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A spatial autoregressive geographically weighted quantile regression to explore housing rent determinants in Amsterdam and Warsaw

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Abstract
A hedonic approach is typically performed to identify housing rental or sales price determinants. However, standard hedonic regression models disregard spatial autocorrelation of prices and heterogeneity of housing preferences across space and over price segments. We developed a spatial autoregressive geographically weighted quantile regression (GWQR-SAR) to address these shortcomings. Using data on the determinants of residential rental prices in Warsaw (Poland) and Amsterdam (The Netherlands) as case studies, we applied GWQR-SAR and rigorously compared its performance with alternative mean and quantile hedonic regressions. The results revealed that GWQR-SAR outperforms other models in terms of fitting accuracy. Compared with mean regressions, GWQR-SAR performs better, especially at the tails of the dependent variable distribution, where non-quantile models overestimate low rent values and underestimate high ones. Policy recommendations for the development of private residential rental markets are provided based on our results, which incorporate spatial effects and price segment requirements.

Keywords
Geographically weighted regression, quantile regression, housing market, hedonic model, spatial autocorrelation

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Introduction

The residential market is central to society and the economy. The housing market often accounts for more than 30% of a country's economy (Fotheringham and Park, 2018) and affects vital macroeconomic indicators (Leung, 2004; Brzezicka et al., 2022). Therefore, housing price determinants play a crucial role for private and public stakeholders (Selim, 2009). The identification of housing rental or sales price determinants is typically performed using a hedonic model (Rosen, 1974). Hedonic price theory values an object based on its utility-bearing characteristics and decomposes the price into its individual value-adding quantitative (Leishman, 2001) and qualitative components (Meese and Wallace, 1991). Multiple linear regression is widely used to estimate the hedonic price function (Goodman, 1998).

In practice, however, three challenges arise in hedonic models. First, hedonic models do not account for spatial autocorrelation of housing prices in their standard forms. Spatial autocorrelation refers to a situation where the price of a given apartment is spatially correlated with the prices of real estate in a neighborhood resulting from the fact that properties in similar locations share various amenities (Dubin, 1998). Also, during the selling or renting process, people generally tend to set the listing price based on the sales prices of nearby properties.

Second, the standard hedonic model does not address spatial heterogeneity, which means that the hedonic price function varies across space (Sunding and Swoboda, 2010; Helbich et al., 2014). It stems from submarkets and variations in household preferences in the residential market (Watkins, 2001). Both spatial dependence and spatial heterogeneity may coexist in the housing market (Yao and Fotheringham, 2016). The omission of spatial effects in hedonic modeling leads to biased and inconsistent parameter estimates and spurious inferences, possibly rendering incorrect marginal prices for housing attributes (Anselin and Lozano-Gracia, 2009; LeSage and Pace, 2009).

Third, the standard hedonic model also ignores that housing characteristics might be valued differently across the housing price distribution due to variations in the residential preferences of buyers in the high-end and low-end property segments (Zietz et al., 2008). Further, it has been reported that the housing preferences of poor and rich households differ (Leung and Tsang, 2012) and are sometimes even opposed (Tomal, 2019). Therefore, it is possible for the marginal prices of dwelling attributes to possess opposite signs at different points of the conditional distribution of the dependent variable. Consequently, the estimated coefficients using the standard hedonic model are not an entirely reliable source of information about the relationship between the price of a housing unit and its characteristics (Liao and Wang, 2012).

Despite the importance of the above aspects for the validity of hedonic house price models, we are not aware of any model capable of simultaneously including spatial autocorrelation of prices and heterogeneity of housing preferences across space and price segments. We propose a new hedonic model called spatial autoregressive geographically weighted quantile regression (GWQR-SAR) to address this research gap. To illustrate the value of GWQR-SAR, we conducted two case studies, one in Amsterdam (The Netherlands) and one in Warsaw (Poland), as had been done previously (McMillen, 2015; Yao and Fotheringham, 2016; Fotheringham and Park, 2018; Tomal, 2022).

Previous works on modeling housing price determinants

Hedonic models considering both spatial effects (i.e., spatial autocorrelation and spatial heterogeneity) have been established in the literature. For example, Yao and Fotheringham (2016) as well as Fotheringham and Park (2018) included a spatiotemporal lag variable as an additional predictor in a geographically weighted hedonic regression (GWR) when analyzing housing prices in Scotland and South Korea, respectively. Similarly, Geniaux and Martinetti (2018) and Li et al. (2019) examined the USA housing market. The simultaneous consideration of both spatial effects in house price modeling was also addressed by Basile et al. (2014), who applied geoadditive models, further relaxing the linearity assumption. In turn, Tomal (2022) investigated the determinants of average house prices in Polish counties using spatial autoregressive multiscale GWR, which allowed relationships between predictors and the dependent variable to operate at different spatial scales. A different approach was proposed by Helbich and Griffith (2016). They used eigenvector spatial filtering to obtain spatially varying regression coefficients and, at the same time, estimates without spatial dependency.

Research on different valuations of housing attributes across housing price segments (Tomal, 2019; Waltl, 2019) was mainly based on quantile regression (QR) (Koenker and Bassett, 1978). Mathur (2019) addressed spatial dependence and heterogeneity across the price distribution when assessing how urban growth boundaries affect housing prices. McMillen (2015) used a conditionally parametric QR approach to model land values, which allowed regression coefficients to change over the distribution of the response variable and spatially. Similarly, Chen et al. (2012) proposed geographically weighted quantile regression (GWQR) to extend the traditional GWR model with quantile regression. In this case, however, the GWQR model was not applied in the context of housing. Expanding on Chen et al. (2012), Wang et al. (2018) proposed the GWQlasso model to simultaneously identify spatially varying coefficients, non-zero constant (global) coefficients, and zero (insignificant) coefficients. We are not aware of any hedonic house price model that has incorporated spatial dependence, spatial heterogeneity, and heterogeneity across price segments within a unified framework.

Methods

Spatial hedonic models

A spatial autoregressive model (SAR) is typically used in housing studies to account for spatial dependence in the data. The model can be formalized as follows (Ord, 1975)

$$y_i = \alpha + \rho \sum\nolimits_j w_{ij} y_j + \sum\nolimits_k \beta_k x_{ik} + \varepsilon_i \tag{1}$$

where y_i is the dependent variable, α denotes the intercept, β_k is a vector of parameters, x_{ik} refers to regressors, ε_i denotes the error term, w_{ij} is the element of a spatial weight matrix W and ρ is the spatial autoregressive parameter. If W is row-standardized, then for each i, $\sum\nolimits_j w_{ij} = 1$, and the spatial autoregressive term, Wy, captures a weighted average of the neighbors. For such settings, the total covariate effect on the response variable is equal to $\beta_k/(1-\rho)$. Because Wy is correlated with the error term and causes an endogeneity bias, the calibration of equation (1) is based on two-stage least squares (2SLS). This procedure starts with estimating an ordinary least squares (OLS) regression, whereas the spatial autoregressive term acts as a dependent variable and the regressors are the set of variables X and WX (Anselin, 2003). Then, the fitted values from the first step are used to estimate the SAR model.

