Unknown Numbers

Rick Nouwen

1 Numerals and Scope

One of the most intriguing properties of cardinal numerals is that they can play a great diversity of grammatical roles. While the numeral in (1) acts like a quantificational determiner, it resembles an adjective in (2) and a proper name in (3).

(1) Twelve students passed the test.

(2) The twelve students that passed the test were happy.

(3) Twelve is not a prime number.

This chameleonic distribution is also reflected in analyses of the semantics of numerals, which tend to focus mainly on the flexibility required to

R. Nouwen(✉)
Utrecht University, Utrecht, The Netherlands
e-mail: r.w.f.nouwen@uu.nl

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accommodate all such examples. See, e.g. Bylinina and Nouwen (2020) and references therein.[1]

Naturally, given the fact that one of the uses of numerals is determiner-like, they can be involved in scope shifting. So, (4) has a de re reading which says that Sue is in compliance with the rules if and only if she reads two particular books by Auster.

(4) Sue should read two books by Auster.

The de re reading is easily derived by assuming that '*two books by Auster*' is a type $\langle 1 \rangle$ quantifier that can be raised. For instance, we could assume the meaning of '*two*' in (5) and the logical form in (6), and thus get the meaning in (7).[2]

(5) $[\![\text{'}\textit{two}\text{'}]\!] = \lambda A.\lambda B.\exists x \exists y[x \neq y \wedge A(x) \wedge B(x)]$

(6) [$[_{\langle\langle e,t\rangle,t\rangle}$ two books by Auster] $[_{\langle e,t\rangle}$ λx Sue should read x]]

(7) $\exists x \exists y[x \neq y \wedge \text{book-by-Auster}(x) \wedge \text{book-by-Auster}(y) \wedge \Box[\text{read}(s, x)] \wedge \Box[\text{read}(s, y)]]$

In this chapter, I will look at examples that involve a mysterious kind of scope shift, one that is not easily captured by standard mechanisms. Consider (8) and (9):

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[1] In a relatively recent development, it is argued by Kennedy (2015) that one of the semantic guises of a numeral is that of a degree quantifier. His motivation is that numerals can be seen to shift scope independent of the noun phrase they combine with, yielding a so-called split scope reading. For instance, (i) has the split reading in (ii), which Kennedy analyses as (iii).

(i) Sue was allowed to eat two biscuits.

(ii) Sue had permission to eat two biscuits, but Sue was not allowed to eat more than two biscuits.

(iii) 3 is the maximum number n such that: Sue has permission to eat n-many biscuits.

In this analysis, numerals are of type $\langle\langle d, t\rangle, t\rangle$. So, they are type $\langle 1 \rangle$ degree generalised quantifiers. From that perspective, it is entirely to be expected that they shift scope—after all, this is what quantifiers *do*. In what follows, I will ignore the degree quantifier guise.

[2] See Bylinina and Nouwen (2020) for much more sophisticated routes to quantificational readings of numerals. For the purpose of this article, the particular implementation of how to get such readings using a $\langle\langle e, t\rangle, t\rangle$ semantics is not relevant.

(8) At this point, the election could have three winners.

(9) Sue had three husbands.

On its most salient reading, (8) says that there are three individuals and each of these individuals is such that it is possible that he or she wins the election. This reading is clearly not a de dicto reading, since (8) does not entail that the elections could result in there being three winners.[3] Crucially, it is not a de re reading either, since the three individuals are not winners. That is, a mechanism like what we used for (4) would yield a de re meaning like (10):

(10) $$\exists x \exists y \exists z[x \neq y \wedge y \neq z \wedge x \neq z \wedge \text{winner}(x) \wedge \text{winner}(y) \wedge \text{winner}(z) \wedge \Diamond[\text{has}(e, x)] \wedge \Diamond[\text{has}(e, y)] \wedge \Diamond[\text{has}(e, z)]]$$

This is not the salient reading of (8). The problem is that if '*three winners*' takes wide scope relative to the modal, the sentence ends up claiming the existence of three winners, which is not what the intended reading does. It claims the existence of three individuals relevant to the election. These are said to be the potential winners, but only one of them will actually end up being a winner.

