Notre Dame Journal of Formal Logic
Volume 63, Number 2, 2022

# Shrieking, Shrugging, and the Australian Plan

Hitoshi Omori and Michael De

**Abstract** We observe that Jc Beall's shrieking and shrugging strategy gives us an opportunity to reflect on the Australian plan for negation in **FDE**, a basic subclassical logic that is used in Beall's argument for subclassical logics. An implication of our observation is applied to a recent defense of the Australian plan for negation by Francesco Berto and Greg Restall.

Jc Beall advances a simple argument for the subclassical logic known as **FDE** (see [2]). Unlike classical logic, **FDE** allows sentences to be both gappy (i.e., neither true nor false) and glutty (i.e., both true and false). Beall's argument in brief is that we lose nothing—inferentially speaking—by adopting **FDE** as our logic, and that what we gain are live theoretical possibilities that would have been excluded by adopting any logic that fails to accommodate both gaps and gluts.

Suppose that our background logic is **FDE** and that we wish to exclude the possibility of gluts for a certain nonlogical predicate $P$. For instance, if we consider arithmetic, then we may believe that the predicate 'is less than' cannot be both true and false of the same pair of natural numbers. If our underlying logic is paraconsistent, then we have no way in general of excluding the possibility of gluts for such a predicate. However, we can force that the predicate behave in a nonglutty way by *shrieking* it, in the terminology of Beall. To shriek a predicate $P$ in the language of theory $T$ is to impose the following constraint on $T$'s closure relation $\vdash_T$:

$$\exists x(Px \wedge \neg Px) \vdash_T \bot,$$

where $\bot$ is true in no model of $T$.[1] To shriek $P$ is to exclude the possibility that something both satisfy $P$ and its negation.

Similarly, if we wish to exclude the possibility of gaps for $P$, then we can shrug the predicate by imposing the following constraint on $T$'s closure relation:

$$\top \vdash_T \forall x(Px \vee \neg Px),$$

Received July 5, 2021; accepted October 11, 2021
2020 Mathematics Subject Classification: Primary 03A05; Secondary 03B53, 03B50
Keywords: negation, **FDE**, contraposition, paraconsistent logic, paracomplete logic
 10.1215/00294527-2022-0009

where $\top$ is true in all models of $T$. To shrug $P$ is to exclude the possibility that something neither satisfy $P$ nor its negation.

We can shriek and shrug an entire theory by shrieking and shrugging all of its predicates. For instance, if we think arithmetic is entirely classical, then we can shriek and shrug it, even if our underlying logic is gappy and glutty. If we think truth is glutty but not gappy, then we can shrug—but not shriek—the truth predicate in our preferred theory of truth. We can do this generally with individual predicates or entire theories. The method of shrieking and shrugging is flexible and allows one to finely tune which parts of a theory are to behave classically or "semiclassically."[2]

The primary aim of this note, however, is cautionary: anyone wanting to employ the shriek-shrug strategy needs to be careful with their treatment of negation in **FDE**. For if contraposition is valid—in the fairly weak form given below—then there is no way to keep shrieking and shrugging apart—that is, shrieking a predicate will imply shrugging it, and conversely. The form of contraposition we have in mind is the following:

**(Contra):** If $A \vdash B$, then $\neg B \vdash \neg A$.

The following is a derivation showing that shrieking implies shrugging when (Contra) is valid:[3]

1. $\exists x(Px \wedge \neg Px) \vdash_T \bot$ [Shriek]
2. $\bot \vdash_T \neg\top$ [$\bot \vdash_T A$ for all $A$ since $\bot$ is true in no model of $T$]
3. $\exists x(Px \wedge \neg Px) \vdash_T \neg\top$ [1, 2]
4. $\top \vdash_T \forall x(Px \vee \neg Px)$ [3, (Contra), de Morgan laws, double neg. elim.]

There are two equivalent semantics for **FDE**, one known as the *American* plan and the other as the *Australian* plan.[4] Both of these semantics validate (Contra) with respect to the usual base language, but there are important subtle differences between the semantics when considering extensions of **FDE**.

It is worth discussing in some detail the validity of (Contra) in the various semantics for **FDE**. We begin with the American plan, following [2]. An interpretation $\rho$ relates sentences to the truth-values truth and falsity without any constraint on the interpretation such as bivalence, namely, that each sentence must be related to at least one of the values.[5] Now assume $\neg B \nvdash \neg A$. Then, there is an **FDE** interpretation $\rho$ such that $\neg B\rho 1$ and not $\neg A\rho 1$. Therefore, by the truth conditions for negations, that is,

1. $\neg A\rho 1$ iff $A\rho 0$,
2. $\neg A\rho 0$ iff $A\rho 1$,

we have that $B\rho 0$ and not $A\rho 0$. As noted by Michael Dunn in [5, p. 165], if $\rho$ is an **FDE** interpretation, then there exists its dual, $\rho^d$, such that, for all formulas $A$,

- $A\rho^d 1$ iff not $A\rho 0$,
- $A\rho^d 0$ iff not $A\rho 1$.

