ 0 & 0 & -i & 1 & 1 & i \end{pmatrix}. \tag{3.3}$$

In terms of two by two blocks, a group element, G, of $SU(2) \times SU(2) \times U(1)$ in this basis then takes the form

$$G = \begin{pmatrix} \mathfrak{g}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathfrak{g}_1 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\mathfrak{g}}_2 & 0 & 0 \\ 0 & 0 & 0 & \mathfrak{g}_3 & 0 \\ 0 & 0 & 0 & 0 & \mathfrak{g}_3 \end{pmatrix}. \tag{3.4}$$

Here $\mathfrak{g}_1$ is an element of the first SU(2), $\tilde{\mathfrak{g}}_2 = \mathrm{diag}(\mathfrak{g}_2, \mathfrak{g}_2^{-1})$ with $\mathfrak{g}_2$ an element of U(1), and $\mathfrak{g}_3$ is an element of the second SU(2).

To pick a κ-symmetry gauge choice that is manifestly compatible with the above bosonic symmetries, let us first write the fermionic element after the above change of basis and upon imposition of the reality condition, $\tilde{\theta}$, as

$$\tilde{\theta} = \frac{1}{2}\begin{pmatrix} \kappa_1 & \kappa_3 & \kappa_5 \\ \kappa_2 & \kappa_4 & \kappa_6 \end{pmatrix},$$

where

$$\kappa_1 = \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} = (\bar{\varkappa}_1, \varkappa_1),$$
$$\kappa_2 = \begin{pmatrix} -f_4^* & -f_3^* \\ f_2^* & f_1^* \end{pmatrix} = (\bar{\varkappa}_2, \varkappa_2) \qquad \kappa_3 = \begin{pmatrix} f_5 & f_6 \\ f_7 & f_8 \end{pmatrix},$$
$$\kappa_6 = -i\sigma_2 \kappa_3^* \sigma_2 = \begin{pmatrix} -if_8^* & if_7^* \\ if_6^* & -if_5^* \end{pmatrix},$$

and where we have labeled the fermions f_i. There are similar relations between κ_4 and κ_5.

The specific κ-symmetry parameter, presented in Appendix B, allows us to remove eight fermions by removing the blocks κ_4 and κ_5 from the fermionic element $\tilde{\theta}$ giving

$$\tilde{\theta}_\kappa = \frac{1}{2}\begin{pmatrix} \kappa_1 & \kappa_3 & 0 \\ \kappa_2 & 0 & \kappa_6 \end{pmatrix}. \tag{3.5}$$

This is a natural choice to make, and most importantly is manifestly compatible with the above bosonic symmetries. We thus consider this a convenient gauge choice and hence will fix it as such.

When the above change of basis is applied to the bosonic coset element it also induces a natural structure there, as seen for the fermionic sector already. The bosonic structure in the $\mathbb{CP}$-sector becomes

$$Y = \sum_{i=1}^{5} y_i T_i \to \tilde{Y} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0_{1\times 2} & -\tilde{y}^\dagger \\ 0 & 0 & -y^\dagger & 0_{1\times 2} \\ 0_{2\times 1} & y & 0_{2\times 2} & -i\sqrt{2} I_{2\times 2} y_5 \\ \tilde{y} & 0_{2\times 1} & -i\sqrt{2} I_{2\times 2} y_5 & 0_{2\times 2} \end{pmatrix}.$$

For completeness, the unaffected bosonic AdS sector is of the form

$$\sum_{i=1}^{3} x_i \Gamma^i = \begin{pmatrix} 0 & X \\ -X & 0 \end{pmatrix}.$$

In the above we have introduced the vectors and matrices

$$X = X^\dagger = \begin{pmatrix} x_1 & x_2 - ix_3 \\ x_2 + ix_3 & -x_1 \end{pmatrix}$$
$$y = \begin{pmatrix} iy_1 + y_2 + y_3 - iy_4 \\ -y_1 + iy_2 - iy_3 - y_4 \end{pmatrix}$$
$$\tilde{y} = \begin{pmatrix} iy_1 - y_2 + y_3 + iy_4 \\ y_1 + iy_2 + iy_3 - y_4 \end{pmatrix}.$$

This natural structure can now be used in the construction of a manifestly covariant Lagrangian, but for clarity let us first explicitly give the action of the manifest symmetry group on the matrices and vectors just introduced. With the group element G of the form (3.4), its action on the bosonic matrices defined above is explicitly given by

$$X \to \mathfrak{g}_1 X \mathfrak{g}_1^{-1}, \qquad y \to \mathfrak{g}_3 y \mathfrak{g}_2, \qquad \tilde{y} \to \mathfrak{g}_3 y \mathfrak{g}_2^{-1},$$

while its action on the fermionic blocks on the other hand is given by

$$\bar{\varkappa}_{1,2} \to \mathfrak{g}_1 \bar{\varkappa}_{1,2} \mathfrak{g}_2^{-1}, \qquad \varkappa_{1,2} \to \mathfrak{g}_1 \varkappa_{1,2} \mathfrak{g}_2, \qquad \kappa_{3,6} \to \mathfrak{g}_1 \kappa_{3,6} \mathfrak{g}_3^{-1}.$$

With these it is easy to construct various covariant couplings, which will be used in the next section.

C. Manifestly covariant Lagrangian

The explicit form of the Lagrangian, given by (2.3), calculated to quartic order in the fields, is here cast into a manifestly covariant form using the above introduced block structure. For presentation purposes the Lagrangian is split into the purely bosonic sector and the fermionic sector, where the Wess-Zumino term is presented separately. Concretely,

$$\mathcal{L} = \gamma^{\alpha\beta}(\mathcal{L}_b + \mathcal{L}_{bf} + \mathcal{L}_f)_{\alpha\beta} + \epsilon^{\alpha\beta}(\mathcal{L}_{bf\mathrm{WZ}} + \mathcal{L}_{f\mathrm{WZ}})_{\alpha\beta} \tag{3.6}$$

Because of the special character of the y_5 field, as mentioned above and discussed in Appendix A, it is expanded around $\frac{\pi}{4}$. Below we have written $v_5 = y_5 - \frac{\pi}{4}$.