Similar considerations apply to (9). We can once more observe that the most salient reading is not de re. There are no three husbands *now* such that Sue 'had' them earlier. The de dicto reading makes Sue a polygamist, but (9) does not entail that Sue ever had more than one husband. The salient reading is clearly different: there are three individuals such that each of them was Sue's husband at some point in the past.

Examples like these have received some attention in the literature, albeit hardly in the literature targeting the semantics of numerals.[4] Examples like (8) and (9) are discussed under the heading of *summative readings* (Szabó 2010, 2011; Francez 2018). The authors that discuss these examples think of these examples in terms of *quantifier scope*. That is, examples like these are taken to be evidence either for quite specific interpretations of

[3]Also, it is not a split reading along the lines of (i) in Footnote 1 above. The split scope would say something about the (maximum) number of people that can (simultaneously) win the elections. This illustrates why I can safely ignore this option for the remainder of this article.

[4]While I will exclusively look at numerals in this paper, I have no reason to believe that what I say here does not extend to vague quantifiers like '*many*' and '*few*'.

the numeral—purpose-built to yield the desired readings (Francez 2018) or of novel mechanisms of how quantifiers take scope (Szabó 2011). Here, I focus on the latter.[5]

Szabo introduces the notion of *bare quantification*. Central to his proposal is the idea that quantifier raising takes one of two forms: either the whole DP moves (an example of this is what we see in (6)) or just the determiner. In the latter case—the case of *bare quantification*—the determiner pairs up with a general restrictor (Ø in (11)) of type $\langle e, t\rangle$, denoting the domain of entities. For (8), this works as in (11), assuming a type $\langle 1, 1\rangle$ quantifier interpretation for '*three*' as before. This results in the truth conditions in (12).

(11) Three Ø λx [could [the election has [t_x winner]]]

(12) $\exists x \exists y \exists z[x \neq y \wedge y \neq z \wedge x \neq z \wedge \Diamond[\text{the election has winner } x] \wedge \Diamond[\text{the election has winner } y] \wedge \Diamond[\text{the election has winner } z]]$

Similarly, (9) can be interpreted using (13), which results in (14).

(13) Three Ø λx [PAST [Sue had t_x husband]]

(14) $\exists x \exists y \exists z[x \neq y \wedge y \neq z \wedge x \neq z \wedge \text{Past}[\text{Sue has husband } x] \wedge \text{Past}[\text{Sue has husband } y] \wedge \text{Past}[\text{Sue has husband } z]]$

Although Szabo's bare quantification proposal is definitely not standard, the actual scope mechanism that it uses, that of quantifier raising, certainly is. Not every time we observe variations in scope, however, can we point to QR as the source of that variation. In particular, not every sentence that is ambiguous with respect to scope involves a quantifier. Numerals are an example of this. For instance, Solt (2009) points out that numerals in attributive position can be non-restrictive, which means that their contribution falls outside the scope of other operators, such as for example negation in (15). On its most salient reading, this sentence entails that

[5] Francez's account targets summative readings in existential constructions and is thus less general than Szabo's proposal. But see Sect. 3 for discussion.

there are three dogs and it denies that the dogs barked menacingly. On that reading, it does not deny that there are three dogs.

(15) It is not true that the three dogs barked menacingly.

Scope, in other words, is not necessarily quantifier scope. Given that numerals play a multitude of different roles in sentences, it would therefore make sense to ask whether the weird scope effects in (8) and (9) are due to the numeral's quantifier guise or whether they are due to its more adjective-like guise. In this paper, I will explore the latter option.