Therefore, we have that there is a dual **FDE** interpretation $\rho^d$ of $\rho$ such that $A\rho^d 1$ and not $B\rho^d 1$, whence $A \nvdash B$, as desired. Contrapositively, (Contra) is valid on the American plan.

This is worrisome for anyone wanting to employ the shriek-shrug strategy who does not want to conflate shrieking with shrugging. However, (Contra) is not "built into" the semantics on the American plan. As an illustration, let us think of a popular extension of **FDE**, namely, **LP**. With the American plan, we can obtain this extension by adding an exhaustivity condition to **FDE** interpretations: for all atoms $p$, either

$p\rho 1$ or $p\rho 0$. Let us refer to such interpretations as **LP** interpretations which form a subclass of the **FDE** ones. It is easy to see that any **LP** interpretation validates the law of excluded middle (LEM), $A \vee \neg A$, which corresponds to shrieking. However, relative to **LP** interpretations, (Contra) fails. Indeed, the dual interpretation—recall from our discussion just above—is not an **LP** interpretation but rather a **K3** interpretation! This shows that (Contra) is not built into the semantics on the American plan, in the sense that (Contra) can fail when the logic is extended. This allows one to keep shrieking and shrugging apart on the American plan.

Let us now move onto the Australian plan. A model $M = (W, *, V)$ consists of a nonempty set of worlds $W$, the Routley star $* : W \to W$ such that for all $w \in W$, $w^{**} = w$, and a valuation $V : W \times \text{Atoms} \to \{0, 1\}$. The semantic consequence relation may be defined *locally*, following the terminology in modal logic: $\Gamma$ entails $A$ locally, written $\Gamma \models_l A$, iff for each model $M = (W, *, V)$ and each state $w \in W$ we have that if $(M, w) \Vdash B$ for each $B \in \Gamma$, then $(M, w) \Vdash A$. The truth conditions for negation on the Australian plan are given in terms of the Routley star:

**(Neg):** $(M, w) \Vdash \neg A$ if and only if $(M, w^*) \nVdash A$.

To see that (Contra) holds, suppose $\neg B \not\models_l \neg A$. Then there is a model $M$ such that $(M, w) \Vdash \neg B$ and $(M, w) \nVdash \neg A$. Therefore, $(M, w^*) \nVdash B$ and $(M, w^*) \Vdash A$, whence $A \not\models_l B$.

Unlike the American plan, (Contra) *is* built into the semantics. If we use **LP** again as our example, then we can obtain **LP** interpretations by requiring that, for all states $x$, if $(M, x^*) \Vdash p$, then $(M, x) \Vdash p$. But the addition of this condition will also imply that, for all states $x$, if $(M, x^{**}) \Vdash p$, then $(M, x^*) \Vdash p$. From this it follows that *ex contradictione quodlibet* (ECQ) is also valid, and, therefore, the resulting semantics yields classical logic, not **LP**. Therefore, attempting to extend the logic on the Australian plan—*understood in terms of local consequence*—preserves (Contra), thereby conflating shrieking with shrugging.

(Contra) remains valid if we work with pointed models, *but it is not built into the semantics* in the same way that it is on the local Australian plan, as we will explain shortly. First, to show that (Contra) is valid over pointed models, let us distinguish a unique state $g$ of each model, now of the form $M = (W, g, *, V)$, relative to which truth in a model is defined, as Richard Routley (later Sylvan) and Valerie Routley (later Plumwood) did in their seminal paper [7]. Let us also define consequence in the usual way as preservation of truth in such a "pointed model," denoted by $\models_p$. Then (Contra) remains valid. Say $\Gamma \models_p A$ iff, for each model $M = (W, g, *, V)$, we have that if $(M, g) \Vdash B$ for each $B \in \Gamma$, then $(M, g) \Vdash A$. Now assume $\neg B \not\models_p \neg A$. Then, there is a pointed model $(M, g)$ such that $(M, g) \Vdash \neg B$ and $(M, g) \nVdash \neg A$. Therefore, $(M, g^*) \nVdash B$ and $(M, g^*) \Vdash A$. Note now that, in general, if $(M, g)$ is a pointed model for **FDE**, then so is $(M, g^*)$. This is because none of the truth conditions for the connectives depend on the distinguished point, in which case $(M, g^*)$ serves as our counterexample to $A \models_p B$.

