THE ROLES OF ARITHMETIC FLUENCY AND EXECUTIVE FUNCTIONING IN MATHEMATICAL PROBLEM-SOLVING

ABSTRACT

This study is conducted to further understand the direct and indirect contributions of executive functioning (visuospatial updating, verbal updating, inhibition, shifting) and arithmetic fluency to mathematical problem-solving in 458 fourth-grade students. Arithmetic fluency along with visuospatial and verbal updating were significant predictors of mathematical problem-solving at the end of grade 4. When the growth in mathematical problem-solving during grade 4 was analyzed, only arithmetic fluency directly and strongly contributed to students' problem-solving at the end of grade 4. Inhibition and shifting (in combination with inhibition) were indirectly connected to mathematical problem-solving at the end of grade 4 via their arithmetic fluency. Arithmetic fluency plays a critical role and continues to do this in mathematical problem-solving. Furthermore, a decline in importance for visuospatial and verbal updating and increasing importance of inhibition and shifting (combined with inhibition) were found with regard to students' ability to solve mathematical problems during grade 4.

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ANALYTICAL thinking and mathematical reasoning abilities contribute to the development of problem-solving skills (Gravemeijer et al., 2017). Both arithmetic fluency (Fuchs et al., 2006, 2016; Geary, 2004; Swanson & Beebe-Frankenberger, 2004) and executive functioning (Lee et al., 2009; Viterbori et al., 2017) have been shown to be predictive for mathematical problem-solving. In some studies, mathematical problem-solving has been understood as solving nonroutine mathematical problems that challenge children to come up with their own solution strategy or strategies (Doorman et al., 2007; Polya, 1957). Mathematical problem-solving has mostly been assessed using single-step or multistep word problems "that are better simulations of the modeling problems people encounter in their personal or professional lives" (Verschaffel et al., 2020, p. 2). The scope of the present research is mathematical problem-solving defined as solving problems with mathematical notation, text, and/or pictures, which have been commonly seen in mathematics education.

However, most of the relevant research has focused on only the mathematical problem-solving of relatively young students (up to the age of about 7 years; e.g., Rasmussen & Bisanz, 2005; Swanson et al., 2008). As a result, only the solution of simple, single-step math problems has been studied (e.g., Fuchs et al., 2006; Swanson & Beebe-Frankenberger, 2004; Zheng et al., 2011). Relatively little is known about the predictive roles of arithmetic fluency and executive functioning for advanced mathematical problem-solving (e.g., multistep problems in the domains of fractions, ratio, and percentage). However, both of these are important in light of the complexity of problem-solving tasks requiring advanced mathematical problem-solving and multistep calculations for their solution. In addition, in grade 4 new domains of mathematics are being taught that also include certain necessary knowledge and skills (e.g., mastery of multiplication and fractions). Development of advanced mathematical reasoning and analytic thinking may not be a matter of simply mastering the required mathematical knowledge; it is possible that there is also a need for sufficient arithmetic fluency and executive cognitive functioning. Additional research on the roles of arithmetic fluency and executive functioning in the mathematical problem-solving skill of older elementary school children is thus needed.

Arithmetic Fluency and Mathematical Problem-Solving

During early elementary school, teachers focus on number, counting, and simple arithmetic competence (Geary, 2011). Students gradually master key arithmetic facts for quick and accurate responding (Andersson, 2008; Fuchs et al., 2006). When solving more advanced mathematical problems, students must be able to quickly retrieve these arithmetic facts from long-term memory and store this information in short-term memory (Baddeley, 2000). To be able to solve mathematical problems, it is necessary that students understand mathematical concepts (conceptual knowledge), know the procedural steps to solve a problem (procedural knowledge), and have sufficient knowledge of basic facts (factual knowledge; Geary, 2004, 2011; Geary & Hoard, 2005). Cragg et al. (2017) offered a framework presenting a refined hierarchical structure for mathematical development, based on the framework of Geary (2004). In that framework, the underlying cognitive system that supports factual knowledge, procedural knowledge, and conceptual understanding also plays a crucial role in advanced mathematical problem-solving. In light of that hierarchical structure, studies presenting both simple, single-step mathematical problems and more complex, multistep problems have demonstrated clear associations between arithmetic fluency and mathematical problem-solving (Fuchs et al., 2006; Viterbori et al., 2017; Zheng et al., 2011). In other words, arithmetic fluency or knowing key arithmetic facts accurately and quickly (addition, subtraction, multiplication, division) has been shown to be crucial for more advanced mathematical problem-solving.

Role of Executive Functioning

Along with domain-specific factual knowledge, procedural skill, and conceptual understanding, domain-general cognitive skills also contribute to mathematics achievement. Many studies involving primary school-aged children have shown consensus on at least three components of executive cognitive functioning that are critical for advanced mathematical problem-solving: updating of information, inhibition of information, and shifting of attention (Bull & Lee, 2014; Miyake et al., 2000).

With regard to the updating of information, a distinction can be made between visuospatial and verbal updating (see also Baddeley, 2000). Visuospatial updating refers to the ability to monitor, manipulate, and retain information presented in a visual form or as objects in space, whereas verbal updating involves the ability to monitor, manipulate, and retain information presented in a verbal auditory form. Inhibition is the ability to suppress irrelevant information and/or inappropriate responses. Shifting is the capacity for flexible thinking and adeptly switching between alternative tasks or strategies (Miyake et al., 2000).

Executive functioning has been found to be linked to both arithmetic fluency and mathematical problem-solving in several ways. During the mathematical problem-solving process, information must be held in memory, manipulated, and regularly updated (Best & Miller, 2010; Bull & Lee, 2014). A representation of the required problem-solving strategy must be formed and stored in working memory. Irrelevant information or inappropriate, misleading responses must be ignored at times, and alternative strategies must be considered and switched to, on occasion. Just how—and the extent to which—visuospatial and verbal updating, inhibition, and shifting (i.e., three important components of executive functioning) contribute to students' developing mathematical problem-solving is not completely clear.

Executive Functioning in Relation to Arithmetic Fluency

With practice, the arithmetic fluency of elementary school students increases, and their mathematical problem-solving becomes more efficient and sophisticated as a result (Geary, 2004). Arithmetic fluency requires not only the quick and accurate retrieval of arithmetic facts from long-term memory but also the efficient updating of information, the suppression of incorrect responding (inhibition), and accurate shifts between operations (+, –, ×, ÷; Bull & Scerif, 2001; Bull et al., 1999; Swanson & Beebe-Frankenberger, 2004). Consider, for example, a student who has to solve 6×8 and needs an intermediate step. The student is able to use the strategy of splitting the problem into subproblems (5×8, 1×8). The well-known arithmetic fact that

$5 \times 8 = 40$ has to be retrieved from memory, and the student has to keep the answer in mind. Then, the student has to complete the other subproblem ($1 \times 8 = 8$) and switch operations by adding the outcomes ($40 + 8$) to produce the answer to 6×8. During this process, the student has to inhibit responses that may have already been activated or other irrelevant stimuli (e.g., suppressing the answer 14 for the number combination of 6 and 8).