To start with the purely bosonic Lagrangian, not expanded for sake of brevity, is given by

$$\begin{aligned}(\mathcal{L}_b)_{\alpha\beta} = & -\frac{1}{2}(\partial_\alpha t \partial_\beta t - \partial_\alpha \phi \partial_\beta \phi) - \frac{1}{2}\cosh 4\rho \partial_\alpha t \partial_\beta t \\ & - \frac{((\cos 4\psi + 3)\psi^4 - 8(\cos 2\psi + 2)\sin^2\psi y_5^2 \psi^2 + 8\sin^4\psi y_5^4)\partial_\alpha\phi\partial_\beta\phi}{8\psi^4} \\ & - \frac{i\sin^2\psi(\psi^2\cos^2\psi - \sin^2\psi y_5^2)\partial_\alpha\phi}{\psi^4}(\partial_\beta y^\dagger y - \mathrm{H.c.}) + \frac{(8\rho^2 - \cosh(4\rho) + 1)\partial_\alpha\rho\partial_\beta\rho}{2\rho^2} + \frac{\sinh^2 2\rho}{2\rho^2}\mathrm{Tr}(\partial_\alpha X \partial_\beta X) \\ & - \frac{4((\sin^2\psi - \psi^2)\partial_\alpha\psi\partial_\beta\psi - \sin^2\psi\partial_\alpha y_5 \partial_\beta y_5)}{\psi^2} + \frac{2\sin^2\psi}{\psi^2}(\partial_\alpha y^\dagger \partial_\beta y) + \frac{\sin^4\psi}{4\psi^4}(\partial_\alpha y^\dagger y - \mathrm{H.c.})(\partial_\beta y^\dagger y - \mathrm{H.c.}),\end{aligned}$$

where we have introduced the invariant quantities $\rho = \sqrt{x_i x^i}$ and $\psi = \sqrt{y_j y^j}$, with $i = 1, \ldots, 3$ and $j = 1, \ldots, 5$. In the above, H.c. denotes Hermitian conjugation of the term within the enclosing brackets; below where there are no brackets this hence stands for the Hermitian conjugate of the complete term. This bosonic Lagrangian can be compared to the one presented in [18,23] where the bosonic string was studied upon imposition of a uniform light-cone gauge; the following parts of the action should be of more novel interest. The boson-fermion interactions in the kinetic term of the Lagrangian are at quartic order given by (continued on the next page)

$$
\begin{aligned}
(\mathcal{L}_{bf})_{\alpha\beta} ={}& \frac{1}{2}\partial_\alpha t\partial_\beta t(1+2\,\mathrm{Tr}(X^2))\,\mathrm{Tr}(\kappa_1^\dagger\kappa_1) - 2i\partial_\alpha t\,\mathrm{Tr}(\kappa_1^\dagger X\partial_\beta X\kappa_1) + \frac{8i}{\pi^2}\partial_\alpha t y^\dagger\partial_\beta y(\bar\varkappa_1^\dagger\bar\varkappa_1 - \varkappa_1^\dagger\varkappa_1)\\
&+ \frac{1}{\sqrt{2}\pi^2}\partial_\alpha(2t+\phi)((2\pi-8\upsilon_5)(\partial_\beta\tilde y^\dagger\kappa_3^\dagger\bar\varkappa_1 - i\partial_\beta y^\dagger\kappa_3^\dagger\varkappa_1) + 4\pi(\upsilon_5\partial_\beta\tilde y^\dagger\kappa_3^\dagger\bar\varkappa_1 - i\partial_\beta\upsilon_5 y^\dagger\kappa_3^\dagger\varkappa_1)\\
&+ (-8\partial_\beta\upsilon_5 + i(-\pi+4\upsilon_5)\partial_\beta\phi)(\tilde y^\dagger\kappa_3^\dagger\bar\varkappa_1 - iy^\dagger\kappa_3^\dagger\varkappa_1) + 2\pi i\upsilon_5\partial_\beta\phi(\tilde y^\dagger\kappa_3^\dagger\bar\varkappa_1 + 2iy^\dagger\kappa_3^\dagger\varkappa_1))\\
&+ \partial_\alpha(t+\phi)\bigg(-2i\,\mathrm{Tr}(\kappa_3^\dagger X\partial_\beta X\kappa_3) + \bigg(\frac{1}{2}\partial_\beta t(1+2\,\mathrm{Tr}(X^2)) + \bigg(\frac{1}{2} - 2\upsilon_5^2 - \frac{2}{\pi^2}y^\dagger y\bigg)\partial_\beta\phi\bigg)\mathrm{Tr}(\kappa_3^\dagger\kappa_3)\\
&- \frac{4i}{\pi^2}(\partial_\beta y^\dagger\kappa_3^\dagger\kappa_3 y + \partial_\beta\tilde y^\dagger\kappa_3^\dagger\kappa_3\tilde y)\bigg) - 2\,\mathrm{Tr}(\kappa_1^\dagger(X\partial_\alpha X - \partial_\alpha XX)\partial_\beta\kappa_1) + i\partial_\alpha t(1+2\,\mathrm{Tr}(X^2))\,\mathrm{Tr}(\partial_\beta\kappa_1^\dagger\kappa_1)\\
&- 2\,\mathrm{Tr}(\kappa_1^\dagger\partial_\alpha(t+iX)\partial_\beta\kappa_2) + \frac{8}{\pi^2}(\partial_\alpha\varkappa_1^\dagger\varkappa_1(y^\dagger\partial_\beta y - \partial_\beta y^\dagger y) + \partial_\alpha\bar\varkappa_1^\dagger\bar\varkappa_1(\tilde y^\dagger\partial_\beta\tilde y - \partial_\beta\tilde y^\dagger\tilde y))\\
&+ \frac{2\sqrt{2}i}{\pi^2}(\pi + 2(\pi-2)\upsilon_5)\partial_\alpha\tilde y^\dagger(\partial_\beta\kappa_3^\dagger\bar\varkappa_1 - \kappa_3^\dagger\partial_\beta\bar\varkappa_1) + \frac{2\sqrt{2}}{\pi^2}(\pi - 4\upsilon_5)\partial_\alpha y^\dagger(\partial_\beta\kappa_3^\dagger\varkappa_1 - \kappa_3^\dagger\partial_\beta\varkappa_1)\\
&+ \frac{\sqrt{2}}{\pi^2}(-8i\partial_\alpha\upsilon_5 + (\pi - 2(2+\pi)\upsilon_5)\partial_\alpha\phi)\tilde y^\dagger(\partial_\beta\kappa_3^\dagger\bar\varkappa_1 - \kappa_3^\dagger\partial_\beta\bar\varkappa_1)\\
&+ \frac{\sqrt{2}}{\pi^2}(4(\pi-2)\partial_\alpha\upsilon_5 - (i\pi + 4(\pi-1)i\upsilon_5)\partial_\alpha\phi)y^\dagger(\partial_\beta\kappa_3^\dagger\varkappa_1 - \kappa_3^\dagger\partial_\beta\varkappa_1)\\
&+ i\bigg((1+2\,\mathrm{Tr}(X^2))\partial_\alpha t + \bigg(1 - 2\upsilon_5^2 - \frac{4}{\pi^2}y^\dagger y\bigg)\partial_\alpha\phi\bigg)\mathrm{Tr}(\partial_\beta\kappa_3^\dagger\kappa_3) - 2\,\mathrm{Tr}(\kappa_3^\dagger(X\partial_\alpha X - \partial_\alpha XX)\partial_\beta\kappa_3)\\
&+ \frac{8}{\pi^2}(y^\dagger\partial_\alpha\kappa_3^\dagger\kappa_3\partial_\beta y - \partial_\alpha y^\dagger\partial_\beta\kappa_3^\dagger\kappa_3 y) - \frac{4}{\pi^2}(y^\dagger\partial_\alpha y - \partial_\alpha y^\dagger y)\,\mathrm{Tr}(\kappa_3^\dagger\partial_\beta\kappa_3) + \text{H.c.}
\end{aligned}
$$