Conventional SAR assumes a unitary housing market across space that can be modeled using a single price function representing the entire study area (Wilhelmsson, 2002). To relax the assumption of a unitary housing market, geographically weighted regression has been introduced to model spatially varying associations between covariates and a response variable (Brunsdon et al., 1996). Specifically, GWR can be expressed as follows

$$y_i = \alpha(u_i, v_i) + \sum\nolimits_k \beta_k(u_i, v_i) x_{ik} + \varepsilon_i \tag{2}$$

where (u_i, v_i) represents geographic coordinates of the location i, $\alpha(u_i, v_i)$ denotes the intercept at location i, $\beta_k(u_i, v_i)$ is the vector of parameters at location i. The estimation of the GWR model parameters is via weighted least squares

$$\widehat{\beta}(u_i, v_i) = \left[X^T M(u_i, v_i) X\right]^{-1} X^T M(u_i, v_i) Y \tag{3}$$

where $M(u_i, v_i) = diag(\alpha_{i1}, \ldots, \alpha_{in})$ and n denotes the number of observations. The model estimation depends on the proximity of point i to the other data points in space, whereas an observation i closer in space receives a higher weight. Fotheringham et al. (2002) recommend using the bi-square kernel function to model the spatial distance decay, as it produces a continuous weighting function up to a certain distance and zero weights for the remaining observations. Considering the Euclidean spatial distance and the bi-square kernel, α_{ij} is represented as

$$\alpha_{ij} = \left[1 - \left(\frac{d_{ij}}{h_o}\right)^2\right]^2 \text{ if } d_{ij} < h_o \text{ and 0 otherwise} \tag{4}$$

where $d_{ij} = \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2}$ and h_o is the bandwidth either predefined (fixed bandwidth) or determined through a nearest neighbor approach (adaptive bandwidth). The latter is frequently used as the density of observations often varies across the study area (Fotheringham et al., 2002). An optimal h_o can be determined using cross-validation (CV), minimizing the sum of the squared error

$$CV(h_r) = \sum_i \left(y_i - \widehat{y}_{(-i)}(h_r)\right)^2 \tag{5}$$

where h_r denotes the rth bandwidth and $\widehat{y}_{(-i)}$ refers to the fitted value from GWR with the ith location being omitted during model calibration. Finally, h_r for which the CV score is the lowest serves as a h_o.

GWR can be supplemented with a spatially lagged dependent variable to also capture spatial autocorrelation (Brunsdon et al., 1998). Spatial autoregressive geographically weighted regression (GWR-SAR) is expressed as

$$y_i = \alpha(u_i, v_i) + \rho(u_i, v_i) \sum_j w_{ij} y_j + \sum_k \beta_k(u_i, v_i) x_{ik} + \varepsilon_i \tag{6}$$

where symbols are defined as previously. The spatial autoregressive parameter ρ serves as a local measure of spatial autocorrelation that also captures the influence of other covariates in the model (Fotheringham and Park, 2018). To deal with the endogeneity of Wy, 2SLS is applied for GWR-SAR estimation as is done for SAR: first, the endogenous variable is modeled, and, second the fitted values are used to calibrate the GWR-SAR model (Cho et al., 2008, 2009; Shoff et al., 2014; Ingram and Marchesini da Costa, 2017).

Spatial quantile hedonic models

Mean regressions like SAR, GWR, and GWR-SAR only allow examination of relationships for the conditional mean of the response. Quantile regression (QR) (Koenker and Bassett, 1978) eases this restrictive assumption by modeling associations at any point of the conditional distribution of the dependent variable. QR takes the following form

$$y_i = \alpha^\tau + \sum_k \beta_k^\tau x_{ik} + \varepsilon_i^\tau \tag{7}$$

where τ denotes the quantile level for which the estimation is performed. The estimation of the model (7) for an assumed level τ minimizes the following equation

$$\sum\nolimits_{i:y_i \geq \widehat{y_i}} \tau \left| y_i - \alpha - \sum\nolimits_k \beta_k x_{ik} \right| + \sum\nolimits_{i:y_i < \widehat{y_i}} (1-\tau) \left| y_i - \alpha - \sum\nolimits_k \beta_k x_{ik} \right| \tag{8}$$

where symbols are defined as previously. Spatial autocorrelation can also be incorporated into QR. The QR-SAR model can be written as

$$y_i = \alpha^\tau + \rho^\tau \sum\nolimits_j w_{ij} y_j + \sum\nolimits_k \beta_k^\tau x_{ik} + \varepsilon_i^\tau \tag{9}$$

where symbols are defined as previously. QR-SAR is also estimated in a two-stage procedure to consider the endogeneity of Wy, whereas quantile regression is employed in the first stage rather than OLS (Kim and Muller, 2004).

Geographically weighted quantile regression (GWQR) (Chen et al., 2012) integrates GWR and QR. Through GWQR, one can obtain spatially varying parameters for different points of the dependent variable conditional distribution. GWQR can be expressed as follows

$$y_i = \alpha^\tau(u_i, v_i) + \sum\nolimits_k \beta_k^\tau(u_i, v_i) x_{ik} + \varepsilon_i^\tau \tag{10}$$

where symbols are defined as previously. Equation (10) can be calibrated using a local constant or local linear estimator (Chen et al., 2012). The former leads to results only at the regression points; the latter can also predict the value of the dependent variable in the neighborhood of the observations (Hallin and Šiman, 2017) while being computationally more demanding (Yu and Jones, 1997). However, as evidenced by Yu and Jones (1997), the performance of both methods is comparable for observed locations. The local constant estimator of the model (10) involves the minimization of the following problem

$$\sum_i \upsilon_\tau \left(y_i - \alpha^\tau(u_i, v_i) - \sum_k \beta_k^\tau(u_i, v_i) x_{ik} \right) M \tag{11}$$

where $\upsilon_\tau(z) = z(\tau - I(z<0))$ denotes the check loss function.

Local constant and local linear GWQR can also be estimated using a bootstrap approach instead of an asymptotic approximation (Chen et al., 2020). The bootstrap approach provides reliable estimates of model parameters and standard errors based on the bootstrap distribution of $\widehat{\beta}(u_i, v_i)$. Specifically, the bootstrapped mean value and the standard deviation are calculated, which serve as the estimated parameter and its standard error, respectively.