Considerable insight has been gained into the associations between executive functioning and arithmetic fluency. In particular, a number of studies have shown that visuospatial and verbal updating are significant predictors of arithmetic fluency (e.g., Cragg et al., 2017; Lee & Bull, 2016; LeFevre et al., 2013; Van de Weijer-Bergsma et al., 2015). However, studies have shown inconsistent findings with regard to the role of visuospatial and verbal updating in relation to age/school grade. In two studies involving only verbal updating, no significant associations with arithmetic fluency were found (Balhinez & Shaul, 2019; Fuchs et al., 2006). In the study by Balhinez and Shaul (2019), moreover, verbal updating was not related to arithmetic fluency in third grade but was in the grades before. Their explanation was that young students who have to solve simple arithmetic problems possibly use different procedures that rely particularly on verbal updating. During the first years of school, arithmetic is based on the representation of a given number quantity through serial counting. Verbal updating plays an important role in arithmetic performance. When strategies become more efficient and students keep practicing, they get faster and more accurate. Arithmetic fluency mastery relies mainly on automatic retrieval and to a lesser extent on verbal updating.

In a study in which visuospatial and verbal updating were included in the analyses, Andersson (2008) found that verbal updating contributed to arithmetic fluency. Longitudinal studies have shown associations between visuospatial and verbal updating and arithmetic fluency, but the studies have not shown consistent findings. In a study by LeFevre et al. (2013), visuospatial and verbal updating jointly predicted arithmetic fluency in grades 2 through 4. Van de Weijer-Bergsma et al. (2015) showed visuospatial and verbal updating to be equally strong predictors of arithmetic fluency through grade 4 with verbal updating later prevailing in grades 5 and 6. In this same study, however, the updating of information showed no significant connections to individual differences in the development of arithmetic fluency within one school year. Finally, Lee and Bull (2016) also showed visuospatial and verbal updating to jointly and strongly predict arithmetic fluency through grade 4 but only weakly thereafter (i.e., in grades 5 through 9). Assuming that arithmetic fluency has fully developed by the end of grade 4, the authors suggest that updating also then has a less prominent role to play.

With regard to the contribution of inhibition and shifting to arithmetic fluency, previous research showed mixed findings. Several studies found relationships between inhibition and arithmetic fluency (Bull & Scerif, 2001; Cragg et al., 2017; LeFevre et al., 2013; Van der Sluis et al., 2007), but a study by Balhinez and Shaul (2019) did not. In the study by Bull and Scerif (2001), shifting was shown to contribute to arithmetic fluency, but in other studies, shifting was not shown to be related to arithmetic fluency (Cragg et al., 2017; Van der Sluis et al., 2007). The mixed findings with regard to particularly the roles of inhibition and shifting in arithmetic fluency may be due to the increasingly quick and easy retrieval of stored arithmetic facts from long-term memory, making inhibition less needed and facilitating the shifting required for more complex mathematical problem-solving (Bull & Scerif, 2001; Bull et al., 1999; Cragg et al., 2017).

Executive Functioning in Relation to Mathematical Problem-Solving

Mathematical problem-solving requires the following skills, among others: identification of relevant information and key words after the reading of a problem and selection and application of most suitable strategies, operations, and algorithms across multiple contexts (Boonen et al., 2013; Fuchs et al., 2008; Verschaffel et al., 2020). School textbooks typically have students solve mathematical problems involving real-world contexts depicted using mathematical notation, text, and/or pictorial representations (Verschaffel et al., 2020).

Visuospatial updating and verbal updating have indeed been found to help students integrate the information identified as relevant to thereby solve advanced mathematical problems requiring multiple steps (Cragg et al., 2017). Inhibition and shifting may also be required when learning new concepts and mastering the procedures needed for new domains of mathematics and for solving more complex mathematical problems, as is the case in grade 4. To prevent irrelevant information from interfering with a new and otherwise unfamiliar problem-solving process, for example, inhibition is needed. In addition, students must be able to readily shift between various procedures for more advanced mathematical problem-solving, such as applying conceptual knowledge of fractions and factual knowledge of addition and multiplication when solving a multistep problem (Lee et al., 2009).

The roles of visuospatial and verbal updating in mathematical problem-solving appear to be most consistent. Studies consistently report significant associations of visuospatial and verbal updating with not only simple, single-step mathematical problem-solving (Swanson, 2011; Swanson & Beebe-Frankenberger, 2004; Swanson et al., 2008; Zheng et al., 2011) but also more complex, multistep mathematical problem-solving (Agostino et al., 2010; Cragg et al., 2017; Fuchs et al., 2016; Passolunghi & Pazzaglia, 2004). In addition, Cragg et al. (2017) found both visuospatial and verbal updating to play similar roles across different components of mathematics and different age groups. In contrast, St. Clair-Thompson and Gathercole (2006) found only visuospatial updating to be strongly related to mathematical problem-solving performance.

The few studies examining inhibition and/or shifting as executive functions in relation to children's mathematical problem-solving have shown mixed results (Jacob & Parkinson, 2015). Regarding inhibition, Lee et al. (2009) found no significant associations for multistep problem-solving. In two other studies, in contrast, significant associations were found between inhibition and the solving of both single-step and multistep mathematical problems (Passolunghi & Pazzaglia, 2004; Swanson, 2011). Specifically, students showing better inhibition of irrelevant information showed better mathematical problem-solving. To date, the evidence regarding the role of shifting in students' mathematical problem-solving is limited and mixed. Some studies (Andersson, 2007; Cantin et al., 2016) found shifting to be a significant predictor of mathematical problem-solving, whereas Cragg et al. (2017) did not.

Finally, the possible associations of updating, inhibition, and shifting—considered together—with students' advanced mathematical problem-solving have only been examined in a few studies (Agostino et al., 2010; Cragg et al., 2017; Viterbori et al., 2017). The findings have again been consistent with regard to the predictive role of updating but not about the roles of inhibition or shifting. Agostino et al. (2010) found not only visuospatial and verbal updating but also inhibition (and not shifting) to be significant predictors, whereas Cragg et al. (2017) found only visuospatial and verbal updating (and not inhibition and shifting) to be significantly related to mathematical problem-solving. Viterbori et al. (2017) found inhibition and shifting to play a role while third graders devised a problem-solving plan and selected the required calculations but not during their actual problem-solving. When the accuracy of their actual mathematical problem-solving was examined, only verbal updating played a role. And similarly, in a very recent study in which only updating was included, Allen and Giofrè (2021) found verbal updating to play a more important role than visuospatial updating in the mathematical problem-solving of third-grade students (7–8 years old).

Overall, updating is most frequently identified as a significant predictor of mathematical problem-solving performance and thus students' ability to update, hold, and manipulate information deemed to be essential. Verbal updating is judged to be of particular importance for grade 4 students. Not all studies distinguish between visuospatial and verbal updating, however. And the findings regarding inhibition and shifting in relation to student's mathematical problem-solving are less consistent than those for updating. It should be noted that when both updating and inhibition were examined in the same study, updating played a more prominent role in the children's mathematical problem-solving (Wiley & Jarosz, 2012).

Relationships between visuospatial and verbal updating, inhibition, and shifting and mathematical problem-solving performance and growth in grade 4 are not yet clear. Most of the relevant research has included only young students and only simple as opposed to more complex mathematical problems. Very little is known about the direct and indirect contributions of executive functioning and arithmetic fluency to mathematical problem-solving in grade 4 when the degree of mathematical complexity and abstraction increases.