Here we note that couplings containing t or ϕ will upon imposition of a light-cone gauge reduce in order, the Lagrangian presented here contains all terms which afterwards will be of quartic order or lower. The rather unpleasant looking coefficients arise due to the expansion of y_5 around $\pi/4$, the terms from which they originate before expanding y_5 look quite natural. Continuing, the quartic fermionic couplings are given by (continued on the next page)

$$
\begin{aligned}
(\mathcal{L}_f)_{\alpha\beta} ={}& \frac{1}{4}\,\mathrm{Tr}(\partial_\alpha\kappa_2^\dagger\kappa_1\partial_\beta\kappa_2^\dagger\kappa_1 - \partial_\alpha\kappa_2^\dagger\partial_\beta\kappa_1\kappa_2^\dagger\kappa_1 + \partial_\alpha\kappa_1^\dagger\partial_\beta\kappa_2\kappa_2^\dagger\kappa_1 - \partial_\alpha\kappa_1^\dagger\kappa_1\partial_\beta\kappa_1^\dagger\kappa_1 + \partial_\alpha\kappa_1^\dagger\partial_\beta\kappa_1\kappa_1^\dagger\kappa_1\\
&- \partial_\alpha\kappa_2^\dagger\partial_\beta\kappa_2\kappa_1^\dagger\kappa_1 + \kappa_2^\dagger\partial_\alpha\kappa_2\kappa_1^\dagger\partial_\beta\kappa_1 - \kappa_2^\dagger\partial_\alpha\kappa_1\kappa_1^\dagger\partial_\beta\kappa_2) + \frac{1}{4}\,\mathrm{Tr}(\kappa_1^\dagger\partial_\alpha\kappa_3\partial_\beta\kappa_3^\dagger\kappa_1 + \partial_\alpha\kappa_1^\dagger\kappa_3\kappa_3^\dagger\partial_\beta\kappa_1\\
&- 4\partial_\alpha\kappa_1^\dagger\kappa_3\partial_\beta\kappa_3^\dagger\kappa_1 + 2\partial_\alpha\kappa_1^\dagger\partial_\beta\kappa_3\kappa_3^\dagger\kappa_1 + 2\kappa_2^\dagger\kappa_3\partial_\alpha\kappa_3^\dagger\partial_\beta\kappa_2 - 2\kappa_2^\dagger\partial_\alpha\kappa_3\kappa_3^\dagger\partial_\beta\kappa_2)\\
&+ \frac{i}{2}(\bar\varkappa_1^\dagger\partial_\alpha\kappa_3\partial_\beta\kappa_6^\dagger\bar\varkappa_2 + \partial_\alpha\bar\varkappa_1^\dagger\kappa_3\kappa_6^\dagger\partial_\beta\bar\varkappa_2 - \partial_\alpha\bar\varkappa_1^\dagger\kappa_3\partial_\beta\kappa_6^\dagger\bar\varkappa_2 - \bar\varkappa_1^\dagger\partial_\alpha\kappa_3\kappa_6^\dagger\partial_\beta\bar\varkappa_2)\\
&+ \frac{i}{6}\partial_\alpha t(\bar\varkappa_1^\dagger\bar\varkappa_1\bar\varkappa_1^\dagger\partial_\beta\bar\varkappa_1 + \varkappa_1^\dagger\varkappa_1\varkappa_1^\dagger\partial_\beta\varkappa_1 + 4(\bar\varkappa_1^\dagger\bar\varkappa_1\varkappa_1^\dagger\partial_\beta\varkappa_1 + \varkappa_1^\dagger\varkappa_1\bar\varkappa_1^\dagger\partial_\beta\bar\varkappa_1) - \bar\varkappa_1^\dagger\varkappa_1\varkappa_1^\dagger\partial_\beta\bar\varkappa_1 - \varkappa_1^\dagger\bar\varkappa_1\bar\varkappa_1^\dagger\partial_\beta\varkappa_1\\
&+ 8\,\mathrm{Tr}(\partial_\beta\kappa_1^\dagger\kappa_3\kappa_3^\dagger\kappa_1 - \kappa_1^\dagger\partial_\beta\kappa_3\kappa_3^\dagger\kappa_1) + 3\,\mathrm{Tr}(\kappa_2^\dagger\partial_\beta\kappa_3\kappa_3^\dagger\kappa_2)) + \frac{1}{4}\partial_\alpha(2t+\phi)(\varkappa_1^\dagger\partial_\beta\kappa_3\kappa_6^\dagger\varkappa_2 + \varkappa_1^\dagger\kappa_3\partial_\beta\kappa_6^\dagger\varkappa_2\\
&- \partial_\beta\varkappa_1^\dagger\kappa_3\kappa_6^\dagger\varkappa_2 - \varkappa_1^\dagger\kappa_3\kappa_6^\dagger\partial_\beta\varkappa_2) + i\partial_\alpha(t+\phi)\bigg(\frac{1}{2}\,\mathrm{Tr}(\partial_\beta\kappa_2^\dagger\kappa_3\kappa_3^\dagger\kappa_2) + \frac{1}{3}\,\mathrm{Tr}(\partial_\beta\kappa_3^\dagger\kappa_3\kappa_3^\dagger\kappa_3)\bigg)\\
&+ i\partial_\alpha\phi\,\mathrm{Tr}\bigg(\kappa_1^\dagger\kappa_3\kappa_3^\dagger\partial_\beta\kappa_1 + \frac{1}{3}\kappa_1^\dagger\partial_\beta\kappa_3\kappa_3^\dagger\kappa_1\bigg)\\
&+ \partial_\alpha t\partial_\beta t\bigg(\frac{1}{12}\,\mathrm{Tr}(2\kappa_1^\dagger\kappa_1\kappa_1^\dagger\kappa_1 + \kappa_1^\dagger\kappa_1\kappa_2^\dagger\kappa_2) + \frac{1}{8}\,\mathrm{Tr}(\kappa_1^\dagger\kappa_1)\,\mathrm{Tr}(\kappa_2^\dagger\kappa_2)\bigg)\\
&+ \frac{1}{4}\partial_\alpha(2t+\phi)\partial_\beta(2t+\phi)\bigg(\frac{i}{2}\bar\varkappa_2\kappa_6\kappa_3^\dagger\bar\varkappa_1 + \frac{7}{12}\,\mathrm{Tr}(\kappa_1^\dagger\kappa_3\kappa_3^\dagger\kappa_1) - \frac{1}{4}\,\mathrm{Tr}(\kappa_1^\dagger\kappa_1)\,\mathrm{Tr}(\kappa_3^\dagger\kappa_3)\bigg)\\
&+ \partial_\alpha\phi\partial_\beta\phi\bigg(-\frac{1}{24}\,\mathrm{Tr}(\kappa_1^\dagger\kappa_3\kappa_3^\dagger\kappa_1) + \frac{1}{16}\,\mathrm{Tr}(\kappa_1^\dagger\kappa_1)\,\mathrm{Tr}(\kappa_3^\dagger\kappa_3)\bigg) + \frac{1}{6}\partial_\alpha(t+\phi)\partial_\beta(t+\phi)\,\mathrm{Tr}(\kappa_3^\dagger\kappa_3\kappa_3^\dagger\kappa_3) + \text{H.c.}
\end{aligned}
$$

