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Luhuan Huang

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te Jiangxi, China

Promotor:
Prof. dr. W.R. van Joolingen

Copromotor:
Dr. L.M. Doorman

Assessment committee:
Prof. dr. M. Bílek (Charles University Prague)
Prof. dr. Y. Cao (Beijing Normal University)
Prof. dr. P.H.M. Drijvers (Utrecht University)
Prof. dr. M.J. Goedhart (Groningen University)
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Prof. dr. L.T.G. Theunissen (Utrecht University)

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Chapter 1

Introduction

1. An Example of Two Mathematics Lessons

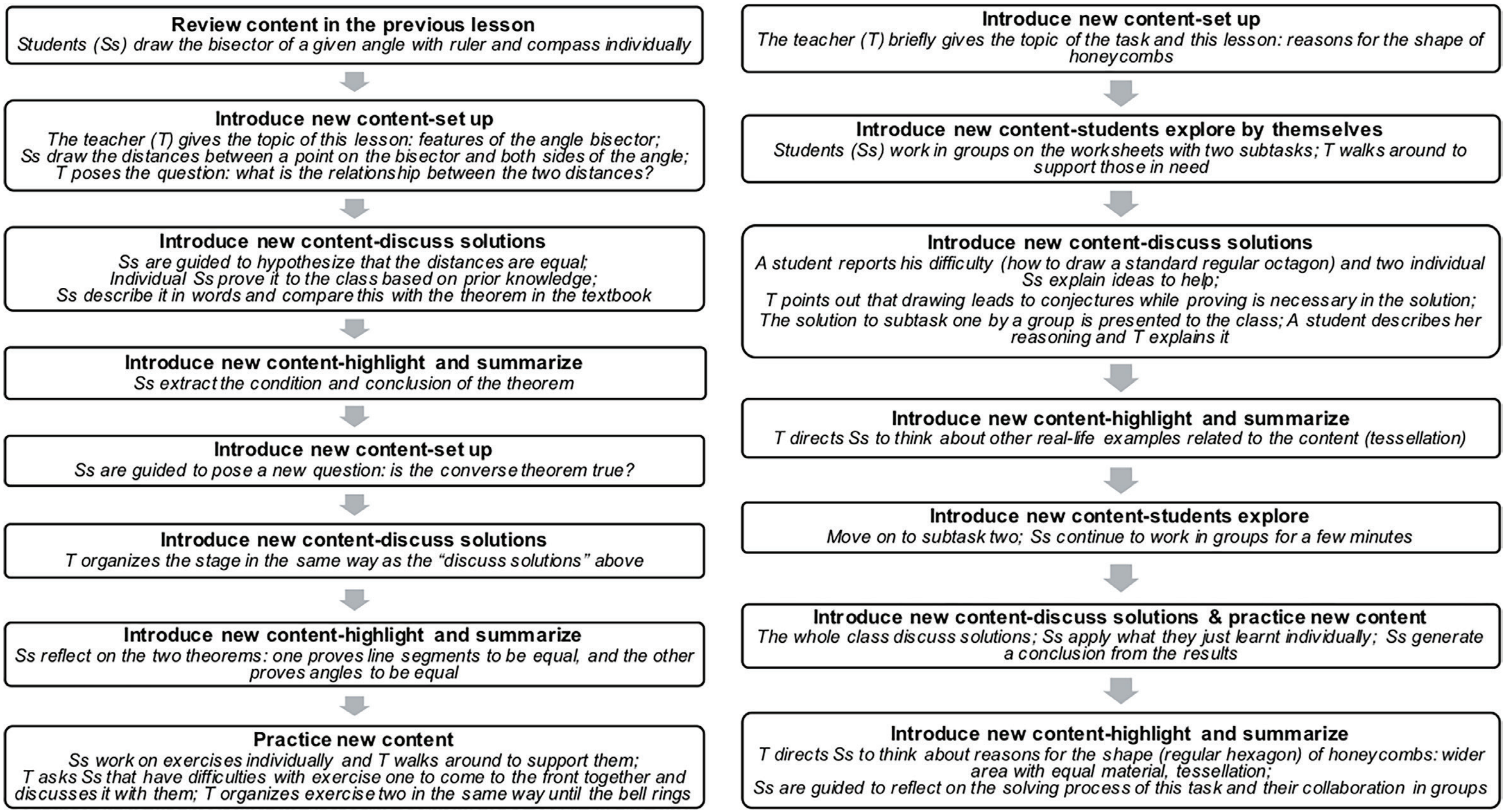
A main concern of the international community in mathematics education is to improve practices by teachers and researchers learning from each other (Clarke et al., 2007). This learning refers to teachers and researchers within one educational context reflecting on classroom practices, but can also refer to communities from different cultures learning from comparing and contrasting their practices. Attention has been paid to comparative studies within and across cultural contexts, including international large-scale projects and small-scale in-depth research (Cai et al., 2016). Comparative studies can focus on students (e.g., student experience and achievement), teachers (e.g., teacher knowledge and beliefs), curriculum (e.g., textbooks) and classroom practices (e.g., lesson structure, distribution of responsibility, collaboration). A well-known one is the TIMSS project (Trends in International Mathematics and Science Study) in 1999 that tried to identify national teaching patterns of seven countries based on videos of their mathematics lessons (Hiebert et al., 2003).

Before continuing, it is interesting to think about existing conceptions of mathematics lessons around the world. How do you suppose mathematics lessons in different cultural contexts to be? To what extent do they tend to be similar or different? Below is an example (see Fig. 1.1) of two lower-secondary mathematics lessons in geometry. Lesson A (the lesson on the left) is about theorems related to features of angle bisectors, and Lesson B (the lesson on the right) is about features of regular polygons in a context of honeycombs. Could you predict where these lessons might happen, for example, more possible to appear in East Asia or in the West based on your impressions?

The challenge here is to see whether you can identify culture-related elements in the short descriptions. Do you already have an idea whether the two lessons happened in East Asia or in the West? The answer is related to the topic of this thesis and will be revealed in Chapter 6.

2. Background: Teaching Cultures in Mathematics Education in East Asia and the West

Many comparative studies have found differences in teachers' beliefs about mathematics education and their classroom practices. The differences can in some cases be connected with cultures in East Asia and the West (e.g., Bryan et al., 2007; Cai & Wang, 2010). This seems to have led to stereotypes related to teaching cultures in mathematics education in the two groups. For example, East



Lesson A

Lesson B

Fig. 1.1 Two mathematics lessons in geometry for grade 8 students

Asia is usually considered to value well-structured learning content, conduct whole-class lecture and emphasize teachers' roles in the lesson (Bryan et al., 2007; Leung, 2001; Norton & Zhang, 2018). In contrast, the West is considered to value learning processes based upon real-life contexts, organize individual and group work and emphasize students' autonomy (Bryan et al., 2007; Leung, 2001; Liu & Feng, 2015; Norton & Zhang, 2018). More details on these stereotypes are provided in Chapter 2.

Although these stereotypes exist and impact people's images of mathematics education in the two groups, studies also found diversities within each group of teaching culture (Clarke, 2013; Felbrich et al., 2012) and within a country. The way of teaching was considered to vary based on the learning content and individual factors of teachers (Seidel & Prenzel, 2006). In addition, many shared elements of teaching between the groups were identified and emphasized (Bryan et al., 2007; LeTendre et al., 2001). Teaching in the two groups are probably not that different (Tweed & Lehman, 2002). Maybe differences within countries in East Asia or the West are bigger than between these countries.

Moreover, recent years have witnessed changes brought about by international communication and education reforms. Countries in East Asia and the West started to collaborate and learn from each other (Zhao et al., 2016). For example, being inspired by large-scale projects such as TIMSS and PISA (Programme for International Student Assessment), some western countries became more interested in East Asian education with an outstanding performance (Mok, 2019). East Asian countries reflected on their own practices and started to encourage student engagement, collaboration and communication to foster creativity in education. Mathematics educators in various cultural contexts are learning from each other and using and adapting each other's best practices. This can be promoted by collaboration in international communities such as ICME (International congress on mathematical education) and PME (International group for the psychology of mathematics education).

3. What to Focus: Inquiry-Based Learning (IBL) in Mathematics Education

The stereotypes of teaching cultures above include characteristics that might impact the understanding and use of approaches such as inquiry-based learning (IBL), which has been considered to be rooted in the Western teaching culture.

Looking into the lessons in Fig. 1.1, we can identify some elements related to IBL, examples of students being stimulated to inquire a certain mathematical idea, and situations in which opportunities for inquiry are lacking. Lesson A lacks opportunities for students to explore features of angle bisectors by themselves before the whole-class discussion. The task can be solved applying prior knowledge and the teacher provides guiding remarks. Nor do these students get chances to collaborate and exchange initial ideas. Lesson B does not require students to question or hypothesize mathematical issues that emerge during the lesson. These aspects are emphasized in IBL, which is possibly still a challenge for mathematics teachers and is the focus of this study.

3.1 Inquiry as a Pedagogical Concept

As pointed out by Artigue and Blomhøj (2013), Dewey started to use the concept of inquiry in pedagogical practices in his work as early as 1916 and 1938. Schwab (1962) proposed to include similar processes of scientific inquiry in school curriculum (Schwartz, 2004; Turner et al., 2018). Another fundamental work was the document by National Research Council (NRC) (1996, 2000) that considered inquiry in science education as a multifaceted activity characterized by five essential features. These are related to processes like questioning, experimenting, analyzing, evaluating and communicating. Inquiry in the NRC document was often interpreted to have three elements: a content area for students to understand what scientists do, a way of learning for students by conducting scientific inquiry, and a student-centered teaching approach to use in classrooms (Capps & Crawford, 2013; Minner et al., 2010). In this study, we took the last interpretation.

3.2 IBL as a Teaching Approach

IBL as a teaching approach invites students to involve in processes similar to what scientists and mathematicians do, such as the processes proposed by NRC (1996, 2000) (McNew-Birren & van den Kieboom, 2017).

Diverse interpretations exist considering elements of IBL (Capps & Crawford, 2013) and its use in classroom practices (Bybee, 2000; Nunnally, 2019). Some studies built upon NRC (1996, 2000) and focused on processes of IBL (Brandon et al., 2008; Danipog, 2018). Although presented to be a set of similar phases, the processes usually involve cycles instead of being linear (Artigue & Blomhøj, 2013; Fry, 2015).

Researchers paid attention to levels of IBL as well. A well-known framework was proposed by Schwab (1962), which divided four levels according to whether questions, methods and results are provided by the teacher or left open to students (Nadelson et al., 2010). Following studies used this framework and named the four levels. For example, Banchi and Bell (2008) took the four levels as confirmation, structured, guided and open inquiry, and Fang et al. (2010) described them as no inquiry, low, moderate and high level of inquiry.

Another fundamental framework was again the one proposed by NRC (2000). The variations in inquiry were regarded as a continuum based on the amount of responsibility taken by students themselves or support from the teacher and material. This framework distinguished four levels, varying from no to high, in each of the five phases of inquiry. A following framework by Capps and Crawford (2013) built upon the NRC framework and provided a detailed rubric for evaluation.

3.3 IBL in Mathematics Education

IBL first emerged in science education and then was taken up in mathematics education inspired by projects that involved both fields (Artigue & Blomhøj, 2013; Minner et al., 2010), thus a larger part of existing literature focused on IBL in science education (Nunnally, 2019). The adoption of IBL in mathematics education is supported by the view that mathematics is not a purely deductive discipline with standard procedures (Artigue & Blomhøj, 2013; Maass et al., 2017). Instead, it is essential for students to get opportunities to explore meaningful problem situations, make hypotheses, set up representations and models, try out multiple ways of solution methods, collaborate, communicate, and reflect on the whole process at the end.

We interpreted IBL in mathematics as a teaching approach which encourages students to learn in ways similar to how mathematicians work (Maaß & Doorman, 2013; Siegel & Borasi, 1994). Students actively experience and learn to take responsibility in mathematical processes like questioning, mathematizing, exploring procedures and communicating (Artigue & Blomhøj, 2013; Pedaste et al., 2015; Treffers, 1987). Practices in these phases can be supported by the teacher or teaching materials or left open to students, which is represented by a varying level from no IBL to high IBL (Bruder & Prescott, 2013).

As for the example in Fig. 1.1, IBL can be identified when students are guided to pose a question and to formulate a hypothesis in Lesson A, and when students explore representations and solution procedures by themselves in Lesson B. For both lessons, individual students have opportunities to explain ideas to the whole class, which involves IBL in communicating. IBL also happens in reflecting when students are asked to reflect on the content (two theorems in Lesson A) or on the solving process (at the end of Lesson B).

In Lesson B in Fig. 1.1, the teacher built upon a textbook task (see the left part in Fig. 1.2) and adapted it into a new version (see the right part in Fig. 1.2) as the worksheet for students in the lesson. She left out the three “experiments” that can be hints to indicate possible directions for approaching the problem. The instruction to try out with papers was also deleted. This allows students to make decisions by themselves to which directions and in what ways to explore. At the end of the task, the new version leaves the question to students to formulate a conclusion based on their findings and reasoning. The revision makes this task more open and provides more opportunities for students to inquire.

4. Where to Compare: China (Beijing) and the Netherlands

We took China, specifically Beijing (BJ), and the Netherlands (NL) as examples of East Asia and the West. Although Beijing and the Netherlands have their own regional characteristics, they are part of and share characteristics of their overarching teaching culture in East Asia and in the West, respectively.

In the past, Chinese education was often labeled as “teacher-centered, rote learning and passive learners” (Zhao et al., 2016). However, the education reform in China was regarded to include more elements of IBL (Dai et al., 2011). The mathematics curriculum standard since 2001 advocated changes in pedagogy and encouraged teachers to organize classroom activities related to inquiry and collaboration (Dai et al., 2011; Wang et al., 2018). The revised document in 2011 emphasized students’ experience and abilities of mathematical thinking and valued the role of students in lessons (Lv & Cao, 2018; Zhao et al., 2016).

Mathematics education in the Netherlands was impacted by Realistic Mathematics Education (RME)until the beginning of this century (van den Heuvel-Panhuizen, 2010). Ideas of problem-solving and modeling were kept in the curriculum reform in 2015. However, Dutch mathematics teachers were considered to be highly textbook dependent that hindered the implementation of the initial ideas in daily practice (Van den Heuvel-Panhuizen & Wijers, 2005). Besides the role of textbooks, the need for more attention for basic knowledge and skills became more central in reform discussions (Schoenfeld, 2014).

China and the Netherlands have different traditions in mathematics education, while ongoing changes can lead to more shared features, which together form the context of this study.

Honeycombs and regular hexagons

(a picture of a honeycombs is attached)

A beehive consists of many honeycombs. The cross section of a honeycomb is a regular hexagon. Polygons with equal sides and equal inner angles were defined as regular polygons.

Question 1: Which type of congruent regular polygons can cover a surface with no gaps and no overlaps? This is called the tessellation of plane figures.

Experiment 1: Can congruent regular triangles make a tessellation? Try out with scraps of paper.

Experiment 2: Can congruent squares make a tessellation? Try out with scraps of paper.

Experiment 3: Can congruent regular pentagons make a tessellation? Try out with scraps of paper.

Think: Any other regular polygons can make a tessellation?

Question 2: Which of the figures have the largest area: a regular hexagon, a square, a regular triangle, each with a perimeter of 6.

The area of a regular hexagon with a perimeter of 6 is______________

The area of a square with a perimeter of 6 is______________

The area of a regular triangle with a perimeter of 6 is______________

As is shown in the content above, there are three types of regular polygons that can make a tessellation. They are regular hexagons, squares and regular triangles. When the perimeter is a set value, the area of a regular hexagon (among the three figures) is the largest. Conversely, when the area is a set value, the perimeter of a regular hexagon among the three figures is the smallest.

Regular hexagons were taken as the cross sections of honeycombs to save materials.

Inquire reasons for the shape of honeycombs

A beehive consists of many honeycombs. The cross section of a honeycomb is a regular hexagon. Polygons with equal sides and equal inner angles were defined as regular polygons.

Question 1: Which type of congruent regular polygons can cover a surface with no gaps and no overlaps? This is called the tessellation of plane figures.

Think: Which type of regular polygons can make a tessellation?

Question 2: Which of the figures have the largest area: a regular hexagon, a square, a regular triangle, each with a perimeter of 24.

The area of a regular hexagon with a perimeter of 24 is______________

The area of a square with a perimeter of 24 is______________

The area of a regular triangle with a perimeter of 24 is______________

What can be learnt from the content above (what can be concluded from your research findings)?

Fig. 1.2 The task of honeycombs in the original form and in the new version by the teacher

5. Content of this Study

There exist stereotypes related to teaching cultures in East Asia and the West, which might have impact on the impressions of people considering mathematics education in the two groups (Leung, 2001). However, views and voices against the stereotypes also emerged from literature. We tried to move beyond these stereotypes and explore the current situations with open views to see what to learn from each other. The opportunities for inquiry by students, considered to be rooted in the West while advocated in policy documents around the world, was selected as the entry point.

In this study, we took Beijing and the Netherlands as examples and investigated what students, teachers, textbook tasks and classroom practices expressed or reflected in regard to IBL. Main questions of the study are: 1) What are the current situations of inquiry-based learning (IBL) in lower-secondary mathematics education in Beijing and the Netherlands? 2) What can Beijing and the Netherlands learn from each other considering IBL in lower-secondary mathematics education?

Comparing the current situations of IBL in different cultural contexts can identify particular elements in line with each teaching culture and shared elements beyond cultural boundaries. It helps to better understand the implementation of IBL and provides learning opportunities to each other. This study can be a starting point for professional development projects considering IBL in mathematics education.

6. Structure of the Thesis

We explored from multiple perspectives considering the current situations of IBL in mathematics education in Beijing and the Netherlands. An overview of the thesis can be seen in Table 1.1. There are connections among data of this study. Chapter 2 and Chapter 3 were based on one investigation, in which we interviewed teachers and surveyed one of the classes of each teacher. Textbooks in Chapter 4 are still in use by the Beijing teachers and most of the Dutch teachers, respectively. The five Beijing teachers in Chapter 5 were among participants of the interview study in Chapter 3.

Chapter 2 is from a students' perspective based on reports from students about the experience of classroom activities related to IBL and their preference. The questionnaire was built upon items from two international projects, i.e., PISA and PRIMAS (Promoting IBL in mathematics and science education across Europe).

Chapter 3 is from a teachers' perspective based on teacher interviews about their understanding and implementation of IBL. Teachers were not provided with a pre-set definition of IBL. Two mathematical tasks with features of IBL were included to provide contexts and provoke the discussion.

Chapter 4 is from a textbook perspective based on algebra and geometry tasks in two Beijing textbooks and a Dutch textbook. The analysis framework evaluated four IBL levels in each of the seven phases of IBL. Examples of tasks with and without opportunities for IBL were shown to illustrate how the opportunities can be provided or limited.

Chapter 5 is from a classroom practice perspective, which involves interactions among students, the teacher and textbook tasks. This chapter is based on usual lessons and the required IBL lessons of five Chinese teachers. Additional data from post-lesson teacher interviews and student survey about the IBL lessons was included. The chapter was only based on data in Beijing because the plan to observe lessons at Dutch schools was hindered by the Covid-19 pandemic.

In Chapter 6, we returned to the main questions and connected findings among the four sub studies about the current situations of IBL in Beijing and the Netherlands, and what can be learnt from each other. We also discussed the findings and the stereotypes about teaching cultures based on evidence from this study. Implications were provided for teachers, teacher educators as well as textbook and curriculum designers in mathematics education. Finally, we reflected on the study and provided insights for further research.

Table 1.1 Structure of the thesis

Chapter	Research questions (RQ)	Data collection
1. Introduction		
2. Inquiry-based learning practices in lower-secondary mathematics education reported by students from China and the Netherlands	*RQ 2.1*: What do students in Beijing and the Netherlands report about their experience and preference with respect to IBL in lower-secondary mathematics education? *RQ 2.2*: What are the shared and particular features on this issue between the two areas? *RQ 2.3*: To what extent can the particular features be explained by the stereotypes about the two teaching cultures?	Survey (858 Beijing and 441 Dutch students)
3. Beliefs and practices related to inquiry-based learning: Chinese and Dutch lower-secondary mathematics teachers' perspectives	*RQ 3.1*: What do lower-secondary mathematics teachers in Beijing and the Netherlands highlight when they describe their IBL beliefs and IBL practices? *RQ 3.2*: In what ways are the IBL beliefs and IBL practices mentioned in the two areas in line with expectations based on the stereotypes about the two teaching cultures?	Semi-structured interview (30 Beijing and 19 Dutch teachers)
4. Opportunities for inquiry-based learning in Chinese and Dutch lower-secondary school mathematics textbook tasks	*RQ 4.1*: To what extent are opportunities for IBL provided by tasks in lower-secondary mathematics textbooks in Beijing and the Netherlands? *RQ 4.2*: In what ways are the IBL-related features of these textbooks in line with expectations based on the stereotypes about the two teaching cultures?	Textbook tasks (404 Beijing and 244 Dutch tasks)
5. Inquiry-based learning practices in lower-secondary mathematics education: An analysis of Chinese teachers' lessons	*RQ 5.1*: To what extent are structures of the required IBL lessons different from those of the observed usual lessons? *RQ 5.2*: To what extent are opportunities for IBL provided in the two types of lessons, and what are the differences?	Observation (24 lessons of five Beijing teachers); Teacher interview and student survey as additional data
6. Conclusion and discussion		

Chapter 2

Inquiry-Based Learning Practices in Lower-secondary Mathematics Education Reported by Students from China and the Netherlands

This chapter is based on:

Huang, L., Doorman, M., & van Joolingen, W. R. (2021). Inquiry-based learning practices in lower-secondary mathematics education reported by students from China and the Netherlands. *International Journal of Science and Mathematics Education*, *19*(7), 1505–1521. https://doi.org/10.1007/s10763-020-10122-5

Abstract Inquiry-based learning (IBL) emphasizes learning through experiencing and constructing. Where IBL is often applied in science education, the conceptualization of IBL practices in mathematics education is less obvious. We compared students' reports on IBL practices in two different teaching cultures as an attempt to better understand IBL practices in connection with overarching teaching cultures. In this study, we investigated IBL practices in lower-secondary mathematics lessons in Beijing and the Netherlands through a survey about the experiences and preferences of 858 Chinese students and 441 Dutch students. Results show that students from the Beijing sample reported experiencing IBL activities in most mathematics lessons, while students from the Dutch sample reported them in some lessons, and both preferred the same amount of IBL activities as they experienced. The Dutch sample reported little experience with posing questions to tackle. The study also suggests a correlation between IBL experience and IBL preference of each class: students with more IBL experience are likely to show a higher preference for IBL activities. Results of this study do not confirm expectations based on stereotypes about the two teaching cultures. The students' perspective in both samples suggests that providing complex problems and organizing group work have potential for further encouraging IBL in mathematics.

Keywords Comparative study; Inquiry-based learning; Lower-secondary education; Mathematics education; Student perspective

1. Introduction

Inquiry-based learning (IBL) is a teaching approach which emphasizes learning through experiencing and constructing. IBL encourages students' autonomy in the learning process and involves student-centered learning activities such as problem-solving, investigation, and collaboration. Since "inquiry" used to be employed almost exclusively to describe science (Ibrahim et al., 2017), and IBL emerged in science education, the conceptualization of IBL practices in mathematics education is less obvious (Artigue & Blomhøj, 2013). Teaching is considered as a cultural activity (Cai et al., 2016), and as IBL is a teaching approach, the use of IBL may be impacted by teaching cultures. Comparing IBL practices in contexts of different teaching cultures tends to reveal particular features that can be explained by each teaching culture and the shared features crossing cultural boundaries, which leads to a better understanding of the current IBL practices. Teaching cultures in East Asia and in the West are considered to be markedly different and have led to stereotypes. We took Beijing and the Netherlands as examples of these two teaching cultures to investigate IBL practices in mathematics reported by students. Although Beijing and the Netherlands have their own regional characteristics, they are also part of and share characteristics of their overarching teaching culture. The research questions are: What do students in Beijing and the Netherlands report about their experience and preference with respect to IBL in lower-secondary mathematics education? What are the shared and particular features on this issue between the two areas? To what extent can the particular features be explained by the stereotypes about the two teaching cultures?

2. Background: IBL in East Asian and Western Education

IBL is an intentional student-centered pedagogy that challenges learners to explore problem situations before formal explanations and solution procedures are provided (Marshall et al., 2017). These explorations are intended to involve students in processes inspired by the inquiry cycle, such as questioning, hypothesizing, designing, investigating, analyzing, evaluating, and reflecting (Swan et al., 2013). Instructions considered as IBL vary a lot in different interpretations (Ibrahim et al., 2017; Turner et al., 2018), especially on the degree that students direct and monitor the learning process (Modrek et al., 2017) and on the amount of guidance that the teacher and teaching materials provide (Bruder & Prescott, 2013). A distinction has been made between open inquiry where students can choose a topic and are fully responsible for inquiry processes, guided inquiry where the teacher is responsible for topics and guides the inquiry processes, and structured inquiry where the teacher structures and exemplifies inquiry processes that students are expected to follow (Bruder & Prescott, 2013). In this study, we interpreted IBL as a teaching culture and classroom practices in which students take responsibility in inquiry processes (Dobber et al., 2017; Maaß & Doorman, 2013). For students to take this responsibility, the teacher is responsible for guiding inquiry by creating problem situations and providing support and organizing student collaboration and communication (Artigue & Blomhøj, 2013).

IBL was originally envisioned in science education (Ibrahim et al., 2017) and consequently a large part of existing research focused on IBL in science. Although advocated in policy documents, IBL does not seem to be a routine in daily teaching (Dobber et al., 2017). According to the results of PISA 2015, one in four students or even fewer reported designing their own experiments or doing laboratory experiments (Organisation for Economic Co-operation and Development [OECD], 2016). The PISA 2015 index of IBL in lower-secondary science education turned out to be rather similar for China[1] (−0.25) and the Netherlands (−0.27) and both are below the OECD average (0.16) (OECD, 2016). A European study presented that mathematics teachers reported less use of IBL than science teachers (PRIMAS[2], 2013). Besides large-scale international studies and studies focusing on science education, more attention also needs to be paid to in-depth studies on IBL in mathematics education. For instance, 986 US teachers reported in a study that they typically spent 34% of the time on IBL during a mathematics lesson, which is quite significant (Marshall et al., 2009). However, researchers also have pointed out the limitation of reporting IBL practices solely from a teachers' perspective, which may result in IBL practices being over-reported (Capps et al., 2016). Consequently, a study into a students' perspective on IBL is expected to enrich our understanding of current practices.