The Present Study

To date, the vast majority of studies have been directed at performance in mathematical problem-solving, not at changes over time (growth), and most studies have not included the executive functions of visuospatial and verbal updating, inhibition, and shifting. There is a marked need for further understanding of the direct and indirect contributions of executive functioning to mathematical problem-solving in grade 4 to extend previous research. The present study therefore takes the following into account when studying performance and growth of students' mathematical problem-solving abilities: (*a*) the specific roles of visuospatial and verbal updating, inhibition, and shifting (i.e., aspects of their executive functioning) in their mathematical problem-solving and (*b*) the possibly mediating role of arithmetic fluency in their mathematical problem-solving. In light of what is known to date, the following research questions then arise.

1. Is students' mathematical problem-solving performance at the end of grade 4 predicted by their arithmetic fluency and executive functioning?

2. Is the association between executive functioning and growth in mathematical problem-solving, if any, mediated by students' arithmetic fluency?

For the present study, a longitudinal design was adopted to monitor students' mathematical problem-solving from the start to the end of fourth grade of elementary school, with nonverbal reasoning ability controlled for as a critical factor underlying mathematical problem-solving ability (Fuchs et al., 2006).

With regard to the first research question, we hypothesized that arithmetic fluency would directly predict mathematical problem-solving when measured at the end of grade 4. Being arithmetically fluent and capable of applying factual math knowledge is clearly necessary to solve advanced mathematical problems. We also hypothesized that both visuospatial and verbal updating would directly and significantly predict mathematical problem-solving at the end of grade 4. In light of the literature, verbal updating might prove more important than visuospatial updating. The roles to be expected for inhibition and shifting were not clear but nevertheless of interest.

With regard to the second research question, we hypothesized that arithmetic fluency would mediate the associations between executive functioning and growth in the students' mathematical problem-solving during fourth grade. We specifically expected both visuospatial and verbal updating to contribute to the mediating function of arithmetic fluency and thus indirectly to the growth in mathematical problem-solving during grade 4 but also directly. We had no specific hypotheses about the direct influences of inhibition and shifting on growth in mathematical problem-solving or possibly indirect influences via associations with arithmetic fluency. The roles of these aspects of executive cognitive functioning are nevertheless of great interest in light of the gradually more advanced mathematics presented during the fourth grade of elementary school.

The present study will pinpoint the different contributions of visuospatial and verbal updating and of inhibition and shifting (in combination with inhibition) to mathematical problem-solving in fourth-grade students, along with the roles of arithmetic fluency and prior mathematical problem-solving achievement. Knowing more about the contributions of these different components may lead to more insight into how upper elementary students can be supported in their mathematics learning.

Method

Participants were 458 fourth-grade students from 27 mainstream elementary schools in the Netherlands. Schools were recruited via social media (Twitter) and direct mailing to the school principals and fourth-grade teachers (contact information gathered via public websites from schools). Twenty-seven schools signed up to participate for two school years. Due to internal school affairs, 22 schools participated throughout the 2-year study period. The participating schools were located in rural and urban areas spread across the Netherlands and were diverse in terms of school size, pupil population, and mathematics curriculum used.

As part of a larger longitudinal research project, the data for this study were collected from a randomly selected sample of 458 out of 1,062 students. This sample comprises an even distribution of low, average, and high math achieving students (based on standardized national mathematics test scores). Of the 458 students composing the sample, 50.3% were male and 49.7% female. The mean age of the students was 9; 1 years ($SD = 0.43$), with a range of 8; 02 to 10; 10 (years; months). The spread in age was due to either having skipped a year of school or repeating a year. For 89.9% of the students, Dutch was the language used in the home. Due to absences or incomplete task performance, the amount of data collected varied from $N = 388$ to $N = 453$ per test. Only complete responding was included in the data analyses.

The Raven's Standard Progressive Matrices were administered at the start of the school year to check on participants' nonverbal reasoning and ensure that none of the participants had scores 2 or more standard deviations below the mean (Raven, 2000; Raven et al., 1998). None of them did. The mean nonverbal reasoning score found for the students at the beginning of fourth grade ($N = 450$) was 36.58 ($SD = 6.99$), skewness −0.73, and kurtosis 1.37. The sample was treated in accordance with institutional guidelines as well as American Psychological Association ethical standards.

Procedure

After recruitment of participants, an information meeting was organized in two different regions of the Netherlands. During the meeting, the schools were presented both verbal and printed information about the purpose of the study, duration of the study, and data collection methods to be used. The parents of the recruited students were provided information about the study by the school. Both the schools and the parents provided their written consent for the participation of the students prior to data collection.

The Cito (Dutch national standardized mathematics test) mathematics achievement data were obtained from the schools. Measures of arithmetic fluency (start of grade 4) and nonverbal intelligence (start of grade 4) were administered in class using paper and pencil. The students sat in a test setup, so they could not copy from each other. The first author gave test instructions and stayed in the classroom. The teacher also remained in the classroom. The testing took about 45 minutes, excluding a short break between the administration of the two measures.

The executive functioning of each student (visuospatial and verbal updating, inhibition, and shifting in combination with inhibition) was tested individually in a quiet room in the student's school by an educational psychologist (i.e., the first author) at the start of grade 4.

Baseline Measure (Start of Grade 4) and Outcome Measure (End of Grade 4)

To measure students' mathematical problem-solving performance the criterion-based mathematics test at the end of grade 4 was adopted as the outcome measure. Standardized Dutch national tests are commonly administered at the middle and end of each school year to monitor student progress (Cito; Janssen et al., 2005). The mathematics test is made up of a mixture of computation problems (e.g., 7,500 ÷ 250 =) and word problems. Some translated examples of word problems are as follows: "The zookeeper has 75 fish. Each penguin gets 3 fish. How many penguins can the zookeeper feed?" "Elsa wants to paint the wall of her room a different color. To know how much paint she needs, she must know the surface area of the wall. The wall is 6 yards long and 2.50 yards wide. What is the surface area of the wall?" Mathematical problems are presented using mathematical notation, text, and/or text with pictures. These pictures are not just decorations but provide additional information needed to solve the problem. The majority of the mathematical problems have a picture in combination with text: How many jars of powdered milk are in this box? __ jars (accompanying picture depicts a full box in which only some of the jars are visible); Dad's birthday is on June 28. He will celebrate his birthday on the following Saturday. That is on __ (accompanying picture depicts the calendar for the month of June).

The following mathematics domains are covered: (1) numbers, number relations, and operations (addition, subtraction, multiplication, and division); (2) proportions and fractions; and (3) measurement and geometry. The reliability coefficients for the different versions of the test (middle and end) ranged from 0.91 to 0.97 (Janssen et al., 2010); in the present study, $\alpha = 0.86$. The test scores at the end of grade 4 were used as the outcome measure (T2); the test scores at the start of grade 4 were used as a baseline measure (T1). It must be noted that the baseline measure was actually included as part of standardized testing at the end of grade 3, but for clarity and consistency, we are using this as the level at the start of grade 4. The mathematical problem-solving measure was a longitudinal measure (T1 and T2), whereas all other measures were collected before T2.