2 Drawing Inspiration from Epistemic Adjectives

Scope effects with attributive modifiers go beyond ambiguities between restrictive and non-restrictive readings. Morzycki (2016) provides an overview of adjectival scope puzzles, including well-known examples like (16)–(19). As the paraphrases indicate, the contribution made by the adjectives here doesn't appear to be in situ.

(16) An occasional sailor strolled by.
$\rightsquigarrow$ *Occasionally, a sailor strolled by.*

(17) Floyd drank a quick cup of coffee.
$\rightsquigarrow$ *Floyd drank a cup of coffee quickly.*

(18) The average American has 2.3 children.
$\rightsquigarrow$ *On average, an American has 2.3 children.*

(19) She dialed the wrong number.
$\rightsquigarrow$ *The number she dialed was wrong.*

It is uncertain whether these interpretations form a unified phenomenon (but see Morzycki 2016). What is clear, though, is that there is an abundance of non-trivial ways in which adjectives appear to take scope.

Here, I would like to focus on one type of example, involving epistemic adjectives like *unknown*, as in (20), first discussed in detail by Abusch (1997). The most salient reading for (20) is not that the suspect stayed in a hotel that has the property of being unknown, but rather that it is unknown which hotel this is.

(20) The suspect stayed in an unknown hotel.
$\rightsquigarrow$ *It is unknown which hotel the suspect stayed in.*

Abusch and Rooth account for examples like (20) using an update semantics based on Groenendijk et al. (1996). In their approach, the sentence is interpreted as setting up a discourse referent for the hotel that the suspect stayed in. The adjective acts as a test over this dref.

To sketch the idea of their analysis, consider (21) as the dynamic interpretation of (20) minus the adjective.

(21) $\exists x$; stayed-in(s, x); hotel(x)

Given some state, an update with (21) results in a set of pairs of possible worlds and assignment functions $\langle w, f \rangle$ that are such that $f(x)$ is a hotel the suspect stayed in world w. The proposed meaning of the adjective '*unknown*' is now as in (22).

(22) $\forall y \Diamond [y \neq x]$

In the framework of Groenendijk et al. (1996), $\Diamond\varphi$ is interpreted as a test that passes on the input information state if and only if φ is supported in at least one of the world-assignment pairs in that input state. So, '*unknown*' expresses the condition that there is no entity that is assigned to x across all worlds. If the hotel the suspect stayed in is known in the input state, then after updating with (21), all world-assignment pairs will have x pointing to the same hotel. If it is unknown, there will be worlds, where the entity assigned to x differs, thus satisfying the test in (22).

Abusch and Rooth propose that the epistemic participle is interpreted as a non-restrictive modifier, which means that it composes as a conjunct to the interpretation of the host sentence:

(23) $\exists x$; stayed-in(s, x); hotel(x); $\forall y \Diamond [y \neq x]$

This late interpretation of the adjective ensures that the contribution of '*unknown*', i.e. (22), can test the outcome of an update with (21), resulting in the observed reading.

In a way, this makes Abusch and Rooth's '*unknown*' a kind of *post-supposition* avant la lettre: the contribution of the adjective is a condition on the result of the interpretation of the rest of the sentence. In the remainder of this article, I want to explore to what extent summative numerals are like Abusch and Rooth's epistemic participles. If some adjectives have this special mode of interpretation and if numerals have adjective-like interpretations, then perhaps this mode of interpretation is also available to numerals. Note that a non-restrictive or post-suppositional analysis of numerals is not that novel. As I remarked above, Solt (2009) argues for non-restrictive readings of numerals in attributive position. Moreover, Brasoveanu (2010) argues that the cardinality restrictions contributed by modified numerals are post-suppositions.

Here's what a post-suppositional account could look like. Consider (8), repeated here.

(8) At this point, the elections could have three winners.

Let's now say that we interpret (23) minus the numeral as (24).