The asymmetry in these terms between t and ϕ results from the different dimensionalities of the AdS and $\mathbb{CP}^3$ spaces;

explicitly t can couple to all fermions directly, whereas ϕ cannot. Continuing, the contributions from the Wess-Zumino term are divided in the same fashion as the above terms, first we have the boson-fermion interaction terms

$$\begin{aligned}(\mathcal{L}_{bf}^{\rm WZ})_{\alpha\beta} = {} & \partial_\alpha t\partial_\beta\phi\frac{2\sqrt{2}}{\pi}(\bar{\varkappa}_1^\dagger X\kappa_3\tilde{y} - i\varkappa_1^\dagger X\kappa_3 y) - i(\mathrm{Tr}(X^2) - \tilde{y}^\dagger\tilde{y})\partial_\beta t(\partial_\alpha\bar{\varkappa}_1^\dagger\bar{\varkappa}_2 - \bar{\varkappa}_1^\dagger\partial_\alpha\bar{\varkappa}_2)\\ & + \frac{4\sqrt{2}}{\pi}(\partial_\beta t + \partial_\beta\phi)(\partial_\alpha\bar{\varkappa}_1^\dagger X\kappa_3\tilde{y} + i\partial_\alpha\varkappa_1^\dagger X\kappa_3 y) + \frac{4\sqrt{2}}{\pi}\partial_\beta t(\varkappa_1^\dagger X\partial_\alpha\kappa_3 y + i\bar{\varkappa}_1^\dagger X\partial_\alpha\kappa_3\tilde{y})\\ & - \frac{1}{4}(1 - 2v_5^2)\,\mathrm{Tr}(\kappa_6^\dagger\partial_\alpha\kappa_3)(\partial_\beta t + \partial_\beta\phi) + \frac{1}{2}i(\bar{\varkappa}_1^\dagger\partial_\alpha\bar{\varkappa}_2 - \varkappa_1^\dagger\partial_\alpha\varkappa_2 + \bar{\varkappa}_2^\dagger\partial_\alpha\bar{\varkappa}_1 - \varkappa_2^\dagger\partial_\alpha\varkappa_1)\partial_\beta t\\ & + 2v_5(\partial_\beta t + \partial_\beta\phi)\,\mathrm{Tr}(\partial_\alpha\kappa_3^\dagger X\kappa_3 + \partial_\alpha\kappa_6^\dagger X\kappa_6) - 2(\partial_\alpha\bar{\varkappa}_1^\dagger X\bar{\varkappa}_1 - \partial_\alpha\varkappa_1^\dagger X\varkappa_1)\partial_\beta t + \sqrt{2}i(\partial_\beta t + \partial_\beta\phi)(\partial_\alpha\varkappa_2^\dagger\kappa_3 y)\\ & - \sqrt{2}i\partial_\beta t(\varkappa_2^\dagger\partial_\alpha\kappa_3 y) - \left(\frac{2\sqrt{2}}{\pi} + \frac{4\sqrt{2}}{\pi^2}(\pi - 2)v_5\right)((\partial_\beta t + \partial_\beta\phi)\partial_\alpha\bar{\varkappa}_2^\dagger\kappa_3\tilde{y} - \partial_\beta t\bar{\varkappa}_2^\dagger\partial_\alpha\kappa_3\tilde{y})\\ & - i(1 - 2v_5^2)\,\mathrm{Tr}(\partial_\alpha\kappa_6^\dagger\partial_\beta\kappa_3) + \frac{1}{2}(\partial_\alpha\bar{\varkappa}_1^\dagger\partial_\beta\kappa_2 - \partial_\alpha\varkappa_1^\dagger\partial_\beta\varkappa_2) - \frac{1}{2}(\partial_\alpha\bar{\varkappa}_2^\dagger\partial_\beta\kappa_1 - \partial_\alpha\varkappa_2^\dagger\partial_\beta\varkappa_1)\\ & - 4iv_5\,\mathrm{Tr}(\partial_\alpha\kappa_3^\dagger X\partial_\beta\kappa_3) + 2i(\partial_\alpha\bar{\varkappa}_1^\dagger X\partial_\beta\bar{\varkappa}_1 - \partial_\alpha\varkappa_1^\dagger X\partial_\beta\varkappa_1) + \frac{4\sqrt{2}}{\pi^2}(\pi - 4v_5)(\partial_\alpha\bar{\varkappa}_1^\dagger\partial_\beta\kappa_6\tilde{y})\\ & + i\frac{4\sqrt{2}}{\pi^2}(\pi + 2(\pi - 2)v_5)(\partial_\alpha\bar{\varkappa}_1^\dagger\partial_\beta\kappa_6\tilde{y}) + \mathrm{H.c.}\end{aligned}$$