In contrast to GWR-based models, GWQR selects the optimal bandwidth via a V-shaped check function to determine the CV scores (Chen et al., 2012). The CV value for a given bandwidth h_r is given as

$$CV(h_r) = \sum_i \upsilon_\tau \left(y_i - \widehat{y}_{\tau,(-i)} \right) \tag{12}$$

where $\widehat{y}_{\tau,(-i)}$ is the fitted value from GWQR, with the ith location being omitted during model calibration and the optimal bandwidth h_o has the lowest CV score.

Spatial autoregressive geographically weighted quantile regression

GWQR-SAR now extends GWQR with a spatial autoregressive term as covariate. The model generates parameters that vary across space and over the response variable distribution while accounting for spatial autocorrelation in the data. The GWQR-SAR model has the following notation

$$y_i = \alpha^\tau(u_i, v_i) + \rho^\tau(u_i, v_i)\sum\nolimits_j w_{ij}y_j + \sum\nolimits_k \beta_k^\tau(u_i, v_i)x_{ik} + \varepsilon_i^\tau \tag{13}$$

where symbols are defined as previously. To fit the model, we propose a two-stage procedure to account for the endogeneity of Wy. First, quantile regression obtains $\widehat{Wy}$ using X and WX as instruments. Second, the estimation procedure follows the assumptions presented for GWQR. Importantly, GWQR-SAR estimates the spatial autoregressive parameter for each location for a particular percentile of the dependent variable allowing the coefficient to be treated as a local spatial quantile dependence index considering other covariates.

The R package GWQR

The estimation of GWQR can be done using the rq function (Wang et al., 2018) from the R package quantreg (Koenker et al., 2018). For a given observation i and a given quantile level τ, the estimation of the local constant GWQR is as follows: rq(Y~X,tau = τ,weights = M) where M is a vector of weights calculated depending on the kernel function adopted and the way the distance is measured. In turn, local linear GWQR requires changing X to $X_t = [X, U(t)X, V(t)X]$ where $U(t) = diag[u_1 - u_i, \ldots, u_n - u_i]$ and $V(t) = diag[v_1 - v_i, \ldots, v_n - v_i]$. For local linear GWQR, three parameters are generated for each location i, that is, $\widehat{\beta}(u_i, v_i)$, $\widehat{\beta}^{(u)}(u_i, v_i)$, and $\widehat{\beta}^{(v)}(u_i, v_i)$.

In line with Wang et al.'s (2018) guidelines, we developed the R package GWQR to fit either the local constant or the local linear estimator. For the former, the function gwqr_lc(formula=,data=,q=,ind=,b=,bb=) can be applied; for the latter gwqr_ll() can be used with the same parameters. The GWQR package also allows the selection of the optimal bandwidth for local constant GWQR using the function gwqr_lc_bw(formula=,data=,q=,ind=,tol=). The function requires the following parameters: formula: model formula, for example, Dependent~X1+X2, data: data used, q: the quantile level, ind: number of independent variables including a constant (note: do not create a vector of ones in the data), b: bandwidth, bb: number of bootstrap replications, tol: convergence parameter. The bootstrap approach has been implemented with the bi-square kernel function based on the Euclidean norm, and an adaptive bandwidth optimized employing cross-validation (CV) to obtain both local constant and local linear GWQR. An example of the use of the GWQR package to estimate the local constant GWQR for quantile 0.5 is as follows:

1. gwqr_lc_bw(formula = Dependent ~ X1 + X2,data = house_prices,q=0.5,ind = 3,tol = 0.00001) # selecting the optimal bandwidth
2. gwqr_lc(formula = Dependent~X1+X2,data=house_prices,q = 0.5,ind = 3,b = b*,bb = 500) # local constant GWQR estimation for quantile 0.5 using 500 bootstrap replication, b* denotes the bandwidth indicated in step 1

A detailed description of the functions and data preparation is provided in the R package GWQR available from https://figshare.com/s/ae6e81c08772593ce627.

Study area and data

We selected housing rental markets in Warsaw (Poland) and Amsterdam (The Netherlands) for an empirical case study. These real estate markets are of interest in our analyses for two reasons. First, while there are numerous publications on drivers of housing sales prices to date

(Wilkinson and Archer, 1973; Peek and Wilcox, 1991; Égert and Mihaljek, 2007; Helbich, 2015; Kopczewska and Ćwiakowski, 2021), little attention has been paid to identifying rent determinants, mainly due to lack of data. This dearth of data relates, in turn, to the paucity of studies on European residential rental markets. Examples of the few extant studies include Efthymiou and Antoniou (2013), Crespo and Grêt-Regamey (2013), McCord et al. (2014), Egner and Grabietz (2018), Tomal (2020), and Tomal and Helbich (2022). Apart from Trojanek et al. (2021) and Trojanek and Gluszak (2022), to the best of our knowledge, no other studies have addressed the determinants of rents either in Warsaw or Amsterdam. Second, the markets in Amsterdam and Warsaw are characterized by different development levels. According to Eurostat (2022), only 3.3% of the Polish population met their housing needs by renting in 2020, compared to 30.1% in the Netherlands.[1] From a macroeconomic perspective, rental housing markets are central because they mitigate fluctuations in the residential sector, contribute to economic stability, and increase housing affordability (Rubaszek and Rubio, 2020). These reasons render Warsaw and Amsterdam ideal case studies to demonstrate the value of GWQR-SAR across different housing markets.

We obtained rental listings from www.otodom.pl for Warsaw and www.funda.nl for Amsterdam using web scraping techniques on January 15, 2021, and April 2, 2021. Next, we removed outliers and repetitive observations (Table S1). We included 583 observations for Amsterdam and 967 for Warsaw (Figure S1). All rental listings were described by asking price per square meter and by 19 additional variables characterizing the property's physical characteristics, surrounding area, and location. Table S2 provides descriptive statistics for Warsaw and Amsterdam.

Results and discussion

Model comparison

We first estimated OLS, SAR, GWR, and GWR-SAR models for comparative purposes. We log-transformed the quantitative variables to stabilize the variance and to be able to interpret obtained parameters as elasticities. Moreover, our log-log model specification reduced the problem of non-linear associations between the dependent variable and the predictors. For SAR and GWR-SAR, we used a row-standardized binary k-nearest-neighbor matrix to calculate the spatial autoregressive term. The exact number of neighbors was determined by minimizing the residual sum of squares. For Amsterdam, $k = 18$, and for Warsaw, $k = 8$ were selected (Table S3). SAR and GWR-SAR were estimated using the 2SLS method, which ensured no correlation between the spatial autoregressive term and the residuals (Table S4).[2] For GWR and GWR-SAR, we applied an adaptive bandwidth along with the bi-square kernel function for model calibration.