IBL is a teaching approach rooted in the Western teaching culture. The East Asian teaching culture seems to differ in many ways from that in the West, which probably has effects on the implementation of IBL. Features of each teaching culture have been identified, gradually leading to stereotypes about teaching cultures in East Asia and the West (Leung, 2001, 2005), also for the subject of mathematics (summarized in Table 2.1). These stereotypes mainly include dimensions of content versus process, whole-class versus individualized, teacher-centered versus student-centered, rote-like versus meaningful, and externally motivated versus internally motivated (Leung, 2001). Beijing is an East Asian city and the Netherlands is part

1 Four provinces (Beijing, Shanghai, Jiangsu, Guangdong) in China took part in PISA 2015.
2 The PRIMAS project: Promoting inquiry-based learning (IBL) in mathematics and science education across Europe

Table 2.1 Stereotypes about teaching cultures in mathematics in East Asia and the West

East Asia		The West	
Content-oriented	Teachers emphasize learning content and related procedures or skills; they value the purity, generality, and logic of mathematics and take mathematics as the product of structured in-depth knowledge separated from real life (Cai, 2006; Leung, 2001; Norton & Zhang, 2018)	Process-oriented	Teachers emphasize learning through experiencing and constructing; they value the pragmatism of mathematics to solve problems and take mathematics as the process of dealing with reality, which involves practical knowledge related to real life (Cai, 2006; Leung, 2001; Norton & Zhang, 2018)
Whole-class instruction	Under the impact of collectivist orientation (Leung, 2001), teachers consider the general needs of students that they usually interact with the whole class and seldom organize group work	Individualized learning	Under the impact of individualist orientation (Leung, 2001), teachers consider particular needs of students that they usually interact with individual students and frequently organize group work
Teacher-centered instruction	Teachers act as respectable authority, expert in mathematics, and role model; students are responsible for keeping attention and suiting the learning environment (Leung, 2001; Liu & Feng, 2015; Tan, 2015). Teachers feel responsible for preparing students for high-stake examinations. Typical classroom activities include well-organized directive instruction with lecturing, drill and practice (Leung, 2001)	Student-centered learning	Teachers act as facilitator of learning and cater to the demands of students (Leung, 2001; Mok, 2006); students act as active learners to construct their own knowledge (Liu & Feng, 2015). Teachers are responsible for providing a suitable learning environment and preparing students for mathematically literate citizens. Typical classroom activities include problem-solving, investigation, and collaboration (Leung, 2001; Liu & Feng, 2015)
Repetitive rote-like learning	Students learn by memorizing mathematical facts and doing a considerable amount of repetitive mathematical exercises (Liu & Feng, 2015; Tan, 2015)	Meaningful learning	Students learn by thoroughly understanding mathematical facts first (Leung, 2001) through experiencing problem situations and classroom activities
Externally motivated	Students are driven by extrinsic factors such as competitive high-stake examinations, and learning is considered to be through hard work (Leung, 2001)	Internally motivated	Students are driven by intrinsic factors such as interests, and learning is considered as more effective in a pleasurable way (Leung, 2001)

of the Western teaching culture. Based upon the existing stereotypes, it can be expected that Dutch students would report much experience and preference related to IBL, while students in Beijing would not. It can also be expected that not many shared IBL-related features could be identified, and the differences would be in line with and be explained by the two teaching cultures.

Recent curriculum changes in both countries might also have an impact on teaching practices. The Chinese mathematics curriculum standard since 2001 required textbooks to provide space for students to investigate and communicate and encouraged teachers to organize inquiry and collaboration in lessons (Wang et al., 2018). The revised curriculum standard in 2011 paid attention to students' experience in mathematics activities and mathematics thinking and encouraged students to pose questions themselves (Lv & Cao, 2018). Dutch mathematics education was influenced by Realistic Mathematics Education (RME) until the beginning of this century, when criticism and debates emerged (Van den Heuvel-Panhuizen, 2010). The latest mathematics curriculum reform implemented in 2015 mentioned "thinking activities" including problem-solving and modeling, while textbooks and examinations were also impacted by requests for more attention for basic knowledge and skills in algebra (Schoenfeld, 2014).

Taking Beijing and the Netherlands as examples of the two teaching cultures and students' reports on IBL-related activities as data, we investigated the current situation of IBL practices in mathematics. We also looked for shared features and particular features between the two areas to provide more insight into the current practice of IBL in mathematics, and into to what extent this practice is related to the overarching teaching culture.

3. Method

3.1 Participants

Eight hundred sixty-seven students from 30 classes in Beijing and four hundred forty-two students from 19 classes in the Netherlands participated in this study. All of them were in grade 7, 8, or 9. With ten invalid questionnaires taken out, the distribution of samples can be seen in Table 2.2.

To get students filling in the questionnaires, we contacted teachers and surveyed one of the classes of each teacher. In Beijing, generally, permission from school leaders makes it convenient to enter a school; therefore, we first contacted school leaders through interpersonal networks, as well as a few local administrations, and some mathematics teachers directly. We ensured a balanced selection of urban and suburban schools in Beijing. In the Netherlands, we invited individual teachers through an electronic newsletter for mathematics teachers and through personal contacts, and included all teachers who showed an interest to participate. This created a bias in the larger than average number of classes at the level of pre-university education (VWO) in the survey. Dutch schools in different districts are quite similar, with the main differences caused by the differentiated school system; therefore, we also ensured the presence of classes at the level of pre-higher vocational secondary education (HAVO) and pre-vocational secondary education (VMBO) (see Table 2.2). We were aware that the study used convenient sampling and we took the samples as examples for the two teaching cultures.

Table 2.2 Distribution of samples in the study

	Valid N	Class size	Class type	Grade of class	Student gender	Average age
Beijing Sample	858	29 ± 7	Urban: 56.6%	Grade 7: 36%	Male: 51.6%	13.2
			Suburban: 43.4%	Grade 8: 43.2%	Female: 48.4%	
				Grade 9: 20.7%		
Dutch Sample	441	23 ± 6	VWO: 72.8%	Grade 7: 16.6%	Male: 53.8%	14.4
			HAVO: 17.2%	Grade 8: 47.4%	Female: 46.2%	
			VWO/HAVO: 6.3%	Grade 9: 36.1%		
			VMBO: 3.6%			

Note. In the Netherlands, after primary school (grade 6), students choose one of three types of secondary education: pre-university education (VWO), senior general secondary education (HAVO), or pre-vocational secondary education (VMBO) (source: https://www.government.nl/topics/secondary-education)

3.2 Instrument

The questionnaire consists of three parts. The first part asks students' basic information, namely the gender, year of birth and grade for mathematics in the last report.

The second part contains an IBL experience scale (see Table 2.3), which measures a student's experience of IBL activities in mathematics lessons with 13 items: items 1–3, 5–7, and 9–15. Eight items were derived from PISA, referring to the background questionnaires from 2012 and 2015 (OECD, 2013, 2016). PISA 2015 used a selected set of nine IBL activities to test the index of IBL in science. Four items were derived from the student questionnaire and teacher questionnaire of PRIMAS[3] (PRIMAS, 2013), which was an international project based on PISA and it added to PISA items.

3 15 items from the "students' interaction," "reference to application," "hands-on experience," and "investigation" scales of PISA 2006 student questionnaire were adapted into 10 items and used in PRIMAS project for teachers and students to report IBL activities.

Both PISA and PRIMAS have proved to be reliable and validated. Some items were changed from a science context to a mathematics one. All items were put in first person to fit the perspective of students. Item 14, which was self-made, was added because we considered communicating solutions with peers as an essential aspect of IBL. These 13 items are about IBL activities that represent the two categories of IBL in mathematics: students take responsibility in inquiry processes (items 3, 5, 7, 11, 13, 15) and the teacher guides the inquiry processes. The latter one can be divided into the teacher providing suitable problem situation and support (items 1, 12) and the teacher organizing collaboration and communication (items 2, 6, 9, 10, 14). This scale uses a four-point Likert scale, according to the frequency of each activity happening in mathematics lessons, students were asked to choose one from "never or hardly ever," "in some lessons," "in most lessons," and "in almost all lessons."

Table 2.3 Items of the questionnaire and their sources

	Items of the questionnaire	Original items and sources
1	The teacher presents mathematical problems for which there is no immediately obvious solution procedure	The teacher presents problems for which there is no immediately obvious method of solution (PISA 2012)
2	We are required to discuss mathematical problems	Students are required to argue about science questions (PISA 2015)
3	We have the opportunity to pose questions to tackle	I give my students the opportunity to choose which questions they tackle (PRIMAS-teacher)
4	The teacher shows how problems should be solved	Self-made
5	We are allowed to design our own procedures for solving complex problems	• The teacher asks us to decide on our own procedures for solving complex problems (PISA 2012) • Students are allowed to design their own experiments (PISA 2015)
6	We are given opportunities to explain our own ideas	Students are given opportunities to explain their ideas (PISA 2015)
7	We spend time doing investigations to test out our own ideas	• Students spend time in the laboratory doing practical experiments (PISA 2015) • Students are asked to do an investigation to test ideas (PISA 2015)
8	We solve problems by following example solution procedures	Self-made and inspired by items from PRIMAS project: • When we do experiments/ investigations by following the instructions of the teacher (PRIMAS-student) • The students do experiments by following my instructions (PRIMAS-teacher)
9	The teacher lets us work in pairs or small groups to come up with joint solutions	• The teacher has us work in small groups to come up with joint solutions to a problem or task (PISA 2012) • The students work collaboratively in pairs or small groups (PRIMAS-teacher)

	Items of the questionnaire	Original items and sources
10	The teacher asks us to explain how we have solved a problem	The teacher asks us to explain how we have solved a problem (PISA 2012)
11	We are encouraged to ask questions when they emerged during investigations	I have students ask questions about math/ scientific phenomena addressed during experiments (PRIMAS-teacher)
12	The teacher gives us extra help when we need it	I give students extra help, when they need it (PRIMAS-teacher)
13	We draw conclusions from investigations we have conducted	Students are asked to draw conclusions from an experiment they have conducted (PISA 2015)
14	We explain our solutions of the problem to other students	Self-made
15	We have the possibility to influence on how things are done during the lesson	• We have the possibility to decide how things are done during the lesson (PRIMAS-student) • We have an influence on what is done in the lesson (PRIMAS-student)

Note. PRIMAS-teacher refers to the teacher questionnaire of the PRIMAS project, and PRIMAS-student refers to the student questionnaire

The second part also includes two additional items (items 4, 8) related to stereotypes about teaching cultures. The items were self-made to test whether these stereotypes exist in Beijing and Dutch mathematics lessons.

The third part is an IBL preference scale, in which five items (items 1, 3, 5, 7 and 9) were selected from the IBL experience scale to measure a student's preference for IBL activities. With the use of a three-point Likert scale, it asked if a student would prefer these activities to happen "less," "the same," or "more" in mathematics lessons.

When translating the original questionnaire from English into Dutch and Chinese, we tried to ensure the equivalence through peer check about possible discrepancies by researchers and postgraduates and pilot surveys. During pilot surveys in each area, we asked students if they had questions about items and we optimized the questionnaires for them. We also asked two Chinese postgraduates to translate the Chinese version back into English, then compared their versions with the original questionnaire and adjusted a few words. For example, we carefully thought over the translation of "investigation" in the context of the item in both languages ("onderzoek" in Dutch and "探究" in Chinese[4]).

To test the quality of the questionnaire, we performed an analysis after the pilot surveys and surveys. For the quality analysis of the surveys, firstly we checked

4 The word "investigation" in Chinese is literally "调查", which came from the West and entered into the Chinese school context. It usually refers to big projects with a complete research cycle. We considered investigations in mathematics during classroom teaching, and translated it as "探究", referring to exploring problems or issues deeply, like "inquiry" in English.

missing values and took out ten invalid questionnaires with more than one item (5%) missing. Then, we calculated “item discrimination”; we distinguished a high-score group (27%) and a low-score group (27%) based on the average scores on the IBL experience scale and the IBL preference scale respectively, and through an independent samples *t* test, we found significant differences between the two groups on each item and on the scale, both for the Beijing sample and the Dutch sample. We also calculated “item-total correlation”; all the correlations between each item and the scale are significant. Furthermore, we calculated the internal consistency reliability (Cronbach’s alpha) of each scale. For the Beijing sample, it is 0.89 on the IBL experience scale and 0.67 on the IBL preference scale; for the Dutch sample, it is 0.74 and 0.56, respectively. The results of the quality analysis are reasonably acceptable.

3.3 Data Collection and Analysis

Dutch data were collected from April to June of 2017, and Beijing data from October to November of the same year. We asked the mathematics teachers who were willing to participate whether one of their lower-secondary classes of students could fill in the questionnaires, which were in Dutch for Dutch students and in Chinese for Beijing students, and if we could be present when handing out the questionnaires. For all the classes that were accessible, the first author was present to give a brief introduction and answer potential questions. The language used for oral communication was English at Dutch schools and Chinese at Beijing schools. For the classes that were not accessible (5 of 19 in the Netherlands and 12 of 30 in Beijing), the mathematics teacher helped to hand out and collect questionnaires in the classroom. Filling in the questionnaires usually took about 10 min.

Based on the data from student questionnaires, we performed descriptive analysis and difference analysis using SPSS 24. Firstly, we scored all the questionnaires. The IBL experience scale as well as items 4 and 8 were scored from one to four, and the IBL preference scale from one to three. Then, we analyzed the scales. We calculated the average scores on scales as well as on each item for both samples, and ranked items within the scale based on the average scores. We also calculated the average scores of each teacher/class. We were aware that we took categorical variables from four-point and three-point Likert scale as continuous variables, and the results need to be interpreted cautiously. To make sure whether significant differences exist between groups, we did an independent samples *t* test based on “mathematics grade” (low-achievers and high-achievers, namely, students with the lowest 5% and highest 5% mathematics grade in each class). We also performed a correlation analysis between IBL experience and IBL preference based on each teacher/class and based on each student. We further identified similarities and differences for students' reported IBL experience and IBL preference between the two samples and compared the differences with the stereotypes about the two teaching cultures, in which results of items 4 and 8 were also reported.

4. Results

4.1 IBL Practices in Lower-secondary Mathematics Education Reported by Students from Beijing

The Beijing sample gets an overall average score of 3.05 (SD = 0.55) on the 13 IBL experience items (ranging from one to four; see Table 2.4) that students generally reported experiencing in most mathematics lessons. They most experienced explaining their own ideas (item 6, M = 3.53), and least being presented complex mathematical problems (item 1, M = 2.31).

As for results on the five IBL preference items (ranging from one to three; see Table 2.4), the Beijing sample gets an overall average score of 2.45 (SD = 0.38), i.e., that the students generally preferred the same amount of IBL activities as they experienced. They most preferred group work (item 9, M = 2.53) to happen more, and least being presented complex mathematical problems (item 1, M = 2.23).

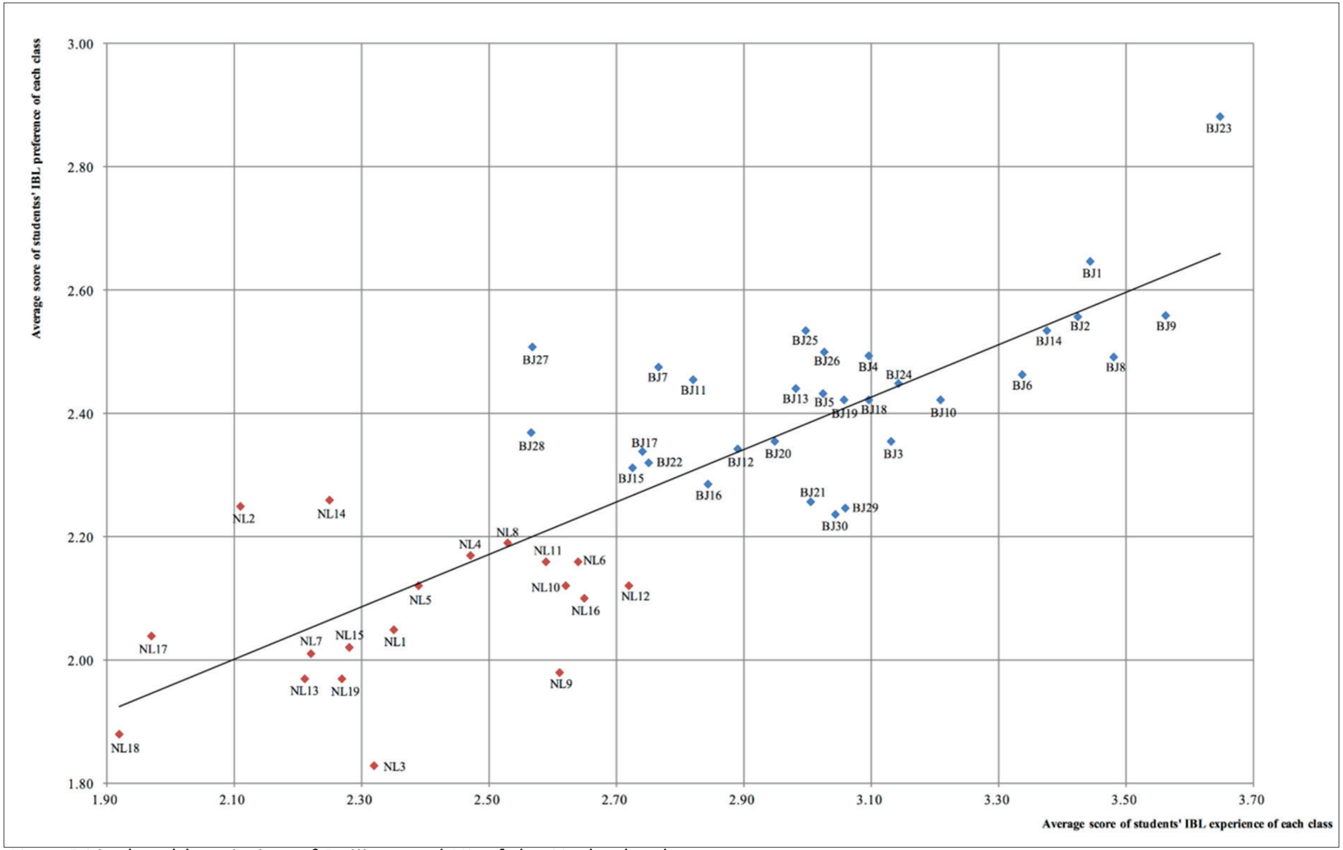
By analyzing the five shared items of IBL experience and IBL preference (see Table 2.4, or Fig. 2.3 and Fig. 2.4), we found item 1 to be a special aspect for the Beijing sample, that is, although the students experienced less on being presented complex mathematical problems than on the other activities, they showed no preference for it to happen more.

Correlation may exist between students' IBL experience and IBL preference (indicated in Fig. 2.1). For the Beijing sample, the correlation coefficient is 0.61 (p = 0.00) if based on the average scores of each class, and 0.26 (p = 0.00) if based on the average scores of each student. We also compared the reports of Beijing low-achievers and high-achievers and found no significant difference ($t(88)$ = 1.71, p = 0.09) for their IBL experience, while low-achievers (M = 2.24) reported significantly ($t(88)$ = 3.69, p = 0.00) less IBL preference than high-achievers (M = 2.55).

Table 2.4 Average scores on IBL experience and IBL preference for both samples

	Scale	Value	Item													Overall
			1	2	3	5	6	7	9	10	11	12	13	14	15	
BJ sample	IBL experience	M	2.31	2.68	3.33	3.30	3.53	2.91	2.62	3.36	3.46	3.41	3.01	3.10	2.65	3.05
		SD	0.82	0.96	0.82	0.82	0.70	0.89	1.05	0.76	0.74	0.76	0.88	0.87	0.95	0.55
	IBL preference	M	2.23	\	2.52	2.45	\	2.50	2.53	\	\	\	\	\	\	2.45
		SD	0.63	\	0.53	0.57	\	0.57	0.59	\	\	\	\	\	\	0.38
NL sample	IBL experience	M	2.02	2.80	1.14	2.20	2.81	2.29	2.07	2.90	2.85	3.41	2.72	2.36	1.51	2.39
		SD	0.86	0.95	0.39	0.96	0.95	1.01	0.98	0.88	0.93	0.76	0.85	0.95	0.72	0.43
	IBL preference	M	1.87	\	1.89	2.12	\	2.11	2.42	\	\	\	\	\	\	2.08
		SD	0.58	\	0.73	0.62	\	0.70	0.69	\	\	\	\	\	\	0.40

Note. BJ is the abbreviation of Beijing, and NL of the Netherlands, \ means the item was not included in the scale



Note. BJ is the abbreviation of Beijing, and NL of the Netherlands

Fig. 2.1 Teachers' IBL practices based on the average scores of students' IBL experience and IBL preference

4.2 IBL Practices in Lower-secondary Mathematics Education Reported by Students from the Netherlands

The Dutch sample gets an overall average score of 2.39 (*SD* = 0.43) on the 13 IBL experience items (ranging from one to four; see Table 2.4) that students generally reported experiencing in some mathematics lessons. They most experienced getting extra teacher help (item 12, *M* = 3.41), and least posing questions to tackle (item 3, *M* = 1.14).

As for results on the five IBL preference items (ranging from one to three; see Table 2.4), the Dutch sample gets an overall average score of 2.08 (*SD* = 0.40), i.e., that they generally preferred the same amount of IBL activities as they experienced. They most preferred group work (item 9, *M* = 2.42) to happen more, and they preferred two activities to happen even less: being presented complex mathematical problems (item 1, *M* = 1.87) and posing questions to tackle (item 3, *M* = 1.89).

Item 3 is a special aspect for the Dutch sample in that, although the students experienced little (*M* = 1.14) on posing questions to tackle, which never or hardly ever happened in their mathematics lessons, they preferred it to happen even less (*M* = 1.89).

For the Dutch sample, the correlation coefficient between students' IBL experience and IBL preference (indicated in Fig. 2.1) is 0.35 ($p = 0.15$) if based on the average scores of each class, and 0.14 ($p = 0.00$) if based on the average scores of each student. We also compared the reports of Dutch low-achievers and high-achievers and found low-achievers (*M* = 2.25) reported significantly ($t(45) = 2.11$, $p = 0.04$) less IBL experience than high-achievers (*M* = 2.54), but no significant difference exists ($t(45) = 0.81$, $p = 0.42$) as for the IBL preference.

4.3 Comparison of IBL Practices in Lower-secondary Mathematics Education in Both Samples

Based on the average scores of each class of students on the IBL experience scale and the IBL preference scale, the relative position of each teacher/class in both samples can be seen in Fig. 2.1, which presents the overview of IBL practices for all 49 teachers participating in this study. The figure clearly shows a cluster of Beijing teachers and a cluster of Dutch teachers and indicates a possible correlation between IBL experience and IBL preference of each teacher/class. We compared IBL practices reported by students in both samples and identified the shared features and particular features below.

4.3.1 Shared Features of IBL Practices in both Samples

Students' reports on IBL practices show similar patterns on certain IBL activities. As is shown in Fig. 2.2, students in both samples share the four most frequent IBL experience with items 6, 10, 11, and 12 (explain own ideas, explain solution strategies, ask questions during investigations and get extra teacher help); they also share three of the four least frequent IBL experience with items 1, 9, and 15 (being presented complex mathematical problems, group work and influencing the lesson).

In addition, students in both samples score between 2 and 3 on the IBL preference scale (see Table 2.4) and preferred the same amount of IBL activities as they experienced. As is shown in Fig. 2.4, they share the highest preference for item 9 (group work) to happen more, and lowest for item 1 (being presented complex mathematical problems).

Moreover, the correlation between IBL experience and IBL preference is strong with data of the two samples taken together (as indicated by Fig. 2.1), the correlation coefficient is 0.83 (p = 0.00) if based on the average scores of IBL experience and IBL preference of each class, and 0.39 (p = 0.00) if based on the average scores of these two variables of each student.

4.3.2 Particular Features of IBL Practices in each Sample

The Beijing sample reported experiencing IBL activities in most mathematics lessons while the Dutch sample in some lessons. Students in the Beijing sample only experienced less on discussing mathematical problems than students in the Dutch sample. Our Dutch students experienced little on posing questions to tackle and preferred it to happen even less. As for the correlation between IBL experience and IBL preference, it only exists in the Beijing sample if based on the average scores of each class, and the correlation is higher in the Beijing sample than in the Dutch sample if based on the average scores of each student. Low-achievers significantly reported less IBL preference than high-achievers in the Beijing sample, while low-achievers significantly reported less IBL experience than high-achievers in the Dutch sample.