(24) $\Diamond[\exists x;\ \text{election-winner}(x)]$

The semantics for (24) would involve looking at worlds in which there's a winner of the election and testing whether there's at least one such world. Imagine that this test yields access to the full set of world-assignment pairs compatible with $\exists x;\ \text{election-winner}(x)$. For instance, if we have a universe of five worlds with differing election outcomes, as in (25), and all of these are compatible with the input state then an update with (24) provides access to (26). If only w_1, w_2 and w_3 are compatible with the input state, then we gain access to (27).

(25)

world	winner
w_1	c_1
w_2	c_2
w_3	c_3
w_4	c_4
w_5	c_5

(26)

w_1	$\{\ldots\langle x, c_1\rangle\ldots\}$
w_2	$\{\ldots\langle x, c_2\rangle\ldots\}$
w_3	$\{\ldots\langle x, c_3\rangle\ldots\}$
w_4	$\{\ldots\langle x, c_4\rangle\ldots\}$
w_5	$\{\ldots\langle x, c_5\rangle\ldots\}$

(27)

w_1	$\{\ldots\langle x, c_1\rangle\ldots\}$
w_2	$\{\ldots\langle x, c_2\rangle\ldots\}$
w_3	$\{\ldots\langle x, c_3\rangle\ldots\}$

What now if summative numerals were conditions on resulting information states like (26) and (27)? For instance, the numeral '*three*' could be interpreted as a test such that (27) passes this test and (26) fails it. (This is assuming the numeral gets an exact interpretation, which is expected when interpreted as an adjectival predicate; Bylinina and Nouwen 2020.)

To sketch an implementation of this idea, I use a framework based loosely on that of Brasoveanu (2010). I use a dynamic logic where logical sentences are interpreted with respect to a world of evaluation and pairs of assignment functions. These assignments range over both world and entity variables (resp. i, j, ...and x, y, ...). Worlds can be atomic or plural.

Here's how existential quantification, predication and dynamic conjunction is interpreted[6]:

(28) $[\![\exists x]\!]^{w,\langle f,g\rangle} = 1$ iff f and g differ at most at $f(x)(w)$ and $g(x)(w)$.

(29) $[\![P(x)]\!]^{w,\langle f,g\rangle} = 1$ iff $f = g \;\&\; \forall w' \leq_{\mathrm{A}} w : f(x)(w') \in I_{w'}(P)$

(30) $[\![\varphi;\psi]\!]^{w,\langle f,g\rangle} = 1$ iff there exists a function k such that: $[\![\varphi]\!]^{w,\langle f,k\rangle} = [\![\psi]\!]^{w,\langle k,g\rangle} = 1$

For example, a formula like $\exists x; P(x)$ is now true at w, $\langle f, g\rangle$ whenever $g(x)(w)$ has the property expressed by P in world w and while $f(x)(w)$ may differ from $g(x)(w)$, f and g agree on everything else.

We can now define the interpretation of modals, which will introduce discourse referents over worlds. In (30), i is the name of the discourse referent that is introduced by this modal and R is the accessibility relation relevant to the modal operator.

(31) $[\![\Diamond^i[\varphi]]\!]^{w,\langle f,g\rangle} = 1$ iff $g(i)$ is the maximum plural world v such that $\forall w' \leq_{\mathrm{A}} v : R(w,w') \wedge [\![\varphi]\!]^{v,\langle f[i\to v],g\rangle} = 1$ and $|g(i)| \geq 1$.

Let me illustrate how this works with an example:

(32) Sue might have a husband.

(33) $\Diamond^i[\exists x;\ \text{husband-of}(x,s)]$

[6]Here, I use $\leq_{\mathrm{A}}$ as the atomic-part-of relation on pluralities.