Mediator Measure (Start of Grade 4)

To measure students' arithmetic fluency performance the Speeded Arithmetic Test (TempoTest Automatiseren; De Vos, 2010) was used. This is a standardized paper-and-pencil test frequently used in Dutch education to measure speeded arithmetic skill (arithmetic fluency). The test consists of four categories of 50 fact problems: addition (tasks with a range of difficulty level from $6 + 0$ to $29 + 28$), subtraction (range from $4 - 2$ to $84 - 38$), multiplication (range from 4×1 to 5×9), and division (range from $8 \div 2$ to $72 \div 9$). Students are given 2 minutes to solve as many problems as possible within a given category. Each correct answer yields 1 point, for a total of 50 possible points per category and a total possible score of 200. The number of problems answered correctly for each category was adopted as the domain score. The total for the four domains was used in the analyses. The test was administered at the start of grade 4. And the reliability and validity of testing were judged to be good ($\alpha = 0.88$; De Vos, 2010); in the present study, $\alpha = 0.92$.

Predictor Measures (Start of Grade 4)

The dot matrix and backward dot matrix subtests from the Alloway Working Memory Assessment (AWMA) were used to assess so-called visuospatial updating (Alloway, 2012; Van Berkel & Van der Zwaag, 2015). The AWMA is an online assessment tool for use with students 9–17 years of age; the dot matrix is a span task that calls upon visuospatial updating. In the dot matrix, the student is required to watch a red dot in a sequence of locations on a four-by-four square matrix on a computer screen. The student is then asked to indicate the sequential order of locations of the red dot on a blank square on the computer screen. The number of red dots presented increases from one to nine red dots on subsequent trials and had to be recalled in the order they were presented. In the backward dot matrix subtest, sets of three geometrical shapes arranged in three square frames are presented. The respondent must identify the odd-one-out shape by pointing to it and then must memorize its location (left, middle, or right). Following presentation of one or more sets of three shapes (i.e., a block composed of a minimum of one and maximum of seven sets of three shapes), the locations of the odd-one-out shapes must be recalled in the same order as presented. The subtest starts with a block containing one set of shapes and increases to a block containing seven sets of shapes. When a student made three or more mistakes within a block, the test stopped automatically. The total number of correct answers for the two AWMA subtests was used as a measure of visuospatial updating. The reliability coefficients for the dot matrix (0.83) and backward dot matrix (0.82) were judged to be good in the past and also in the present study ($\alpha = 0.85$ and 0.84).

The digit span subtest from the Wechsler Intelligence Scale for Students (WISC-III) was used to measure verbal updating (Wechsler, 2003). First, the student is asked to repeat a sequence of digits in forward order as read aloud by the examiner. The number of digits of a sequence increases from two to nine on subsequent trials. Then, the student is asked to repeat different sequences of digits in backward order. This task increases in difficulty from two to eight digits on subsequent trials. Every item on the digit span consists of two trials, each of which is scored 1 or 0 points. The test was completed when the student failed both trials of the same length. The sum of scores was calculated. Higher scores indicate better performance. The reliability coefficient for this test was found to be .88 in the past (Kaufman, 1993) and was 0.65 in the present study, which is acceptable. Forward digit span requires rote memory and auditory sequential processing, whereas backward digit span also requires the use of working (i.e., short-term) memory for the transformation and manipulation of information.

To assess inhibition and shifting, the Color Word Interference Test (CWIT) was used. This test is part of the Delis-Kaplan Executive Function System (DKEFS; Delis et al., 2001), an age-normed battery of tests designed to measure executive functions in students and adults, ages 8–89. The CWIT has four conditions: color naming (condition 1), word reading (condition 2), inhibition (condition 3), and shifting + inhibition (condition 4). Condition 1 involves naming the color of colored squares and condition 2 involves reading words (names of colors) aloud. Conditions 3 and 4 were used to measure inhibition and shifting. In the inhibition condition (condition 3), students must suppress a prepotent response (i.e., predisposition) by stating the color of the ink used to present a word rather than reading the word itself (which may be a color word). For example, the word "green" is printed in red ink. The correct answer in this case is "red," not "green." This task is based on the Stroop (1935) procedure. In the shifting + inhibition condition (condition 4), the student is presented with a page containing the words red, green, and blue written in red, green, or blue ink. Half of the words are presented in boxes. The respondent is asked to state the color of the ink in which the word is printed (just as in the inhibition condition) or, when the word appears within a box, instead to read the word aloud (and not name the ink color). The student has to switch between reading the word and naming the color of the ink.

This must be done as quickly and accurately as possible. Each condition has two practice rows, with a total of 10 items. The 50 items were presented in five rows of 10 items each. The student has to complete each condition in a maximum of 180 seconds. When the student completed each condition in less than 180 seconds, the completion time for each condition is noted in seconds. Raw scores were used as measures for inhibition and shifting + inhibition, consisting of completion time and correct words named for each of the two conditions. For both the inhibition and shifting + inhibition conditions, faster completion times and fewer errors indicate better performance; the lower the score, the better. In the present study, the reliability for the CWIT (all four conditions) was found to be generally acceptable (0.76) but questionable to acceptable for both the inhibition (0.62) and shifting + inhibition (0.68) conditions.

Data Analyses

The data and descriptive statistics for all of the measures were first screened for potential errors and outliers. We discovered five outliers when checking for normality. We used boxplots as well as z-scores with a standard cutoff value of ±3.00 from 0. Outliers were then removed from the data (one nonverbal reasoning score, one inhibition, three shifting + inhibition). All of the variables were normally distributed with acceptable values of skewness and kurtosis (Field, 2009). We next computed the Pearson's correlations between the predictor and outcome measures.

To address the first research question, a multiple hierarchical regression analysis was conducted with mathematical problem-solving at the end of grade 4 as the outcome variable. Arithmetic fluency and the measures of executive functioning were the independent variables.

To address the second research question, we computed mediation analyses using the PROCESS add-on by Hayes, version 3.5, model 4, with a default bootstrapping at 5,000 cycles (Hayes, 2018). Mathematical problem-solving at the end of grade 4 was the outcome variable. The four measures of executive functioning were the independent variables, arithmetic fluency at the start of grade 4 was a mediating variable, and mathematical problem-solving at the start of grade 4 was included as a covariate. We estimated the direct, indirect, and total effects for each of the independent variables. The direct effects are the influences of the measures of executive functioning on mathematical problem-solving at the end of grade 4 without inclusion of the mediator arithmetic fluency. The indirect effects are the influences of the measures of executive functioning when arithmetic fluency is included as a mediating variable. The total effect is the impact of the measures of executive functioning on mathematical problem-solving at the end of grade 4 without inclusion of the mediator and not controlled for mathematical problem-solving performance at the start of grade 4.