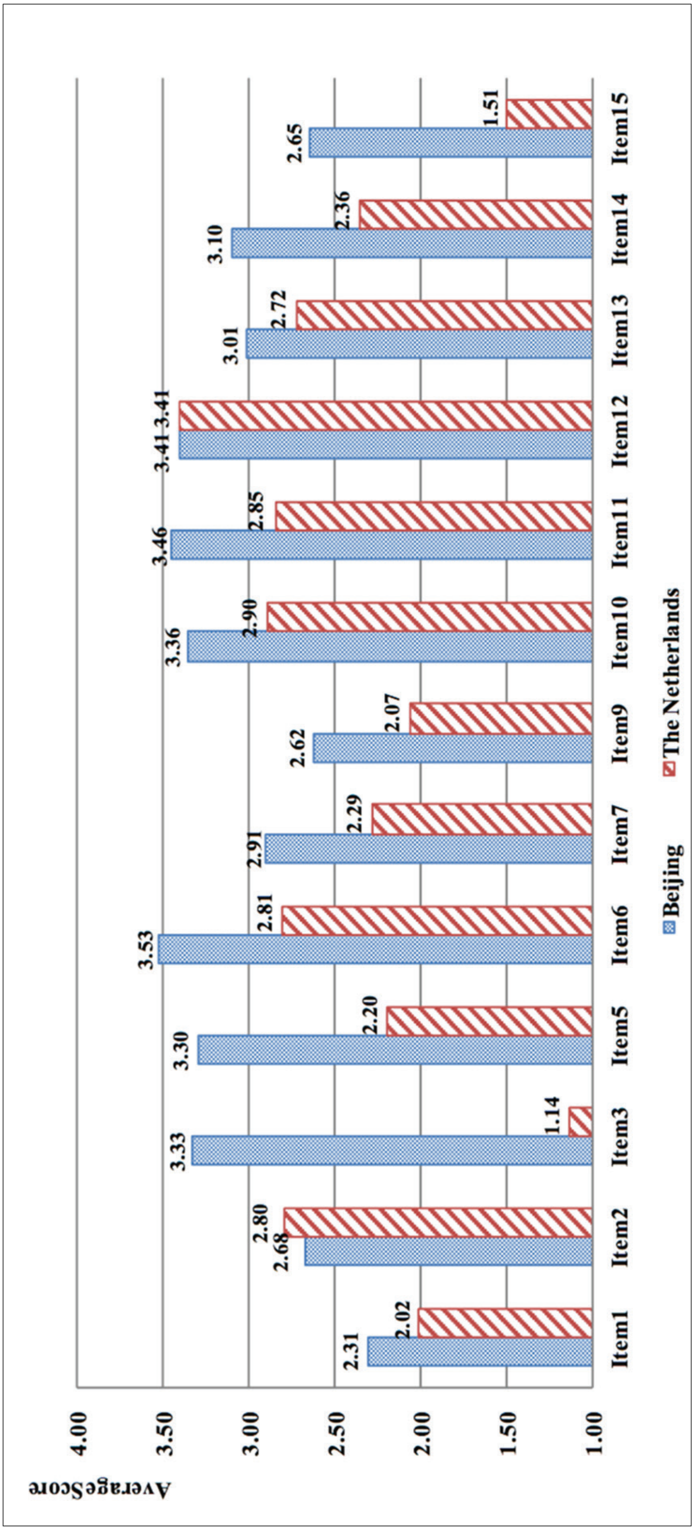


Fig. 2.2 Average scores of both samples on IBL experience items

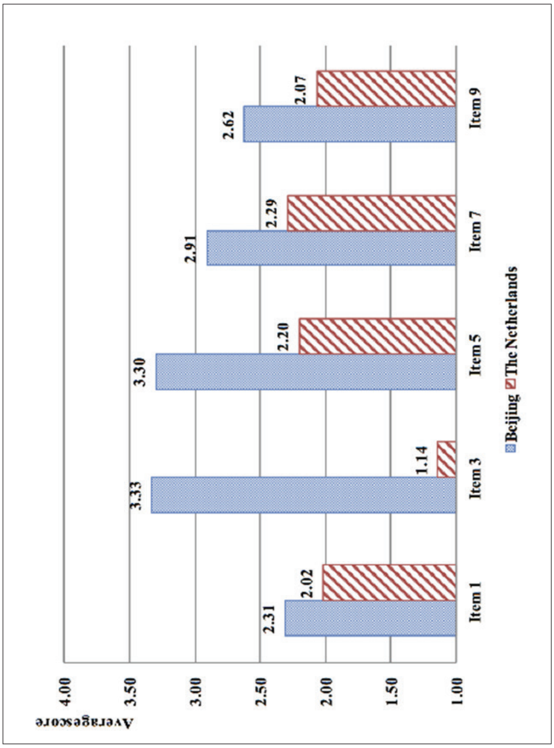


Fig. 2.3 Average scores on five IBL experience items

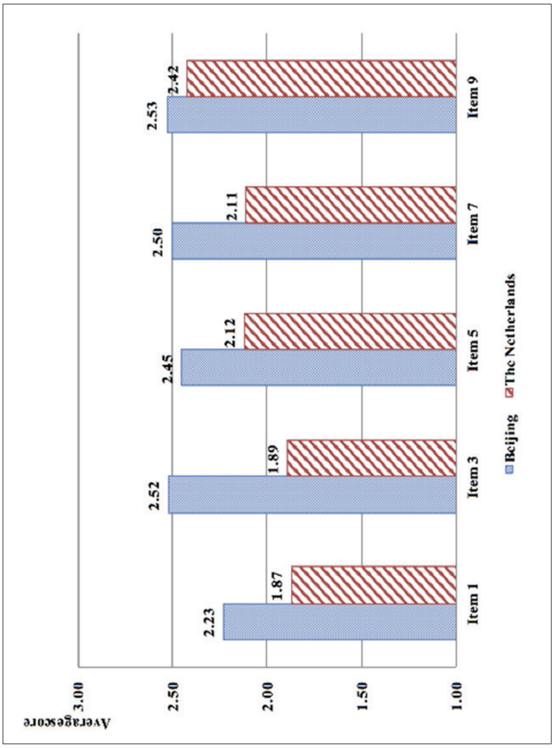


Fig. 2.4 Average scores on IBL preference items

4.4 Connecting Particular Features of IBL Practices in Both Samples with Stereotypes about the Overarching Teaching Culture

In the Beijing sample, students reported less experience with discussing mathematical problems than our Dutch students, and low-achievers significantly reported less IBL preference than high-achievers. These results are in line with the whole-class instruction and externally motivated aspects of the perceived teaching culture in East Asia (Leung, 2001). This might explain our findings that low- and high-achievers do not get much opportunity to differentiate their involvement in IBL activities, while high-achievers recognize the benefits of IBL on solving complex mathematical problems in fiercely competitive examinations. In addition, students in the Beijing sample experienced much on item 4 ($M = 3.59$, the teacher shows how problems should be solved) and it supports the teacher-centered aspect. However, they reported experiencing IBL activities in most mathematics lessons, which is not in line with the teacher-centered and rote learning aspects of perceived teaching culture in East Asia.

Low-achievers in the Dutch sample significantly reported less IBL experience than high-achievers, which is in line with the individualized learning aspect of perceived Western teaching culture. However, students in the Dutch sample reported experiencing IBL activities in only some mathematics lessons, and they experienced less on posing questions to tackle than on other IBL activities. In addition, they experienced much on item 8 ($M = 3.22$, solve problems by following example solution procedures). These findings do not match the student-centered and process-oriented aspects of perceived Western teaching culture.

5. Discussion

Our findings show that the Beijing sample reported students experienced IBL activities in most mathematics lessons, while the Dutch sample of students reported them in some lessons, and both preferred the same amount of IBL activities as they experienced. Students' report in both samples show similar patterns on certain activities, sharing the four most frequent and three of the four least frequent IBL experience. Particular features also exist for both samples, in that the Beijing sample experienced less on discussing mathematical problems while the Dutch sample experienced little on posing questions to tackle. Parts of the results are not in line with stereotypes about the teacher-centered and rote learning aspects of the perceived East Asian teaching culture, and the student-centered and process-oriented aspects of the perceived Western teaching culture. The study also suggests a positive correlation between IBL experience and IBL preference of each class.

Results of this study do not confirm expectations that could be based on stereotypes about the two teaching cultures. Students in the Dutch sample did not report much experience and preference related to IBL, while students in the Beijing sample did. The IBL practices reported by students in the two samples share quite a lot, and particular IBL-related features in each sample cannot be fully explained by stereotypes about the two teaching cultures.

Findings above are based on the samples in this study. We are aware that the study took the two samples as examples instead of representatives for the two teaching cultures, and we adopted convenient sampling, thus the results cannot be generalized to broader contexts. Most Dutch teachers in this study had relations with universities and research institutes, and a bias existed in the percentage of VWO classes that participated in the survey, which possibly led to more IBL experience reported in the Dutch sample than that in the Dutch situation, while the pattern of this result in comparison with the Beijing sample is not impacted. We also focused on reports of students without interviewing them or observing the actual practice in mathematics lessons. Moreover, we asked about the frequency of activities related to IBL in mathematics lessons, but did not evaluate the level or quality of IBL in these activities.

Nevertheless, the findings of this study challenge stereotypes about teaching cultures in East Asia and the West, especially for the dimension of so-called teacher-centered versus student-centered approaches in mathematics education (Cai & Wang, 2010). Results support the argument from previous research that a label like "teacher-centered" does not accurately reflect East Asian classrooms, and Chinese mathematics teachers may have their own practices of student-centeredness through a framed exploratory experience (Huang, 2002; Leung, 2005; Mok, 2006), they involved students to think through questioning and variation (Gu et al., 2004). The PRIMAS survey showed that the lessons of Dutch mathematics teachers could also be considered as teacher-centered (Engeln et al., 2013; PRIMAS, 2013).

In addition, those particular IBL-related features which are not in line with stereotypes about the two teaching cultures may be explained by factors within specific context of Beijing and the Netherlands. Chinese education seemed to have borrowed some ideas, concepts, and practices from the West (Liu & Feng, 2015; Tan, 2015). The revised mathematics curriculum standard encouraged teachers to organize inquiry in lessons (Wang et al., 2018). Students in the Beijing sample may experience more classroom activities with elements of IBL than in the past, although these activities might be closer to structured or guided inquiry in the inquiry continuum. Dutch mathematics teaching is considered to have a textbook-oriented culture, i.e., teachers seem to spend much time reviewing textbook problems, and choices for learning content and lesson design are highly textbook dependent (Van den Heuvel-Panhuizen & Wijers, 2005). Limited by tasks and solutions from textbooks, students in the Dutch sample may be not used to posing questions by themselves and requests for more attention for basic knowledge and skills in algebra (Schoenfeld, 2014) might also have an impact on their IBL experience.

The findings of this study are in line with studies showing that classroom practices between the two groups of teaching cultures could also share some elements (Hiebert et al., 2003; OECD, 2014) and with studies eliciting differences within a teaching culture that are ignored in such comparative studies (Clarke & Xu, 2008; Shimizu & Williams, 2013). Stereotypes about the two groups of teaching cultures need to be treated carefully.

The findings seem to match the PISA 2015 results that the two samples share a lot reported IBL practices. A surprising difference is that the Beijing sample reported IBL experience in most mathematics lessons while the Dutch sample only in some lessons.

This study also suggests a correlation between students' IBL experience and IBL preference of each class. It seems that students with more IBL experience are likely to show a higher preference, or that, when students prefer more IBL activities, teachers will adjust their teaching to include more IBL activities.

An implication for practice is that, when taking a students' perspective into account, providing complex mathematical problems, organizing group work, and encouraging students to have an influence on the lesson have potential for implementing IBL in mathematics. Mathematics teachers in Beijing need to provide more opportunities for students to discuss mathematical problems and to participate in IBL activities at their own pace, while Dutch mathematics teachers need to encourage students to pose questions to tackle. Further research can test the direction of the potential correlation between IBL experience and IBL preference of each class, and investigate why the correlation in Beijing is not present in the Netherlands.

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Chapter 3

Beliefs and Practices Related to Inquiry-based Learning: Chinese and Dutch Lower-secondary Mathematics Teachers' Perspectives

This chapter is based on:

Huang, L., Doorman, L.M. & van Joolingen, W.R. Beliefs and practices related to inquiry-based learning: Chinese and Dutch lower-secondary mathematics teachers' perspectives. (Manuscript submitted for publication)

Huang, L., Doorman, L.M. & van Joolingen, W.R. (2019). Chinese and Dutch mathematics teachers' beliefs about inquiry-based learning. In U.T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the eleventh congress of the European society for research in mathematics education (CERME11)* (pp. 3913–3920). Utrecht University and ERME. Retrieved from https://hal.archives-ouvertes.fr/hal-02430470

Abstract The understanding and implementing of inquiry-based learning (IBL) might be impacted by teaching cultures, which seem very different in East Asia and the West, and have been identified with stereotypes in literature. We took Beijing and the Netherlands as examples of these two teaching cultures, and investigated the beliefs and practices related to IBL of 30 Beijing and 19 Dutch lower-secondary mathematics teachers through semi-structured interviews. Results show the two groups of teachers mentioned many shared IBL beliefs and IBL practices; they both consistently emphasized students taking responsibility by themselves, teachers providing support and student communication. Compared with the Beijing group, the Dutch group did not indicate a more frequent use of IBL, but seemed to describe a lower level of teacher support as for their beliefs and practices. Some particular IBL beliefs and IBL practices mentioned by each group do not match the stereotypes about each teaching culture. Most expectations based on the stereotypes are not confirmed in this study.

Keywords Comparative study; Inquiry-based learning; Mathematics education; Teacher belief; Teaching practice

1. Introduction

As an intentional student-centered pedagogy, inquiry-based learning (IBL) encourages students to explore by themselves before possible formal explanations by the teacher, and to actively take more responsibility in the learning process through questioning, hypothesizing, designing, investigating, analyzing and reflecting, which also involves collaboration and communication (Marshall et al., 2017; Swan et al., 2013). As teaching is considered to be a cultural activity (Stigler & Hiebert, 1998), and the beliefs and practices of teachers are related to the cultural contexts that they are embedded (Correa et al., 2008; Xenofontos & Andrews, 2012), the understanding and implementing of IBL as a teaching approach may be impacted by teaching cultures. Comparing IBL beliefs and IBL practices in contexts of different teaching cultures can reveal particular features that match each teaching culture, as well as shared features beyond cultural boundaries. Looking into these relationships helps to better understand the implementation of IBL in mathematics. Teaching cultures in East Asia and the West are recognized to be remarkably different (Cai & Wang, 2010). In a previous study (Huang et al., 2021), we took Beijing and the Netherlands as examples of these two teaching cultures, and investigated IBL practices in mathematics from a students' perspective. Students in the Dutch sample did not report much IBL experience while students in the Beijing sample did, and their reports showed similar patterns on certain activities; the results challenge expectations based on the stereotypes about the two teaching cultures. In this study, we continued to look into the teachers of these students and explored the IBL beliefs and IBL practices of teachers in the Beijing sample and the Dutch sample. The research questions are: 1) What do lower-secondary mathematics teachers in Beijing and the Netherlands highlight when they describe their IBL beliefs and IBL practices? 2) In what ways are the IBL beliefs and IBL practices mentioned in the two areas in line with expectations based on the stereotypes about the two teaching cultures?

Presenting the beliefs and practices from a teacher's perspective, and connecting IBL with the cultural context that might shape its construction, we expect the analysis to provide in-depth insights into the current situation of IBL in mathematics, and to be a basis for further research on mathematics teachers' professional development of IBL.

2. Background: IBL Beliefs and IBL Practices Related to Cultural Contexts

Inquiry-based learning (IBL) approaches are characterized by student-centered learning activities in which students are invited to work in similar ways to how mathematicians and scientists work (Artigue & Blomhøj, 2013; Van Joolingen et al., 2007) indicating a major educational trend. We go back to the origin of inquiry as a pedagogical concept in the work of Dewey (e.g., 1916, 1938). These approaches emphasize learning by doing and discovering scientific relationships while performing experiments in contrast to following cookbook recipes for experiments. Students are invited to create their own scientifically oriented questions, to think of experiments for investigating these questions, to think of ways to systematically collect and analyze data. However, the degree of openness in the tasks for students and the responsibility given to them in IBL practices is quite diverse (Barrow, 2006), and the implementations in daily practice are varied (Furtak et al., 2012). A distinction can be recognized between structured inquiry, guided inquiry and open inquiry, which differ in the degree that students can explore and take responsibility by themselves, and the degree of teacher support during the process (Bruder & Prescott, 2013).

The views and uses of IBL also depend on the subject. IBL emerged in science education with experiments and empirical evidence playing a prominent role. The conceptualization of IBL in mathematics is less obvious, since mathematics is usually considered as deductive, less related to empirical findings and more axiomatically oriented on proofs and abstract procedures (Engeln et al., 2013; Maass et al., 2017). Inquiry processes such as questioning and experimenting systematically in mathematics involve activities such as modelling, mathematizing, reasoning, problem solving and proving (Artigue & Blomhøj, 2013). Similar to IBL in science education, students can be invited to take responsibility in these inquiry processes in mathematical practices. Nevertheless, a large part of existing research focused on implementing IBL in science, and mathematics teachers were found to report less IBL practices than science teachers (Engeln et al., 2013). A better understanding of teachers' IBL practices and views on IBL in mathematics needs attention.

Following insights from cognitive theories, teachers' beliefs have been studied in educational research (Civitillo et al., 2018). Beliefs are taken as a filter to interpret teachers' experience and as a normative framework to guide their intention and action in teaching practice (Fives & Buehl, 2012), and thus have an influence on student learning (Correa et al., 2008). Some research has focused on teachers' underlying beliefs, such as beliefs about knowledge, teaching, learning, teachers, students and these aspects in regard to the subject (Xie & Cai, 2021). These underlying beliefs are considered to have an influence on teachers' understanding and implementing of IBL in their daily practice (Engeln et al., 2013), and need to be taken into account in professional development projects focusing on IBL (Voet & De Wever, 2019). If teachers take a subject as a set of facts and concepts, or believe students should be at a certain level of maturity or ability, they may not adopt a more student-centered pedagogy (Wallace & Kang, 2004). However, there is still a lack of research on teachers' beliefs specifically about IBL and how this connects to their perceived and actual classroom teaching practice, especially in mathematics education (Maass et al., 2017). To enrich this branch of literature, this study focuses on mathematics teachers' beliefs about IBL and their reported use of IBL.

IBL tends to be challenging for teachers and not yet a routine in daily teaching (Dobber et al., 2017; McNew-Birren & van den Kieboom, 2017), and decisions about and ways to use IBL may be shaped by teachers' beliefs about it (Song & Looi, 2012; Wallace & Kang, 2004). Teachers may have difficulties in getting a complete understanding of IBL approach with varied interpretations. Their beliefs about the detailed content of IBL were found to be diverse (Chan, 2010). Teachers' beliefs about IBL tend to be consistent with their practices in some studies, for example, the beliefs about the effectiveness of IBL were found positive with teachers' reported use of IBL (Wilkins, 2008), and teachers who considered their role in inquiry lessons as facilitators adopted approaches closer to open inquiry, while teachers who considered themselves as 'shepherds' adopted approaches closer to structured inquiry (Correia & Harrison, 2020). However, some studies found an inconsistency (Engeln et al., 2013). Mathematics teachers showing a positive attitude towards IBL may have expository-oriented practices. There are also studies that pointed out a more complicated impact than a linear connection (Chan, 2010; Correa et al., 2008; Xenofontos & Andrews, 2012). A deep investigation of teachers' beliefs and practices related to IBL makes sense.

Teachers' beliefs and practices are related to their cultural contexts (Correa et al., 2008; Xenofontos & Andrews, 2012). For example, Cai and Wang (2010) found differences between Chinese and American mathematics teachers regarding effective mathematics teaching. The Chinese teachers of their study emphasized the importance of abstract reasoning after using concrete examples, while the American teachers described the use of concrete real-life examples to encourage students' understanding of mathematics. The Chinese group believed memorization can come before understanding, whereas the American group provided attention for memorization only after students' understanding of procedures. These results were traced back to the cultural values raised by Confucius and Socrates. In that study, the two countries were taken as representatives of the East Asian and Western cultures.

Researchers recognized distinctive differences in classroom teaching in East Asia and the West, and they connected these differences to values and beliefs in the two cultures (Cai & Wang, 2010). Identification of teaching cultures in East Asia and the West has resulted in stereotypes with features often presented as contrasting dichotomies, which make the comparison feasible but ignore relative positions of the two cultures on a continuum (Leung, 2001) and might be at the risk of oversimplification (Clarke, 2006). These stereotypes include several characteristics that might be related to the interpretation and implementation of IBL.

According to these stereotypes, East Asia tends to value learning content and related skills (Leung, 2001) and to emphasize the structure of in-depth knowledge (Bryan et al., 2007; Norton & Zhang, 2018), while the West tends to value learning process (Leung, 2001) and to emphasize practical knowledge connected to real life (Bryan et al., 2007; Norton & Zhang, 2018). East Asia usually involves whole-class activities in lessons, while the West usually conducts individualized learning and organizes group work (Leung, 2001). East Asia highly emphasizes the role of teachers and adopts well-organized directive instruction to deliver knowledge (Bryan et al., 2007; Leung, 2001), while the West highly emphasizes the role of students and believes they can construct knowledge by themselves (Bryan et al., 2007; Liu & Feng, 2015). East Asia usually makes students learn by memorizing and doing exercises repetitively (Liu & Feng, 2015; Tan, 2015), while the West usually provides situations and activities for students to understand first (Leung, 2001). Finally, East Asia tends to value the important role of external factors such as examinations to motivate students to learn, while the West tends to value internal factors such as interests in students' learning (Leung, 2001, 2014).

These stereotypes might apply to mathematics education. However, some studies also found the presence of variations within teaching cultures (Andrews, 2016; Clarke et al., 2010; Clarke & Xu, 2008; Kim, 2018; Shimizu & Williams, 2013) and similarities between different teaching cultures like shared basic ingredients of mathematics lessons (Hiebert et al., 2003). Moreover, the large-scale Teaching and Learning International Survey (TALIS) showed similar beliefs between Chinese (Shanghai) and Dutch teachers with two Likert items in the questionnaire. Almost all participants agreed their role as a teacher is to facilitate students' own inquiry (96% for Shanghai[1] and 98% for Dutch teachers), and students should be allowed

1 The two values of results for Shanghai teachers were calculated based on TALIS 2013 database (http://www.oecd.org/education/talis/talis-2013-data.htm).

to think of solutions to practical problems themselves before the teacher shows them (99% for Shanghai and 96% for Dutch teachers) (OECD, 2014). It is necessary to further look into beliefs and practices related to IBL to see to what extent teachers from different cultural contexts might highlight different aspects of IBL when being interviewed about their views and uses of IBL with tasks that have potential for promoting IBL.

We took Beijing and the Netherlands as examples of the two teaching cultures. The two areas are part of and share characteristics of their overarching teaching culture. According to the existing stereotypes above, the context of Western teaching culture would tend to be more supportive for the IBL approach. We expect that teachers in the Dutch sample would describe more aspects related to a lower level of teacher support (with students taking more responsibility by themselves), and a more frequent use of IBL, while teachers in the Beijing sample would not. We also expect that not many shared IBL beliefs and IBL practices would be expressed, and the particular beliefs and practices mentioned in each area would match the stereotypes about each teaching culture.

3. Method

This study was based on a teachers' perspective. The challenge was to create an environment for teachers to talk about their views freely. Often, Likert scale surveys are used to measure teachers' beliefs (e.g., Lotter et al., 2018). However, this method has its limitations for involving contextual factors and getting a deep understanding of reasons for or against implementing IBL (Safrudiannur & Rott, 2020; Xenofontos, 2018). We chose to conduct semi-structured interviews that help to better capture, probe and make sense of the beliefs and experiences of participants (Luft & Roehrig, 2007; Safrudiannur & Rott, 2020), especially for teachers from different cultural contexts (Cai & Wang, 2010). Inspired by research with tasks to elicit and capture teachers' beliefs (Ambrose et al., 2004), we included two tasks with potential for IBL in the interviews, and teachers were asked to express how they perceived them as instructional tasks and how they would use them in lessons. The study is qualitative in nature, but also has a quantitative component where we calculated and presented high-ranking statements which most represent the IBL beliefs and IBL practices mentioned by teachers.

3.1 Participants

We interviewed a total of 49 lower-secondary mathematics teachers in Beijing and the Netherlands. The information of participants can be seen in Table 3.1.

Table 3.1 Information of participants in the study

	Number of teachers	Number of schools	Gender (F, M)	Average age	Average years of experience
Beijing group	30	15	(93%, 7%)	38	15
Dutch group	19	13	(47%, 53%)	42	11

The two areas have similar populations (21.7 million in Beijing[2] and 17.1 million in the Netherlands[3] in 2017) and both are well-developed in aspects such as urbanization and education. Participants were contacted mainly through an interpersonal network. In Beijing, we contacted school principals or a few local administrations first, and some mathematics teachers directly, while in the Netherlands we invited individual teachers with an interest. As there may be differences between schools in urban and suburban districts in Beijing, we ensured a balanced selection of eight urban schools and seven suburban schools, while Dutch schools in different districts are quite similar.

3.2 Instrument

We constructed an interview outline (shown in Table 3.2) starting with a general question "What is your understanding of IBL," followed by two example tasks with potential for IBL from PRIMAS project[4] (PRIMAS, 2013). Participants are not provided with a pre-set definition of IBL, and the two tasks are not defined as IBL tasks, but they provide contexts and serve as a common ground to provoke the discussion of IBL about the prerequisites, activities and outcomes. One of the example tasks can be seen in Fig. 3.1.

The next part further asks questions about practices (frequency of using IBL[5], a recent IBL lesson, and the role as teacher in the lesson) and beliefs (attitude, reasons for, difficulties, strategies) related to IBL. These topics were inspired by information from the PRIMAS project (PRIMAS, 2013).

2 Source: http://nj.tjj.beijing.gov.cn/nj/main/2019-tjnj/zk/indexch.htm.

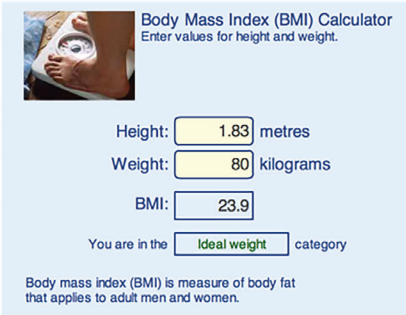
3 Source: https://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=demo_pjan&lang=en.

4 The PRIMAS project: Promoting inquiry-based learning (IBL) in mathematics and science education across Europe. Example task one was taken from the handout of Module 3 about learning concepts, and example task two from Module 2 about unstructured problems. The materials can be viewed through the link, https://primas-project.eu/modules/modules-english/.

5 For participants that report they never implemented IBL in mathematics lessons, we designed different subsequent questions about whether and how they would use IBL, and their role in an IBL lesson. The last part includes questions about reasons, difficulties and attitude related to IBL.

Calculating Body Mass Index

This calculator is used to help adults find out if they are overweight.



1. Fix the height at 2 metres - a very tall person!
Complete the table below and draw a graph to show your results.

Weight (kg)	60	70	80	90	100	110	120	130	140
BMI									

(a) What is the largest BMI for which someone is underweight?
(b) What is the smallest BMI for which someone is overweight?
(c) When you double the weight, what happens to the BMI?
(d) Can you find a rule for calculating BMI from the weight?

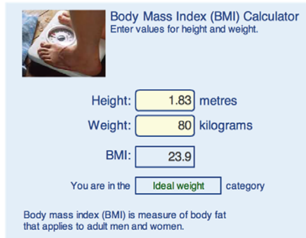
2. Fix the weight at 80 kilograms and try varying the height.

(a) When you double the height, what happens to the BMI?
(b) Can you find a rule for calculating BMI from the height?
(c) Draw a graph to show the relationship between the height and the BMI.

Calculating Body Mass Index

This calculator shown is used on websites to help an adult decide if he or she is overweight.
What values of the BMI indicate whether an adult is underweight, overweight, obese, or very obese?

Investigate how the calculator works out the BMI from the height and weight.



Note for pupils: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

Fig. 3.1 One of the two example tasks used in the interviews

The interview outline and example tasks were translated from English to Chinese, and the translation was ensured through peer check by researchers and postgraduates. Then the instruments were piloted with two Dutch teachers and two Beijing teachers to make sure they were clear enough and led to information we expected to collect.

3.3 Data Collection and Analysis

We conducted interviews in the Netherlands from April to June of 2017, and in Beijing from October to November of the same year. English was used for Dutch teachers[6] and Chinese for Beijing teachers. Each teacher was interviewed for around 40 minutes.

All the interview recordings were turned into transcripts, imported into NVivo 11 and divided into sets of fragments[7]. Each fragment represents an issue related to IBL beliefs or IBL practices. A mixture of theory driven and data driven was used.

Fragments were first marked with main codes based on questions from the interview outline (see Table 3.2). Most sub codes with example quotations were developed from participants' responses to create a more nuanced understanding of the main codes and to help organize information. The sub codes of "Students' responsibility" and "Teachers' responsibility" came from literature (Bruder & Prescott, 2013). The final coding scheme resulted from several rounds of coding.

When a fragment was related to more than one main code, it was assigned to multiple codes. For example, in *"All the questions, actually they really have to do things to find the answer, I think that is the basic thing of inquiry-based learning"*, the teacher considered students "do things to find the answer" to be what makes it IBL, and he took students "do things to find the answer" as a kind of IBL activities, he also expressed views about IBL tasks that they offer opportunities for students to "do things". This fragment was coded into main codes "general views", "prerequisites" and "activities".

After organizing all fragments with main codes and possible sub codes, we further grouped fragments with similar meanings, and extracted representative statements from them. To present an overview of what teachers highlighted in interviews, all the statements were ranked within each main code (except "Attitude" and "Frequency") based on the number of teachers that mentioned each statement,

6 The first author conducted most of the interviews with Dutch teachers, but she does not speak Dutch, while Dutch teachers have a proficiency in English.

7 For Beijing data, we coded and analyzed the original transcripts in Chinese. Only high-ranking statements and quotations used in results were translated into English.