Estimates of the coefficient of determination (R^2) and the residual sum of squares (RSS) indicated that the best model was GWR-SAR for both case studies (Table 1). Furthermore, GWR and GWR-SAR eliminated the problem of residual spatial autocorrelation to the greatest extent possible. The combined results of the preceding models highlight the importance of accounting for both spatial autocorrelation and heterogeneity when modeling housing markets. Our results are consistent with previous studies comparing OLS, SAR, GWR, and GWR-SAR (Fotheringham and Park, 2018; Tomal, 2022).

We found that our covariates better explained rental prices in Amsterdam than in Warsaw. Three possible explanations might contribute to this discrepancy. First, it might be due to the quality of the data available for each city. Unlike Amsterdam, in Warsaw, properties are not street numbered and only identified by the name of the street. While advertisers may optionally indicate the geographic coordinates of an apartment, if they fail to do so the property listing system defaults to coordinates for the middle of the street on which an apartment is located. Second, the residential rental market is

Table 1. Comparison of OLS, SAR, GWR, and GWR-SAR performance.

	Amsterdam			Warsaw		
Model	RSS	R^2	Moran's I (residuals)	RSS	R^2	Moran's I (residuals)
OLS	20.48	0.52	0.08 ($p < 0.01$)	49.68	0.34	0.11 ($p < 0.01$)
SAR	20.16	0.53	0.07 ($p < 0.01$)	48.99	0.35	0.09 ($p < 0.01$)
GWR	9.74	0.77	−0.04 ($p < 0.01$)	38.37	0.49	0.01 ($p = 0.24$)
GWR-SAR	9.43	0.78	−0.04 ($p < 0.01$)	35.63	0.53	0.00 ($p = 0.44$)

underdeveloped in Poland, creating the possibility that prices may be affected by random fluctuations. Third, it is possible that specific price determinants could exist that are not accounted for by our model of the Warsaw residential market.

We then calibrated QR, QR-SAR, GWQR, and GWQR-SAR for the dependent variable's fifth, 25th, 50th, 75th, and 95th percentiles. For GWQR and GWQR-SAR, a local constant estimator was used with an adaptive bandwidth and the bi-square kernel to ensure comparability to the GWR and GWR-SAR settings. Moreover, a two-stage estimation of QR-SAR and GWQR-SAR successfully eliminated the endogeneity of Wy (Table S4). Local parameters for GWQR and GWQR-SAR were generated based on 500 bootstrapped replications. Using a computer with an Intel Core i5 1135G7 processor and 16 GB DDR4 memory, the estimation for a particular quantile took about 12 h. Table 2 shows the RSS values for the calibrated quantile models. Consistently across the study areas and for all percentiles tested, GWQR-SAR resulted in the lowest RSS.

Mean and quantile regressions cannot be directly compared in terms of traditional goodness-of-fit measures because the latter has a local character for particular quantiles, while mean regression refers to the entire conditional distribution of the response variable (Koenker and Machado, 1999). Therefore, following Khattak et al. (2016), to compare the performance of GWQR-SAR with other mean models, we selected observations from the sample that were the fifth, 25th, 50th, 75th, and 95th percentiles of the dependent variable and 10 observations below and above these percentiles (105 observations in total). We determined the fitted values based on sets of parameter estimates across the percentiles. Table 3 presents the in-sample fitting accuracy for GWQR-SAR against regressions without quantile effects. GWQR-SAR outperformed the other competitive models, particularly in the case of the residential rental market in Warsaw, with an RMSE decrease of >0.10. In both cities, the correlation between the observed and the GWQR-SAR fitted values exceeded 0.95. Figure 1 shows that GWR-SAR resulted in pronounced inaccuracies in estimating the cheapest and the most expensive rents (i.e., GWR-SAR overestimated low rent values and underestimated high ones). As a robustness test, we checked the fitting accuracy of GWR-SAR and GWQR-SAR for the whole sample. Again, GWQR-SAR achieved better performance (Figure S2). Notably, the performance of GWQR-SAR can be significantly improved by generating model parameters for more percentiles of the dependent variable. Finally, we examined uncertainty of local parameters of GWQR-SAR and GWR-SAR using 95% confidence intervals (Table 4). With the exception of the 75th percentile case for Warsaw, GWR-SAR estimates had higher levels of uncertainty than GWQR-SAR estimates.

Spatial variations and quantile effects in local parameters

Table 5 presents the median of local parameters of GWQR-SAR for Warsaw and Amsterdam across studied percentiles (minimum and maximum values in Tables S5 and S6). The optimal bandwidths are highest when examining the fifth and 95th percentiles of the rental price distribution, consistent

Table 2. RSS values for quantile models.

	Amsterdam					Warsaw				
Model	P5	P25	P50	P75	P95	P5	P25	P50	P75	P95
QR	74.55	30.82	23.09	26.84	73.68	195.45	70.04	50.33	70.75	189.89
QR-SAR	72.73	31.36	22.64	26.91	73.59	188.21	70.66	49.80	69.41	192.49
GWQR	47.91	17.51	12.82	15.82	74.15	179.21	56.59	44.37	54.75	187.57
GWQR-SAR	46.52	16.99	12.70	15.50	73.29	178.89	55.97	44.27	52.70	181.19

Notes: P represents percentile

Table 3. Comparison of GWQR-SAR and non-quantile models.

Panel A: Amsterdam							
Correlation analysis							
Fitted values							
Model	OLS	SAR	GWR	GWR-SAR	GWQR-SAR	Observed values	RMSE
OLS	1	0.994	0.881	0.882	0.851	0.732	0.212
SAR	0.994	1	0.890	0.891	0.856	0.740	0.209
GWR	0.881	0.890	1	0.997	0.954	0.909	0.135
GWR-SAR	0.882	0.891	0.997	1	0.960	0.914	0.131
GWQR-SAR	0.851	0.856	0.954	0.960	1	0.952	0.107
Panel B: Warsaw							
Correlation analysis							
Fitted values							
Model	OLS	SAR	GWR	GWR-SAR	GWQR-SAR	Observed values	RMSE
OLS	1	0.987	0.894	0.871	0.800	0.658	0.282
SAR	0.987	1	0.903	0.881	0.802	0.658	0.281
GWR	0.894	0.903	1	0.993	0.875	0.758	0.249
GWR-SAR	0.871	0.881	0.993	1	0.892	0.788	0.237
GWQR-SAR	0.800	0.802	0.875	0.892	1	0.951	0.118

with Chen et al. (2020). When analyzing the local parameters obtained for the structural variables, the apartment area variable, in line with other studies (Helbich et al., 2014), is significant in nearly all locations in both markets. The relationship between this variable and rental price changes in both cities depended on the examined response variable percentile confirming Fitzenberger and Fuchs (2017). For example, Figure 2 illustrates the spatial and quantile variation in the relationship between the apartment living area variable and rental prices in Amsterdam. Exploring the 95th percentile of the dependent variable found that the weakest impact of the studied covariate was observed in the southeast, which is spatially disconnected from the rest of Amsterdam. This finding aligns with other analyses suggesting that this area has a unique residential character (Kauko, 2005). Conversely, the relationship is most marked in the north of Amsterdam. Of note, the estimation results for the two cities are divergent in terms of the other structural variables. As opposed to