However, the "pointed model" version of the semantics does not build in (Contra). Working again with **LP** as our example, we can obtain an **LP** interpretation by adding the condition that if $(M, g^*) \Vdash p$ then $(M, g) \Vdash p$. With this additional condition, LEM is valid, but (Contra) now fails. Note that the move above from $(M, g^*) \Vdash A$ but $(M, g^*) \nVdash B$ to the failure of (Contra) now fails because even though $(M, g)$ is a model for **LP**, $(M, g^*)$ is not, but it is instead a model for **K3**. This shows that (Contra) is not built into the semantics on the Australian plan semantics with a distinguished point, and that (Contra) can fail when extending the logic. This allows one to keep shrieking and shrugging apart on the Australian plan.[6]

In sum, we considered one American plan and two Australian plan semantics for **FDE** and showed that (Contra) is valid with respect to each when we confine ourselves to the usual base language. However, if we consider extending the logic, for example, by adding constraints to the interpretation (or by adding additional vocabulary, an option we did not consider), then the three semantics fail to agree on the validity of (Contra), which is valid only on the Australian plan with local validity (i.e., without a designated basepoint). The addition of these constraints corresponds to shrieking and shrugging in an obvious way: LEM (i.e., $\top \vdash A \vee \neg A$) corresponds to shrug rules—except that LEM is a logical rather than a nonlogical rule—and ECQ (i.e., $A \wedge \neg A \vdash \bot$) corresponds to shriek rules. Therefore, anyone wishing to employ the shriek-shrug strategy on the Australian plan should define truth in a model as truth at a distinguished state.

This cautionary note bears on a recent defense of the Australian plan for negation by Francesco Berto and Greg Restall in [4].[7] There, Berto and Restall employ a local consequence relation, but this will be less flexible if one wishes to employ the shriek-shrug strategy within a more general setting. One friendly suggestion, therefore, is to define truth in a model relative to a distinguished state. One, we think, positive consequence of doing so is that Berto and Restall's account of negation will then be able to accommodate the negations of Strong Kleene logic **K3** and the Logic of Paradox **LP** in a simple manner, as this is not possible in the original framework because of (Contra) holding. Since the negation of **K3** and **LP** is a close cousin of **FDE**'s—indeed, in one sense it is the very same negation considered in a three-valued setting—this would seem to be a desirable consequence for anyone defending the Australian plan.

## Notes

1. For simplicity, it is assumed that $P$ is unary.

2. An earlier application of shrieking to the so-called just-true problem can be found in Beall [1].

3. Proof of the other direction is similar, whence shrieking and shrugging are conflated in the presence of (Contra).

4. A proof of the equivalence for the propositional language can be found, for example, in Priest [6, Chapter 8], and it is easy to observe that this carries over to the first-order language.

5. An equivalent four-valued semantics assigns subsets of the two truth-values to sentences, construed as the values to which the sentence is related.

6. As a referee rightly points out, on some ways of adding a relevant conditional to the language, (Contra) remains valid, which spells trouble for the shriek-shrug strategy. So the point we are making applies to a broader range of logics than just **FDE**. But since the failure of (Contra) in a larger context depends on the details of the semantics, and

since we are primarily interested in the application of this observation to the shriek-shrug strategy as Beall conceives it, we have confined ourselves to the usual arrow-free base language of **FDE**.

7. See also [3] where Berto defends (Contra).

## References

[1] Beall, Jc, "Shrieking against gluts: The solution to the 'just true' problem," *Analysis*, vol. 73 (2013), pp. 438–45. Zbl 1311.03008. MR 3090917. DOI 10.1093/analys/ant057. 140

[2] Beall, Jc, "The simple argument for subclassical logic," *Philosophical Issues*, vol. 28 (2018), pp. 30–54. 137, 138

[3] Berto, F., "A modality called 'negation,'" *Mind*, vol. 124 (2015), pp. 761–93. MR 3413351. DOI 10.1093/mind/fzv026. 141

[4] Berto, F., and G. Restall, "Negation on the Australian plan," *Journal of Philosophical Logic*, vol. 48 (2019), pp. 1119–44. Zbl 1457.03007. MR 4036483. DOI 10.1007/s10992-019-09510-2. 140

[5] Dunn, J. M., "Intuitive Semantics for First-Degree Entailments and 'Coupled Trees'," *Philosophical Studies*, vol. 29 (1976), pp. 149–68. Zbl 1435.03043. MR 0491026. DOI 10.1007/bf00373152. 138

[6] Priest, G., *Introduction to Non-classical Logics: From Ifs to Is*, 2nd ed., Cambridge University Press, Cambridge, 2008. MR 2412933. DOI 10.1017/CBO9780511801174. 140

[7] Routley, R., and V. Routley, "The semantics of first-degree entailment," *Noûs*, vol. 6 (1972), pp. 335–90. Zbl 1366.03122. MR 0505279. DOI 10.2307/2214309. 139

## Acknowledgments

The research of Hitoshi Omori was supported by a Sofja Kovalevskaja Award of the Alexander von Humboldt-Foundation, funded by the German Ministry for Education and Research. We would like to thank the referees for their careful comments.

*Omori*
Department of Philosophy I, Ruhr University Bochum, Bochum, Germany;
Hitoshi.Omori@rub.de; https://www.ruhr-uni-bochum.de/philosophy/nklogik/

*De*
Department of Philosophy, Utrecht University, Utrecht, Netherlands; m.j.de@uu.nl;
http://michaelde.com/