Table 3.2 Structure of the interview outline and the corresponding main codes

Section	Questions		Main codes
Orientation	• What is your understanding of IBL		• General views
Discussion based on two tasks	Task one	Task two	
	• What do you think of the task? Can it be used in an IBL lesson?	• What do you think of both versions of the task? Which one would you prefer, Why? Do both versions represent IBL?	• Prerequisites
	• What would students learn from this task?	• Do they have the same learning goal for students to achieve?	• Outcome
	• How will you use it? What kind of support will you offer to the students?	• How will you use it (or them)? What kind of support will you offer to the students?	• Activities
Further questions about IBL	• How often do you implement IBL in your lessons?		• Frequency
	For teachers that said they implemented IBL	For teachers that said they did not implement IBL	
	• Could you describe an IBL lesson that you implemented recently?		• IBL lesson practices
	• What do you think about your role as teacher in that lesson?		• Teacher role in IBL practices
	• Personally, are you in favor of using IBL frequently in lessons?	• Would you consider implementing IBL? Personally, are you in favor of using IBL frequently in lessons?	• Attitudes
		• How would you implement IBL in your lesson? What do you think about the role of the teacher in an IBL lesson?	• Activities
	• What do you see as reasons for using IBL?	• What do you see as reasons for using IBL?	• Reasons For
	• What are the main difficulties?	• What about main difficulties?	• Difficulties
	• If your colleagues want to implement IBL, what tips would you give?		• Strategies

for Beijing teachers and Dutch teachers respectively. The top four high-ranking statements were kept, while those mentioned by less than 30% of the teachers within each group (i.e., 9 teachers in the Beijing group and 6 teachers in the Dutch group) were removed.

To ensure the reliability of results, we performed an audit trail, which allows external evaluators to check the documentation about the research process and choices, and verify the findings (Lub, 2015). The first author kept detailed record of the way she coded transcripts, grouped fragments, extracted statements and ranked statements. An external researcher, who is Chinese and was a visiting researcher in mathematics education at a Dutch university, randomly selected two main codes- “Reasons For” and “Difficulties”, then checked the method through the trail, for Beijing data and Dutch data respectively. He traced the detailed process and evaluated the choices through a series of checking, which include whether all coded fragments fit the selected codes, whether there are better ways to arrange or merge the groups of fragments, whether each extracted statement fully presents what is originally expressed. Then he discussed possible questions with the first author, mainly about arranging specific fragments (e.g., why a fragment has been put into that group, or would it be better to be also into another group). Based on the external researcher’s advice during discussion, we made some adjustments, such as rephrasing “IBL develops mathematical thinking” to a more accurate statement, “Students learn the way to solve mathematical problems (in IBL),” which better reflected what teachers expressed. The external researcher agreed the coding process as a reliable way to condense interview data into lists of statements.

For main codes “Attitude” and “Frequency” with results in a few mutually exclusive categories, we did not extract statements, but presented the percentage of teachers that expressed each category. Results of “Attitude” ended up with three categories: “Positive”, “Conditionally positive” and “Not positive”. “Conditionally positive” refers to situations that a teacher is positive towards IBL while willing to use it under certain conditions or emphasizing the difficulties. Options were provided with “Frequency” for participants to report their usage of IBL during interviews. The results ended up with four categories: “Frequently used” (Every lesson or weekly), “The use depended” (on the learning content), “Sometimes used” (Monthly or occasionally), or “Never used”.

To better understand these percentages of categories and high-ranking statements, we added quotations with reference to the participant, for example, BJ18 means Beijing teacher 18, and NL13 means Dutch teacher 13. To connect the IBL beliefs with reported IBL practices, we compared high-ranking statements related to beliefs and practices within each group of teachers, connected the attitude of each teacher with their frequency of using IBL, and analyzed possible misunderstandings that might hinder the use of IBL. We also analyzed the answers of one Beijing and one Dutch teacher who seem typical in the way that they targeted the more high-ranking statements within each group of teachers, and they had similar years of teaching experience in mathematics. Moreover, we compared the IBL beliefs and IBL practices mentioned by the two groups of teachers with expectations based on the stereotypes about the two teaching cultures.

4. Results

4.1 Beijing and Dutch Teachers' Beliefs about IBL and Reported Practices of IBL

4.1.1 Teachers' Attitude towards and Frequency of Using IBL

As is shown in Table 3.3, the Beijing group showed a relatively more positive attitude towards IBL than the Dutch group. 15 (50%) of the 30 Beijing teachers and 4 (21%) of the 19 Dutch teachers were positive towards IBL. As for teachers' report on "Frequency" (see Table 3.3), our Dutch teachers did not indicate a more frequent use of IBL in mathematics lessons than our Beijing teachers. 15 (50%) of the 30 Beijing teachers and 7 (37%) of the 19 Dutch teachers expressed a frequent use (every lesson or weekly), and 4 (21%) Dutch teachers responded they never used IBL in mathematics.

Table 3.3 Results of teachers' report on "Attitude" and "Frequency"

	Attitude			Frequency			
	Positive	Conditionally positive	Not positive	Frequently used	The use depended	Sometimes used	Never used
Beijing group	15 (50%)	15 (50%)	0	15 (50%)	8 (27%)	7 (23%)	0
Dutch group	4 (21%)	12 (63%)	3 (16%)	7 (37%)	0	8 (42%)	4 (21%)

4.1.2 High-ranking Statements of Teachers' IBL Beliefs

Table 3.4 shows what teachers highlighted when describing their IBL beliefs on seven important aspects through the top four high-ranking statements within each aspect that also represent at least 30% of the teachers within each group. Statements in bold are shared between the two groups. For example, teachers in both groups (17 of the 30 Beijing teachers and 12 of the 19 Dutch teachers) talked about "students explore, think and find a way to the problem by themselves".

Table 3.4 High-ranking statements of IBL beliefs that teachers mentioned

Aspect	BJ statements	n	NL statements	n
General Views	• **Students explore, think and find a way to the problem by themselves**	17	• **Students explore, think and find a way to the problem by themselves**	12
	• The teacher supports students through guidance including asking questions	15	• **IBL is one of the ways to teach and learn mathematics**	9
	• Students collaborate or communicate with peers	13	• IBL adds to only content-learning, it also benefits students in thinking, general skills or motivation	7
	• **IBL is one of the ways to teach and learn mathematics**	11		
Prerequisite	• **IBL tasks do not give away much information or provide guidance through small steps**	27	• **IBL tasks do not give away much information or provide guidance through small steps**	18
	• The teacher should make a good design of IBL activities before the lesson	24	• IBL is more suitable for motivated students	14
	• Open problems with different solutions can be used as IBL tasks	22	• It requires more time to prepare or to do IBL	11
	• The level of students' intelligence or performance should be considered before IBL activities	21	• IBL tasks to be used in the lesson should not be too open (clear questions or some structures can be provided by the tasks)	10
Activities	• **Students explore, think and find a way to the problem by themselves**	30	• **Students explore, think and find a way to the problem by themselves**	19
	• **Students collaborate and communicate with peers**	29	• **Students collaborate and communicate with peers**	19
	• **Students express their own ideas to the whole class**	29	• **The teacher supports students** through providing hints or asking questions	19
	• **The teacher supports students** through providing guidance	28	• **Students express their own ideas to the whole class**	14
Outcomes	• **Students learn the way to solve mathematical problems**	29	• **Students learn the way to solve mathematical problems**	17
	• **Students understand the learning content better**	24	• Students develop general skills that can also be used outside school and in future life	16
	• **Students may not reach all the expected learning goals**	21	• **Students are motivated to think and explore**	13

Aspect	BJ statements	n	NL statements	n
	• **Students are motivated to think and explore**	20	• **Students understand the learning content better**	10
			• **Students may not reach all the expected learning goals**	10
Reasons For	• **IBL helps students to learn the way to solve mathematical problems**	19	• **IBL develops general skills that can also be used outside school and in future life**	11
	• **IBL leads to a better understanding of the learning content**	16	• **IBL motivates students to think and explore**	9
	• **IBL motivates students to think and explore**	14	• **IBL leads to a better understanding of the learning content**	8
	• **IBL develops general skills that can also be used outside school and in future life**	11	• **IBL helps students to learn the way to solve mathematical problems**	6
Difficulties	• **Context: lack of enough support for IBL from school culture**	16	• **Context: lack of enough support for IBL from school culture**	10
	• **Context: lack of time to prepare and do IBL**	13	• Task: lack of suitable tasks at hand for IBL	10
	• **Perceived IBL feature**: IBL is difficult to design and organize in lessons	12	• **Context: lack of time to prepare and do IBL**	8
	• Student: students lack motivation to do IBL	11	• **Perceived IBL feature**: IBL is less predictable and may not lead to good results	8
Strategies	• Design IBL activities carefully before the lesson	13	• **The teacher withdraws to give students space to explore**	6
	• Organize IBL activities properly during the lesson	10		
	• **The teacher withdraws to give students space to explore**	9		

Note. "BJ" is the abbreviation of Beijing, and "NL" of the Netherlands. "n" means the number of teachers who expressed this view during interviews. Statements in bold are shared between the two groups

The Beijing group and the Dutch group both emphasized students taking responsibility by themselves in their general views on IBL. They also both took IBL as a way to learn mathematics. Our Beijing teachers paid more attention to teacher support during the IBL process, and they connected IBL with student collaboration or communication. The Dutch teachers mentioned the added value of IBL approach to content-learning.

As for factors considered before implementing IBL, both groups regarded IBL tasks as without much information or intermediate steps. Teachers in the Beijing group thought open problems can be IBL tasks, while teachers in the Dutch group pointed out that tasks suitable for IBL activities cannot be too open. The Beijing group mentioned more about the consideration of students' cognition and the Dutch group more about students' motivation. In addition, our Beijing teachers emphasized the input of teachers with a good design of IBL activities and our Dutch teachers emphasized the input of context with more time for IBL.

When talking about activities related to IBL, almost all teachers paid attention to students taking responsibility by themselves, student collaboration and communication as well as the need for teacher support. The Beijing group often used "guidance" and the Dutch group often used "hints" to describe teacher support. Almost all Beijing teachers and most Dutch teachers mentioned students explaining ideas to the whole class.

As for outcomes of IBL, teachers in both groups highlighted its positive side in motivation and two sides in cognition (positive in learning the content and negative in not reaching all learning goals). Almost all teachers highly emphasized the benefits of IBL on getting acquainted with the process of solving mathematical problems. However, the Beijing teachers mainly considered the benefits of IBL within mathematics, while most Dutch teachers highlighted its benefits on general skills that are broader than the field of mathematics, such as creativity, critical thinking and communication skills.

Both groups took the four benefits that they mentioned in outcomes as reasons to implement IBL. However, the rankings of these benefits are opposite for the two groups. Our Beijing teachers paid relatively more attention to the solving process and learning content, and our Dutch teachers to general skills and motivation.

With respect to difficulties in using IBL, teachers in both groups included factors related to context, such as low priority for IBL in school culture and lack of time. NL10 (Dutch teacher 10) talked about *"But I also have to follow the curriculum, so I have to reach certain goals, they need to master certain knowledge and certain techniques"*. Both groups mentioned factors related to a perceived feature of IBL. The Beijing group considered IBL as difficult to design and organize, like what BJ20 (Beijing teacher 20) described, *"I don't know how to design it in a better way, how to involve all the children to play a role, how to make them benefit in this kind of training"*. The Dutch group focused on the less predictable feature of IBL, NL3 expressed although IBL can be a pleasant challenging mathematics experience for some students, teachers cannot control its risk to keep other students away from mathematics. The Dutch group also focused on the lack of suitable IBL tasks at hand and the Beijing group mentioned the lack of student motivation.

As for the strategies to implement IBL, both groups talked about the importance of providing space for students to explore mathematical problems. The Beijing teachers emphasized the efforts of teachers to design and organize IBL activities.

Table 3.5 High-ranking statements of IBL practices that teachers mentioned

Aspect	BJ statements	n	NL statements	n
IBL lesson practices	• **Students explore, think and find a way to the problem by themselves**	27	• **Students explore, think and find a way to the problem by themselves**	13
	• **The teacher uses open problems with different solutions**	25	• **The teacher uses open problems with different solutions**	13
	• **The teacher supports students** by providing guidance	25	• **The teacher supports students** by stimulating them to think (such as through questions or games)	9
	• **Students express their own ideas to the whole class**	24	• **Students express their own ideas** in groups or **to the whole class**	9
Teacher role in IBL practices	• **The teacher supports students** by providing guidance to help them proceed in the right direction	15	• **The teacher supports students when necessary** (such as when they get stuck)	9
	• **The teacher withdraws to give students space to explore, and supports students when necessary**	14	• **The teacher withdraws to give students space to explore**, instead of explaining much	9
	• The teacher is responsible for the learning content, he/she adjusts student explanation during the process and makes summaries at the end	9	• **The teacher supports students** by stimulating them to think (such as through questions or encouragement)	7
	• The teacher organizes IBL activities	9		

Note. "BJ" is the abbreviation of Beijing, and "NL" of the Netherlands. "n" means the number of teachers who expressed this view during interviews. Statements in bold are shared between the two groups

4.1.3 High-ranking Statements of Teachers' Reported IBL Practices

Table 3.5 shows what teachers highlighted when describing their IBL practices through the top four high-ranking statements within "IBL lesson practices" and "teacher role in IBL practices" that also represent at least 30% of the teachers within each group. Statements in bold are shared between the two groups.

As is shown in Table 3.5, when describing practices related to IBL (e.g., a specific recent lesson), both groups emphasized that they provided space for students to explore and take responsibility by themselves, used open problems, provided support to students and allowed students to express own ideas. The Dutch group also mentioned student communication in groups. Our Beijing teachers used "guidance" and our Dutch teachers used "stimulation" to describe their support in practice.

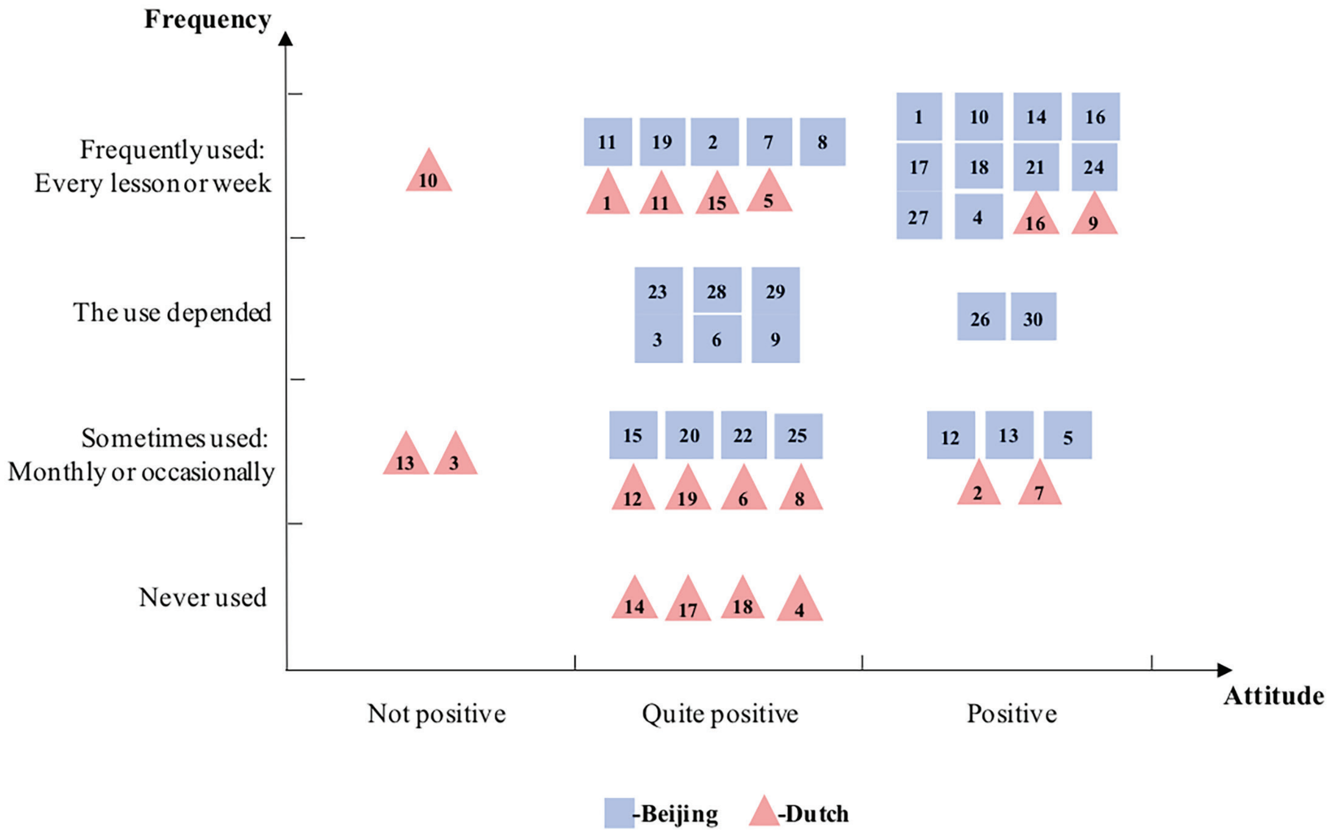
As for the role of teachers in an IBL lesson, teachers in both groups talked about providing support to students when necessary and providing space for students to take responsibility. The Beijing group talked more about guidance and the Dutch group talked more about stimulation to describe teacher support. Our Beijing teachers also paid attention to teachers' role for the learning content, such as what BJ6 mentioned: *"(I need to) summarize and promote at the end, (because) what students have expressed may be separate ideas. How can they combine these ideas and emphasize the mathematical knowledge that I expected to target or to review? This is beyond what they can achieve"*. The Beijing group mentioned organizing IBL activities as well.

4.1.4 Connecting IBL Beliefs with IBL Practices Mentioned by Teachers in Each Group

When considering high-ranking statements of IBL beliefs (in Table 3.4) and of reported IBL practices (in Table 3.5), we found teachers in both groups consistently mentioned students exploring and taking responsibility by themselves, teachers providing support and student communication. The Beijing group consistently paid attention to teacher support through guidance, student communication to the whole class, learning content and open problems as IBL tasks, and the Dutch group consistently mentioned both student communication to the whole class and in groups, which show possible match between IBL beliefs and reported IBL practices.

When considering individual teachers, we connected each teacher's response on the aspect of "Attitude" (overall tendency towards IBL) with his/her report on "Frequency" (of using IBL in mathematics lessons), and presented relative positions of all the 49 teachers that participated in this study in Fig. 3.2.

Part of the Beijing group expressed that their use of IBL depended on the learning content, and five of the teachers considered geometry as suitable to do IBL. Four Dutch teachers showed they were conditionally positive towards IBL while they never used it in mathematics lessons. Although NL13 and NL3 responded they used IBL every month, and NL10 even every lesson, they did not show positive attitudes



Note. The number in each square or triangle refers to the code number of a Beijing or Dutch participant, such as BJ18 or NL13

Fig. 3.2 Attitude towards IBL and frequency of using IBL reported by teachers towards IBL. NL10 and NL13 were neutral that they thought it necessary to do both IBL and non-IBL, and NL3 was hesitant towards IBL: *“it (IBL) would not be the main thing for me, it would be like once in a while, as a kind of a challenging task…I find aspects of it very nice and interesting, but as a whole I see a lot of problems.”*

In addition, as is shown in the responses of participants in this study, there were misunderstandings of IBL that possibly led to teachers’ perceived difficulties with IBL, and might hinder them from incorporating IBL into daily teaching. Some of our teachers took IBL as additional to the required learning content and learning goals, and felt a lack of support from school culture, time or tasks to do the “additional” IBL. They indicated that they only used IBL under certain conditions, such as when encountering suitable tasks for IBL or in lessons for demonstration. Also, some of our teachers only recognized IBL as a complete process or cycle to go through, and did not consider small activities with elements of inquiry in a part of the lesson also as IBL. In addition, teachers’ views varied about whether students can do IBL before they are fully prepared (in knowledge, skills, experience or motivation) for it. For example, NL18 thought IBL would not work without enough preparation and he chose to wait, *“you have to introduce it (IBL), you have to… I cannot do it right now, it would not work, but in the end, next year, it would be nice to play with it, to give them this kind of exercises”.* While BJ27 thought students could do a lower level of IBL at the starting stage and she let them do it: *“In the first semester of Grade 7, IBL was usually quite simple and low level. After students know different ways to solve problems, when he/she acquires a certain amount of methods, he/she can do IBL independently.”* Moreover, a few of our teachers ignored possible benefits of IBL and showed worries that students “gain nothing” when they cannot solve an IBL task.

4.1.5 Typical Cases of Teachers in the Two Groups

To better understand the results above, we present two case analysis of BJ18 and NL13, who seem typical for the two groups of participants in the way that they mentioned all the highest ranking statements within each aspect in Table 3.4 and Table 3.5. BJ18 was 30 years old with 7 years of teaching experience in mathematics, she taught grade 7 at an urban school in Beijing when being interviewed. NL13 was 39 years old with 5 years of teaching experience in mathematics, she taught grade 8 at VWO, HAVO and VMBO[8] level.

8 In the Netherlands, after primary school (grade 6) students choose one of three types of secondary education: pre-university education (VWO), senior general secondary education (HAVO) or pre-vocational secondary education (VMBO) (source: https://www.government.nl/topics/secondary-education).

As for BJ18, she took IBL as an essential way of teaching in mathematics, and considered the IBL approach to be better than its opposite, i.e., directly delivering knowledge to students. As she said, "*No matter in what type of lessons, no matter how you organize the lessons, you should use inquiry…If not in the way that students repeat and memorize what the teacher tells them, I think it is kind of IBL*". She considered her practice in line with this understanding and not telling students directly (about the conclusion or knowledge). She distinguished different levels of IBL that she recognized, small IBL activities possibly with teacher support as lower level, and complete IBL activities conducted more by students as higher level, which is partly reflected in the excerpt below.

> **Interviewer**: What do you think about your role as teacher in that lesson with elements of IBL? What is the function of the teacher?
>
> **BJ18**: Mainly to guide I think, maybe students play the key role, as what is emphasized nowadays…Although I tried to follow it in practice, sometimes, like what I mentioned just now, if the content is very difficult, the teacher can be quite worried because the time issue needs to be considered, and the teacher might intervene to some degree, actually it is not autonomous inquiry fully by students. For example, the teacher might guide students to observe through 'maybe look at the symmetry' or 'does it have a highest or lowest point', then it is semi-inquiry[9] instead of fully autonomous inquiry. I think it might be the case in usual lessons.

She mentioned small IBL activities that require students to observe, guess, compare or summarize, and gave an example that she asked students to look for patterns in a series of numbers by themselves and express their ideas to the whole class. She also recognized a higher level of IBL in well-designed lessons for demonstration with more student interactions like group work and presentations and emphasized the design for IBL lessons.

As for NL13, she considered IBL as students experiencing and finding out the theory by themselves (individually or in groups) instead of getting all the information from the teacher. She thought it necessary to do both non-IBL and IBL, while IBL was usually skipped because more time should be spent on "basic things" (see the excerpt below).

9 The original word that BJ18 used in Chinese is "半探究"; we translated it literally into "semi-inquiry" to try to capture the meaning she expressed.

> **NL13**: Because they have to choose the way by themselves, so that's why I think it's very good for IBL.
>
> **Interviewer**: In our current lessons or our textbooks, they do not have the opportunity to choose the way by themselves?
>
> **NL13**: Sometimes they are, but most of time it is extra, it is not in the normal things… And I think it's important to do things like this, but they also have to do the basic...We are three (hours a week for mathematics lessons), that's very short, so just basic. And IBL, it's mostly not the basic, that's why most of the time it is skipped.

She viewed IBL as mostly "not the basic" and "extra", and tasks for IBL as "more difficult," "not very standard," or "with different solutions." She implemented IBL once a month, mostly by using an IBL task from the textbook and doing more activities. She gave an example that she asked individual students to solve a textbook task which included a question that they had never done before. She emphasized supporting through questions before providing further help about the learning content. She also regarded whole-class discussion after group work important for students to communicate different solutions. In addition, she felt if an IBL task is too difficult, it would only benefit a small group of interested students, while *"the biggest largest group isn't learning anything…because they cannot solve it, they are stuck."* And she admitted liking structure in her lessons, while the IBL way is less predictable.

These two cases show that IBL can be interpreted very broad and be considered as essential in all mathematics lessons (BJ18). She took small IBL activities as a lower level of IBL and frequently conducted IBL in her daily practice, maybe with a certain amount of teacher support. In contrast, NL13 interpreted IBL as related to something extra, not basic and not normal. She only implemented IBL when encountering a textbook task that she considered suitable for IBL.

4.2 Connecting IBL Beliefs and IBL Practices Mentioned in the Two Areas with Expectations Based on the Stereotypes about the Two Teaching Cultures

As for the particular IBL beliefs and IBL practices identified in teachers' responses, some of them match the stereotypes about teaching cultures in East Asia and the West. The attention in the Beijing group on teachers' responsibility for learning content, teacher support through guidance, teacher efforts in designing and organizing IBL activities as well as student cognition as input matches the stereotypes that East Asia tends to emphasize learning content and the role of teachers in instruction (Bryan et al., 2007; Leung, 2001). The attention in the Dutch group on the added value of IBL to content-learning, benefits on general skills, student communication in groups, teacher support through hints/stimulation and student motivation matches the stereotypes that the West tends to emphasize learning process and practical knowledge, individualized learning, student-centered learning and internal motivations (Bryan et al., 2007; Leung, 2001; Liu & Feng, 2015; Norton & Zhang, 2018).

However, some of the particular IBL beliefs and IBL practices mentioned by the two groups of teachers do not match the stereotypes about each teaching culture. The attention in the Beijing group on open problems as IBL tasks, student collaboration, communication and motivation does not match the stereotypes that East Asia usually involves whole-class instruction, teacher-led directive instruction, rote learning and ignores students' internal motivations (Bryan et al., 2007; Leung, 2001, 2014; Liu & Feng, 2015; Tan, 2015). The attention in the Dutch group on more time as input, not too open tasks, difficulties related to IBL tasks and to the perceived less predictable feature of IBL does not match the stereotypes that the West usually focuses on student-centered learning (Bryan et al., 2007; Liu & Feng, 2015) and tends to be supportive for a teaching approach like IBL.

As is shown in the results above, the Dutch teachers did not describe a more frequent use of IBL than the Beijing teachers, and many shared IBL beliefs and IBL practices were expressed by the two groups of teachers. In addition, some particular beliefs and practices they mentioned do not match the stereotypes about each teaching culture. These do not confirm expectations based on the stereotypes. However, the Dutch teachers seem to mention a lower level of teacher support than the Beijing teachers, which tends to confirm a part of the expectations.

5. Conclusion

Teachers in the Beijing group and the Dutch group highlighted many shared IBL beliefs and IBL practices. Both groups consistently emphasized students taking responsibility by themselves, teacher support and student communication. They also both considered IBL as one of the ways of mathematics teaching and learning, mentioned IBL tasks as without much information or intermediate steps, paid attention to student collaboration, took the four positive outcomes of IBL as reasons to do it, listed factors related to context and a perceived feature of IBL as difficulties, and used open problems in their practices. Compared with the Beijing group, the Dutch group did not indicate a more frequent use of IBL in practices.