Results

Descriptives

Descriptive statistics are displayed in Table 1. The correlation results showed all of the measures to correlate highly significantly with each other; see Table 2. Each of

Table 1. Descriptive Statistics

Measures	N	M (SD)	Skewness	Kurtosis
Visuospatial updating	388	19.54 (6.20)	.01	.11
Verbal updating	454	11.64 (2.44)	.35	.72
Inhibition	452	87.02 (19.73)	.46	1.61
Shifting + inhibition	451	85.38 (18.70)	.44	1.72
Arithmetic fluency	452	106.90 (34.49)	.18	−.58
Mathematical problem-solving T1	453	215.67 (28.27)	.05	.51
Mathematical problem-solving T2	446	239.41 (25.82)	−.09	.25

Note.—T1 = start of grade 4; T2 = end of grade 4.

the predictor measures correlated significantly with the outcome measure. The correlations between arithmetic fluency and mathematical problem-solving were moderate. The other correlations were low but significant. Some of the correlations showed up negative, given that for some of the measures, a lower score indicated better performance (e.g., inhibition and shifting + inhibition speed and number of errors).

Predicting Mathematical Problem-Solving Performance

To answer the first research question, namely, whether students' mathematical problem-solving at the end of grade 4 is predicted by their arithmetic fluency and their executive functioning at the start of grade 4 (or not), the results of the multiple regression analyses were examined (see Table 3). As can be seen, 23.6% of the variance in the students' mathematical problem-solving at the end of grade 4 could be explained by their arithmetic fluency alone. When the components of executive functioning were added to the model, 31.4% of the variance in mathematical problem-solving was accounted for. Examination of the individual contributions of the predictors in Model 2 showed arithmetic fluency, visuospatial updating, and verbal updating to be significant predictors. Inhibition and shifting + inhibition at the start of grade 4 did not predict mathematical problem-solving at the end of grade 4.

Predicting Mathematical Problem-Solving Growth

Our second research question was whether or not any association between executive functioning and growth (i.e., changes) in the students' mathematical problem-solving during grade 4 was mediated by students' arithmetic fluency (measured at the

Table 2. Correlations between Measures

Measures	1	2	3	4	5	6	7
1. Visuospatial updating	–						
2. Verbal updating	.218	–					
3. Inhibition	−.206	−.311	–				
4. Shifting + inhibition	−.268	−.230	.600	–			
5. Arithmetic fluency	.258	.238	−.366	−.349	–		
6. Mathematical problem-solving T1	.374	.383	−.235	−.203	.547	–	
7. Mathematical problem-solving T2	.334	.316	−.172	−.177	.490	.759	–

Note.—$p < .001$ for all correlations.

Table 3. Multiple Regression Analysis for Contributions of Components of Executive Functioning and Arithmetic Fluency to Mathematical Problem-Solving at the End of Grade 4

	B	*SE*	ß	*t*
Model 1 $F(1{,}363) = 112.310$, $p < .001$, $R^2 = .236$				
Arithmetic fluency	.362	.034	.486***	10.598
Model 2 $F(4{,}360) = 33.028$, $p < .001$, $R^2 = .314$, $\Delta R^2 = .079$				
Arithmetic fluency	.311	.036	.418***	8.603
Visuospatial updating	.821	.189	.203***	4.351
Verbal updating	1.989	.493	.189***	4.032
Inhibition	.093	.075	.071	1.244
Shifting + inhibition	.010	.076	.008	.132

*** $p < .001$.

start of grade 4) after controlling for the level of mathematical problem-solving at the start of grade 4. The mediation results are presented in Figure 1.

The mediation (see Fig. 1) with visuospatial updating, verbal updating, inhibition, and shifting + inhibition as predictors and arithmetic fluency at the start of grade 4 as a mediator, and initial level of mathematical problem-solving as control explained 57.2% of the variance in the growth of the students' mathematical problem-solving during grade 4. The indirect effects of visuospatial and verbal updating via arithmetic fluency on mathematical problem-solving at the end of grade 4 were not found to be significant, $a_1b_1 = 0.027$, 95% CI = [–0.014, 0.084], $a_2b_2 = 0.043$, 95% CI = [–0.064, 0.185]; the bootstrapped 95% confidence intervals did cover zero. Similarly, the direct effects of visuospatial and verbal updating on mathematical problem-solving at the end of grade 4 (c) were not found to be significant, $\beta_1 = 0.270$, $SE = 0.150$, $t = 1.797$,

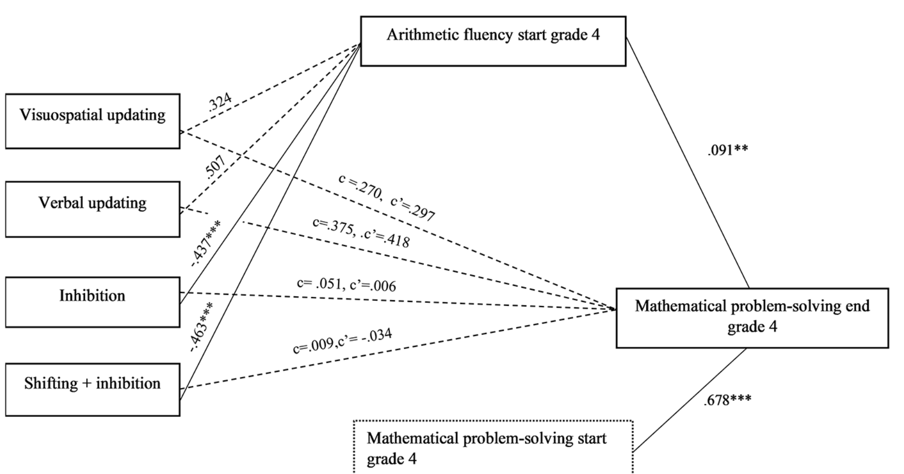

Figure 1. Results of mediation analyses with measures of executive functioning (at the start of grade 4) as predictors, arithmetic fluency (at the start of grade 4) as mediator, mathematical problem-solving (at the start of grade 4) as covariate, and mathematical problem-solving at the end of grade 4 as outcome. **$p < .01$, ***$p < .001$.

$p = .073$; $\beta_2 = 0.375$, $SE = 0.359$, $t = 1.044$, $p = .297$. The total effects of visuospatial and verbal updating on mathematical problem-solving at the end of grade 4 (c′) were also not found to be significant, $\beta_1 = 0.297$, $SE = 0.151$, $t = 1.962$, $p = .051$; $\beta_2 = 0.418$, $SE = 0.362$, $t = 1.155$, $p = .249$.

In contrast, the indirect effects of inhibition and shifting + inhibition via arithmetic fluency on the students' mathematical problem-solving at the end of grade 4 were significant, $a_3b_3 = -0.045$, 95% CI = [–0.078, –0.017]; $a_4b_4 = -0.043$, 95% CI = [–0.078, –0.014]. The bootstrapped 95% confidence intervals did not cover zero. The direct effects of inhibition and shifting + inhibition on mathematical problem-solving (c) were not found to be significant, $\beta_3 = 0.051$, $SE = 0.043$, $t = 1.172$, $p = .242$; $\beta_4 = 0.009$, $SE = 0.046$, $t = 0.196$, $p = .845$. The total effects of inhibition and shifting + inhibition on mathematical problem-solving at the end of grade 4 (c′) were also not found to be significant, $\beta_3 = 0.006$, $SE = 0.042$, $t = 0.134$, $p = .893$; $\beta_4 = -0.034$, $SE = 0.044$, $t = -0.774$, $p = .439$.