Warsaw, the number of rooms in an apartment is a vital rent determinant in Amsterdam, following Daams et al. (2019) and confirming Tomal (2020). Contrary to the preferences of Amsterdam residents, results suggested that the housing preferences of Varsovians are dependent on building age. Renters in Warsaw are mainly looking for new apartments, for which they are willing to pay a premium. These differences may be because the older housing stock in Warsaw is built with prefabricated technology (Tofiluk et al., 2019), and the buildings themselves are characterized by unattractive appearances, with units lacking modern appliances and few available building amenities. Finally, in both markets, while what story an apartment is on does not significantly impact the rental price, the presence of elevators in buildings does affect the price, with rental prices in buildings possessing elevators being significantly higher.

The distance of an apartment to a public transport stop should be examined first in terms of the locational variables. In Amsterdam, this variable was insignificant in each examined percentile, as a significant share of the trips people took were by bicycle (Rietveld and Daniel, 2004). In the case of Warsaw, the impact was variable over space and the percentiles. Quantile effects related to the distance to the city center were primarily observed in Warsaw. In Warsaw, the locational characteristic most strongly affected the rental prices of the most luxurious apartments. Similar results for both cities were found for apartments' distances to main roads. We found a positive relationship

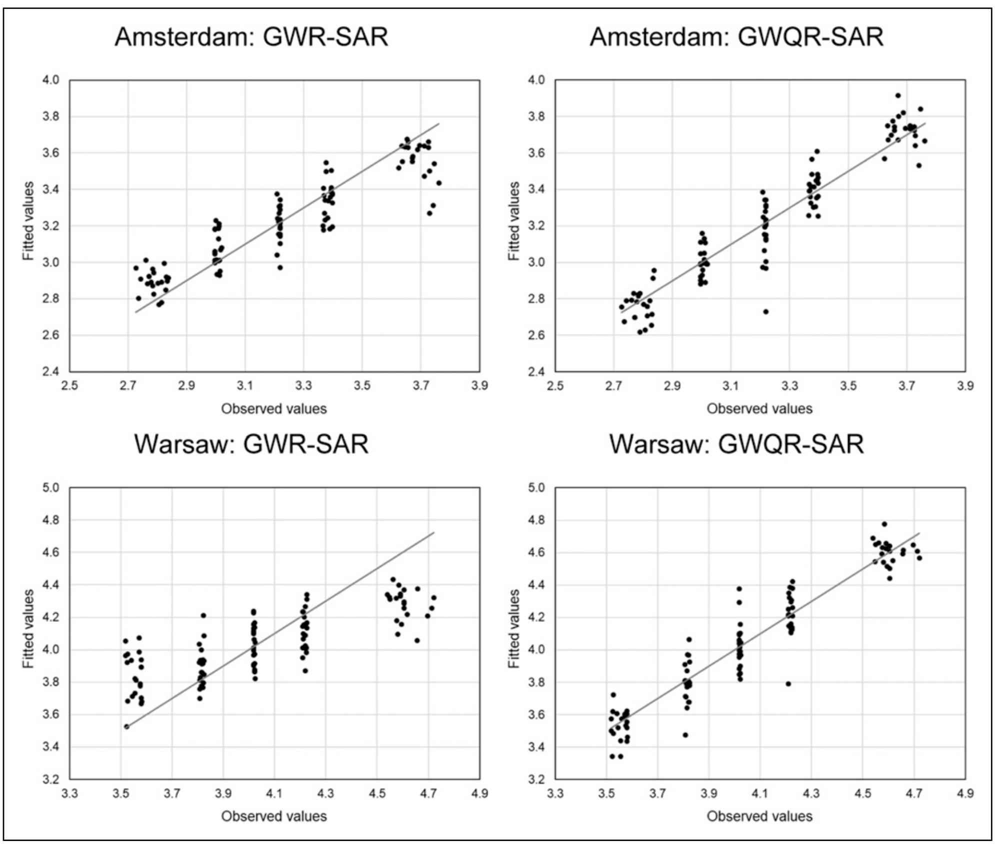

Figure 1. Accuracy in fitting of GWR-SAR and GWQR-SAR. *Notes*: Values ranked in ascending order based on observed rents.

Table 4. Average lengths of 95% confidence intervals.

Variable	Amsterdam						Warsaw					
	GWR-SAR	GWQR-SAR					GWR-SAR	GWQR-SAR				
		P5	P25	P50	P75	P95		P5	P25	P50	P75	P95
Intercept	7.543	4.202	5.171	4.365	6.942	5.316	5.157	3.226	3.692	3.107	6.193	4.349
Floor area [S1]	0.239	0.175	0.196	0.170	0.220	0.220	0.233	0.188	0.184	0.153	0.248	0.205
Number of rooms [S2]	0.317	0.250	0.257	0.223	0.288	0.292	0.256	0.215	0.204	0.167	0.271	0.240
Floor level [S3]	0.117	0.088	0.095	0.087	0.105	0.115	0.110	0.089	0.086	0.070	0.110	0.106
Age of the building in years [S4]	0.074	0.058	0.066	0.057	0.073	0.070	0.061	0.047	0.045	0.041	0.065	0.049
Availability of elevator in the building [S5]	0.188	0.153	0.158	0.143	0.181	0.184	0.139	0.117	0.106	0.092	0.146	0.129
Distance to nearest bus, tram or train stop [L1]	0.100	0.071	0.081	0.071	0.091	0.091	0.080	0.075	0.064	0.053	0.084	0.072
Distance to city center [L2]	0.416	0.200	0.300	0.232	0.368	0.211	0.179	0.131	0.114	0.099	0.233	0.127
Distance to nearest primary or secondary road [L3]	0.078	0.060	0.064	0.057	0.072	0.076	0.046	0.041	0.038	0.030	0.052	0.041
Distance to nearest local government building [N1]	0.153	0.095	0.125	0.108	0.145	0.135	0.081	0.074	0.061	0.049	0.091	0.069
Distance to nearest work center [N2]	0.123	0.092	0.101	0.090	0.116	0.102	0.066	0.052	0.049	0.041	0.073	0.055
Distance to nearest kindergarten [N3]	0.113	0.079	0.094	0.080	0.107	0.100	0.087	0.068	0.064	0.058	0.093	0.076
Distance to nearest school [N4]	0.104	0.082	0.083	0.075	0.098	0.099	0.077	0.064	0.060	0.048	0.084	0.068
Distance to nearest university [N5]	0.204	0.102	0.156	0.130	0.195	0.123	0.082	0.068	0.059	0.055	0.093	0.069
Distance to nearest pharmacy [N6]	0.117	0.085	0.098	0.085	0.110	0.104	0.077	0.066	0.059	0.049	0.084	0.066
Distance to nearest shopping mall [N7]	0.166	0.117	0.147	0.114	0.163	0.129	0.100	0.080	0.079	0.064	0.113	0.079