However, differences also existed. Our Beijing teachers paid particular attention to support through guidance, learning content, open problems as IBL tasks, teacher efforts in designing and organizing lessons as well as student collaboration, communication, cognition and motivation. Our Dutch teachers particularly highlighted support through hints/stimulation, added value especially on general skills, not too open problems as IBL tasks, more time in context, student communication in groups, student motivation as well as lack of suitable tasks and the less predictable feature of IBL.

Only a part of expectations based on the stereotypes is confirmed in that the Dutch group seems to describe a lower level of teacher support as for their beliefs and practices than the Beijing group. Most expectations are not confirmed. This study presents examples contradicting and challenging these stereotypes and is in line with studies showing that teachers' beliefs or classroom practices between countries or areas in the two groups of teaching cultures could also share some elements (Hiebert et al., 2003; OECD, 2014), and with studies eliciting variations within a teaching culture that are ignored in such comparative studies (Andrews, 2016; Clarke et al., 2010; Clarke & Xu, 2008; Kim, 2018; Shimizu & Williams, 2013).

The framework of stereotypes about teaching cultures makes it feasible to describe and compare in both cultures, although it has been pointed out that presenting features in the form of dichotomies ignores relative positions of the two cultures on a continuum (Leung, 2001), and it simplifies the variety of teaching present in each area. We encourage the development of more rich frameworks that treat dichotomies as complementary and interrelated instead of oppositional (Clarke, 2006).

The findings are based on two relatively small groups of participants from convenient sampling. It would go too far to generalize the findings to represent the whole of these two areas. Besides, the study was limited to reported data without observing actual lessons. Nevertheless, this study provides more in-depth insights into teachers' responses about how they understood IBL and used IBL in their lessons, and reveals the differences in viewpoints in terms of factors to consider before doing IBL, the way to organize lessons and support students in IBL, and difficulties during the process.

Particular IBL beliefs and reported IBL practices that do not match the stereotypes about each teaching culture might be explained by factors related to the specific context of each area. Chinese education has borrowed some ideas, concepts and practices from the West (Liu & Feng, 2015; Tan, 2015). The Chinese mathematics curriculum standard since 2001 required textbooks to provide space for students to investigate and communicate, and encouraged teachers to organize inquiry and collaboration in lessons (Wang et al., 2018). The attention in our Beijing group on open problems as IBL tasks, student collaboration, communication and motivation is in line with this trend. Dutch mathematics teaching is considered to have a textbook-oriented culture, and choices for learning content and lesson design are highly textbook dependent (Van den Heuvel-Panhuizen & Wijers, 2005). The attention in our Dutch group on not too open tasks, the mentioned difficulties related to IBL tasks and to the perceived less predictable feature of IBL might reflect this textbook-oriented culture.

The study found both consistencies and inconsistencies between teachers' beliefs and reported practices related to IBL, which shows the complexity of their relationship (Chan, 2010; Correa et al., 2008; Xenofontos & Andrews, 2012). The study also identified possible misunderstandings of IBL that might lead to teachers' perceived difficulties of IBL and hinder their practices. Reasons for not engaging with IBL in usual mathematics lessons can be that teachers (1) take IBL as something additional to the required learning content and learning goals, (2) think IBL should be a complete process/cycle to go through, (3) do not believe students can do IBL before they are fully prepared for it and (4) think students do not benefit at all if they cannot solve an IBL task. Further study can investigate actual practices about whether and how teachers incorporate IBL into ordinary mathematics lessons.

Teachers in the Beijing group mainly described their support to students through guidance and teachers in the Dutch group mainly described support through hints/stimulation, which seems to indicate different levels of teacher support. Connecting this finding to our previous study (Huang et al., 2021) from a student's perspective, a conjecture emerges that although Dutch students in previous samples did not report much IBL experience while Beijing students did, the IBL experience of Beijing students might be under a higher level of teacher support. It highly depends on the context in which the teacher makes choices and cannot be fully answered by data of this study, and observation about classroom practices will help to provide more insights. Further studies can investigate to testify this conjecture and look into the space students get to explore by themselves before possible teacher explanations in actual mathematics lessons.

Chapter 4

Opportunities for Inquiry-based Learning in Chinese and Dutch Lower-secondary School Mathematics Textbook Tasks

This chapter is based on:

Huang, L., Doorman, L.M., & van Joolingen, W.R. Opportunities for inquiry-based learning in Chinese and Dutch lower-secondary school mathematics textbook tasks. (Manuscript submitted for publication)

Huang, L., Doorman, L.M., & van Joolingen, W.R. *Opportunities for inquiry-based learning provided by Chinese and Dutch lower-secondary school mathematics textbook tasks*. (Paper presented at the 14th international congress on mathematical education [ICME 14])

Abstract The practices of mathematics teaching, including those related to inquiry-based learning (IBL), are in general highly impacted by textbooks. We investigated to what extent opportunities for IBL are provided by mathematics textbooks in Beijing and in the Netherlands, which were taken as examples of East Asia and the West. Using a framework to evaluate textbook tasks with respect to phases and levels of IBL, we coded 648 algebra and geometry tasks from three lower-secondary textbooks. Many shared IBL features between the two Beijing and one Dutch textbooks were identified. The results also show that the textbooks allow students to inquire into mathematizations and solution procedures. However, they rarely involve higher levels of IBL that challenge students to explore problem situations or design solution procedures themselves. The textbooks seem to define a learning trajectory that does not involve phases of questioning, hypothesizing, collaborating, communicating or reflecting. Some particular IBL features of the textbooks do not match the stereotypes about teaching cultures in East Asia and the West. The Dutch textbook involves relatively fewer opportunities to organize mathematically and to inquire into solution procedures than the Beijing textbooks. Most expectations based on the stereotypes are not confirmed in this study.

Keywords Mathematics education; Inquiry-based learning; Comparative study; Mathematics textbook; Textbook analysis; Lower-secondary education

1. Introduction

A common misconception about mathematics is that it is a purely deductive and well-defined discipline (Maass et al., 2017; Stodolsky & Grossman, 1995) where given problems can be solved using standard procedures. However, decades ago, several authors emphasized that mathematics is a human activity and the process of constructing mathematical knowledge involves complexity, uncertainty and ambiguity (Freudenthal, 1971; Siegel & Borasi, 1994). An essential part of mathematics is to explore multiple ways of approaching mathematical problems, as well as to create mathematical representations and models. These process-related aspects of mathematics are addressed and fostered with teaching approaches such as problem-based learning and inquiry-based learning (IBL) (Artigue & Blomhøj, 2013; Hmelo-Silver et al., 2007; Siegel & Borasi, 1994).

IBL appeared in the longstanding discussion of the nature of teaching and learning (Minner et al., 2010). A definition often quoted is the one by National Research Council (NRC) (2000) that defined inquiry in science education as a multifaceted activity characterized by processes like questioning, experimenting systematically, analyzing, evaluating and communicating results (shown later in Table 4.1). The research community in mathematics education is more familiar with using problem-solving to describe mathematical activities than these processes of inquiry, although they were considered strongly connected by some scholars (Dorier & Maass, 2014). Also differences have been pointed out through emphasizing the deductive feature of mathematics compared with that of science education. The main difference was thought to be the procedural character of the result of mathematical inquiry "which is necessarily presented as a deduction from what was given in the problem to what was to be found or proved" (Schoenfeld & Kilpatrick, 2013). Inquiry processes in mathematics education tend to include different ways of experimentation and validation when students explore problem situations that are meaningful for them (Artigue & Blomhøj, 2013). This exploration activity encompasses mathematization either of a real-life problem or of a mathematical problem situation. Treffers (1987) developed the concept of mathematization and distinguished horizontal and vertical mathematization. Horizontal mathematization describes the transition from realistic contexts into (tentative or temporary) mathematical terms and models. Vertical mathematization describes the process of reflecting on your mathematical activity and the further development of mathematics from generalizing, structuring and formalizing these earlier mathematical results (Artigue & Blomhøj, 2013; Treffers, 1987).

We interpret IBL in mathematics as classroom practices that encourage and support students to learn in ways similar to how mathematicians work (Maaß & Doorman, 2013; Siegel & Borasi, 1994). Students actively take responsibility in processes like questioning, hypothesizing, designing, investigating, analyzing, collaborating, communicating and reflecting (Chapman & Heater, 2010; Pedaste et al., 2015). We use "Procedure" to refer to phases of designing, investigating and analyzing, and include a phase of "Mathematization" in our interpretations of the processes of IBL in mathematics education (shown later in Table 4.3). These practices can be implemented with a varying level of guidance (Bruder & Prescott, 2013). With the IBL approach, students are expected to develop mathematical competences and inquiry habits of mind (Artigue & Blomhøj, 2013), and become mathematically literate citizens (Engeln et al., 2013). However, the implementation of such practices requires social and mathematical classroom norms that, for instance, allow for students' taking responsibility and teachers guiding negotiation of meaning (Makar & Fielding-Wells, 2018; Yackel & Cobb, 1996).

Teaching is considered a cultural activity (Stigler & Hiebert, 1998), and teaching cultures may have an impact on teachers' understanding and implementation of IBL. East Asia and the West are considered to have distinctively different teaching cultures (Cai & Wang, 2010), which has led to stereotypes with features often described by contrasting dichotomies. According to the stereotypes, East Asia seems to emphasize learning content (Leung, 2001) and the structure and depth of knowledge (Bryan et al., 2007; Norton & Zhang, 2018), while the West emphasizes learning process (Leung, 2001) and the practicality of knowledge related to real-life contexts (Bryan et al., 2007; Norton & Zhang, 2018). East Asia usually organizes classroom activities for the whole class, while the West for individual students or students in groups (Leung, 2001). East Asia tends to value the role of teachers and conducts well-designed directive instruction (Bryan et al., 2007; Leung, 2001), while the West values the role of students as active learners to construct knowledge on their own (Bryan et al., 2007; Liu & Feng, 2015). East Asia usually makes students learn by rote-like learning through memorizing and practicing repetitively (Liu & Feng, 2015; Tan, 2015), while the West provides problem situations and organizes activities for students to understand first and achieve meaningful learning (Leung, 2001). East Asia seems to value external factors like examinations to motivate students, while the West values internal factors like students' interests (Leung, 2001, 2014).

In previous studies, we took Beijing and the Netherlands as examples of the two teaching cultures and investigated their IBL practices in mathematics education through students' and teachers' perspectives. More shared than particular IBL practices in the two samples were found. In addition, students in the Dutch sample did not report much experience of IBL, while students in the Beijing sample did (Huang et al., 2021). These findings do not confirm expectations based on the stereotypes about the two teaching cultures. To further understand the current situation, we continued to investigate to what extent IBL practices in mathematics education are supported by teaching and learning resources, of which school textbooks tend to be the main part (Fan et al., 2017). In this study, lower-secondary mathematics textbooks used in Beijing and the Netherlands were analyzed.

Textbooks represent an existing view on mathematics learning and required practices in school mathematics (Love & Pimm, 1996; Van Dormolen, 1986). Textbooks are considered to highly influence the learning content and the learning opportunities for students (Dogan, 2021; Wijaya et al., 2015). This role of textbooks has been recognized, and there has been abundant research on mathematics textbooks. Most of the research analyzed how different mathematics content and topics were presented, or the thinking activities and cognitive demands required by various types of textbook tasks, or social and cultural issues such as gender equality and values reflected in textbooks. Mathematics textbooks from different cultures were compared in regard to these aspects (Fan et al., 2013).

Students' learning can be dominated by copying procedures when textbooks are assertive with exposition of given ideas, while the learning tends to be different when students develop understanding through their own constructions of knowledge with textbook tasks involving IBL (Love & Pimm, 1996; Watson & Thompson, 2015). Textbook tasks that are IBL-oriented have potential to support the implementation of IBL in lessons (Yang et al., 2019). Some research investigated the current situation of IBL in mathematics or science textbooks. A few studies found that textbooks include elements of IBL (Campanile et al., 2015) or can potentially be used to support the development of IBL (Dunne et al., 2013). However, more studies found only a few IBL in general and/or in phases such as questioning, hypothesizing, communicating and reflecting (Aldahmash et al., 2016; Kahveci, 2010; Li et al., 2018; Ma et al., 2021), or only a few high-level IBL in textbooks (Ma et al., 2021; Park & Lavonen, 2013). Textbooks from different countries were also compared in regard to the inclusion of IBL (Park & Lavonen, 2013; Xu, 2012).

An analytical framework is needed to evaluate IBL reflected in textbooks or instruction. The research base of IBL is stronger in science education than in mathematics education (Marshall et al., 2010), therefore frameworks from both fields were examined. Some frameworks seem not that focused on IBL for paying more attention to general features of tasks, such as the one from Xu (2012) that includes aspects related to context and information, question, group work, type of activities (solving, experiment, reading, writing, project), and the connection with other content in the textbook. A few frameworks focus on the educational functions that IBL-oriented tasks need to serve, such as ITAI (Inquiry-Based Tasks Analysis Inventory), a rubric to analyze the quality of IBL-oriented tasks in science education through evaluating the achievement of essential functions: understanding scientific concepts and scientific inquiry, using inquiry process skills and developing higher order thinking skills (Yang & Liu, 2016). However, the aim of our study is to present opportunities for IBL provided by textbook tasks, not to look into the functions they achieve. What fits our aim better are frameworks (see Table 4.1) connecting curriculum, textbook tasks or classroom activities with phases of IBL, and their levels concerning the openness and responsibility for students.

Table 4.1 Frameworks for evaluating levels of IBL

Source	Framework	Phases of IBL	Levels of IBL
Germann et al. (1996)	Evaluate the degree to which laboratory materials promote science process skills in scientific inquiry	Problems, Variables, Methods, Performance, Solutions, Extensions	Seven levels (Level 0 means all given in the six phases, and Level 6 means all open in the six phases)
National Research Council (NRC) (2000)	Essential features of inquiry and the variations in science lessons	Questions, Evidence, Explanation, Evaluation, Communication	Four levels (from more teacher/material directed to more student self-directed in each phase)
Capps and Crawford (2013)	Evaluate who initiated aspects of doing inquiry in science lessons	Question, Investigation, Evidence, Explanation, Evaluation, Communication, Use tools and techniques, Use mathematics	Four levels (from teacher-initiated to student-initiated in each phase)

Compared to the model of Germann et al. (1996), the framework of NRC (2000) presents a detailed variation of the levels of IBL in each phase and has been used or built upon by many studies. These include a study by Capps and Crawford (2013) with a rubric including more extensive phases of IBL in science education, which might be helpful to analyze IBL in mathematics after being adjusted to the discipline with the phase of "Mathematization" included.

A study by Calleja (2013) analyzed mathematical investigations in tasks through evaluating the levels of real-life contexts. Attention can also be paid to contexts in tasks for IBL, because real-life contexts might demand mathematization and modeling to be transformed into mathematical means (Wijaya et al., 2015). In addition, the real-life context match "relevant and essential context" and is distinct from "camouflage context" (De Lange, 1995), in which the mathematical operations needed to solve the tasks are explicit and the context information can be neglected (Wijaya et al., 2015).

Textbook task analysis of similar topics such as problem-solving and context-based tasks (van Zanten & van den Heuvel-Panhuizen, 2018; Wijaya et al., 2015) in mathematics education provide insights into the analysis of tasks with potential for IBL as well.

According to the history of IBL and the existing stereotypes above, the context in the West might be more supportive for the IBL approach than the context in East Asia. We expect that mathematics textbooks in the Netherlands would show a large percentage of tasks with opportunities for higher levels of IBL in various phases, while mathematics textbooks in Beijing would not. We also expect that not many shared features on this issue between the two areas would be identified, and the particular IBL-related features of Beijing and Dutch textbooks would match the stereotypes about each teaching culture.

However, studies also pointed out variations within a teaching culture (Andrews, 2016; Clarke et al., 2010; Clarke & Xu, 2008; Kim, 2018; Shimizu & Williams, 2013) or some shared elements between teaching cultures (Hiebert et al., 2003). Chinese mathematics textbooks were required to provide opportunities for investigations by the curriculum standard since 2001 (Wang et al., 2018), while Dutch mathematics textbooks were impacted by requests for more attention on basic knowledge and skills in algebra (Schoenfeld, 2014).

This study explores whether tasks in Beijing and Dutch mathematics textbooks have potential for IBL and contain possible insights into the design of IBL-oriented tasks in mathematics. The research questions are: 1) To what extent are opportunities for IBL provided by tasks in lower-secondary mathematics textbooks in Beijing and the Netherlands? 2) In what ways are the IBL-related features of these textbooks in line with expectations based on the stereotypes about the two teaching cultures?

2. Method

2.1 Selection of Textbooks and Chapters

Lower-secondary (Grade 7 to 9) mathematics textbooks that are most commonly used in the two areas were included for analysis. In Beijing (BJ), five of the six urban districts use the textbook published by the People's Education Press (PEP) (2013), the other urban district and all the ten suburban districts use a local version of a textbook published by the Beijing Publishing House (BP) (Beijing Academy of Educational Sciences, 2013)[1]. Considering the coverage of the PEP textbook in urban districts, and possible differences between schools in urban and suburban districts in Beijing, we included both textbooks for analysis. For the lower-secondary mathematics textbooks in the Netherlands (NL), we chose Numbers & Space (NS) (Noordhoff Publishers, 2019), which is the English translation for bilingual (VWO, i.e., pre-university) classes with the same content as the most used textbook[2] in the Netherlands: Getal & Ruimte. The abbreviations BJ-PEP, BJ-BP and NL-NS are used to refer to the three textbooks in this study.

In addition, to make comparison feasible, we compared content in the three textbooks and included chapters with similar topics and matching numbers of pages. In all the textbooks, statistics takes up only a small part. We verified the presence of tasks in algebra and geometry in case there can be differences in opportunities for IBL. As a result, an algebra chapter about quadratic equations including factorization[3] and a geometry chapter about similarity were selected for analysis. Detailed information is displayed in Table 4.2.

Also, we looked into the context of each task because it might be related to P3 (Mathematization) and impact opportunities for IBL. We did it in a similar way as Wijaya et al. (2015) that used the three categories by De Lange (1995). A category of "mathematical context" was added by us to distinguish tasks that require formulating a problem from a mathematical setting (e.g., the task in Fig. 4.2) from those involving only calculations (e.g., Fig. 4.3). We defined the context of a task with four categories: No context, mathematical context, camouflage context, real-life context.

1 This is based upon oral information from a leading teacher educator in Beijing

2 Source: https://www.noordhoff.nl/voortgezet-onderwijs/methoden/wiskunde/getal-en-ruimte The publisher did not state the percentage of teachers that were using this version, but it is believed to be above 70%.

3 The topic about factorization is contained in the chapter on quadratic equations only for the NL-NS textbook. To get corresponding topics for analysis and comparison, we also included a chapter of the BJ-BP textbook and a section (and relevant exercises at the end of the chapter) of the BJ-PEP textbook related to factorization. We used "the algebra chapter" to refer to the selected algebra content in each textbook for convenience.

Table 4.2 Information about the selected chapters of the three textbooks

	BJ-PEP				BJ-BP				NL-NS			
	G	V	C	Topic	G	V	C	Topic	G	V	C	Topic
Algebra	8	1	14 (Section3)	Factorization	7	2	8	Factorization	8	2	7	Quadratic Equations (including Factorization)
	9	1	21	Quadratic Equations	8	2	16	Quadratic Equations				
Geometry	9	2	27	Similarity	9	1	18	Similar Figures	9	1	2	Similarity

Note. “G” refers to Grade, “V” refers to Volume, and “C” refers to chapter

2.2 Framework for Coding Tasks

To get a framework evaluating opportunities for IBL in mathematics textbook tasks, we built upon the rubric of Capps and Crawford (2013) and the IBL processes from literature (Chapman & Heater, 2010; Pedaste et al., 2015) and included a phase of “Mathematization”. As a result, our framework has seven phases (P1 to P7, shown in Table 4.3) of the IBL cycle. The phases do not tend to be linear in the cycle. P3 (Mathematization) refers to situations where students are invited to organize mathematically, i.e., to describe a problem situation or a solution procedure in a mathematical way. P4 (Procedure) focuses on whether a task addresses a given “recipe” for solving the problem and includes processes of inquiry such as designing or analyzing a procedure or algorithm. In addition, the framework uses four levels to indicate to what extent students get opportunities from tasks to explore and invent by themselves, of which Level 0 is the lowest level and Level 3 is the highest level. Generally, Level 0 indicates that this phase is not necessary and not involved in the task; Level 1 indicates that all the needed information (such as solution procedures) has been provided for students to apply; Level 2 indicates that the needed information is partly provided for students to choose from or to adapt; Level 3 as a higher level of IBL indicates that the needed information is not provided and students have to figure it out by themselves. Considering that textbook tasks might not indicate much for P5 (Collaboration), P6 (Communication) and P7 (Reflection), only two levels (not present/present) were set up with P5, and levels in P6 and P7 focus on the activities involved in each phase. Level 2/Level 3 in P1 to P7 or Level 1 in P7 indicates opportunities for IBL in the phase.

Table 4.3 Framework in this study about the level of IBL in phases of the IBL cycle

Phase	Level 0	Level 1	Level 2	Level 3
P1: Question	Not present; the task provides commands	The task provides questions	The task requires students to choose from provided questions	The task offers opportunities for students to pose questions
P2: Hypothesis	Not present and not needed	The task provides a hypothesis	The task requires students to choose from provided hypotheses or to adapt a pattern into a hypothesis for a general rule	The task offers opportunities for students to formulate a hypothesis
P3: Mathematization	Not present and not needed	The textbook provides means for organizing mathematically (by the task, or by the explanation section/the worked example previously in this chapter)	The task requires students to choose from provided means or to adapt a given means for organizing mathematically	The task offers opportunities for students to design the means for organizing mathematically
P4: Procedure	Not present and not needed	The textbook provides solution procedures (by the task, or by the explanation section/ the worked example previously in this chapter)	The task requires students to choose from provided solution procedures, or to adapt a given solution procedure to a new problem situation, or to refer to a prior knowledge before this chapter	The task offers opportunities for students to design the solution procedures
P5: Collaboration during the process	Not present (Level 0)/ Present (Level 3)			

Phase	Level 0	Level 1	Level 2	Level 3
P6: Communication during the process	Not present	The task requires students to talk about and check their answers	The task requires students to talk about their solution procedures or other phases in the IBL cycle	The task requires students to discuss different solution procedures or other phases in the IBL cycle and justify them
P7: Reflection during the process	Not present	The task requires students to reflect on the answers (mathematics results)	The task requires students to reflect on part of (like one or two phases in P1 to P7) the IBL cycle	The task requires students to reflect on the whole IBL cycle

Inspired by Jupri and Drijvers (2016), we developed a scheme (see Fig. 4.1) to facilitate the understanding and coding of P3 (Mathematization). For a textbook task, we considered whether it involves horizontal mathematization through visualizing the problem situation or formulating the mathematical problem from a context, and whether it involves vertical mathematization through visualizing the solution procedure or generalizing the solution. If so, then we considered whether the needed information for mathematization is all given, partly given or not given, and coded the task with Level 1, Level 2 or Level 3.

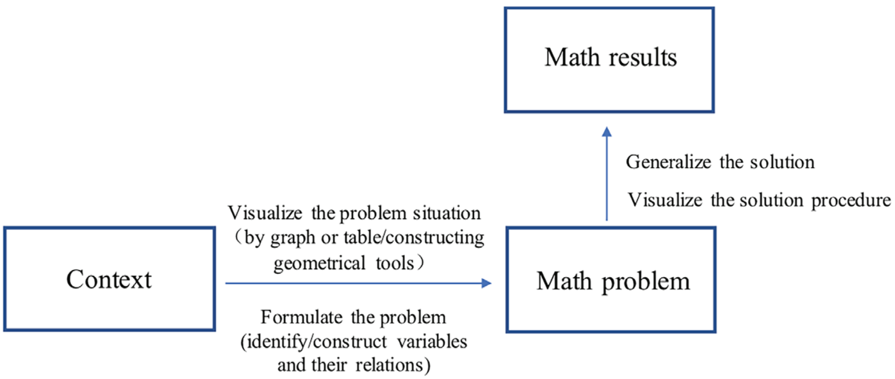


Fig. 4.1 A scheme to facilitate the understanding and coding of P3 (Mathematization)

Fig. 4.2 shows an example of coding P3 with the scheme above. Graphs have been provided by the task, which indicates Level 1 for “visualize the problem situation”; the equations for solving are not provided directly and students have to adapt the given means of graphs to formulate the equations, which indicates Level 2 for “formulate the problem”; the means to visualize solution procedure is not provided by the task, but it has been shown in a worked example before (see the red parts in Fig. 4.3) and can be applied by students, thus it was coded Level 1; it is not necessary to generalize the solution of the task and indicates Level 0. As the highest level of IBL in the four aspects, Level 2 was kept to code IBL in P3 for the task.

A 5 Figure 7.2a shows horizontal line *DE* at a height of 5 and figure 7.2b shows horizontal line *PQ* at a height of −13.

a Calculate the coordinates of points *D* and *E*.

b Calculate the coordinates of points *P* and *Q*.

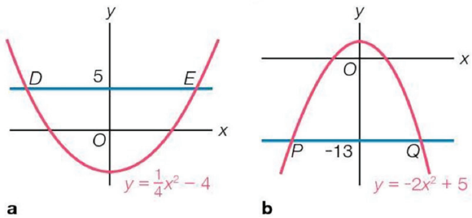


figure 7.2

Fig. 4.2 A task from the NL-NS textbook (Grade 8, Volume 2, p. 105)

Example

Solve. If necessary, round to two decimal places.

a $x^2 + 16 = 0$ **b** $2x^2 = 30$ **c** $5x^2 - 4 = 16$

How to work it out

a $x^2 + 16 = 0$
$-16 \quad -16$
$x^2 = -16$
no solution

b $2x^2 = 30$
$\div 2 \quad \div 2$
$x^2 = 15$
$x = \sqrt{15}$ or $x = -\sqrt{15}$
$x \approx 3.87$ or $x \approx -3.87$

c $5x^2 - 4 = 16$
$+4 \quad +4$
$5x^2 = 20$
$\div 5 \quad \div 5$
$x^2 = 4$
$x = 2$ or $x = -2$

Fig. 4.3 A worked example from the NL-NS textbook (Grade 8, Volume 2, p. 104)

2.3 Coding

All tasks except worked examples in selected chapters were included and coded by the first author using the framework in Table 4.3. As the unit for analysis, a “task” may include multiple subtasks that share the same problem situations and information. For example, Fig. 4.2 shows a “task” which includes two subtasks (a & b), and the subtasks include commands (“Calculate the coordinates of…”) but not questions. If there are differences in coding between or among subtasks, we coded the task with the higher level of IBL. Each task was coded with a level of IBL in each of the seven IBL phases.

Tasks that were difficult to code were discussed by the authors and the decisions regarding such tasks were taken as rules for coding, which helped to better interpret the framework and ensure the consistency of the coding.

2.4 Reliability

A check was performed to ensure the reliability of the coding. Firstly, the first author introduced an external researcher to the framework and the way to code with it.

For different levels of IBL in each phase, typical examples of tasks from two chapters that were not in our selection were provided. Also, a Beijing task and a Dutch task were coded for practice and discussed.