The association between arithmetic fluency at the start of grade 4 and mathematical problem-solving at the end of grade 4 was significant ($\beta = .091$, $p < .01$). The association between mathematical problem-solving at the start of grade 4 and mathematical problem-solving at the end of grade 4 was also significant ($\beta = 0.678$, $p < .001$).

In sum, visuospatial updating, verbal updating, and arithmetic fluency significantly predicted mathematical problem-solving at the end of grade 4. At least some of the growth in mathematical problem-solving during the fourth grade was mediated by the students' arithmetic fluency (as measured at the start of grade 4 and after controlling for mathematical problem-solving at the start of grade 4). Inhibition and shifting + inhibition related directly and significantly to arithmetic fluency and therefore only indirectly with the growth in the student's mathematical problem-solving during grade 4. Only arithmetic fluency directly affected the growth in students' mathematical problem-solving during grade 4.

Discussion

The purpose of the present study was to identify the roles of student's arithmetic fluency and executive cognitive functioning—including visuospatial updating, verbal updating, inhibition, and shifting—in students' fourth-grade mathematical problem-solving. Arithmetic fluency, visuospatial updating, and verbal updating proved predictive of mathematical problem-solving at the end of grade 4, whereas inhibition and shifting (in combination with inhibition) did not. With regard to the changes (i.e., growth) in the students' mathematical problem-solving during fourth grade, only arithmetic fluency showed a strong and direct effect on performance at the end of grade 4 and after controlling for mathematical problem-solving at the start of grade 4. Inhibition and shifting (in combination with inhibition) were now found to indirectly relate to the students' mathematical problem-solving at the end of grade 4 via arithmetic fluency and to thus play a role in the growth of the students' mathematical problem-solving.

Mathematical Problem-Solving Performance

The present finding that arithmetic fluency is predictive of mathematical problem-solving at the end of grade 4 is consistent with previous findings (Fuchs et al., 2006;

Viterbori et al., 2017; Zheng et al., 2011). Being arithmetically fluent and thus able to quickly access and apply factual knowledge is clearly necessary for the solution of advanced mathematical problems. Of the components of executive cognitive functioning, visuospatial updating and verbal updating were predictive for the student's mathematical problem-solving at the end of grade 4; inhibition and shifting (in combination with inhibition) were not. This finding is also consistent with the findings of previous studies showing principal roles for visuospatial and verbal updating in mathematical problem-solving (e.g., Andersson, 2007; Cragg et al., 2017; Passolunghi & Pazzaglia, 2004; Zheng et al., 2011). Indeed, mathematical problems with more abstract and predominantly verbal information are increasingly presented in grade 4. Verbal updating gains importance, in addition to visuospatial updating (Andersson, 2007; Van de Weijer-Bergsma et al., 2015). It should be noted that we did not have specific expectations about the possible contributions of inhibition and shifting (in combination with inhibition) to the prediction of the fourth-grade students' mathematical problem-solving and did not find significant contributions. In a meta-analysis of previous studies that included visuospatial updating, verbal updating, inhibition, and shifting to examine students' mathematical problem-solving, the executive functions of visuospatial updating and verbal updating were also found to predominate—just as in the present study—over inhibition and shifting in the prediction of mathematical problem-solving (Friso-van den Bos et al., 2013).

Mathematical Problem-Solving Growth

With regard to the changes/growth in the students' mathematical problem-solving during grade 4, we hypothesized—on the basis of a more recent study by Fuchs et al. (2016)—that starting arithmetic fluency would mediate any associations between the executive functioning of the students and changes in their mathematical problem-solving. This was indeed found to be the case. Unexpectedly, however, the executive functions of inhibition and shifting (in combination with inhibition) as opposed to visuospatial updating and verbal updating were found to indirectly contribute to mathematical problem-solving at the end of grade 4 via starting arithmetic fluency and after controlling for the students' mathematical problem-solving at the start of grade 4. Declining importance for visuospatial updating and verbal updating has also been found in a few other studies when mastery of the relevant mathematical content within a given domain can be assumed to have increased (e.g., mastery of basic arithmetic in grade 4; Balhinez & Shaul, 2019; Fuchs et al., 2006). In the present study, we nevertheless expected both visuospatial and verbal updating to continue to play both direct and indirect roles in the changes/growth of students' mathematical problem-solving during grade 4, which did not prove to be the case. The finding of significant roles for inhibition and shifting + inhibition was unexpected. The students in our study had to solve increasingly more advanced, multistep mathematical fact and word problems, with and without pictures, requiring a variety of calculations within a single problem. To solve such multiple step problems, inhibition and shifting may be more critical than visuospatial and verbal updating (Bull & Scerif, 2001; Cantin et al., 2016; Verschaffel et al., 2020). For example, when students confront a new domain of mathematics entailing increasingly complex and abstract mathematical problems, inhibition may be increasingly needed to suppress irrelevant information (e.g., irrelevant textual information) and prior learning experiences (e.g., ignoring a counting on strategy when applying a multiplication strategy is more appropriate). In addition, shifting is increasingly needed to switch between procedures (e.g., going from addition to multiplication, a shift to another strategy; Wiley & Jarosz, 2012). At this point in the student's learning, visuospatial and verbal updating may still be important but not as important as when the student is less arithmetically fluent. In other words, the roles of inhibition and shifting in mathematical problem-solving may increase in grade 4 but remain indirect as they still depend on arithmetic fluency (Cragg et al., 2017). As students learn to solve a wider variety of mathematical problems in grade 4, greater flexibility in the determination of solution strategies and conduct of calculations is needed (Fuchs et al., 2006, 2016; Geary, 2011; Wiley & Jarosz, 2012). The executive function of inhibition and/or shifting comes to play an increasingly important role in students' mathematical problem-solving as found in the present study.

Finally, the results of the present study indicate that although the level of mathematical problem-solving at the start of grade 4 is predictive for the development of mathematical problem-solving ability (and therefore used as a control variable in some of our analyses), the level of arithmetic fluency is equally important and continues to be important. These findings are in line with the hierarchical frameworks for understanding changes in students' mathematics achievement over time and the assumption that the influences of various aspects of students' executive functioning are mediated during their development by the concomitant development of domain-specific mathematical competencies (Cragg et al., 2017; Geary, 2004; Geary & Hoard, 2005).

Study Strengths, Limitations, and Directions for Future Research

A major strength of the present study is the large and representative sample size of 458 students from 27 elementary schools, with also control for the students' nonverbal reasoning capacities. Another strength of the study is the use of students from grade 4 or, in other words, students facing the challenge of solving increasingly complex and more abstract mathematical problems but also expanding their knowledge and skills to include new domains of mathematics. Direct measures of executive functioning were used and important aspects of executive functioning were distinguished in doing this: visuospatial updating, verbal updating, inhibition, and shifting (in combination with inhibition). Two mathematics tests that have been proven to be reliable were also used: one for arithmetic fluency and one for more advanced fact and contextual mathematical problem-solving.