As the final piece, the fermionic quartic terms are given by

$$\begin{aligned}(\mathcal{L}_{f}^{\rm WZ})_{\alpha\beta} = {} & -\frac{1}{8}\partial_\alpha t\partial_\beta\phi((\varkappa_1^\dagger\kappa_6\kappa_6^\dagger\varkappa_2 - \bar{\varkappa}_1^\dagger\kappa_6\kappa_6^\dagger\bar{\varkappa}_2) + i(\bar{\varkappa}_1^\dagger\kappa_3\kappa_6^\dagger\bar{\varkappa}_1 + \varkappa_1^\dagger\kappa_3\kappa_6^\dagger\varkappa_1)) + \frac{5}{6}i(\varkappa_2^\dagger\varkappa_1\varkappa_2^\dagger\partial_\alpha\varkappa_2 - \bar{\varkappa}_2^\dagger\bar{\varkappa}_1\bar{\varkappa}_2^\dagger\partial_\alpha\kappa_2)\partial_\beta t\\ & + \frac{1}{6}i(\varkappa_2^\dagger\partial_\alpha\varkappa_1\varkappa_1^\dagger\varkappa_1 - \bar{\varkappa}_2^\dagger\partial_\alpha\bar{\varkappa}_1\bar{\varkappa}_1^\dagger\bar{\varkappa}_1)\partial_\beta t + \frac{7}{6}i(\partial_\alpha\bar{\varkappa}_2^\dagger\bar{\varkappa}_1\varkappa_2^\dagger\varkappa_2 - \partial_\alpha\varkappa_2^\dagger\varkappa_1\bar{\varkappa}_2^\dagger\bar{\varkappa}_2)\partial_\beta t\\ & + \frac{1}{6}(\varkappa_2^\dagger\kappa_3\partial_\alpha\kappa_6^\dagger\varkappa_2 + \bar{\varkappa}_2^\dagger\kappa_3\partial_\alpha\kappa_6^\dagger\bar{\varkappa}_2)(\partial_\beta t - \partial_\beta\phi) - \frac{1}{3}i(\bar{\varkappa}_2^\dagger\partial_\alpha\kappa_6\kappa_6^\dagger\bar{\varkappa}_1 - \varkappa_2^\dagger\partial_\alpha\kappa_6\kappa_6^\dagger\varkappa_1)\partial_\beta t\\ & + \frac{1}{6}i(\bar{\varkappa}_2^\dagger\partial_\alpha\kappa_3\kappa_3^\dagger\bar{\varkappa}_1 - \varkappa_2^\dagger\partial_\alpha\kappa_3\kappa_3^\dagger\varkappa_1)\partial_\beta t + \frac{1}{12}(\varkappa_2^\dagger\partial_\alpha\kappa_3\kappa_6^\dagger\varkappa_1 - \bar{\varkappa}_2^\dagger\partial_\alpha\kappa_3\kappa_6^\dagger\bar{\varkappa}_2)(8\partial_\beta t - 5\partial_\beta t)\\ & + \frac{1}{6}(\partial_\beta t + \partial_\beta\phi)(2(\partial_\alpha\bar{\varkappa}_1^\dagger\kappa_3\kappa_6^\dagger\bar{\varkappa}_1 + \partial_\alpha\varkappa_1^\dagger\kappa_3\kappa_6^\dagger\varkappa_1) - (\partial_\alpha\bar{\varkappa}_1^\dagger\kappa_6\kappa_3^\dagger\bar{\varkappa}_1 + \partial_\alpha\varkappa_1^\dagger\kappa_6\kappa_3^\dagger\varkappa_1))\\ & + \frac{1}{6}i(\bar{\varkappa}_2^\dagger\kappa_3\kappa_3^\dagger\partial_\alpha\bar{\varkappa}_1 - \varkappa_2^\dagger\kappa_3\kappa_3^\dagger\partial_\alpha\varkappa_1)\partial_\beta t - i(\partial_\alpha\bar{\varkappa}_2^\dagger\kappa_3\kappa_3^\dagger\bar{\varkappa}_1 - \partial_\alpha\varkappa_2^\dagger\kappa_3\kappa_3^\dagger\varkappa_1)\left(\frac{8}{3}\partial_\beta t + \partial_\beta\phi\right)\\ & - \frac{1}{6}(\partial_\alpha\bar{\varkappa}_1^\dagger\kappa_6\kappa_3^\dagger\bar{\varkappa}_1 + \partial_\alpha\varkappa_1^\dagger\kappa_6\kappa_3^\dagger\varkappa_1)(\partial_\beta t - \partial_\beta\phi) - \frac{1}{6}(\partial_\alpha\bar{\varkappa}_1^\dagger\partial_\beta\bar{\varkappa}_1\varkappa_2^\dagger\varkappa_1 - \partial_\alpha\varkappa_1^\dagger\partial_\beta\varkappa_1\bar{\varkappa}_2^\dagger\bar{\varkappa}_1)\\ & + \frac{1}{6}(\partial_\beta t + \partial_\beta\phi)\,\mathrm{Tr}(2\kappa_6^\dagger\kappa_3\kappa_6^\dagger\partial_\alpha\kappa_6 + \kappa_6^\dagger\partial_\alpha\kappa_3\kappa_3^\dagger\kappa_3 + \kappa_6^\dagger\kappa_3\kappa_3^\dagger\partial_\alpha\kappa_3) - \frac{1}{3}(\partial_\alpha\bar{\varkappa}_1^\dagger\kappa_1\partial_\beta\bar{\varkappa}_2^\dagger\bar{\varkappa}_1 + \partial_\alpha\bar{\varkappa}_1^\dagger\bar{\varkappa}_1\partial_\beta\varkappa_2^\dagger\varkappa_1\\ & - \partial_\alpha\varkappa_1^\dagger\varkappa_1\partial_\beta\bar{\varkappa}_2^\dagger\bar{\varkappa}_1 - \partial_\alpha\varkappa_1^\dagger\varkappa_1\partial_\beta\varkappa_2^\dagger\varkappa_1) + \frac{1}{3}(\partial_\alpha\bar{\varkappa}_2^\dagger\bar{\varkappa}_2\partial_\beta\bar{\varkappa}_2^\dagger\bar{\varkappa}_1 + \partial_\alpha\bar{\varkappa}_2^\dagger\bar{\varkappa}_2\partial_\beta\varkappa_2^\dagger\varkappa_1 - \partial_\alpha\varkappa_2^\dagger\varkappa_2\partial_\beta\bar{\varkappa}_2^\dagger\bar{\varkappa}_1\\ & - \partial_\alpha\varkappa_2^\dagger\varkappa_2\partial_\beta\varkappa_2^\dagger\varkappa_1) + \frac{1}{2}(\partial_\alpha\varkappa_1^\dagger\partial_\beta\varkappa_2\varkappa_2^\dagger\varkappa_2 + \partial_\alpha\varkappa_1^\dagger\partial_\beta\varkappa_2\bar{\varkappa}_2^\dagger\bar{\varkappa}_2) + \frac{1}{6}i\,\mathrm{Tr}(\kappa_1^\dagger\kappa_1\partial_\alpha\kappa_6^\dagger\partial_\beta\kappa_3)\\ & + \frac{1}{3}i\,\mathrm{Tr}(\partial_\alpha\kappa_6^\dagger\kappa_1\partial_\beta\kappa_1^\dagger\kappa_3) + \frac{1}{3}(\partial_\alpha\bar{\varkappa}_2^\dagger\kappa_3\partial_\beta\kappa_3^\dagger\bar{\varkappa}_1 - \partial_\alpha\varkappa_2^\dagger\kappa_3\partial_\beta\kappa_3^\dagger\varkappa_1) - \frac{1}{6}(\partial_\alpha\bar{\varkappa}_2^\dagger\partial_\beta\kappa_3\kappa_3^\dagger\bar{\varkappa}_1 - \partial_\alpha\varkappa_2^\dagger\partial_\beta\kappa_3\kappa_3^\dagger\varkappa_1)\\ & - \frac{1}{6}i(\partial_\alpha\bar{\varkappa}_1^\dagger\partial_\beta\kappa_6\kappa_3^\dagger\bar{\varkappa}_1 + \partial_\alpha\varkappa_1^\dagger\partial_\beta\kappa_6\kappa_3^\dagger\varkappa_1) - \frac{1}{6}(\partial_\alpha\varkappa_2^\dagger\kappa_6\kappa_6^\dagger\partial_\beta\varkappa_1 - \partial_\alpha\bar{\varkappa}_2^\dagger\kappa_6\kappa_6^\dagger\partial_\beta\bar{\varkappa}_1)\\ & + \frac{1}{3}i\,\mathrm{Tr}(\kappa_3^\dagger\partial_\alpha\kappa_3\kappa_3^\dagger\partial_\beta\kappa_6) + \mathrm{H.c.}\end{aligned}$$

These terms combined presents the quartic Lagrangian in a manifest covariant form with respect to the bosonic light-cone symmetries. In the next section we will discuss an integrable reduction of the full model, and we would like to note here that the resulting Lagrangian presented below can almost directly be read of from the above upon taking the reduction into account.

IV. INTEGRABLE REDUCTIONS FOR STRINGS ON $AdS_4 \times \mathbb{CP}^3$

The full model given by the Lagrangian (2.3) above, admits a consistent truncation, which upon imposition of the uniform light-cone gauge yields a purely fermionic Lagrangian containing two complex fermions. Being a consistent truncation, this model inherits the integrable structure possessed by the full Lagrangian. A remarkable and interesting fact is that this system is identical to the integrable system found in [28] for the $AdS_5 \times S^5$ background.