(continued)

Table 4. (continued)

Variable	Amsterdam GWR-SAR	Amsterdam GWQR-SAR P5	P25	P50	P75	P95	Warsaw GWR-SAR	Warsaw GWQR-SAR P5	P25	P50	P75	P95
Distance to nearest supermarket [N8]	0.106	0.082	0.087	0.076	0.100	0.100	0.084	0.071	0.066	0.059	0.093	0.072
Distance to nearest forest [N9]	0.112	0.086	0.090	0.078	0.100	0.108	0.091	0.071	0.065	0.058	0.103	0.080
Distance to nearest park [N10]	0.114	0.077	0.090	0.080	0.105	0.095	0.071	0.055	0.054	0.044	0.074	0.060
Distance to nearest river or reservoir [N11]	0.092	0.065	0.072	0.065	0.082	0.083	0.095	0.078	0.072	0.062	0.111	0.082
Wy	1.541	0.936	1.052	0.874	1.412	1.096	0.950	0.638	0.711	0.578	1.063	0.717

between this covariate and rents for each percentile, which may have resulted from increasing proximity to the main road being correlated to increasing exposure to road noise (Kim et al., 2015).

For neighborhood variables, the most noteworthy differences between the two cities were observed for the effect of distances to work centers and kindergartens. In the case of the Warsaw rental market, these factors did not determine rental prices, except in the case of the most expensive apartments. This may have resulted from the fact that in Warsaw, the rental market only acted as a supplement to the housing sales market, and for a significant percentage of tenants, renting was only a temporary solution (Głuszak, 2015). However, in Amsterdam, these neighborhood variables significantly influenced rental prices in, on average, 40% of the observations. In both cities, distance to a university was another factor that impacted residential rental markets, mainly due to the housing demand generated by students (Sirmans and John, 1991).

Finally, in Amsterdam, except for the model of the 95th percentile, the spatial autoregressive term was significant in about 20%–30% of cases, primarily positive, and characterized by a significant spatial variation. In the 75th percentile analysis, negative spatial autocorrelation could be observed, implying that observations close to one another possessed significantly different values (Fotheringham, 2009). Such a situation does not often occur in the housing market, but it can be supported under certain circumstances, for example, when cheaper properties surround more expensive rental apartments. In the case of Warsaw, the significant and global impact of the spatial autoregressive term is observed only for the least expensive properties. Furthermore, in Warsaw, while the value of the spatial autoregressive parameter was high at the tails of the dependent variable distribution, it was low for the 25th, 50th, and 75th percentiles. Liao and Wang (2012) also found this U-shaped pattern.

Implications for rental housing market development

We formulated recommendations in Table 6 for rental market development based on the GWQR-SAR estimation results (Wu et al., 2019). In Amsterdam, local renter preferences should be considered when creating housing stock for the private rental market. The exception is the fifth

Table 5. Median values of local parameters generated by GWQR-SAR.

	Amsterdam					Warsaw				
Variable	P5	P25	P50	P75	P95	P5	P25	P50	P75	P95
Intercept	3.956 (92.8) [No]	4.686 (99.8) [Yes]	5.053 (100) [Yes]	6.557 (98.1) [Yes]	7.460 (100) [Yes]	3.472 (100) [No]	4.827 (99.9) [No]	4.976 (100) [No]	6.410 (88.9) [No]	5.122 (100) [No]
Floor area [S1]	−0.591 (100) [No]	−0.574 (100) [Yes]	−0.545 (100) [No]	−0.520 (92.62) [No]	−0.363 (100) [Yes]	−0.325 (100) [No]	−0.112 (65.8) [No]	−0.059 (38.4) [No]	−0.024 (31.4) [Yes]	−0.075 (0) [No]
Number of rooms [S2]	0.260 (74.1) [No]	0.226 (75.6) [Yes]	0.233 (85.3) [Yes]	0.260 (80.0) [Yes]	0.137 (45.3) [Yes]	0.143 (80.8) [No]	0.033 (0) [No]	0.013 (0) [No]	0.019 (13.3) [Yes]	0.088 (0) [No]
Floor level [S3]	−0.020 (12.2) [No]	−0.009 (15.4) [Yes]	−0.018 (12.3) [Yes]	−0.007 (0) [Yes]	0.011 (10.0) [No]	0.002 (0) [No]	0.002 (0) [No]	0.018 (0) [No]	0.002 (5.1) [Yes]	−0.033 (0) [No]
Age of the building in years [S4]	0.042 (82.3) [No]	0.005 (6.4) [Yes]	0.015 (24.0) [Yes]	0.006 (6.5) [Yes]	0.016 (4.0) [Yes]	−0.098 (100) [No]	−0.091 (100) [No]	−0.082 (100) [No]	−0.063 (98.6) [Yes]	−0.057 (100) [No]
Availability of elevator in the building [S5]	0.066 (48.5) [Yes]	0.023 (24.5) [Yes]	0.044 (33.3) [Yes]	0.049 (24.7) [Yes]	0.123 (75.9) [No]	0.035 (33.3) [No]	0.054 (67.7) [No]	0.046 (55.2) [No]	0.038 (10.7) [Yes]	−0.012 (0) [No]
Distance to nearest bus, tram or train stop [L1]	−0.008 (0) [No]	−0.002 (0.7) [No]	0.009 (0) [No]	0.014 (2.4) [No]	−0.012 (0) [No]	0.056 (68.9) [Yes]	0.005 (0) [No]	−0.019 (37.0) [Yes]	0.010 (13.4) [Yes]	−0.023 (0.2) [No]
Distance to city center [L2]	−0.001 (0.5) [No]	0.005 (28.6) [Yes]	0.002 (50.0) [Yes]	−0.053 (52.7) [Yes]	−0.277 (95.9) [Yes]	−0.062 (51.4) [No]	−0.087 (97.1) [No]	−0.078 (99.3) [No]	−0.203 (66.8) [Yes]	−0.124 (100) [No]
Distance to nearest primary or secondary road [L3]	0.043 (72.6) [No]	0.035 (57.5) [Yes]	0.028 (51.1) [Yes]	0.023 (26.0) [Yes]	0.037 (52.5) [Yes]	0.026 (65.7) [No]	0.027 (70.0) [No]	0.005 (0) [No]	0.002 (2.0) [Yes]	−0.013 (0) [No]
Distance to nearest local government building [N1]	0.002 (0.5) [No]	−0.007 (35.7) [Yes]	−0.022 (26.4) [Yes]	−0.045 (23.5) [Yes]	0.005 (0) [Yes]	−0.027 (0) [No]	−0.015 (17.0) [Yes]	−0.019 (27.9) [No]	−0.009 (0.9) [Yes]	−0.022 (25.3) [No]
Distance to nearest work center [N2]	−0.009 (5.8) [No]	0.003 (2.4) [Yes]	−0.029 (33.8) [Yes]	−0.027 (37.7) [Yes]	−0.111 (100) [Yes]	0.014 (0) [No]	0.006 (8.7) [Yes]	0.002 (12.1) [Yes]	0.000 (0) [Yes]	0.031 (78.3) [No]
Distance to nearest kindergarten [N3]	0.032 (40.5) [No]	−0.012 (21.8) [Yes]	−0.020 (33.3) [Yes]	−0.021 (21.6) [Yes]	−0.065 (95.0) [Yes]	−0.022 (0) [No]	0.005 (0) [No]	0.003 (0) [No]	−0.003 (2.0) [No]	−0.040 (59.9) [No]
Distance to nearest school [N4]	−0.007 (0) [No]	−0.004 (0) [No]	0.008 (0) [No]	0.008 (1.0) [No]	0.017 (0) [Yes]	0.027 (0) [No]	−0.007 (2.2) [No]	−0.005 (0) [No]	−0.001 (5.2) [No]	0.021 (0) [No]
Distance to nearest university [N5]	−0.008 (9.2) [No]	−0.031 (30.3) [Yes]	−0.005 (50.9) [Yes]	−0.008 (47.5) [Yes]	0.078 (83.3) [Yes]	−0.017 (0) [No]	−0.059 (65.6) [Yes]	−0.059 (78.0) [Yes]	−0.018 (26.8) [Yes]	−0.064 (100) [No]