The present study also has some possible limitations. Multiple measures were not used to assess the four components of executive functioning, although doing this might have yielded more reliable results (e.g., use of two different tests per executive function and use of a measure that focuses exclusively on shifting). Furthermore, for follow-up research, we recommend including the measurement of arithmetic fluency at the end of grade 4 and using a structural equation model to examine the direct and indirect effects over time, in a cross-lagged design. In addition, we did not explore just how the students went about solving the mathematical problems presented to them. Observational methods might therefore be incorporated into future studies to provide a process measure of students' mathematical problem-solving. By doing this, for example, Kotsopoulos and Lee (2012) found that executive updating (with no distinction between visuospatial and verbal updating) was most challenging during the phase of understanding a mathematical problem, inhibition during the planning phase, and shifting during the reflection/evaluation phase. Another possible limitation on the present study is that other potentially relevant factors—such as students' reading comprehension, task approach, and (in)adequate identification of problem-solving strategies—were not included. Consideration of these factors in future research is therefore recommended.

Implications for Practice

Solid mastery of starting mathematical knowledge and skills obviously facilitates later learning and mathematical problem-solving (Watts et al., 2014). Careful attention should therefore be paid in the teaching of mathematics to the establishment of a solid mathematical foundation during the early elementary school years. Students with poor arithmetic fluency especially require explicit instruction and intensive training to improve their arithmetical knowledge and efficient strategy use (Koponen et al., 2018). Students need arithmetic fluency and sufficient prior mathematical knowledge for successful mathematics learning in grade 4 and subsequent grades.

With regard to executive functions, attempts to improve executive functioning have shown limited transfer to other domains, and long-term effects from interventions are largely unknown (Diamond, 2012). Based on a recent study by Gunzenhauser and Nückles (2021), supporting executive functioning during daily mathematics lessons in several ways can be suggested. One suggestion is modeling by the teacher; that is, the teacher can demonstrate how to make a plan and monitor its implementation in solving a complex mathematical problem. Another suggestion is informed training; that is, the teacher provides information about how, when, and why to enact a particular skill. Furthermore, it is important that teachers consider the specific executive functions that might help students to solve mathematical problems and scaffold the students during instruction (e.g., break complex problems into manageable parts and teach strategies to deal with irrelevant information).

Conclusion

The present research findings provide further insight into the roles of arithmetic fluency and specific aspects of executive functioning in the mathematical problem-solving of students. Arithmetic fluency and the visuospatial and verbal updating aspects of executive functioning appear to be most important for mathematical problem-solving measured at the end of grade 4. When mathematical problem-solving measured at the start of grade 4 is controlled for and the growth in students' mathematical problem-solving during grade 4 is considered, the executive functions of inhibition and shifting (in combination with inhibition) are now seen to directly relate to arithmetic fluency and indirectly to growth in mathematical problem-solving. An important finding in this study is the continued and unique contribution of arithmetic fluency to the mathematical problem-solving of students in grade 4, which required a more advanced level of mathematical problem-solving than in previous studies using younger students.

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References

Agostino, A., Johnson, J., & Pascual-Leone, J. (2010). Executive functions underlying multiplicative reasoning: Problem type matters. *Journal of Experimental Child Psychology*, **105**(4), 286–305. https://doi.org/10.1016/j.jecp.2009.09.006

Allen, K., & Giofrè, D. (2021). A distinction between working memory components as unique predictors of mathematical components in 7–8 year old children. *Educational Psychology*, **41**(6), 678–694. https://doi.org/10.1080/01443410.2020.1857702

Alloway, T. P. (2012). *Alloway Working Memory Assessment 2nd Edition (AWMA-2): Manual*. Pearson Education.

Andersson, U. (2007). The contribution of working memory to students' mathematical word problem solving. *Applied Cognitive Psychology*, **21**(9), 1201–1216. https://doi.org/10.1002/acp.1317

Andersson, U. (2008). Working memory as a predictor of written arithmetical skills in students: The importance of central executive functions. *British Journal of Educational Psychology*, **78**(2), 181–203. https://doi.org/10.1348/000709907X209854

Baddeley, A. D. (2000). The episodic buffer: A new component of working memory? *Trends in Cognitive Sciences*, **4**(11), 417–423. https://doi.org/10.1016/S1364-6613(00)01538-2

Balhinez, R., & Shaul, S. (2019). The relationship between reading fluency and arithmetic fact fluency and their shared cognitive skills: A developmental perspective. *Frontiers in Psychology*, **10**, Article 1281. https://doi.org/10.3389/fpsyg.2019.01281

Best, J. R., & Miller, P. H. (2010). A developmental perspective on executive function. *Child Development*, **81**(6), 1641–1660. https://doi.org/10.1111/j.1467-8624.2010.01499.x

Boonen, A. J. H., Van der Schoot, M., Van Wesel, F., De Vries, M. H., & Jolles, J. (2013). What underlies successful word problem solving? A path analysis in sixth grade students. *Contemporary Educational Psychology*, **38**(3), 271–279. https://doi.org/10.1016/j.cedpsych.2013.05.001

Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology*, **15**(3), 421–442. https://doi.org/10.1080/87565649909540759

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, **8**(1), 36–41. https://doi.org/10.1111/cdep.12059

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of students' mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology*, **19**(3), 273–293. https://doi.org/10.1207/S15326942DN1903_3

Cantin, R. H., Gnaedinger, E. K., Gallaway, K. C., Hesson-McInnis, S., & Hund, A. M. (2016). Executive functioning predicts reading, mathematics, and theory of mind during the elementary years. *Journal of Experimental Child Psychology*, **146**(2016), 66–78. https://doi.org/10.1016/j.jecp.2016.01.014

Cragg, L., Keeble, S., Richardson, S., Roome, H. E., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition*, **162**(2017), 12–26. https://doi.org/10.1016/j.cognition.2017.01.014

Delis, D. C., Kaplan, E., & Kramer, J. H. (2001). *Delis-Kaplan executive function system, color-word interference test: Manual*. Pearson Assessment and Information.

De Vos, T. (2010). *TempoTest Automatiseren* [Speeded Arithmetic Test]. Boom Test.

Diamond, A. (2012). Activities and programs that improve children's executive functions. *Current Directions in Psychological Science*, **21**(5), 335–341. https://doi.org/10.1177/0963721412453722

Doorman, M., Drijvers, P., Dekker, T., Van den Heuvel-Panhuizen, M., De Lange, J., & Wijers, M. (2007). Problem solving as a challenge for mathematics education in The Netherlands. *ZDM Mathematics Education*, **39**, 405–418. https://doi.org/10.1007/s11858-007-0043-2

Field, A. P. (2009). *Discovering statistics using SPSS* (3rd ed.). Sage.