We would like to truncate down to purely light-cone bosons, which will subsequently be removed by utilizing the reparametrization invariance of the action. The question then becomes which set of fermions admit such a truncation, but of course the remaining (light-cone) bosonic symmetry group gives a natural structure here. Given the representation of this group, it is again only natural to introduce the following block structure on a generic fermionic block η

$$\eta = \frac{1}{2}\begin{pmatrix} \vartheta_1 & \vartheta_3 & \vartheta_5 \\ \vartheta_2 & \vartheta_4 & \vartheta_6 \end{pmatrix}.$$

In order to truncate down to only light-cone bosons, i.e. the fields t and ϕ, we switch of all fermions besides either ϑ_3 and ϑ_6 or ϑ_4 and ϑ_5. Choosing to switch of ϑ_4 and ϑ_5 we are left with the block

$$\begin{pmatrix} 0 & 0 & \eta^{31} & \eta^{33} & 0 & 0 \\ 0 & 0 & \eta^{32} & \eta^{34} & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta^{61} & \eta^{63} \\ 0 & 0 & 0 & 0 & \eta^{62} & \eta^{64} \end{pmatrix}.$$

This leaves a truncated model with four complex fermionic degrees of freedom.[2] This model admits a further truncation, down to two complex fermions, achieved by switching off either the diagonal or off-diagonal terms in ϑ_3 and ϑ_6. Choosing here to set $\eta^{32} = \eta^{33} = \eta^{62} = \eta^{63} = 0$, we obtain the smallest truncated model possible in this setting. Further truncations would lead to nonlinear constraints, due to couplings in the Wess-Zumino term of the Lagrangian schematically of the form $(t + \phi)\eta^{31}\eta^{34}$, with possible derivatives. Switching either of these fermionic fields off independently would lead to nonlinear constraints on the remaining field and the combination $t + \phi$ through the equation of motion of the removed fermion.

The reduced model thus obtained is manifestly integrable, being the theory of a free two dimensional massive Dirac fermion. It will nonetheless still be interesting to see how the Lax connection for the full model reduces for this specific model as shown further below.

[2]The reality condition on the fermions relates those in ϑ_6 to the ones in ϑ_3.

A. The Lagrangian of the reduced model

The Lagrangian for the truncated model above gives a quadratic action for two complex fermions, equivalent to the integrable model presented in [28] found for the $AdS_5 \times S^5$ background. Given our different choice of coset parametrization however, the fermions are uncharged under the U(1) shifts corresponding to shifts in t and ϕ from the start, doing away with the need for a field redefinition of the fermions as done in [28].

After substituting the truncation into the Lagrangian (2.3) we find the following;

$$\mathcal{L} = \frac{\sqrt{\lambda}}{4\pi}\Big(\gamma^{\alpha\beta}\Big(\partial_\alpha t\partial_\beta t - \partial_\alpha\phi\partial_\beta\phi + \frac{i}{2}\partial_\alpha(t+\phi)\zeta_\beta - \frac{1}{2}\partial_\alpha(t+\phi)\partial_\beta(t+\phi)\Lambda\Big) - \frac{\kappa}{2}\epsilon^{\alpha\beta}\partial_\alpha(t+\phi)\Omega_\beta\Big),$$

where we have introduced the following abbreviations for the fermionic contributions

$$\zeta_\alpha = 2(\eta_i\partial_\alpha\eta^i + \eta^i\partial_\alpha\eta_i), \qquad \Lambda = 2\eta_j\eta^j,$$
$$\Omega_\beta = -2i(\eta^{31}\partial_\beta\eta^{34} + \eta^{34}\partial_\beta\eta^{31} + \eta_{31}\partial_\beta\eta_{34} + \eta_{34}\partial_\beta\eta_{31}).$$

Here the reality condition has been imposed, we write $\eta^{i*} = \eta_i$, and the summations are of course over the relevant two fermions. The factor of $\sqrt{\lambda}/4\pi$ has been included here for comparison purposes. Upon rescaling our fermions as

$$\eta^{31,34} \rightarrow \frac{e^{-i(\pi/4)}}{\sqrt{2}}\eta^{31,34},$$

it should be clear that this action is identical to the one presented in [28], where we note that there the string tension is taken to be $\frac{1}{2\pi\alpha'} = 1$. This model simplifies to the greatest extent upon imposition of a uniform light-cone gauge, which utilizes the reparametrization invariance of the string action by introducing the following light-cone coordinates

$$x_+ = \frac{1}{2}(\phi + t), \qquad x_- = \frac{1}{2}(\phi - t),$$
$$p_+ = p_\phi + p_t, \qquad p_- = p_t - p_\phi,$$

and subsequently imposing the gauge choice

$$x_+ = \tau + \frac{m}{2}\sigma, \qquad p_+ = \text{constant} = P_+ = E + J,$$

where m is the winding number which arises due to periodicity of the ϕ field. E and J are the Noether charges associated with the U(1) isometries of shifts in t and ϕ respectively. This gauge fixing was carefully done in [29] for this Lagrangian, hence the reader is referred there for the exact details of the procedure. The upshot is the gauge fixed Lagrangian

$$\mathcal{L} = -\frac{i}{4}P_+\zeta_\tau + \frac{1}{2}\kappa m\Omega_\tau - \kappa\Omega_\sigma + \frac{1}{2}P_+\Lambda. \tag{4.1}$$

B. Lax representation

The integrability of classical superstrings on $AdS_4 \times \mathbb{CP}^3$ has been demonstrated in [3]; by construction there is in fact no essential difference from the $AdS_5 \times S^5$ coset model. In the above we have shown that the $AdS_4 \times \mathbb{CP}^3$ model allows a reduction to a fermionic Lagrangian containing two complex fermions, yielding a manifestly integrable system of a free two dimensional massive Dirac fermion for the suitable choice of fixing a uniform light-cone gauge. Despite its manifest integrability, it will still be insightful to see how the general Lax connection reduces in the above truncation and uniform light-cone gauge.