(continued)

Table 5. (continued)

	Amsterdam					Warsaw				
Variable	P5	P25	P50	P75	P95	P5	P25	P50	P75	P95
Distance to nearest pharmacy [N6]	0.003 (18.2) [No]	−0.001 (11.5) [Yes]	0.006 (0) [Yes]	0.002 (2.1) [Yes]	0.067 (88.5) [Yes]	0.019 (0) [No]	0.001 (0) [No]	0.007 (0) [No]	−0.001 (13.6) [Yes]	0.045 (84.1) [No]
Distance to nearest shopping mall [N7]	−0.038 (38.2) [No]	0.008 (6.5) [Yes]	−0.012 (22.8) [Yes]	−0.001 (19.6) [Yes]	0.015 (11.0) [Yes]	−0.021 (0) [No]	−0.005 (33.8) [Yes]	0.000 (24.6) [Yes]	−0.003 (48.3) [Yes]	0.014 (22.3) [No]
Distance to nearest supermarket [N8]	−0.009 (0) [No]	0.005 (0) [Yes]	−0.004 (2.9) [Yes]	0.002 (7.2) [Yes]	−0.045 (43.2) [No]	−0.001 (0) [No]	−0.012 (0) [No]	0.008 (0) [No]	−0.001 (13.0) [Yes]	0.015 (0) [No]
Distance to nearest forest [N9]	−0.014 (0) [No]	−0.001 (9.1) [Yes]	0.004 (10.5) [Yes]	0.003 (4.1) [Yes]	−0.055 (50.1) [Yes]	−0.022 (1.0) [No]	−0.018 (0) [Yes]	−0.004 (0) [No]	0.028 (16.3) [Yes]	0.017 (0) [No]
Distance to nearest park [N10]	0.024 (41.7) [Yes]	0.009 (1.4) [No]	0.012 (3.10) [No]	0.016 (7.0) [No]	−0.017 (4.6) [Yes]	−0.020 (27.9) [No]	−0.022 (35.3) [No]	−0.014 (4.4) [No]	−0.017 (11.1) [No]	−0.005 (0) [No]
Distance to nearest river or reservoir [N11]	−0.005 (3.6) [No]	0.013 (3.4) [Yes]	0.008 (13.4) [Yes]	0.001 (0.3) [Yes]	0.037 (44.3) [No]	0.029 (4.1) [No]	0.012 (0) [No]	−0.023 (2.3) [No]	−0.017 (21.9) [Yes]	−0.008 (0) [No]
Wy	0.315 (19.7) [No]	0.294 (32.9) [Yes]	0.263 (30.3) [Yes]	−0.080 (18.7) [Yes]	−0.077 (0) [Yes]	0.510 (78.0) [No]	0.160 (14.9) [No]	0.159 (13.2) [No]	−0.094 (12.1) [Yes]	0.245 (6.9) [No]
h_o	374	219	264	223	477	780	594	674	324	826

Notes: In parentheses, information on the number of significant cases at the 0.05 level in %. In square brackets, information on whether the parameter is significantly varied over space. The spatial heterogeneity of each parameter was assessed by comparing whether its interquartile range is at least twice the standard error generated by the global spatial quantile regression as proposed by Chen et al. (2012). P represents percentile. For comparison purposes, Table S7 contains the results of QR-SAR based estimations