Friso-van den Bos, I., Van der Ven, S. H. G., Kroesbergen, E. H., & Van Luit, J. E. H. (2013). Working memory and mathematics in primary school students: A meta-analysis. *Educational Research Review*, **10**(2013), 29–44. https://doi.org/10.1016/j.edurev.2013.05.003

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., Schatschneider, C., & Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, **98**(1), 29–43. https://doi.org/10.1037/0022-0663.98.1.29

Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem-solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of Educational Psychology*, **100**(1), 30–47. https://doi.org/10.1037/0022-0663.100.1.30

Fuchs, L. S., Gilbert, J. K., Powell, S. R., Cirino, P. T., Fuchs, D., Hamlett, C. L., Seethaler, P. M., & Tolar, T. D. (2016). The role of cognitive processes, foundation mathematical skill, and calculation accuracy and fluency in word-problem solving versus prealgebraic knowledge. *Developmental Psychology*, **52**(12), 2085–2098. https://doi.org/10.1037/dev0000227

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, **37**(1), 4–15. https://doi.org/10.1177/00222194040370010201

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, **47**(6), 1539–1552. https://doi.org/10.1037/a0025510

Geary, D. C., & Hoard, M. K. (2005). Learning disabilities in arithmetic and mathematics: Theoretical and empirical perspectives. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 253–267). Psychology.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F. L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education*, **15**(Suppl 1), S105–S123. https://doi.org/10.1007/s10763-017-9814-6

Gunzenhauser, C., & Nückles, M. (2021). Training executive functions to improve academic achievement: Tackling avenues to far transfer. *Frontiers in Psychology*, **12**, Article 624008. https://doi.org/10.3389/fpsyg.2021.624008

Hayes, A. F. (2018). *Introduction to mediation, moderation, and conditional process analysis: A regression-based approach* (2nd ed.). Guilford.

Jacob, R., & Parkinson, J. (2015). The potential for school-based interventions that target executive function to improve academic achievement: A review. *Review of Educational Research*, **85**(4), 512–552. https://doi.org/10.3102/0034654314561338

Janssen, J., Scheltens, F., & Kraemer, J. M. (2005). *Leerling- en onderwijsvolgsysteem rekenen-wiskunde* [Student monitoring system mathematics]. Cito.

Janssen, J., Verhelst, N., Engelen, R., & Scheltens, F. (2010). *Wetenschappelijke verantwoording voor de toetsen LOVS Rekenen-Wiskunde voor groep 3 tot en met 8* [Scientific jusitification of the mathematics test for grade 1 until grade 6]. Cito.

Kaufman, A. S. (1993). King WISC the third assumes the throne. *Journal of School Psychology*, **31**(2), 345–354.

Koponen, T. K., Sovo, R., Dowker, A., Räikkönen, E., Viholainen, H., Aro, M., & Aro, T. (2018). Does multi-component strategy training improve calculation fluency among poor performing elementary school children? *Frontiers in Psychology*, **9**, Article 1187. https://doi.org/10.3389/fpsyg.2018.01187

Kotsopoulos, D., & Lee, J. (2012). A naturalistic study of executive function and mathematical problem-solving. *Journal of Mathematical Behavior*, **31**(2), 196–208. https://doi.org/10.1016/j.jmathb.2011.12.005

Lee, K., & Bull, R. (2016). Developmental changes in working memory, updating, and math achievement. *Journal of Educational Psychology*, **108**(6), 869–882. https://doi.org/10.1037/edu0000090

Lee, K., Ng, E. L., & Ng, S. F. (2009). The contributions of working memory and executive functioning to problem representation and solution generation in algebraic word problems. *Journal of Educational Psychology*, **101**(2), 373–387. https://doi.org/10.1037/a0013843

LeFevre, J. A., Berrigan, L., Vendetti, C., Kamawar, D., Bisanz, J., Skwarchuk, S. L., & Smith-Chant, B. L. (2013). The role of executive attention in the acquisition of mathematical skills for students in grades 2 through 4. *Journal of Experimental Child Psychology*, **114**(2), 243–261. https://doi.org/10.1016/j.jecp.2012.10.005

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., & Howerter, A. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology*, **41**(1), 49–100. https://doi.org/10.1006/cogp.1999.0734

Passolunghi, M. C., & Pazzaglia, F. (2004). Individual differences in updating in relation to arithmetic problem solving. *Learning and Individual Differences*, **14**(4), 219–230. https://doi.org/10.1016/j.lindif.2004.03.001

Polya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton University Press.

Rasmussen, C., & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology*, **91**(2), 137–157. https://doi.org/10.1016/j.jecp.2005.01.004

Raven, J. (2000). The Raven's progressive matrices: Change and stability over culture and time. *Cognitive Psychology*, **41**(1), 1–48. https://doi.org/10.1006/cogp.1999.0735

Raven, J., Raven, J. C., & Court, J. H. (1998). *Manual for Raven's Progressive Matrices and Vocabulary Scales. Section 3: The Standard Progressive Matrices*. Oxford Psychologists.

St. Clair-Thompson, H., & Gathercole, S. E. (2006). Executive functions and achievements in school: Shifting, updating, inhibition, and working memory. *Quarterly Journal of Experimental Psychology*, **59**(4), 745–759. https://doi.org/10.1080/17470210500162854

Stroop, J. R. (1935). Studies of interference in serial verbal reactions. *Journal of Experimental Psychology*, **18**(6), 643–662.

Swanson, H. L. (2011). Working memory, attention, and mathematical problem solving: A longitudinal study of elementary school students. *Journal of Educational Psychology*, **103**(4), 821–837. https://doi.org/10.1037/a0025114

Swanson, H. L., & Beebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem solving in students at risk and not at risk for serious mathematical difficulties. *Journal of Educational Psychology*, **96**(3), 471–491. https://doi.org/10.1037/0022-0663.96.3.471

Swanson, H. L., Jerman, O., & Zheng, X. (2008). Growth in working memory and mathematical problem solving in students at risk and not at risk for serious mathematical difficulties. *Journal of Educational Psychology*, **100**(2), 343–379. https://doi.org/10.1037/0022-0663.100.2.343

Van Berkel, S. L., & Van der Zwaag, W. D. (2015). *Alloway Working Memory Assessment 2nd Edition (AWMA-2): Dutch manual*. Pearson Assessment and Information.

Van der Sluis, S., de Jong, P. F., & van der Leij, A. (2007). Executive functioning in children, and its relations with reasoning, reading, and arithmetic. *Intelligence*, **35**(5), 427–449. https://doi.org/10.1016/j.intell.2006.09.001

Van de Weijer-Bergsma, E., Kroesbergen, E. H., & Van Luit, J. E. H. (2015). Verbal and visual-spatial working memory and mathematical ability in different domains throughout primary school. *Memory and Cognition*, **43**, 367–378. https://doi.org/10.3758/s13421-014-0480-4

Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM Mathematics Education*, **52**(1), 1–16. https://doi.org/10.1007/s11858-020-01130-4

Viterbori, P., Traverso, L., & Usai, M. C. (2017). The role of executive function in arithmetic problem-solving processes: A study of third graders. *Journal of Cognition and Development*, **18**(5), 595–616. https://doi.org/10.1080/15248372.2017.1392307

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher*, **43**(7), 352–360. https://doi.org/10.3102/0013189X14553660

Wechsler, D. (2003). *Wechsler Intelligence Scale for Students-Third Edition (WISC-III): Administration and scoring manual*. Psychological Corporation.

Wiley, J., & Jarosz, A. F. (2012). How working memory capacity affects problem solving. *Psychology of Learning and Motivation*, **56**(6), 185–227. https://doi.org/10.1016/B978-0-12-394393-4.00006-6

Zheng, X., Swanson, H. L., & Marcoulides, G. A. (2011). Working memory components as predictors of students' mathematical word problem solving. *Journal of Experimental Child Psychology*, **110**(4), 481–498. https://doi.org/10.1016/j.jecp.2011.06.001