The general Lax pair for this model, is a pair of ten by ten matrices, with zero curvature

$$\partial_\sigma L_\tau - \partial_\tau L_\sigma - [L_\sigma, L_\tau] = 0, \tag{4.2}$$

following from the equations of motion and vice versa. For the reduced model, the appropriate Lax connection can be formulated in terms of two by two matrices. To connect with the notation in [29], we have as concise notation for the fermions

$$\psi_1 = \eta_{31}, \qquad \psi_2 = \eta^{34},$$
$$\psi_1^* = \eta^{31}, \qquad \psi_2^* = \eta_{34},$$

and we introduce the even quantities

$$\mathrm{s}_\alpha = \psi_1\partial_\alpha\psi_1^* + \psi_1^*\partial_\alpha\psi_1 - \psi_2\partial_\alpha\psi_2^* - \psi_2^*\partial_\alpha\psi_2$$
$$\xi = \psi_1\psi_1^* + \psi_2\psi_2^*.$$

In terms of these quantities the components of the two by two Lax connection are given as

$$L_\tau = \begin{pmatrix} \frac{i(1+z^2)}{2(-1+z^2)} + \frac{1}{8}\mathrm{s}_\tau + \frac{i}{4}\xi & -\frac{e^{i(\pi/4)}(z(\partial_\tau\psi_2^* - i\psi_2^*) + (-i\partial_\tau\psi_1^* + \psi_1^*))}{\sqrt{2-2z^2}} \\ -\frac{e^{i(\pi/4)}(z(-i\partial_\tau\psi_2 + \psi_2) + (\partial_\tau\psi_1 - i\psi_1))}{\sqrt{2-2z^2}} & -\frac{i(1+z^2)}{2(-1+z^2)} + \frac{1}{8}\mathrm{s}_\tau + \frac{i}{4}\xi \end{pmatrix},$$

$$L_\sigma = \begin{pmatrix} \frac{i(zP_+ + m + mz^2)}{4(-1+z^2)} + \frac{1}{8}\mathrm{s}_\sigma + \frac{im}{8}\xi & -\frac{e^{i(\pi/4)}(z(2\partial_\sigma\psi_2^* - im\psi_2^*) + (-2i\partial_\sigma\psi_1^* + m\psi_1^*))}{2\sqrt{(2-2z^2)}} \\ -\frac{e^{i(\pi/4)}(z(-2i\partial_\sigma\psi_2 + m\psi_2) + (2\partial_\sigma\psi_1 - im\psi_1))}{2\sqrt{(2-2z^2)}} & -\frac{i(zP_+ + m + mz^2)}{4(-1+z^2)} + \frac{1}{8}\mathrm{s}_\sigma + \frac{im}{8}\xi \end{pmatrix},$$

where z is the spectral parameter associated with this Lax pair. This reduced Lax connection contains the essential features of the original ten by ten Lax connection as of course it should; it is constructed directly from it by considering its independent entries and suitably combining them to obtain this equivalent two by two connection. It is easily checked that this Lax connection has zero curvature (4.2) on shell, with the equations of motion following from (4.1) being

$$\partial_\tau\psi_1 = \frac{-2P_+(m\psi_2 - 2i\partial_\sigma\psi_2) + 8m\partial_\sigma\psi_1 - iP_+^2\psi_1}{4m^2 - P_+^2},$$
$$\partial_\tau\psi_2 = \frac{-2P_+(m\psi_1 + 2i\partial_\sigma\psi_1) + 8m\partial_\sigma\psi_2 + iP_+^2\psi_2}{4m^2 - P_+^2}.$$

C. Other truncated models

There are some good indications that other integrable truncations exist, to a sector containing one bosonic field from the AdS space and two complex fermions on top of the light-cone bosons t and ϕ. This would concretely, for example, be a truncation down to the bosonic fields t, ϕ and x_1, and the fermions f_1 and f_4, giving the fermionic block the structure

$$\begin{pmatrix} f_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_4 & 0 & 0 & 0 & 0 \\ -f_4^* & 0 & 0 & 0 & 0 & 0 \\ 0 & f_1^* & 0 & 0 & 0 & 0 \end{pmatrix}.$$

In this truncated model obviously the field ϕ does not couple to the other fields and hence could also be removed, leaving just the AdS sector coupling to two complex fermions. This model has a Lagrangian nonlinear in the x_1 field, which extends till quartic order in the fermionic fields, leading to a considerably less simple Lagrangian than the one obtained above. This makes an explicit check of the integrability more involved and less insightful, hence we present it here as a possibility without explicit proof. Finally, further possible truncations would seem to require inclusion of even more fields and would hence be considerably less concise.

V. CONCLUSIONS

In this paper we have focussed on the fermionic structure of the coset sigma model describing Green-Schwarz superstrings on $AdS_4 \times \mathbb{CP}^3$. This model is classically integrable, however, the question of quantum integrability is considerably more involved and calls for further investigation. With a choice of coset parametrization suitable for imposition of a light-cone gauge, the manifest global bosonic symmetry algebra of the model was found to be $\mathfrak{C} = \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$, and κ-symmetry gauge fixing compatible with this global symmetry was performed. Furthermore, with this knowledge, the quartic Lagrangian of the sigma model has been cast in a covariant form with respect to $\mathfrak{C}$, a result which can be further used to compute the light-cone two-body S-matrix. Analyzing the structure of the latter should present the first step towards understanding quantum integrability of the model, by allowing one to check the factorization property of the S-matrix.

In addition, by exploiting the classical integrable structure of the full coset model, in this paper we have analyzed the question whether there exist integrable truncations of the superstring Lagrangian. We found that this question has an affirmative answer by explicitly providing the (smallest) integrable fermionic model which arises upon consistent truncation of the full coset model. This truncation is again based in part on the manifest symmetry algebra, and yields a model containing two complex fermions and the bosonic light-cone fields $x_\pm$. Perhaps, the most interesting fact about this model is that it is exactly the same as the one arising from consistent truncation of the coset model describing $AdS_5 \times S^5$ superstrings. The reduced model simplifies drastically upon imposition of the uniform light-cone gauge where it coincides with a *free* model of two complex fermions. In the AdS_5/SYM duality this truncated model is directly related to a certain closed sector of the dual gauge theory. As such, given the exact agreement between the two truncated models and the similarities between the two models in general, it would be interesting to investigate whether this is also the case for the current duality.

ACKNOWLEDGMENTS

We would like to thank Gleb Arutyunov for valuable discussions.

APPENDIX A: THE COSET ELEMENT AND LINEAR ISOMETRIES

For reader's convenience, in this appendix we collect the matrices used in the construction of the coset element. The AdS_4 is parametrized by the following set of Γ matrices

$$\Gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \Gamma^1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\Gamma^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \qquad \Gamma^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix},$$

such that these matrices satisfy the Clifford algebra $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$, where $\eta^{\mu\nu}$ is Minkowski metric with signature $(1, -1, -1, -1)$. We also define $\Gamma^5 = -i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$ with the property $(\Gamma^5)^2 = \mathbb{I}$.

The charge conjugation matrix C_4 obeys $(\Gamma^\mu)^t = -C_4\Gamma^\mu C_4^{-1}$ and in the present case it can be chosen as

$$C_4 = i\Gamma^0\Gamma^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}.$$

Moreover we also require $(\Gamma^\mu)^t = K_4\Gamma^\mu K_4^{-1}$, where $K_4 = iI_2 \otimes \sigma_2$ and where I_2 is the 2×2 identity matrix. Lastly we define $\Gamma^{\mu\nu} \equiv \frac{1}{4}[\Gamma^\mu, \Gamma^\nu]$.