If we interpret (33) in a world where we know Sue doesn't have a husband, then we just get *false*. In the interpretation of (32), $g(i)$ ends up the sum of all accessible worlds, where Sue has a husband. If there is no such world: $|g(i)| \not\geq 1$ and the conditions imposed by $\Diamond$ are not met. In a world that *is* compatible with Sue having a husband, $g(i)$ will collect all compatible worlds where she is indeed married to a man. What's more, $g(x)$ will keep track of these men. For instance, say that there are three worlds compatible with the world of evaluation: in w_1 Sue is unmarried, in w_2 she is married to Harry and in w_3 she is married to Frank. Output assignment g will then look like this:

(34)
$$\begin{aligned} g(i) &= w_2 \sqcup w_3 \\ g(x)(w_2) &= \text{harry} \\ g(x)(w_3) &= \text{frank} \end{aligned}$$

The advantage of this setup is that we can now use modal discourse referents as the locus of where the numeral does its summative counting. Brasoveanu (2010) uses this to account for how modified numerals interact with modals. Numerals express cardinality tests. In Brasoveanu (2010), such tests are distributive, defined as follows:

(35) $[\![|x| =_i n]\!]^{w,\langle f,g\rangle} = 1$ iff $f = g$ & $\forall w' \leq_{\text{A}} f(i) : |f(x)(w')| = n$

We could now attempt to analyse our running example as $\Diamond^i[\exists x;$ election-winner$(x)]$; $|x| =_i 3$, but that is wrong. This says that there are three winners in each world where there is a winner. Given that the cardinality constraint is interpreted distributively, we impose the constraint in every world. Summatives, however, are just like epistemic participles like '*unknown*': instead of looking at what holds in individual worlds, they look at what holds over the complete state of possibilities. This means that we need cumulative cardinality constraints. For instance:

(36) $[\![|x| \mathrel{\mathring{=}}_i n]\!]^{w,\langle f,g\rangle} = 1$ iff $f = g$ & $|\{f(x)(w') : w' \leq_{\text{A}} f(i)\}| = n$

This will lead to (37), which corresponds to the observed summative reading of (8).[7]

(37) $\Diamond^i[\exists x;\ \text{election-winner}(x)];\ |x| \mathring{=}_i 3$

3 Scope and Scopelessness

According to Szabo's account, the summative readings are due to a special kind of quantifier scope. This entails that summative readings are dependent on the presence of a second scope-taking operator. Compare, for instance, (9) (repeated here) and (38). While the former allows for a summative interpretation, (38) does not. In Szabo's account, this can be accounted for by assuming that past tense expresses a quantificational operator and that the simple present does not.

(9) Sue had three husbands.

(38) Sue has three husbands.

The post-suppositional account I introduced in the previous section makes a similar prediction. If we assume that past tense is like a modal, then (9) will introduce the collection of past situations in which Sue had a certain husband x. The numeral can be interpreted as a post-suppositional cardinality condition with respect to that plural world. In contrast, (38) introduces no such discourse referent and, thus, the account correctly predicts there is no summative reading.

Now consider (39), which also lacks a summative reading, just like (38).

(39) Sue must have three husbands.

Szabo's account predicts that the numeral should be able to scope above the modal, yielding (40), which amounts to (41):

[7] In Brasoveanu's (2010) account of modified or post-suppositionality. Technically, post-suppositions constrain the assignment functions in the logic in a way that gives the widest scope to these conditions. To keep things simple, I will provide wide scope for the post-suppositional material by syntactic means (Say, as in Abusch 1997). But bear in mind that this is a simplification.

(40) three $\emptyset$ λx [must [Sue has husband x]]

(41) $\exists x \exists y \exists z[x \neq y \wedge y \neq z \wedge x \neq z \wedge \Box[\text{Sue has husband } x] \wedge \Box[\text{Sue has husband } y] \wedge \Box[\text{Sue has husband } z]]$

This says that the set of individuals of which it is an epistemic certainty that the individual is Sue's husband is a set of three individuals. This entails the de dicto reading of (39), which may explain why (39) has no discernible summative reading.[8]

The post-suppositional approach does less well. The most natural interpretation for a universal modal would be something like (42).