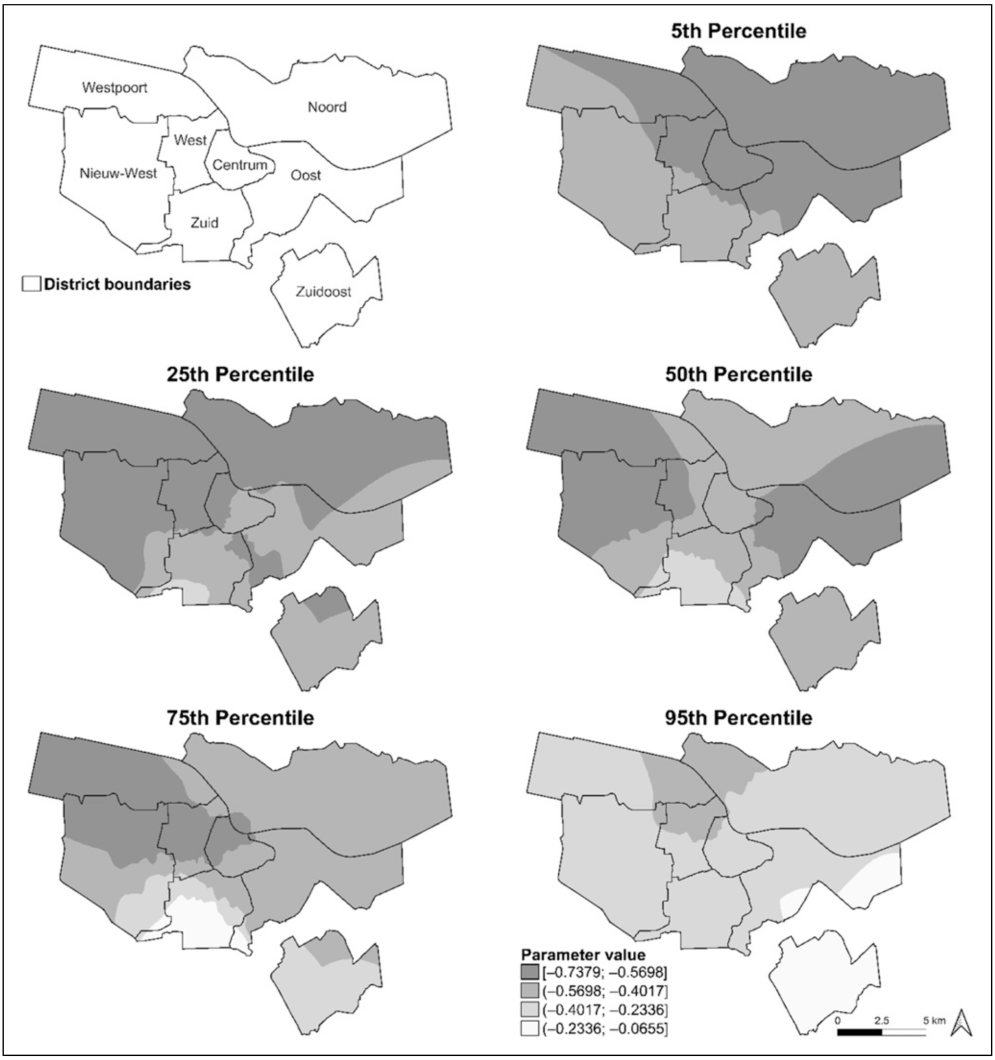

Figure 2. Parameter values for the floor area variable across space and the rental price distribution in Amsterdam. *Notes*: We used the inverse distance weighting (IDW) method to provide parameter surface maps.

percentile representing the least costly price segment primarily influenced by global factors across all neighborhoods. The new housing stock in this price segment should provide apartments having as many rooms as feasible within an adequate space and in conjunction with locations distant from main roads that maximize tranquility. Furthermore, it is recommended to locate new housing close to universities for the below-mid-priced, mid-priced, and above-mid–priced segments. Apartments in the most expensive price segment should possess expansive floor areas, be served by elevators, and be close to kindergartens and natural amenities. Finally, such apartments should be located as far as possible from main roads and close to city and business centers. The least costly, below-average, and average segments in Warsaw should offer new apartments to the market with adequate space, located close to the city center and away from main roads. Local conditions regarding the vicinity of transportation stops and proximity to universities should also be considered for these housing

Table 6. Suggestions for rental housing market development.

Category	Rent price segment									
	Cheapest (P5)		Below average (P25)		Average (P50)		Above average (P75)		Most expensive (P95)	
Spatial dimension	A	W	A	W	A	W	A	W	A	W
Global	S1,S2, S4,L3	S1,S2, S4,L2,L3	—	S1,S4,S5,L2, L3	—	S4,S5, L2	—		S5	S4,L2,N2,N3,N5,N6
Local	—	L1	S1,S2, L3	N5	S1,S2,L3,N5	N5	S1,S2, L2	S4, L2	S1,L2, L3,N2, N3,N5,N6,N9	—

Notes: A represents Amsterdam. W represents Warsaw submarkets. Furthermore, particular attention should be paid to location and neighborhood quality for apartments in the most expensive price segment.

Limitations

This study also has limitations that should be addressed in future research. First, GWQR-SAR does not consider the possibility of spatial autocorrelation in the error term. Therefore, future analyses should focus on incorporating the spatial error model into GWQR-SAR. Second, GWQR-SAR uses only a single bandwidth for each predictor and can be developed by accounting for the impact of determinants at different spatial scales, as pointed out by Fotheringham et al. (2017). Third, the results indicated that some of the estimated local parameters do not change significantly across space. Therefore, GWQR-SAR should be extended toward incorporating global and local relationships along the lines of mixed GWR (Yao and Fotheringham, 2016). Fourth, GWQR-SAR neglects temporal dependencies and may not be effective when exploring determinants of sales or rental prices across time. To capture these effects, GWQR-SAR should account for both temporal and spatial information when locally weighting observations similar to geographically and temporally weighted regression (Huang et al., 2010). Research limitations are also related to the data used. We analyzed listing data that represented only the supply side of the housing market, which could potentially affect the quality of the results obtained (Kolbe et al., 2021). Finally, the presented policy implications might potentially be misleading due to new buildings, generally being constructed on the outskirts of the two cities due to inner-city land scarcity.

Conclusion

This paper explored residential rent determinants in Warsaw and Amsterdam using a novel GWQR-SAR hedonic model, which simultaneously considers spatial autocorrelation of prices and heterogeneity of housing preferences across space and price segments. Our findings showed that the proposed model outperforms other widely applied hedonic regression models. Compared to mean regressions, the better performance of GWQR-SAR is mainly due to better goodness-of-fit at the tails of the distribution of the dependent variable. GWQR-SAR also performs significantly better than global quantile regressions that ignore spatial heterogeneity in housing preferences. The empirical analyses indicated that housing rents in both cities are determined by structural, locational, and neighborhood characteristics and rental prices in the local area. The determinants ultimately varied across space and over the price distribution. The present study's results support recommendations for optimizing rental housing liveability for new housing developments in private residential markets. Rental market development policies in Warsaw should be primarily global for each price segment, whereas Amsterdam should account for local conditions except in the case of the least expensive apartments.

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Data

Due to data protection restrictions, the housing rent data set cannot be made publicly available.

Code availability

The R package GWQR available from https://figshare.com/s/ae6e81c08772593ce627.

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Supplemental Material

Supplemental material for this article is available online.

Notes

1. Rent at market rate.
2. GeoDa (https://geodacenter.github.io/) and MGWR (https://sgsup.asu.edu/sparc/multiscale-gwr) software were used to estimate SAR and GWR-SAR models.

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