To parametrize the $\mathbb{CP}^3$ coset space, we need to consider the space orthogonal to $\mathfrak{u}(3)$ in $\mathfrak{so}(6)$, which is spanned by solutions to the following equation

$$\{K_6, Y\} = 0, \tag{A1}$$

The general solution to Eq. (A1) is six-parametric and is represented by a matrix

$$Y = y_i T_i.$$

where we have introduced the six matrices T_i which are Lie algebra generators of $\mathfrak{so}(6)$ along the $\mathbb{CP}^3$ directions:

$$\begin{aligned} T_1 &= E_{13} - E_{31} - E_{24} + E_{42}, \\ T_2 &= E_{14} - E_{41} + E_{23} - E_{32}, \\ T_3 &= E_{15} - E_{51} - E_{26} + E_{62}, \\ T_4 &= E_{16} - E_{61} + E_{25} - E_{52}, \\ T_5 &= E_{35} - E_{53} - E_{46} + E_{64}, \\ T_6 &= E_{36} - E_{63} + E_{45} - E_{54}, \end{aligned}$$

where E_{ij} are the standard matrix unities, and where the T_i matrices are normalized as $\mathrm{tr}(T_iT_j) = -4\delta_{ij}$. Moreover, the complete set of generators of $\mathfrak{so}(6)$ are given by $T_{ij} = E_{ij} - E_{ji}$.

In section III A we indicated that it would be desirable to extract the coordinates t and ϕ from the general bosonic element, in anticipation of imposing a light-cone gauge choice on these coordinates. This was done by introducing the parametrization of the coset element as given by Eq. (3.1). Here we will illustrate why this form is the appropriate choice.

$\mathbb{CP}^3$ can be parametrized in terms of the coset element

$$g_{\mathbb{CP}} = e^Y, \tag{A2}$$

where

$$Y = \sum_{i=1}^{6} y_i T_i.$$

We parametrize $\mathbb{CP}^3$ by introducing spherical coordinates, $(r, \phi, \theta, \alpha_1, \alpha_2, \alpha_3)$, which are related to the y_i in the following fashion

$$y_1 + iy_2 = r\sin\theta\cos\frac{\alpha_1}{2}e^{((i/2)(\alpha_2+\alpha_3))+(i/2)\phi} = \frac{r}{|w|}w_1,$$
$$y_3 + iy_4 = r\sin\theta\sin\frac{\alpha_1}{2}e^{-((i/2)(\alpha_2-\alpha_3))+(i/2)\phi} = \frac{r}{|w|}w_2,$$
$$y_5 + iy_6 = r\cos\theta e^{i\phi} = \frac{r}{|w|}w_3, \qquad \text{(A3)}$$

where $|w| = \bar{w}_k w_k$ and $\sin r = \frac{|w|}{\sqrt{1+|w|^2}}$. Now we would like to effectively extract the angle ϕ from this element, (A2), in order to write $g_{\mathbb{CP}}$ as

$$g_{\mathbb{CP}} = \Lambda_{\mathbb{CP}}(\phi)\tilde{g}_{\mathbb{CP}},$$

where $\tilde{g}_{\mathbb{CP}}$ no longer depends on ϕ. Inspection of the relations just above (A3), shows that extracting ϕ corresponds to setting y_6 to zero. The remaining five y's in conjunction with ϕ still give a good parametrization of $\mathbb{CP}^3$. It is moreover not hard to verify explicitly that on the coset element (A2), $T_{34} + T_{56}$ generates shifts in ϕ, hence giving the form of $\Lambda_{\mathbb{CP}}(\phi)$.

It is perhaps noteworthy to mention that even though we have introduced ϕ in exchange for y_6, such that we remain with y_i, $i = 1, \ldots, 5$ and ϕ, it is important to remember the coordinates are not all completely independent in the usual sense. As an example, confining oneself to the geodesic circle parametrized by ϕ corresponds to taking $w_3 = e^{i\phi}$, which means $y_i = 0$ for $i = 1, \ldots, 4$, and $y_5 = \pi/4$.

Lastly the Υ matrix used in the definition of the map Ω is given by

$$\Upsilon = \begin{pmatrix} K_4C_4 & 0 \\ 0 & -K_6 \end{pmatrix}.$$

APPENDIX B: THE COSET PARAMETRIZATION AND κ-SYMMETRY PARAMETER

As indicated above, the choice of coset parametrization, does have an effect on the explicit form of the κ-symmetry parameter. As κ-symmetry acts on the coset element by multiplication from the right

$$g \to ge^{\epsilon} = g'g_c,$$

at linear order in χ and ϵ, we have the following transformation for our choice of coset parametrization (3.1)

$$g \to g_o g_\chi g_B e^{\epsilon} = g_o e^{\chi} e^{g_B \epsilon g_B^{-1}} g_B \approx g_o e^{\chi + g_B \epsilon g_B^{-1}} g_B.$$

Thus at the linearized level the fermionic matrix χ undergoes a shift

$$\chi \to \chi + g_B \epsilon g_B^{-1},$$

under a κ-symmetry variation. In the coset parametrization $g = g_\chi g_B$, the light-cone coset element, $g_B \sim \exp(\mathrm{diag}(i\Gamma^0, T_6)$, commutes with ϵ as presented in [16], however for our choice of coset parametrization, we have $g_B \sim \exp(\mathrm{diag}(0, \pi/4T_5))$ which does not commute with ϵ. As a consequence of this the epsilon parameter needs to be modified by exactly this conjugation by g_B. Considering we are interested in gauge choices invariant under the manifest bosonic symmetry, we will also rotate the ϵ parameter by use of the matrix S (3.3) which will then clearly indicate some invariant gauge choices. Concretely the parameter of [3] is calculated from (2.5) by considering a bosonic coset element schematically of the form

$$A^{(2)} = \begin{pmatrix} ix\Gamma^0 & 0 \\ 0 & yT_6 \end{pmatrix}.$$

The Virasoro constraint here reads $\mathrm{str}(A^{(2)}_{\alpha,-}A^{(2)}_{\beta,-}) = 0$, and implies $x^2 = y^2$. Choosing $x = y$ as a solution and doing the just indicated conjugation and basis change (3.3), we find

$$\epsilon^{(1)} \to S g_B \epsilon^{(1)} (S g_B)^{-1} = \begin{pmatrix} 0 & \varepsilon \\ \bullet & 0 \end{pmatrix},$$

where

$$\varepsilon = \frac{1+i}{\sqrt{2}}\begin{pmatrix} 0 & 0 & \varrho \\ 0 & \sigma_2\varrho^*\sigma_2 & 0 \end{pmatrix},$$

and

$$\varrho = \begin{pmatrix} i(\theta_{43} + \theta_{44}) - \theta_{45} - \theta_{46} & -i(\theta_{43} - \theta_{44}) - \theta_{45} + \theta_{46} \\ -\theta_{43} + \theta_{44} - i(\theta_{45} - \theta_{46}) & -\theta_{43} - \theta_{44} + i(\theta_{45} + \theta_{46}) \end{pmatrix}.$$

Here the θ_{ij} correspond to the original entries of the fermionic parameter κ_{++} entering in (2.5). Of course the expression for $\epsilon^{(3)}$ is similar. It should now be clear that this parameter can be used to impose the gauge choice (3.5).

This expression might appear somewhat asymmetrical, however we note here that picking the alternate solution to the Virasoro constraint in the above, $x = -y$, exactly leads to a parameter schematically of the form

$$\varepsilon_- = (1+i)\begin{pmatrix} 0 & \varrho_- & 0 \\ 0 & 0 & -\sigma_2\varrho_-^*\sigma_2 \end{pmatrix}.$$

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