(42) $[\![\Box^i[\varphi]]\!]^{w,\langle f,g\rangle} = 1$ iff $g(i)$ is v, the sum of all worlds accessible from w, and $[\![\varphi]\!]^{v,\langle f[i\rightarrow v],g\rangle} = 1$.

A summative construal of (39) would now be as in (43):

(43) $\Box^i[\exists x; \text{husband-of}(x, s)]; |x| \mathring{=} 3$

This says that each epistemically accessible world is such that Sue has a husband and that if we look at all epistemically accessible worlds, that across these worlds, there are three individuals that act as Sue's husband. In other words, this says that it is known that Sue has a husband, but that it is not known which of three men is that husband. This summative reading is unavailable.

In summary, it appears that Szabo's quantifier theory has an important advantage over the post-supposition alternative I am exploring here. When

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[8] Szabo's main example for bare quantification involves belief:

(i) Alex believes that eleven terrorists live across the street.

Szabo sketches a situation in which the police show Alex lots of pictures and ask him to tell them which of the people in the pictures he suspects to be terrorists. In the end he has identified eleven of his across-the-street neighbours as terrorists. He didn't count them, so while there are eleven people such that Alex believes them to be terrorists living across the street, we cannot ascribe the de dicto belief to Alex that there are eleven terrorists living across the street.

Note that if belief were to be analysed as a universal modal, the summative again entails de dicto reading. So, in order to do justice to (i), Szabo's account will have to make sure that belief is not a universal modal and, in particular, that belief is not closed under logical consequence, so that summative and de dicto can be distinguished.

a bare quantifier (in Szabo's sense) takes scope over a universal quantifier, the result is indistinguishable from a de dicto reading. As such, this account correctly predicts that summative readings are not available for universal modals. On the account I developed above, the numeral does not really interact with the scope of the other operator, it merely takes the output of that operator and imposes a condition on it. As a result, we wrongly predict (43) as a summative reading.

Things are not *so* simple, however. One particularly thorny issue is that summatives are in fact much less common than either approach sketched here will predict. While (8) and (9), repeated here, have summative readings, the summative reading of (44) is not very prominent.[9]

(8) At this point, the election can have three winners.

(9) Sue had three husbands.

(44) (At this point in my investigations), Sue can have three husbands.

This is reminiscent of the discussion in Francez (2018), who argues that summative readings of existential there sentences are much more constrained than summative readings elsewhere. Some of the constraints he mentions, however, seem to me to apply not just to the existential case. For instance, summative readings are unavailable for numeral DPs that do not contain relational nouns, witness the contrast in (45) and (46). While (45) clearly has a summative reading, (46) does not.

(45) There can be three outcomes to the negotiations.

(46) There can be three disguised spy cameras in her purse.

For (46), Francez discusses a scenario where a spy has several objects in her purse and we believe one of these objects to be a disguised spy camera. We narrowed the suspect object down to three: her lipstick, her lighter, her pack of cigarettes. This scenario makes a summative reading for (46) salient: there are three objects such that it is epistemically possible that one of these objects is a spy camera. Nevertheless, this reading is simply unavailable.

[9] An anonymous reviewer reports that they get the summative reading for this sentence, but at least three native informants I consulted disagreed.

Examples like these are one of the reasons why Francez offers a purpose-built interpretation for numerals in summative existential sentences. However, I believe that the constraints he observes are active much more generally. For instance, the non-existential variant (47) of (46) equally lacks a summative reading:

(47) Sue can be hiding three disguised spy cameras in her purse.

What this shows is that there is more to summative readings than just finding a way to provide appropriate scope to the semantic contribution of the numeral. Neither Szabo's account nor the post-supposition account sketched above does justice to the fact that summative readings don't just pop up everywhere. I won't be able to fix this issue in this chapter. Instead, I will focus on the nature of the scope mechanism behind summatives. Can summative readings always be characterised as involving proper quantifier scope, as Szabo's account would have? Here is one case, where I think the post-suppositional account fares better than bare quantification: summative cumulative readings.