Then, the external researcher and the first author individually coded one third of the selected algebra chapter of the BJ-BP textbook and the NL-NS textbook. They first checked the Beijing part of 40 tasks with 280 assigned codes and reached an initial agreement of 81%. The disagreements were discussed, especially about P3 (Mathematization). For example, it was clarified that P3 focuses on whether it involves visualizing the solution procedure or generalizing the solution, and the solving process to get answers from a mathematical problem involves P4 (Procedure) more than P3. The two coders agreed upon 36 more assigned codes and reached an agreement of 94%. Then the Dutch part of 36 tasks with 252 assigned codes was checked and got an initial agreement of 89%. The coders further clarified that visualizing the solution procedure in P3 should be distinguished from the reasoning process with mathematical expressions that belongs to P4. The coders agreed upon 28 more assigned codes and eliminated all the disagreements. These reflections and adjustments were included in the coding that followed.

2.5 Procedure for Analysis

We calculated the frequency and percentages of pages and tasks to present an overview of selected tasks. The context information of each task was also presented.

The variable of IBL levels was taken as categorical but not continuous to keep most of the information. We calculated percentages of IBL levels (Level 0, Level 1, Level 2, Level 3) in the seven phases (P1 to P7) for selected tasks in each textbook and further focused on the phases with some opportunities for IBL. Percentages of IBL levels in P3 and P4 were calculated for tasks separated for different content (algebra, geometry) and different context types (none, mathematical, camouflage, real-life). Shared and particular IBL-related features of the two BJ and one NL textbooks were identified.

In addition, we provided examples of tasks to illustrate opportunities for IBL limited or provided by textbooks. We made sure to include tasks from each textbook and from both algebra and geometry.

Moreover, we compared features of opportunities for IBL provided by BJ and NL textbooks with expectations based on the stereotypes about the two teaching cultures.

3. Results

3.1 Overview of Selected Tasks in the Three Textbooks

A total of 648 tasks on 231 pages were coded and the distribution in algebra and geometry chapters in each textbook is displayed in Table 4.4. Pages included from algebra and geometry are balanced in total, and the ratio of algebra tasks to geometry tasks is almost the same for the three textbooks.

The context information of selected tasks in the textbooks is displayed in Table 4.5. For each textbook, there are relatively more tasks with no or mathematical contexts, and less tasks with camouflage or real-life contexts.

Table 4.4 The distribution of tasks selected from the three textbooks

	Content	BJ-PEP	BJ-BP	NL-NS	Total
Pages	Algebra	34 (47.9%)	46 (56.1%)	38 (48.7%)	118 (51.1%)
	Geometry	37 (52.1%)	36 (43.9%)	40 (51.3%)	113 (48.9%)
	Total	71 (100%)	82 (100%)	78 (100%)	231 (100%)
Tasks	Algebra	103 (55.7%)	124 (56.6%)	136 (55.7%)	363 (56%)
	Geometry	82 (44.3%)	95 (43.4%)	108 (44.3%)	285 (44%)
	Total	185 (100%)	219 (100%)	244 (100%)	648 (100%)

Table 4.5 Context information of task selected from the three textbooks

Context	BJ-PEP	BJ-BP	NL-NS	Total
None	56 (30.3%)	110 (50.2%)	91 (37.3%)	257 (39.7%)
Mathematical	73 (39.5%)	74 (33.8%)	115 (47.1%)	262 (40.4%)
Camouflage	25 (13.5%)	13 (5.9%)	16 (6.6%)	54 (8.3%)
Real-life	31 (16.8%)	22 (10%)	22 (9%)	75 (11.6%)
Total	185 (100%)	219 (100%)	244 (100%)	648 (100%)

3.2 Opportunities for IBL Provided by Selected Tasks in the Three Textbooks

3.2.1 Percentages of IBL levels in Phases in Each Textbook and the Comparison

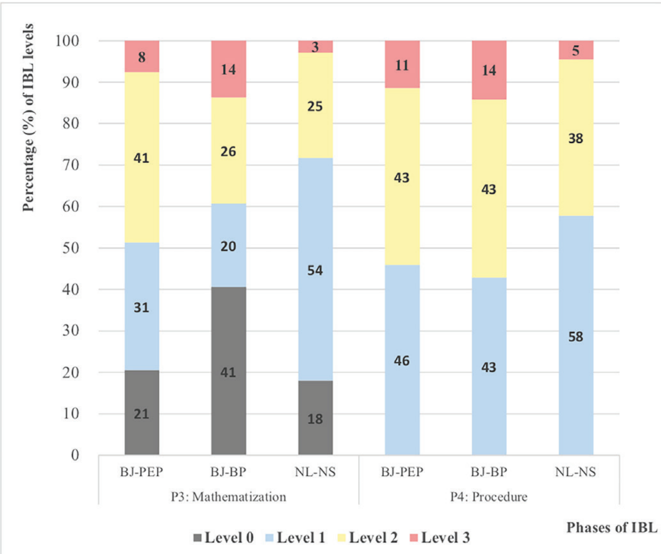
Percentages of the four IBL levels in each phase are displayed in Table 4.6. Almost only Level 0 is observed in P2 (Hypothesis), P5 (Collaboration) and P6 (Communication), and Level 2 or Level 3 are not involved at all in P1 (Question), thus few IBL opportunities are provided in these phases. Only a few opportunities (Level 1) are involved in P7 (Reflection). Some opportunities for IBL can be identified in P3 (Mathematization) and P4 (Procedure), however, Level 3 as a higher level of IBL is only involved with a small percentage.

The distribution of IBL levels in P3 and P4 with some opportunities for IBL is displayed in Fig. 4.4. We compared the features of selected tasks in the three textbooks. For IBL levels in P3, each textbook presents a different pattern. Compared with the other two textbooks, the BJ-PEP textbook involves more Level 2, the BJ-BP textbook involves more Level 0 and relatively more Level 3, and the NL-NS textbook involves more Level 1. Generally the NL-NS textbook shows fewer opportunities (Level 2 and Level 3) for IBL in P3 than the BJ textbooks. For IBL levels in P4, the two BJ textbooks are very similar, while the NL-NS textbook involves relatively fewer opportunities (Level 2 and Level 3) for IBL than them.

Table 4.6 Percentages (%) of four IBL levels in seven phases for selected tasks in each textbook

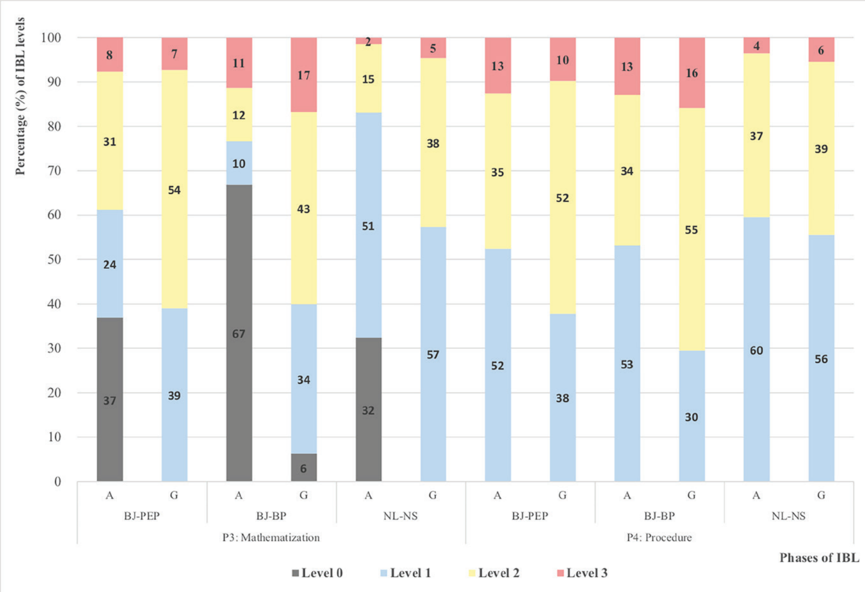
	Level 0			Level 1			Level 2			Level 3		
	BJ-PEP	BJ-BP	NL-NS	BJ-PEP	BJ-BP	NL-NS	BJ-PEP	BJ-BP	NL-NS	BJ-PEP	BJ-BP	NL-NS
P1: Question	44.3	52.5	81.2	55.7	47.5	18.9	0	0	0	0	0	0
P2: Hypothesis	98.4	97.3	99.2	.5	2.3	.4	0	0	.4	1.1	.5	0
P3: Mathematization	20.5	40.6	18.0	30.8	20.1	53.7	41.1	25.6	25.4	7.6	13.7	2.9
P4: Procedure	0	0	0	46.0	42.9	57.8	42.7	42.9	37.7	11.4	14.2	4.5
P5: Collaboration	99.5	99.5	100							.5	.5	0
P6: Communication	98.4	90.9	100	1.1	9.1	0	.5	0	0	0	0	0
P7: Reflection	84.9	90.4	90.2	14.6	8.7	9.0	.5	.9	.8	0	0	0

Note. The percentages (%) are within the textbook; Some percentages do not add up to 100% due to rounding errors



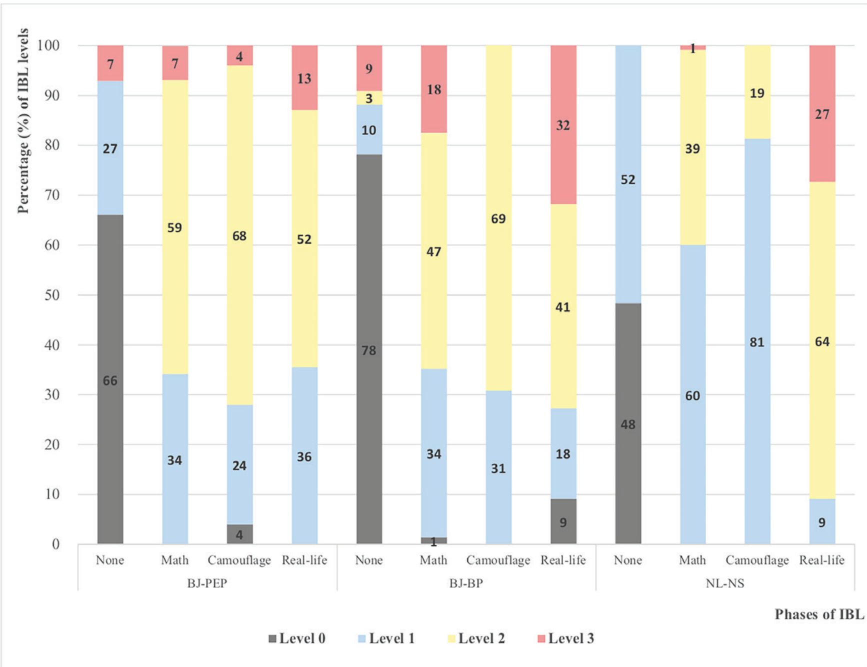
Note. Some percentages do not add up to 100% due to rounding errors

Fig. 4.4 Percentages of IBL levels in P3 and P4 for selected tasks in each textbook



Note. "A" refers to algebra and "G" refers to geometry; Some percentages do not add up to 100% due to rounding errors

Fig. 4.5 IBL levels in P3 and P4 for tasks in selected algebra and geometry chapters in each textbook



Note. Some percentages do not add up to 100% due to rounding errors

Fig. 4.6 IBL levels in P3 (Mathematization) for tasks with four context types in each textbook

Fig. 4.5 further presents the distribution of IBL levels in P3 and P4 separated for algebra and geometry in each textbook. A shared pattern between textbooks can be identified in P3 that more opportunities for IBL are involved in the selected geometry chapters than in the selected algebra chapters. These differences between algebra and geometry mainly lie in Level 2. As for P4, the BJ textbooks show the same pattern as in P3, while the NL-NS textbook provides almost equal opportunities in algebra and in geometry. The differences between algebra and geometry in P3 and P4 are relatively larger in the BJ-BP textbook than those in the other two textbooks.

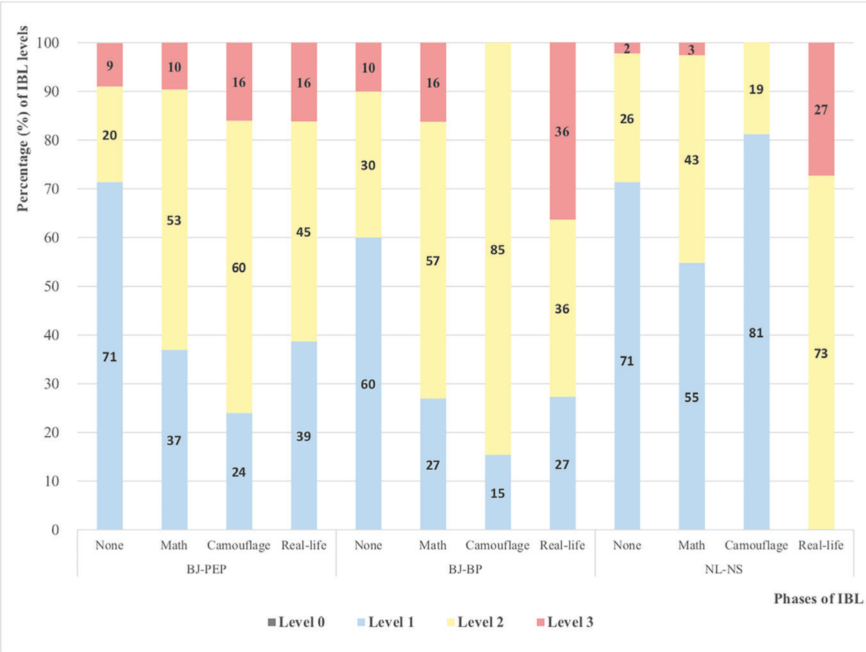
Fig. 4.6 and Fig. 4.7 present the distribution of IBL levels in P3 and P4 respectively, for tasks with four context types (none, mathematical, camouflage, real-life).

For P3, generally the textbooks share a pattern that a lot fewer opportunities for IBL (Level 2 and Level 3) are shown in tasks with no contexts than in tasks with other context types. The two BJ textbooks tend to be more similar, and the NL-NS textbook is different from them with more variations for IBL opportunities among different context types: tasks with no contexts do not involve IBL opportunities at all, while almost all tasks with real-life contexts show IBL opportunities. What lie in between are tasks with mathematical and camouflage contexts.

Compared with those types of tasks in the BJ textbooks, tasks with real-life contexts in the NL-NS textbook provide more IBL opportunities, while tasks with the other types of contexts in the NL-NS textbook show less IBL opportunities. Differences between the BJ textbooks can also be identified: percentages of Level 3 as a higher level are quite close among tasks with different context types for the BJ-PEP textbook, while the percentages are relatively varied for the other two textbooks, in which IBL opportunities are more involved in tasks with real-life contexts.

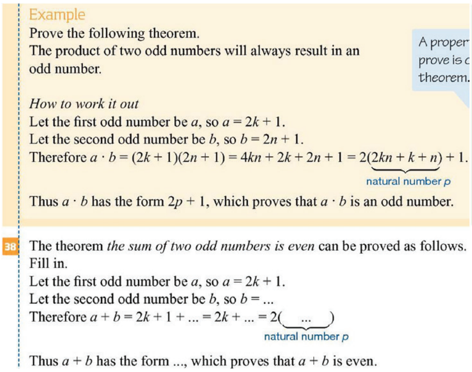
For P4, generally the two BJ textbooks share the same pattern: tasks with no contexts show fewer IBL opportunities (Level 2 and Level 3), while tasks with camouflage contexts involve more IBL opportunities, what lie in between are tasks with mathematical and real-life contexts with similar percentages. The NL-NS textbook is different from them that tasks with no and camouflage contexts involve fewer IBL opportunities, while all the tasks with real-life contexts show IBL opportunities, and what lie in between are tasks with mathematical contexts.

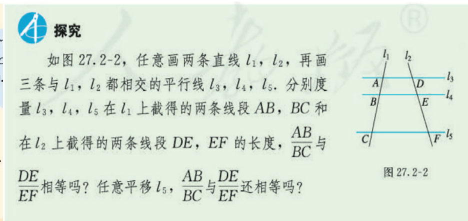
Compared with those types of tasks in the BJ textbooks, tasks with real-life contexts in the NL-NS textbook show more IBL opportunities, while tasks with mathematical and camouflage contexts in the NL-NS textbook show less IBL opportunities. Differences between the BJ textbooks can be identified again, and the patten of Level 3 among tasks with different context types is similar to what was found in P3 (Mathematization) above.



Note. Some percentages do not add up to 100% due to rounding errors

Fig. 4.7 IBL levels in P4 (Procedure) for tasks with four context types in each textbook





Translation: Inquire. As is shown in the figure, l1 and l2 are two random lines, they are intercepted by parallel lines l3, l4, and l5. Measure the length of line segments AB and BC on l1, and DE and EF on l2. Are AB/BC and DE/EF equal? Translate line l5 randomly, are AB/BC and DE/EF still equal?

Fig. 4.8 An algebra task from the NL-NS textbook (Grade 8, Volume 2, p. 115)

Fig. 4.9 A geometry task from the BJ-PEP textbook (Grade 9, Volume 2, p. 29)

3.2.2 Example Tasks to Illustrate Opportunities for IBL in Textbooks

We selected tasks from the three textbooks as examples to illustrate opportunities for IBL limited or provided by textbooks.

The two examples above present how textbooks limit opportunities for students to explore a mathematical situation (Level 3) or make choices (Level 2) by themselves. Fig. 4.8 shows how a task right after a worked example precisely follows the provided means to organize a problem mathematically (Level 1 in P3) and provided solution procedures (Level 1 in P4), which can be applied directly by students without any exploration, and the task itself already provides most of the solutions. Students only need to follow commands (Level 0 in P1) by filling in the empty parts, which does not allow for opportunities to approach the problem. Fig. 4.9 shows a task that provides students with commands to measure and calculate the assigned variables (Level 1 in P3). Although the task seems to involve students in finding a pattern, the commands lead them to the answers without a real exploration of ways to solve the problem (Level 1 in P4). The two tasks would have enhanced opportunities for IBL if they had left an opening for students to hypothesize (P2; about the sum of two odd numbers or quantitative relations among the line segments), to find a way to testify (P4), to generalize a rule in mathematical means (P3), and to reflect on the inquiry cycle (P7). Group work (P5) and whole-class discussion (P6) can be included for students to collaborate and communicate. Fig. 4.9 can also leave it open for students to observe and pose questions to tackle by themselves.

48 There are two different ways to solve the equation $4x^2 - 9 = 0$.

I Use $a^2 - b^2 = (a - b)(a + b)$	II Simplify to the form $x^2 = c$
$4x^2 - 9 = 0$	$4x^2 - 9 = 0$
$(2x - 3)(2x + 3) = 0$	$4x^2 = 9$
$2x - 3 = 0 \vee 2x + 3 = 0$	$x^2 = \frac{9}{4}$
$2x = 3 \vee 2x = -3$	
$x = 1\frac{1}{2} \vee x = -1\frac{1}{2}$	$x = \frac{3}{2} = 1\frac{1}{2} \vee x = -\frac{3}{2} = -1\frac{1}{2}$

Solve. Choose the method that you find easier.

a $x^2 - 25 = 0$ **c** $16x^2 - x = 0$ **e** $x^2 - 121 = 0$
b $25x^2 - 16 = 0$ **d** $9x^2 - 49 = 0$ **f** $16x^2 - 81x = 0$

Fig. 4.10 An algebra task from the NL-NS textbook (Grade 8, Volume 2, p. 119)

10. 观察下列式子：Observe the forms below:

$2\times4+1=9=3^2$；

$6\times8+1=49=7^2$；

$14\times16+1=225=15^2$；

……

What conclusion do you get? Could you prove it?
你得出了什么结论？你能证明这个结论吗？

Note. The first author inserted English translations in the task

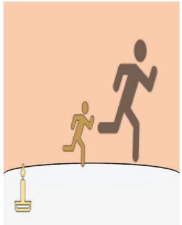
Fig. 4.11 An algebra task from the BJ-PEP textbook (Grade 8, Volume 1, p. 120)

Fig. 4.10 and Fig. 4.11 are two algebra tasks showing how textbooks can provide opportunities for IBL by requiring students to think about means to organize a problem mathematically (P3) and/or solution procedures (P4). In Fig. 4.10, two methods are presented within the task for students to compare and make a choice, which indicates Level 2 in P4. Students also need to identify that the equations c and d cannot be solved with any of the methods, and to choose to apply another method they have learnt before this task. Fig. 4.11 shows an advanced task that asks students to observe, generalize a rule and prove it. The task not only provides opportunities (Level 3) for IBL in P3 and P4, but also in P2 for students to formulate a hypothesis.

09 See the candle in figure 2.7. When the candle is lit, you get a shadow on the wall.

a What happens to the shadow when the burning candle slowly melts away? Does the shadow become longer or shorter or does its length stay the same?

b And what happens to the location of the shadow on the wall when the candle melts away?



Note. The figure is self-made to represent the original figure

Fig. 4.12 A geometry task from the NL-NS textbook (Grade 9, Volume 1, p. 58)

Fig. 4.12 is a geometry task from the NL-NS textbook with opportunities (Level 3) for mathematizing the problem (P3) and inquiring solution procedures (P4). The BJ-BP textbook has a task (Grade 9, Volume 1, p. 32) about calculating the height through the shadow: a group of students measured that the shadow of a 1m tall bamboo was 0.9m, and they found the shadow of a tree was 2.7m on the ground and 1.2m on the wall (shown in an illustration), please join them to calculate the height of the tree. In both tasks, students are expected to construct drawings, such as creating side views and adding lines, and to identify variables and relations to formulate the problem in a mathematical way. The tasks also highlight the role of real-life contexts in connection with these opportunities for IBL.

3.3 Connecting Features of Opportunities for IBL in the BJ and NL Textbooks with Expectations Based on the Stereotypes about the Two Teaching Cultures

As for the particular IBL-related features of the BJ and NL textbooks, some of them match the stereotypes about teaching cultures in East Asia and the West. For IBL levels in P4 and those further separated for algebra and geometry, also for IBL levels in P3 and P4 among different types of contexts, the two BJ textbooks show similar patterns while the NL textbook is different from them, which seems to support the existence of two groups of teaching cultures.

However, some particular IBL-related features of the textbooks do not match the stereotypes. Each textbook presents a different pattern for IBL levels in P3, and there exist differences between the BJ textbooks for IBL levels in P3 and P4 between content and among context types. These findings seem to show the presence of variations within a teaching culture. The NL textbook involves relatively fewer IBL opportunities in P3 and P4 than the BJ textbooks, which does not match the stereotypes that consider mathematics education in East Asia to value learning content and the structure and depth of knowledge, teacher-led instruction as well as rote-like learning (Bryan et al., 2007; Leung, 2001; Liu & Feng, 2015; Norton & Zhang, 2018; Tan, 2015), and stereotypes that consider mathematics education in the West to value learning process and the practicality of knowledge, student-centered learning as well as meaningful learning (Bryan et al., 2007; Leung, 2001; Liu & Feng, 2015; Norton & Zhang, 2018).

As is shown in the results above, the expectation that the NL textbook would show more tasks with higher levels of IBL in various phases is not confirmed. Instead, the NL textbook shows relatively fewer opportunities for IBL in P3 and P4 than the BJ textbooks. Many shared IBL-related features between the BJ and NL textbooks were identified, and some particular IBL-related features in each area do not match the stereotypes about each teaching culture. These findings are not in line with the expectations, either. However, some particular features such as those related to IBL levels in P4 seem to confirm a part of the expectations.

4. Discussion

Our findings show many shared features between the Beijing and Dutch textbooks regarding the opportunities for IBL that they provide. The textbooks tend to define a learning trajectory that does not require students to question or hypothesize mathematical issues, nor to learn to collaborate or communicate or reflect mathematically, while it does allow students to make some choices about mathematizations and solution procedures. However, a higher level of IBL that really provides opportunities for students to inquire by themselves is rarely involved. These findings are quite in line with studies in science textbooks that found only a few IBL in phases like questioning, hypothesizing, communicating and reflecting (Aldahmash et al., 2016; Li et al., 2018; Ma et al., 2021) or only a few high-level IBL (Ma et al., 2021; Park & Lavonen, 2013). Another shared feature is that the textbooks provide more IBL opportunities to organize mathematically in geometry than in algebra, which might explain why a few teachers in our previous study considered geometry as suitable to do IBL. The textbooks also share that many more IBL opportunities to inquire into solution procedures are shown in tasks with contexts than in tasks without contexts.

Particular IBL-related features of the textbooks in each area were identified as well. The Dutch textbook involves relatively fewer opportunities to organize mathematically and to inquire into solution procedures than the Beijing textbooks, which is in line with a finding in our previous study that students in the Dutch sample reported less experience of designing their own procedures for solving complex problems than students in the Beijing sample (Huang et al., 2021). While a further look into context types found that tasks with real-life contexts in the Dutch textbook show more IBL opportunities to organize mathematically and to inquire into solution procedures than this type of tasks in the Beijing textbooks.

For some categories, the two Beijing textbooks tend to be similar while the Dutch textbook is different from them. For example, as for inquiring into solution procedures, the Beijing textbooks involve more opportunities in geometry than in algebra, while the Dutch textbook shows almost equal opportunities in different content. This can be related to possible differences in ways to design mathematics textbooks in the two areas. Topics in the Dutch mathematics textbooks can be treated in the same didactical approach to deal with solution procedures, while it might not be the case in Beijing.

However, for some categories, each textbook presents a different pattern, or differences exist between the two Beijing textbooks. The differences can be more prominent than differences between the Beijing and Dutch textbooks. The BJ-BP textbook is different from the other two textbooks in providing relatively more opportunities for addressing solution procedures in geometry than in algebra.

Findings of this study are only in line with a part of expectations based on the stereotypes, while most expectations are not confirmed. The results challenge these stereotypes and are in line with studies indicating that countries or areas in the two groups of teaching cultures could also share some elements (Hiebert et al., 2003; OECD, 2014), and with studies presenting variations within a teaching culture (Andrews, 2016; Clarke et al., 2010; Clarke & Xu, 2008; Kim, 2018; Shimizu & Williams, 2013).

The study also presents that generally more IBL opportunities to organize mathematically and to inquire into solution procedures are shown in tasks with contexts than in tasks without contexts. This can be empirical evidence for enriching contexts of tasks in mathematics textbooks.

Findings above are based on selected tasks in the three textbooks. Further study needs to testify whether same results would be found in other chapters, content, textbooks, grades, or countries/areas.

The findings need to be interpreted with care. They are based on textbook analyses that are not necessarily indicative for actual classroom practices created by the interactions between students, the teacher and textbook tasks. Students' learning is also mediated by the school context (Mesa, 2004) including social and mathematical classroom norms (Makar & Fielding-Wells, 2018; Yackel & Cobb, 1996). A limitation of the study is that we did not take into account how teachers interpreted the IBL opportunities offered by the textbooks and to what extent teachers supported their students to realize these opportunities in daily practice. Ideally, textbooks are analyzed also in terms of classroom use (Haggarty & Pepin, 2002). Further study can observe mathematics lessons to investigate to what extent the IBL opportunities in textbook tasks are taken by Beijing and Dutch mathematics teachers in their teaching and to evaluate the level of IBL in these activities.