Cumulative readings are the prime example of scopelessness. For instance, Scha's (1981):

(48) 600 Dutch firms have 5000 American computers.

is true whenever 600 Dutch firms have American computers and 5000 American computers are owned by Dutch firms. To reach this interpretation, we cannot assume a generalised quantifier interpretation for the numerals (Krifka 1999). If, for instance, we analyse '*600 Dutch firms*' as a type $\langle 1 \rangle$ quantifier, then it will take scope, as for instance in (49).

(49) [600 Dutch firms] λx [x has 5000 American computers]

Generalised quantifiers count atoms, and as such (49) yields a distributive reading. Obviously, if we analyse '*5000 American computers*' as a generalised quantifier as well, things only get worse, for we predict another distributive reading, where the object quantifier scopes over the subject quantifier.

In Szabo's theory, numerals as bare quantifiers are type $\langle 1, 1 \rangle$ quantifiers, but in combination with the general restrictor (the domain of entities), they yield a scope-taking ⟨1⟩ quantifier. As such, the bare quantifier reading of (48) will yield a distributive reading (equivalent to the subject distributive reading), but not a cumulative one.

Given all this, we come to expect that summative readings are never cumulative. But now consider (50):

(50) Three sisters had eight husbands.

(51) The final four races can have ten winners.

The sentence in (50) has a reading on which there are eight men such that each of them was married to at least one of the sisters, and there are three sisters each of which was married to at least one of the eight men. If we apply Szabo's account, we get (52), which amounts to (53).

(52) eight $\emptyset$ λx [the three sisters had husband x]

(53) $\exists x_1 \exists x_2 \ldots \exists x_8 [x_1 \neq x_2 \wedge \ldots \wedge x_1 \neq x_8$
$\wedge$ Past[the three sisters had husband x_1]
...
$\wedge$ Past[the three sisters had husband x_8]]

This says that there are eight men such that each of these men was married to (each of) the sisters. It does not amount to the salient summative reading in which different women married different men.

Similarly, (51) has a summative cumulative reading: there are ten people and four races; each of these people might win one of the races and each race might have one of these people as winner. Using bare quantification, we won't be able to derive this meaning. Bare quantification involves taking scope and the counting of atoms satisfying the scope.

In the post-suppositional approach I sketched in the previous section, things look a lot more promising. This is not surprising, since Brasoveanu (2010) shows that post-suppositions are an ideal mechanism to deal with the scopelessness of cumulative readings. We could for instance account for (51) as follows:

(54) $\text{PAST}^i[\exists x; \exists y; \text{sister}(x); \text{husband-of}(y, x)]; |x| \mathring{=} 3; |y| \mathring{=} 8$

The upshot is this: Standard type $\langle 1, 1\rangle$ determiners do not create cumulative readings. Given that we observed cumulative summative readings, we can conclude that at least some summatives do not involve type $\langle 1, 1\rangle$ quantifiers, and, so, at least some cases will fall outside the reach of Szabo's mechanism of bare quantification.

4 Conclusion

Let's take stock. Summative readings are surprising creatures. They do not seem to follow straightforwardly from standard accounts of numeral quantification. I've looked at two ways of obtaining the readings: (i) Szabo's bare quantification account, which involves a radical departure from standard ideas on how quantifiers take scope and (ii) an account involving an adjectival and post-suppositional interpretation of numerals. Both approaches end up predicting many more summative readings than are actually observed. The tricky thing about summatives is apparently not how to derive such a reading, but how to derive it only in the (quite rare) occasions where it surfaces. That said, I showed that whatever mechanism is responsible for summative readings, it cannot just involve the numeral as a determiner. This is because summative readings can arise in tandem with cumulative readings of numerals. Cumulative readings are scope-less and thus cannot involve scope-taking determiners. This, I believe, disqualifies Szabo's account, but I would be overstating my case if I claimed that these data support the post-suppositional theory I sketched here. We really first need to understand exactly when the summative reading occurs.

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