We conclude that textbooks for mathematics seem to avoid problems for students in which they do not immediately know what to do, and consequently do not provide problems to students that have a potential for learning to inquire mathematically. Mathematics textbook designers need to consider to provide more attention for questioning, hypothesizing, collaborating, communicating and reflecting. This attention should illustrate teaching opportunities to better prepare students for inquiry in mathematics, and for developing related flexible 21st century skills like creativity, collaboration and communication that are crucial for our current quickly changing society (Gravemeijer et al., 2017). The framework of this study can be used in teacher professional development programs to facilitate them to reflect on, adapt to and design lesson materials with potential for IBL, and promote its implementation in teaching practices.

Chapter 5

Inquiry-based Learning Practices in Lower-secondary Mathematics Education: An Analysis of Chinese Teachers' Lessons

This chapter is based on:

Huang, L., Doorman, L.M., & van Joolingen, W.R. Inquiry-based learning practices in lower-secondary mathematics education: An analysis of Chinese teachers' lessons. (Manuscript submitted for publication)

Abstract Inquiry-based learning (IBL) is advocated but seems not yet incorporated into teaching practices. This study explored actual practices of five Chinese lower-secondary mathematics teachers with respect to the involvement of IBL. Twenty-four lessons of the teachers were analyzed, of which five lessons were required to be designed and implemented with elements of IBL based on their interpretations of it and of pedagogies to promote it. Additional data came from post-lesson teacher interview and student survey. In the analyses we focused on the structures of the lessons and to what extent opportunities for IBL were provided in the two types of lessons. Findings show that the teachers distributed more time in the IBL lessons to introduce new content, while they did not adjust much for specific IBL practices. Generally the IBL opportunities shown in the selected usual lessons were kept or only extended a little in the required IBL lessons. A higher level of IBL was relatively involved for students to organize mathematically and inquire into solution procedures. The teachers seemed not to have a complete picture of the IBL cycle, thus they focused on problem solving processes while ignoring phases like questioning and hypothesizing.

Keywords Mathematics education; Inquiry-based learning; Teaching practice; Lower-secondary education

1. Introduction

The structure of a lesson can strongly influence the way content is being taught and learned, which is also the case in mathematics education. The presence, sequence or organization of specific classroom activities such as presentation of content or inclusion of group work could be “hidden curriculum” and shape the learning opportunities for students (Baldry, 2017; Kelly, 2009; Watson & Evans, 2012). Lesson structures are often described along the dimensions of purposes and forms. These dimensions were analyzed in international comparative studies such as TIMSS 1999 (Trends in International Mathematics and Science Study) (Hiebert et al., 2003) and LPS (Learner’s Perspective Study) (Clarke et al., 2007). Purposes focus on elements of instruction and refer to pedagogical functions of classroom activities. Forms focus on features of organization and refer to forms of classroom interaction (Savola, 2008).

Frameworks of *purposes* of classroom activities were inspired by the four formal stages of learning by Herbart (Savola, 2008). A similar framework was used in TIMSS 1999 and it consisted of three categories: review, introduce new content, practice new content. That study included data in East Asia such as Hongkong SAR and Japan. For Hongkong, 24%, 39% and 37% of lesson time was spent on reviewing, introducing new content and practicing new content respectively (Hiebert et al., 2003). Follow-up studies (e.g., Clivaz & Miyakawa, 2020; Willbergh & Aasebø, 2019) built upon the TIMSS framework and compared features of lesson structure in different countries and identified some differences. As for the situation in China, mathematics lessons are generally well-structured. Mathematics lessons at Beijing were found much more structured than lessons in Hong Kong (Leung, 1995). Another study found mathematics lessons at Shanghai tightly controlled by the teacher and explorations never large and open (Lopez-Real et al., 2004). There seems to be a lack of student exploration in Chinese lessons, and mathematics teachers tend to value guidance through teacher–student conversations (Cao et al., 2018).

As for *forms* of classroom interaction, TIMSS 1999 mainly considered public interaction (the teacher or students present to the whole class) and private interaction (students work individually or in groups and the teacher often interacts with individual students). For Hongkong lessons in that study, only 20% of the interactions were private, of which 95% were individual work and 5% were group or pair work (Hiebert et al., 2003). The framework used in the TALIS project (Teaching and Learning International Survey) (OECD, 2019) consisted of four categories: whole group, small group (three or more students), pairs and individual. 70% of the teachers from the Shanghai sample reported they always or frequently engaged students to work in small groups for a joint solution (OECD, 2019). This finding seems not in line with studies (e.g., Cao et al., 2018) that identified a lack of student collaboration in Chinese lessons.

In addition to the analysis of percentages of purposes and forms, existing research also looked into possible patterns related to the two aspects. Lesson structures are considered to be varied in different cultural contexts (Alexander, 2000). Savola (2010) built upon TIMSS and LPS projects and identified patterns in mathematics lessons in Finland and Iceland. The pattern in Finland is similar to "review → introduce new content → practice new content". The pattern in Iceland is similar to "individual work" and was regarded as more student-centered. Willbergh and Aasebø (2019) analyzed lower-secondary school lessons of all subjects in Norway. They identified a pattern similar to "introduce new content → practice new content" for purposes, and a pattern similar to "whole-class + individual work" for forms. There are ways to visualize the structure of lessons to make it more explicit. Below is an example with bars of timelines to represent two Grade 4 mathematics lessons in Switzerland and Japan (Clivaz & Miyakawa, 2020). According to categories in that study, the Swiss lesson shows a pattern of "introduction → research → sharing" for purposes and the Japanese lesson shows "introduction → research → sharing → synthesis. More time was spent on the activity of "research" in the Swiss lesson than in the Japanese lesson.

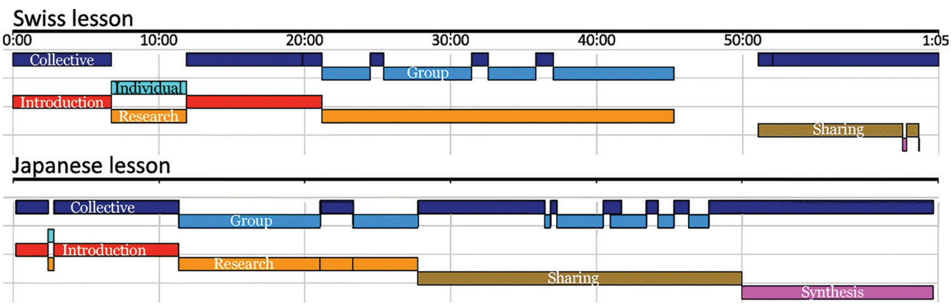


Fig. 5.1 An example of visualizing the structure of two lessons by Clivaz and Miyakawa (2020)

The example in Fig. 5.1 is a useful way to visualize and compare lesson structures, which characterizes lessons in different contexts. This can also be used to characterize lessons adopting different teaching approaches, for examples usual lessons and reform-oriented lessons like inquiry-based lessons. Usual lessons are often regarded as teachers delivering knowledge and students working on exercises to practice algorithms. We are going to research whether structures of lessons adopting the inquiry-based learning (IBL) approach would be different.

Structures of IBL lessons and practices of IBL implemented by teachers might vary based upon their interpretations of IBL and pedagogies to promote it (Engeln et al., 2013; Fang, 2021). The research base of IBL is stronger in science education than in mathematics education (Marshall et al., 2010). A well-known definition of IBL was proposed by National Research Council (NRC) (2000) about inquiry in science education, which was considered through processes like questioning, experimenting systematically, analyzing, evaluating and communicating results. Much research defined IBL as constructing knowledge through processes or a cycle with similar phases (Turner et al., 2018). Different levels of IBL were evaluated based on the extent that students initiate and direct the inquiry processes by themselves (Bruder & Prescott, 2013; Turner et al., 2018).

IBL is not yet a routine in teachers' daily practices (Dobber et al., 2017; Fang, 2021). Barriers include internal factors such as teachers' knowledge and their beliefs about inquiry, and external factors such as time, curriculum and students' knowledge and skills (Turner et al., 2018; Wallace & Kang, 2004). It remains open to what extent IBL has been incorporated into teaching practices (Engeln et al., 2013). Many existing research focused on the use of IBL in science education, and some included IBL practices in mathematics. Mathematics teachers were found to report less use of IBL than science teachers (Marshall et al., 2009; PRIMAS, 2013). Teachers seem often not to include the whole IBL cycle or not involve many phases of it in practice (Capps & Crawford, 2013). Science and mathematics teachers were found to involve IBL more in some phases than in other phases (Capps & Crawford, 2013; Danipog, 2018; Lucero et al., 2013; Turner et al., 2018). However, results vary in different studies concerning the specific phases with relatively more or less practices. For example, Lucero et al. (2013) identified more IBL practices in understanding provided materials, and less practices in phases related to data and conclusion based on evidence, while Danipog (2018) captured more IBL practices in question formulation and communication, and less in phases related to investigation, data and explanation. Research also identified the low level of science and mathematics teachers' IBL practices in general (Capps & Crawford, 2013; Engeln et al., 2013; Samuel & Ogunkola, 2013) or in phases like formulating questions and designing procedures (Lucero et al., 2013). Turner et al. (2018) found that the phase of "verbally interpreting outcomes" is most frequent (87% lessons) in mathematics lessons, and "critiquing others' interpretations" is the most frequent (5% lessons) student-initiated phase.

Previous research employed survey and interview for teachers to report their practices of IBL (Brandon et al., 2009; Engeln et al., 2013; Fang, 2021; Lucero et al., 2013). Students' report and classroom observation were also conducted. It has been pointed out that reporting by teachers or students alone fails to accurately present actual practices, classroom observation can be helpful to show a more detailed picture (Capps & Crawford, 2013). This study intended to evaluate actual practices of IBL in lessons through observation, in which observation frameworks are essential.

We focused on existing observation frameworks related to IBL practices and made a list in Table 5.1. Marshall et al. (2011) criticized that many of the frameworks are "either too general (e.g., consider all elements of effective practice), too granular (e.g., consider one aspect of instruction such as classroom management), or too complex (e.g., necessary to use multiple rubrics over multiple days)". What we listed are mainly frameworks specifically concerning IBL. RTOP (Sawada et al., 2002) was included because it is a common framework in use to observe IBL (Baldry, 2017).

As is shown in Table 5.1, these frameworks vary in the dimensions of IBL, also in ways to measure the degree of IBL (Turner et al., 2018). The dimensions are based on a set of inquiry processes (e.g., STIR, SITOI) or a few key aspects (e.g., RTOP, EQUIP, SIO). As for the degree of IBL, most of the frameworks are rated according to the extent that students initiate or direct the learning process, while RTOP is based on the frequency of occurrence (Turner et al., 2018).

A framework that considers the extent of student initiation in a set of inquiry processes is close to our understanding of evaluating IBL practices. The one by Capps and Crawford (2013) was selected because it meets the criteria and includes more extensive phases of IBL. A rubric was also developed by them to evaluate who initiated aspects of doing inquiry. This framework fits in our interpretations of IBL, except for the subject that it focuses on IBL in science lessons.

We interpreted IBL in mathematics as a teaching approach that invites students to learn in ways similar to how mathematicians work (Maaß & Doorman, 2013; Siegel & Borasi, 1994). Students take responsibility in a cycle of phases such as questioning, hypothesizing, designing, investigating, analyzing, collaborating, communicating and reflecting (Chapman & Heater, 2010; Pedaste et al., 2015). Different levels of IBL exist according to the degree that students initiate the inquiry processes, which tend to include different ways of experimentation and validation in mathematics and science education (Maaß & Artigue, 2013). An empirical study found "creation of graphs/charts" and "visually representing concept or data" with a higher level of inquiry in mathematics than in science education (Turner et al., 2018). In order to adapt the rubric by Capps and Crawford (2013) to better fit the evaluation of

Table 5.1 Observation frameworks for evaluating practices of IBL in classrooms

Source	Framework	Phases of IBL/Aspects	Levels of IBL/Frequency of occurrence
NRC (2000)	Framework for features of inquiry and the variations in science lessons	Questions, Evidence, Explanation, Evaluation, Communication	Four levels (from teacher/material-directed to learner-directed in each phase)
Sawada et al. (2002)	Reformed Teaching Observation Protocol (RTOP); Evaluate constructivist practices in mathematics and science lessons	Lesson design and implementation, Content, Classroom culture	Five-point frequency (from never occurred to very descriptive)
Bodzin and Beerer (2003)	Science Teacher Inquiry Rubric (STIR) ; Evaluate teachers' use of IBL in science lessons	Questions, Evidence, Explanation, Evaluation, Communication (based on NRC 2000)	Five levels (from teacher-centered to learner-centered in each phase)
Marshall et al. (2010)	Electronic Quality of Inquiry Protocol (EQUIP); Evaluate IBL in mathematics and science lessons	Instruction, Curriculum, Discourse, Assessment	Four levels (from pre-inquiry to exemplary inquiry in each aspect)
Capps and Crawford (2013)	Evaluate who initiated aspects of doing inquiry in science lessons	Question, Investigation, Evidence, Explanation, Evaluation, Communication, Use tools and techniques, Use mathematics (adapted from NRC 2000)	Four levels (from teacher-initiated to student-initiated in each phase)
Chin et al. (2016)	Framework for IBL practices in mathematics lessons	Exposition, Discussion, Explanation, Elaboration, Classification, Challenge	Three levels (from teacher-centered to reform-oriented according to the interaction types)
Turner et al. (2018)	Scholastic Inquiry Observation (SIO); Evaluate IBL activities in lessons (not only for mathematics and science)	Hypotheses, Communication, Hands-on inquiry	Five levels (from teacher-led to student-led in each phase)
Danipog (2018)	Scientific Inquiry Teaching Observation Instrument (SITOI); Evaluate IBL practices in science lessons	Question, Investigation, Data collection, Data analysis, Explanation, Communication	Four to eight levels from teacher-centered to student-centered in each phase

IBL in mathematics education, we included a phase of "mathematization". This concept originated from Realistic Mathematics Education (RME) that considers "mathematics as a human activity" and students to be active participants (Van den

Heuvel-Panhuizen & Drijvers, 2020). Mathematization, which refers to the activity of organizing and studying problem situations with mathematical means, includes horizontal mathematization and vertical mathematization (Jupri & Drijvers, 2016). The former describes the transition from realistic contexts into mathematical symbols. The latter describes the process of reorganizing within mathematical symbols through generalizing, structuring and formalizing earlier mathematical results (Artigue & Blomhøj, 2013; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020). The mathematization phase refers to these horizontal transitions and vertical formalizations by students during mathematical inquiry.

In a previous study, we built upon the framework by Capps and Crawford (2013) to investigate IBL practices in mathematics education in Beijing through a textbook perspective and found a higher level of IBL that really provides opportunities for students to inquire by themselves is rarely present. However, the analysis of textbook tasks indicates a potential for but not actual use of IBL. To further understand the current situation, we continued to use a similar framework to investigate from the perspective of actual classroom practices. This study is taken as a starting point to show what might be potential for teachers' professional development programs focusing on IBL.

The research questions of this study are: 1) To what extent are structures of the required IBL lessons different from those of the observed usual lessons? 2) To what extent are opportunities for IBL provided in the two types of lessons, and what are the differences?

2. Method

2.1 Participants

Five Beijing mathematics teachers who had participated in a previous interview study of IBL agreed to participate in this study. Their information is shown in Table 5.2.

2.2 Data Collection

For each of the five teachers, the first author attended and observed his/her lessons for one week, which included usual lessons and a required IBL lesson. Teachers taught like they usually did in usual lessons. For the IBL lesson, the teacher was required to design and implement a mathematics lesson that includes elements of IBL based on his/her understanding of it. Table 5.3 shows information of these lessons.

Table 5.2 Information of teachers in the study and of students that participated in the survey of the IBL lessons

Teacher	Teacher information					Student information	
	Gender	Age	Years of experience	As a leading teacher in the district	School type	Number	Gender (F, M)
T1	F	43	22	No	Suburban	27	(11,16)
T2	F	40	19	No	Suburban	27	(13,14)
T3	F	35	10	No	Urban	22	(13,9)
T4	M	41	20	Yes	Suburban	24	(10,14)
T5	F	37	14	Yes	Urban	29	(15,14)

Note. F of gender refers to female, and M refers to male

Table 5.3 Information of the lessons observed in this study

Teacher	Grade	Number of usual lessons	Content of usual lessons	Content of the IBL lesson
T1	7	4	Application of linear equation with one unknown; Geometric shape	Linear equation with one unknown to solve real-life problems
T2	8	4	Construct with ruler and compass	A problem related to features of regular polygons
T3	8	4	Pythagoras theorem	Pythagoras theorem
T4	7	4	Geometric shape	Angle bisector theorem
T5	8	3	Quadratic radical	A real-life problem related to axial symmetry

The first author recorded the lessons as video or audio (audio only for T3) from the back of classrooms. At the same time, she observed these lessons using the framework in Table 5.4 and identified segments of classroom activities that extended or limited the opportunities for IBL included in the tasks involved. Then these segments of classroom activities were used in the teacher interview after each lesson. Teachers were mainly asked why they designed or organized in that way. There could also be additional questions such as “why did you change textbook tasks for use” and “what issues did you consider when designing the lesson”. After the required IBL lesson, teachers were asked to describe what made the lesson an IBL lesson, what supported or hindered the use of IBL in the lesson. Teachers were also required to report the frequency that a lesson like this lesson happened in his/her practices, to compare this lesson with his/her regular lessons and to indicate to what extent this lesson is an IBL lesson.

Students filled in a short survey after the required IBL lesson. They were asked whether they noticed five activities (see A1 to A5 in Table 5.6) happened in the lesson. The items were taken from the questionnaire in our previous study that focused on students' IBL experiences. These IBL-relevant activities can be related to some phases in our framework for coding lessons (see Table 5.4)[1]. Students were also asked to what degree this lesson is different from their regular mathematics lessons. There was an open question at the end for students to express which part of the lesson they liked or disliked and the reason. The lessons, interviews and surveys were all in Chinese.

2.3 Framework for Coding Lessons

With the videos or audios of these lessons, we coded them on structure and on opportunities for IBL.

The framework to code structure of lessons consists of two parts. The first part focuses on purposes of classroom activities with three mutually exclusive categories from TIMSS: review, introduce new content, practice new content (Hiebert et al., 2003). We further divided "introduce new content" into four sub categories: set up, explore (by students), discuss (solutions or the learning content), summarize (highlight and summarize the task). The second part focuses on forms of classroom interaction and was built upon TALIS (OECD, 2019). We combined the "small group" and "pairs" of categories in TALIS into "group" and got three categories: whole-class, group, individual.

The framework to evaluate opportunities for IBL in lessons was built upon the rubric of Capps and Crawford (2013). We adjusted the IBL processes based on literature (Chapman & Heater, 2010; Pedaste et al., 2015) and included a phase of "mathematization" to better suit the subject of mathematics. We used a similar rubric to code textbook tasks in a previous study.

The framework includes seven phases (see P1 to P7 in Table 5.4) of the IBL cycle. "Question" refers to the main problem to be tackled. "Hypothesis" refers to the conjecture before designing procedures and testifying. "Mathematization" refers to situations where students are invited to organize in a mathematical way, i.e., to visualize a problem situation by graph/table or geometrical tools, to formulate the problem through constructing variables and identifying their relations, to visualize

1 Complex problems are presented (A1) might be related to Mathematization (P3) and Procedure (P4). Pose questions (A2), Design procedures (A3) and Group work (A4) are related to Question (P1), Procedure (P4) and Collaboration (P5) respectively. Explain solutions (A5) can be related to P5 when students talk about solutions within pairs or groups, and to Communication (P6) when individual students explain to the whole class.

Table 5.4 Framework in this study for evaluating the level of IBL in seven phases in lessons

Phase	Level 0	Level 1	Level 2	Level 3
P1: Question	Not present; students follow commands provided by the task or the teacher	Students engage in questions provided by the task or the teacher	Students choose from provided questions or are guided in posing questions to tackle	Students have opportunities to pose questions to tackle by themselves
P2: Hypothesis	Not present	Students engage in a hypothesis provided by the task or the teacher	Students choose from or adapt provided hypotheses, or are guided in formulating a hypothesis	Students have opportunities to formulate a hypothesis by themselves
P3: Mathematization	Not present	Students apply means for organizing mathematically provided by the task or the teacher, or an explanation section/worked example previously in this chapter	Students choose from or adapt provided means, or are guided in designing the means for organizing mathematically or choose to refer to a means in knowledge before this chapter	Students have opportunities to design the means for organizing mathematically by themselves
P4: Procedure	Not present	Students apply solution procedures provided by the task or the teacher, or an explanation section/worked example previously in this chapter	Students choose from or adapt provided solution procedures, or are guided in designing the solution procedures or choose to refer to knowledge before this chapter	Students have opportunities to design the solution procedures by themselves
P5: Collaboration		Not present (Level 0)/ Present (Level 3)		
P6: Communication during whole-class discussion	Not present	Students are required to talk about and check their answers	Students are required to talk about their solution procedures or other phases in the IBL cycle	Students are required to discuss and justify different solution procedures or other phases in the IBL cycle
P7: Reflection at the end	Not present	Students are required to reflect on the answers (mathematics results)	Students are required to reflect on part of (like one or two phases in P1 to P7) the IBL cycle	Students are required to reflect on the whole IBL cycle

solution procedures, to generalize the solution into a rule or theorem. “Procedure” focuses on whether students apply a given “recipe” for solving the problem, or they need to design a procedure to approach the problem. “Collaboration” in this study refers to students working together with peers on a task in groups (including pairs). “Communication” focuses on individual students or a group of students orally explaining ideas or solutions to the whole class, but not include teacher lecture or teacher-class dialogue or students talking during group work. “Reflection” focuses on students rethinking their findings or actions.

The framework evaluates to what extent students have opportunities to explore and invent by themselves through four levels, of which Level 0 is the lowest and Level 3 is the highest. In general, Level 0 means that this phase is not involved in the lesson; Level 1 means that all the needed information has been provided for students to follow or apply; Level 2 means that the needed information is partly provided for students to choose from or adapt or use under guidance; Level 3 means that the needed information is not provided and students have to investigate by themselves. Level 2 or Level 3 in P1 to P7 indicate opportunities for IBL. In P7 we also took Level 1 as an indication of IBL because it already involves students to reflect.

2.4 Coding

The first author did all the coding with the frameworks above. The coding was based on the original videos or audios of lessons in Chinese. Non-math activities (e.g., give out worksheets or hand in homework) at the beginning or end of a lesson were not included for coding because they are not relevant to the focus of this study.

As for structures of all the 24 lessons, she coded the purposes of classroom activities and forms of classroom interaction using NVivo 12. The coding of lesson structure is useful for selecting lessons in a systematic way for further analysis (Watson & Evans, 2012). Based on the percentages of categories in purposes, we selected a usual lesson that is typical (closest to the average percentages on this issue in the teacher’s usual lessons that we observed) for each teacher.

Then the first author coded opportunities for IBL in the five selected usual lessons and the five required IBL lessons, i.e., 10 lessons in total. Each lesson was divided into segments related to different tasks, and each segment was coded according to the level of IBL in the seven phases. After coding the lessons, we used the highest level of IBL in each phase within a lesson as the indicator for the level of inquiry in that phase. Those segments difficult to code were discussed by the authors and the decisions were taken as guidelines for coding.

Moreover, main segments of IBL practices were coded to prepare for the presentation of timelines. There are two criteria for the selection of main segments: show the highest level of IBL in each phase within the lesson, and practices without opportunities for IBL (Level 0 or Level 1 in P1 to P6, or Level 0 in P7) were ignored.

2.5 Reliability

The frameworks for coding structure of lessons came from large-scale international projects and had been validated. In order to assure reliability of coding opportunities for IBL, we invited an external researcher to perform a check. She was a Chinese PhD student in the field of mathematics education at a Dutch university. She was asked to select two lessons (20%)[2] before the check.

The check was conducted online through Teams. The first author introduced the external researcher to the framework in Table 5.4 and the way to code with examples. Next, the video of each lesson segment was played. The two coders individually coded the segment with the framework. Then they compared their coding and discussed the disagreements. After that, they moved on to the next segment.

A lesson segment was first coded and discussed for pilot. Then the two selected lessons were coded. The two coders got an initial agreement of 76%. The disagreements were discussed, mainly about P1 (Question), P2 (Hypothesis), P5 (Collaboration) and P6 (Communication). For example, it was clarified that the question considered in P1 refers to the main problem to be solved rather than questions pointed out by students as difficulties. It was also clarified that a hypothesis comes before students design procedures and it is different from trying out various ways to approach the problem. The external researcher provided advice that P5 and P6 should be further explained in the framework. An agreement of 93% was reached after discussion. These reflections were included in the coding that followed.

2.6 Procedure for Analysis

To begin with, we focused on structure of the 24 lessons and calculated percentages of categories in purposes and forms. For each teacher, the average percentages for the observed usual lessons were calculated.

Then we continued to analyze based on the ten selected lessons (The selection has been explained in "Coding"). We graphically represented the results of coding

2 She was required to randomly choose one lesson in each of two columns, while she was not informed that there are two types of lessons (the five selected usual lessons in one column and the five required IBL lessons in the other column)

opportunities for IBL in lessons (see Fig. 5.2). Features related to the levels of IBL in each phase were identified and compared among teachers.

Next, we analyzed data from teacher interview and student survey to provide more insights, especially about the degree that the required IBL lessons are the same as or different from their usual lessons. Results from the perspective of students, teachers and our coding of lessons were triangulated.

In addition, we presented timelines of structures and main segments of IBL practices in the ten selected lessons to show an overview of the distribution. The figure (Fig. 5.4) was redrawn based on original figures generated from NVivo to make categories belonging to the same topic (purposes, forms, main IBL practices) shown in a line, which is easier to read and compare. Features appeared in these timelines were identified.

For all the results above, we compared between situations in the required IBL lessons and those in the usual lessons that we observed and/or further selected.

Moreover, we presented cases of three teachers with timelines of purposes and details of lessons to show a more complete picture of what happened in their "typical" usual lessons and the required IBL lessons. T2 and T5 were taken as cases because they changed IBL levels in more phases than what the other teachers did when designing the IBL lesson. Besides, their IBL lessons showed relatively more opportunities for IBL in more phases among the ten lessons. T1 was also selected as a case because the opportunities for IBL were relatively less in her usual lesson compared with the lessons of other teachers. We provided details of how these teachers organized lesson structures and conducted inquiry practices, also some relevant information from teacher interview and student survey. Content of lessons and quotations of teachers were translated from Chinese into English.

3. Results

3.1 Structures of Purposes of Classroom Activities and Forms of Classroom Interaction in All the Observed Lessons

Table 5.5 shows the percentages of categories in purposes and forms in all the 24 lessons.

Every teacher spent a greater proportion of time to deal with new content in the required IBL lesson than in the observed usual lessons. However, teachers showed various choices regarding the change of time on the four specific activities of setting up, exploring, discussing and summarizing, of which we did not find a shared feature. T3 and T4 did not distribute more time for students to explore in the required IBL lesson than in their usual lessons. Whole-class activity was the dominating form in both types of lessons. In the IBL lessons, only T2 assigned about 30% of lesson time to conduct group work, while T1 and T3 did not organize any group work.

Table 5.5 Coverage of time (%) of purposes and forms in the observed usual lessons and in the required IBL lesson for each teacher

Categories		T1		T2		T3		T4		T5	
		Usu	IBL	Usu	IBL	Usu	IBL	Usu	IBL	Usu	IBL
Purposes	Review	24	35	13	0	56	0	13	4	23	0
	Introduce new content	13	47	68	97	27	75	53	58	49	98
	Set up	*1*	*3*	*8*	*4*	*2*	*13*	*4*	*11*	*4*	*17*
	Explore	*0*	*6*	*11*	*33*	*4*	*4*	*9*	*5*	*8*	*28*
	Discuss	*10*	*25*	*43*	*47*	*18*	*55*	*32*	*31*	*30*	*49*
	Summarize	*3*	*13*	*5*	*12*	*3*	*3*	*6*	*11*	*8*	*4*
	Practice new content	63	18	19	3	16	25	34	39	27	2
Forms	Whole-class	69	72	65	65	93	95	76	88	91	72
	Group	1	0	3	33	0	0	1	8	0	8
	Individual	29	28	32	1	7	5	23	4	9	20

Note. Usu refers to the observed usual lessons, and IBL refers to the required IBL lesson; Content in italic are four specific activities under the category of "introduce new content" and their proportions in lesson time. Some percentages do not add up correctly due to rounding errors

3.2 Opportunities for IBL in the Ten Selected Lessons

IBL practices of the teachers in the five selected usual lessons and in the five required IBL lessons were analyzed. Fig. 5.2 shows opportunities for IBL characterized by the highest level of IBL in each of the seven IBL phases within each lesson.

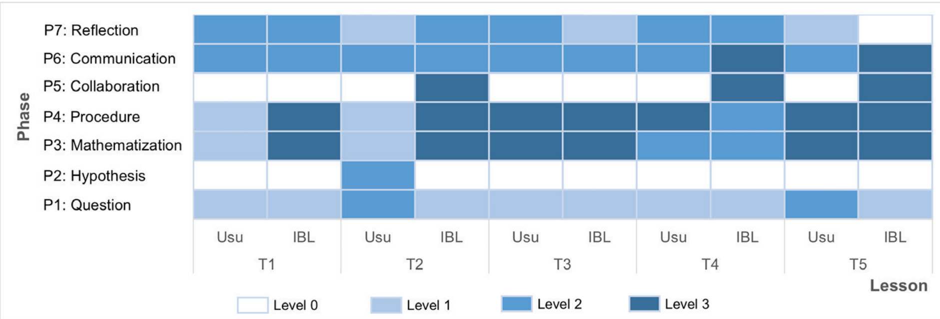
It can be seen that opportunities (Level 2) for IBL in P1 (Question) and P2 (Hypothesis) were only present in the usual lessons of T2 and T5. All the IBL lessons lacked opportunities for IBL in these two phases: questions were directly provided to students and no hypotheses were involved.

There were already opportunities (Level 3) for IBL for students to organize mathematically (P3) and design procedures (P4) in the usual lessons of T3 and T5. These opportunities were kept in their IBL lessons. For T1 and T2, although no opportunities for IBL in P3 and P4 were shown in the usual lessons, the opportunities were highly extended in their IBL lessons. For T4, the opportunities in P3 and P4 were not extended and even narrowed a bit in his IBL lesson.

As for student' collaboration (P5) in group work, no opportunities for IBL were shown in any of the usual lessons, while T2, T4 and T5 conducted P5 in their IBL lessons.

The teachers all provided IBL opportunities for students to communicate during whole-class discussion (Level 2 in P6) and reflect (Level 2 or Level 1 in P7) in the usual lessons. The opportunities in P6 were kept or extended in their IBL lessons. However, the opportunities in P7 were narrowed by T3 and not provided by T5 in the IBL lessons.

The teachers conducted inquiry practices in P3, P4, P6 and P7 in the five selected usual lessons. Generally the opportunities in P3, P4 and P6 were kept or extended when they designed and implemented the IBL lessons, while those in P7 were a bit narrowed. Some teachers paid attention to conduct inquiry practices in P5 in the IBL lessons. Level 3 as a higher level of IBL were mainly shown in P3, P4 in both types of lessons, also in P5 and P6 in some IBL lessons, but not in the other phases in these lessons. More opportunities for IBL are necessary, especially in P1 and P2 that were ignored in both types of lessons. Students need to be invited to think about questions to tackle by themselves, and more conjectures should be involved.



Note. Usu refers to the selected usual lesson, and IBL refers to the required IBL lesson

Fig. 5.2 Opportunities for IBL in the selected usual lesson and in the required IBL lesson for each teacher

Students were asked whether they noticed five activities relevant to IBL (see A1 to A5 in Table 5.6) happened in the required IBL lesson. Most of the results tend to match results above in Fig. 5.2 based on our coding of lessons. For example, more than 80% of students in each class chose “Yes” for designing own procedures (A3), which is in line with what is shown in Fig. 5.2 that opportunities for IBL were provided in Procedure (P4) in the five IBL lessons. However, some results in Table 5.6 seem not in line with our analyses of the actual lessons. Students in each class answered that they got opportunities to pose questions (above 80% Yes in A2), while we did not identify opportunities in this phase (Level 2-3 in P1) in the IBL lessons. This reflects that students might mix posing questions to tackle (A2) with asking questions when they encounter difficulties during working on tasks. Students of T1 reported that group work happened in the IBL lesson (85% Yes in A4), while we did not identify it (P5, Collaboration) in our analysis of that lesson. The reason can be that T1 once required students to discuss in small groups during the lesson, while students did not react and kept silent, thus we did not code the segment as Collaboration (P5). Another interpretation is that students might mix group work (A4) with informal talk with neighbors.

As is shown in Fig. 5.2, among the five selected usual lessons, there were relatively more opportunities for IBL in more phases in the lessons of T3 and T5, while less in the lesson of T1. For the five IBL lessons, those of T2, T4 and T5 showed relatively more opportunities for IBL in more phases. However, three teachers (T3, T4, T5) regarded their IBL lesson as “considerably” an IBL lesson in interviews, and two teachers (T1, T2) regarded it as “moderately”. The reports of T2 and T3 do not match our coding of lessons. T2 seemed to underestimate the level of IBL in her lesson and T3 seemed to overreport it.

After analyzing IBL levels in seven phases in the IBL lesson compared with those in the selected usual lesson for each teacher (see Fig. 5.2), we identified that IBL levels change in six, three and four phases for T2, T4 and T5 respectively. However, the changes involved both increase and decrease in levels for each teacher. Increase was also shown for T1 but only in two phases, and decrease for T3 in one phase. We cannot identify a teacher who made the biggest change or increase in IBL levels.

Differences were considered according to the number of phases with changes in IBL levels in the two selected lessons for each teacher. As for the differences, the rank of teachers based on our coding (see Fig. 5.2) is T2>T5>T4>T1>T3, and the rank based on students’ perceptions (see Fig. 5.3) is T2>T5>T4>T3>T1, and the rank based on teachers’ perceptions (see answers to Q2 in Table 5.7) is T2>T5/T4/T3>T1. Ranks from the perspective of students, teachers and our coding of lessons almost match, except for the position of T1. However, the degree of differences can be not the same.

Table 5.6 Students' reports of whether they noticed five activities happened in the required IBL lesson

Activity	T1(%)			T2(%)			T3(%)			T4(%)			T5(%)		
	No	Not sure	Yes	No	Not sure	Yes	No	Not sure	Yes	No	Not sure	Yes	No	Not sure	Yes
A1: Mathematical problems without obvious solution procedure are presented	41	26	33	11	11	78	23	32	46	17	13	71	14	24	62
A2: Pose own questions to tackle or to work on	0	0	100	0	0	100	5	5	91	0	0	100	7	10	83
A3: Design own procedures for solving complex problems	4	7	89	0	0	100	5	9	86	4	13	83	10	0	90
A4: Pair or group work	7	7	85	0	0	100	46	32	23	8	0	92	7	7	86
A5: Explain solutions to other students	7	7	85	0	0	100	5	0	96	4	8	88	7	3	90

Note. Some percentages do not add up to 100% due to rounding errors

Students' and teachers' perceptions are a bit different in that T2 thought her IBL lesson "quite different" from her usual lessons while students felt "quite the same". It should be noted that "usual lessons" in their perceptions were not limited to the usual lessons that we observed.

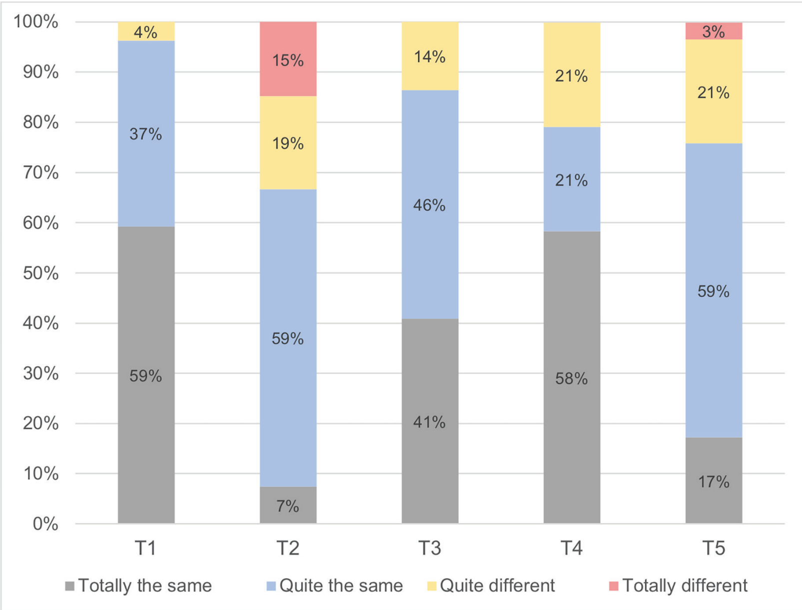
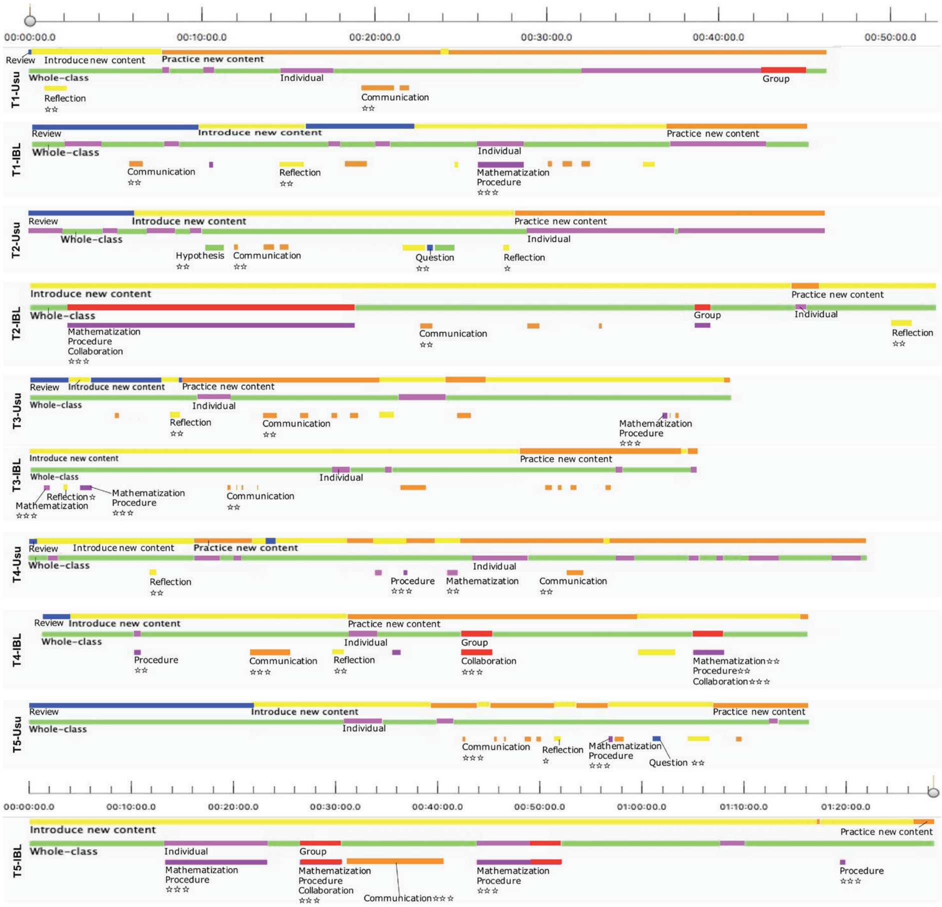


Fig. 5.3 Students' perception of how the required IBL lesson compares to their usual lessons

According to teachers' reports below, the degree of differences between the required IBL lesson and usual lessons seems in line with the frequency of lessons like the IBL lessons: more frequent, more similar. However, this is not the case for T5.

Table 5.7 Teachers' reports of two questions during interviews

Question	T1	T2	T3	T4	T5
Q1: How often do lessons like the required IBL lesson happen in your teaching practices?	Every class	Never or hardly ever	Weekly	Weekly	Occasionally
Q2: To what extent is the required IBL lesson the same as or different from your usual lessons?	Totally the same	Quite different	Quite the same	Quite the same	Quite the same



Note. Usu refers to the selected usual lesson and IBL refers to the required IBL lesson; ☆☆☆ refers to Level 3, ☆☆ refers to Level 2, and ☆ refers to Level 1; T5 spent two lesson periods on the IBL lesson and it lasted for about 90 minutes. The other lessons lasted for around 40 to 50 minutes.

Fig. 5.4 Timeline of structures and main segments of IBL practices in the ten selected lessons

3.3 Overview of Structures and Main Segments of IBL Practices in the Ten Selected Lessons

Fig. 5.4 shows the distribution of purposes and forms as well as main segments of IBL practices in the five selected usual lessons and the five required IBL lessons. For example, the figure begins to show the situation in the usual lesson of T1. The lesson lasted for about 46 minutes. The first line of bars represents purposes. Below are the marks of categories, e.g., yellow bars refer to “introduce new content”. Forms are shown in the second line of bars. The third line consists of discontinuous bars, which represent main segments of IBL practices. Level 2 in P6 (Communication) and P7 (Reflection) were present in this lesson. For purposes, the selected usual lessons of teachers except T3 showed a pattern of "review → introduce new content → practice new content". The form of whole-class played an important role in all the lessons.

Main IBL practices appeared at all stages (beginning, middle, end) of the lessons. Features can be identified considering the period of main segments in the two lessons of the teachers: 1) T2 and T5 spent a much greater proportion of time to involve main segments of IBL practices in the IBL lessons than in their selected usual lessons. The added time was mainly spent for students to figure out means for mathematization (P3), design own solution procedures (P4) and collaborate in groups (P5). T5 also provided more time for students to communicate ideas during whole-class discussion (P6); 2) T3 almost did not adjust the distribution of time on main segments of IBL practices in the IBL lesson compared with her usual lesson.

3.4 Examples to Illustrate Opportunities for IBL in Mathematics Lessons

Below are three cases to show in detail what T1 (see Table 5.8), T2 (see Table 5.9) and T5 (see Table 5.10) did in the selected usual lesson and the required IBL lesson, and how they talked about the lessons in interviews. T2 and T5 made relatively more changes when designing the IBL lesson and their IBL lessons showed relatively more opportunities for IBL in more phases. On the contrary, the usual lesson of T1 showed relatively less opportunities for IBL. Timelines in the three tables are the same as timelines of purposes in their lessons in Fig. 5.4. Here the timelines are presented together with specific content of the lessons.

3.4.1 The Case of T1

T1's usual lesson started with a short review and dealt with a new task related to constructing a given line segment with ruler and compass, which led to summarized procedures about this type of problem. The remaining part was for students to recite and memorize these procedures, and apply the procedures with more exercises. Opportunities for IBL were still provided in some situations. For example, students were required to reflect on why they need to draw a half-line first, which showed Level 2 in P7 (Reflection) and was abbreviated as P7L2.

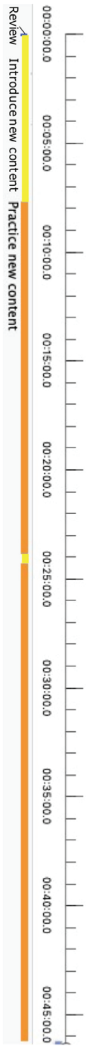
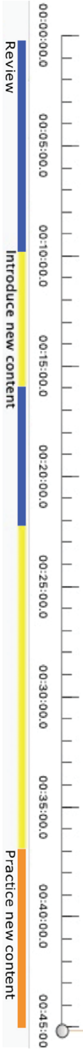
The IBL lesson of T1 was a demonstration lesson observed by several teachers. T1 spent more time to introduce new content and less time to practice in the lesson. She relied on three tasks that are at least partly open. Students cannot apply given variables for organizing mathematically or given procedures for solving. They had to investigate by themselves, which provided opportunities for IBL in P3 (Mathematization) and P4 (Procedure). Students got opportunities to talk about solutions in whole-class discussion (P6L2), for example, about different plans for renting boats to make the expense lower. They were also required to rethink about the rule to consider in solving the problem (P7L2), i.e., finding out the key variable.

As for elements of IBL represented in the required IBL lesson, T1 explained that "Students actively thought about the problems. They could use the experience and mathematical thinking they got in this lesson to solve similar problems in real life. I think this is inquiry". She seemed to connect IBL to the thinking activity of students and the outcome to solve real-life problems. In addition, T1 regarded this lesson as "moderately" an IBL lesson in relation to the openness of tasks: the third task contains a good problem (about expenses of swimming with a membership card or with single tickets, while the times of swimming as the key variable is implicit, students need to figure it out themselves) while there are guidance (with sub questions related to intermediate steps) in the first two tasks.

She talked about how she made choices in the setting of tasks. For example, the first task (see Fig. 5.5) came from the 2018 entrance examination to high schools at Beijing. The original form of the task can be seen in Fig. 5.6. Although T1 took reformed examinations as a factor that inspired her to implement IBL and included such a task to challenge students, she added two sub questions, i.e., (1) and (2). (1) can be easily solved applying what students learnt as procedures in previous lessons. (2) was added as an intermediate step mainly because of an experience in teaching (when T1 provided this task last year, students did not think about the variable of price per person, so she gave the hint this year). The inclusion of subtasks limited the opportunities for students to approach this problem by themselves (P4). It is interesting that T1 added more guidance to this task and used it in an IBL lesson. T1 expressed that this choice was impacted by the intention to connect the task to prior knowledge and by an experience in teaching. This choice might also be impacted by the aim to make students familiar with the steps for solving this type of problems and prepare them for examinations.

It can also be noticed that students kept silent when T1 asked them to talk in pairs after individual thinking, thus P5 (Collaboration) was not really involved. T1 acknowledged in the interview that students were not adjusted to group work. She thought factors related to students hindered more involvement of IBL, "little experience of life, not enough abilities to do hand-on activities…not used to think independently". Students indicated in the survey that what they liked most in this lesson was working on the tasks by themselves, which was reported by 37% of the students.

Table 5.8 Timeline and content of the selected usual lesson and the required IBL lesson of T1

Usual lesson		IBL lesson	
	Review		Review: Procedures to solve real-life problems of linear equation with one unknown
	Set up the task: *construct a given line segment with ruler & compass*		• Give the task: *Plan for renting boats* *The first two subtasks that require students to calculate price and price per person can be solved with prior knowledge* • Students think individually, the whole class checks, individual students talk about solutions (P6L2) • Set up the third subtask that is open: *It asks for a plan with lower expense* • Students think (P3L3, P4L3), discuss solutions during whole-class session, summarize through reflecting on the rule to consider (P7L2)
	Discuss, reflect on why to start with a half-line (P7L2), talk about procedures and summarize procedures		
	Practice new content • Recite and memorize the procedures individually and with the whole class		
	• Give an exercise: *construct a line segment as the sum of two given line segments (a+b) with ruler & compass* • Individual work • Whole-class check, individual students talk about procedures (P6L2), T1 repeats to emphasize • Highlight a term		• Give the task: *Plan for buying tickets* *The first subtask (calculate the number of adults and students based on the expense) can be solved with prior knowledge* • Individual work, individual students talk about solutions to the whole class (P6L2) • Set up the second subtask: *Buy tickets with lower expense;* The opportunities in P4 are underused because T1 gives hints quickly • Students discuss and reflect (P7L2)
	Exercise: *construct 2a+b*		
	Exercise: *construct the difference between two given line segments (a-b) with ruler & compass*		• Set up the task: *Apply for a membership card or buy a ticket each time*; Variables are implicit and need to be figured out • Students explore individually (P3L3, P4L3); T1 organizes group work but students do not talk with each other (P5L0) • Whole-class discussion; Individual students talk about solutions (P6L2) • Students are guided to reflect (P7L2) and summarize how to solve this type of problem
	• Exercise: *construct a, a+b, a-b with ruler & compass* • Students construct the line segments individually to practice • Assign one student to check the worksheets of other students in each group • Collect questions		Practice new content • Students do a similar exercise individually • Summarize the lesson and assign homework

例1. 某公园划船项目收费标准如下：The price for boats in a park is as follows:

船型 Type of boat	两人船 For two person （限乘两人）	四人船 For four people （限乘四人）	六人船 For six people （限乘六人）	八人船 For eight people （限乘八人）
Price per hour 每船租金（元/小时）		100		150
Price per person 人均				

某班18名同学一起去该公园划船，若每人划船的时间均为1小时，
18 students go to the park for boating for one hour

（1） 若选择两人船3条和六人船2条，共需花费530元，已知每条两人船比六人船每小时租金少40元，则两人船租金________元，六人船租金________元.
(1) If they choose five boats for two person and two boats for six people, the expense is 710. Price for the two-person boat is 40 lower than the six-person boat per hour, what is the price for each type of boat?

（2）求出每条船的人均花费并填入表格中.
(2) Calculate the price per person for each type of boat and fill in the table

（3）你有更低的费用选择选择方案吗？需花费________元.
(3) Do you have plans with lower expense? The expense would be ?

Fig. 5.5 Task one in the worksheet of T1’s IBL lesson

（第 15 题）某公园划船项目收费标准如下：The price for boats in a park is as follows:

船型 Type of boat	两人船 For two person (限乘两人)	四人船 For four people (限乘四人)	六人船 For six people (限乘六人)	八人船 For eight people (限乘八人)
每船租金 Price per hour (元/小时)	90	100	130	150

某班 18 名同学一起去该公园划船，若每人划船的时间均为 1 小时，则租船的总费用最低为____元. 18 students go to the park for boating for one hour, the lowest expense would be?

Fig. 5.6 The original form of task one before being adapted by T1

3.4.2 The Case of T2

For T2, her usual lesson showed a standard pattern of “review → introduce new content → practice new content”. When dealing with new content (the theorem and converse theorem), students were guided to observe, hypothesize, clarify the hypothesis, explain their mathematical proof, describe finding in words, compare with the theorem and extract the theorem. These activities seemed well-designed and coherent. The lesson was special in showing IBL opportunities in P1 (Question) and P2 (Hypothesis). However, students did not get opportunities to explore by themselves before the whole-class discussion, and the solving process was supported by T2 and can be achieved using prior knowledge. Nor did students get chances to collaborate with peers.

The IBL lesson of T2 was almost all about introducing new content. It was based on a task with a context of honeycombs to inquire reasons for the shape of regular hexagons and features of regular polygons. The task came from a textbook task (see the left part in Fig. 5.7) that is optional and was usually skipped in teaching.

T2 adapted it into a new version (see the right part in Fig. 5.7) to use in the lesson. She thought the intermediate steps of "experiments" would limit students' own ideas, thus she left them out and allowed students to make decisions by themselves about how to approach the problem. Also, she required students to formulate a conclusion based on their findings and reasoning at the end. These revisions made the task more open and provided more opportunities for students to inquire. The new version of the task itself contains many opportunities (Level 3) in P3 (Mathematization) and P4 (Procedure). Students need to identify the key variable to formulate the mathematical problem, figure out procedures for solving, and design means for visualizing solution procedures. The question at the end seems potential for students to reflect on the findings (P7L1).

T2 expressed in the interview that at first she considered to let students explore the task without sub questions provided. However, that is too difficult for these starters without much experience of IBL. Therefore, guiding questions are still necessary. She did not like "Question 2" in that it might indicate the answer of "Question 1", while she had to keep it to support students. She also changed the perimeter from 6 to 24 to make all length of the sides whole numbers, which is easier for students to calculate and compare.

Before the IBL lesson, students' seats were adjusted to be in groups and worksheets were given out to each student. In the lesson, students had the chance to explore the task on worksheets by themselves in groups. T2 did not explain in advance while she supported students in need during the process. She mainly listened to students' ideas first and used questions or provided hints to help them to move forward. T2 achieved the IBL opportunities included in the task, and she extended those in P5 (Collaboration), P6 (Communication), also increased the level of IBL in P7 (Reflection). However, she did not require students to question or hypothesize mathematical issues that emerged during the lesson.

Honeycombs and regular hexagons

(a picture of a honeycombs is attached)

A beehive consists of many honeycombs. The cross section of a honeycomb is a regular hexagon. Polygons with equal sides and equal inner angles were defined as regular polygons.

Question 1: Which type of congruent regular polygons can cover a surface with no gaps and no overlaps? This is called the tessellation of plane figures.

Experiment 1: Can congruent regular triangles make a tessellation? Try out with scraps of paper.

Experiment 2: Can congruent squares make a tessellation? Try out with scraps of paper.

Experiment 3: Can congruent regular pentagons make a tessellation? Try out with scraps of paper.

Think: Any other regular polygons can make a tessellation?

Question 2: Which of the figures have the largest area: a regular hexagon, a square, a regular triangle, each with a perimeter of 6.

The area of a regular hexagon with a perimeter of 6 is__________

The area of a square with a perimeter of 6 is__________

The area of a regular triangle with a perimeter of 6 is__________

As is shown in the content above, there are three types of regular polygons that can make a tessellation. They are regular hexagons, squares and regular triangles. When the perimeter is a set value, the area of a regular hexagon (among the three figures) is the largest. Conversely, when the area is a set value, the perimeter of a regular hexagon among the three figures is the smallest.

Regular hexagons were taken as the cross sections of honeycombs to save materials.

Inquire reasons for the shape of honeycombs

A beehive consists of many honeycombs. The cross section of a honeycomb is a regular hexagon. Polygons with equal sides and equal inner angles were defined as regular polygons.

Question 1: Which type of congruent regular polygons can cover a surface with no gaps and no overlaps? This is called the tessellation of plane figures.

Think: Which type of regular polygons can make a tessellation?

Question 2: Which of the figures have the largest area: a regular hexagon, a square, a regular triangle, each with a perimeter of 24.

The area of a regular hexagon with a perimeter of 24 is__________

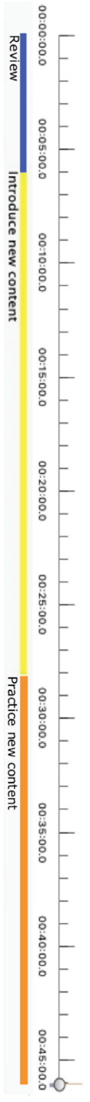
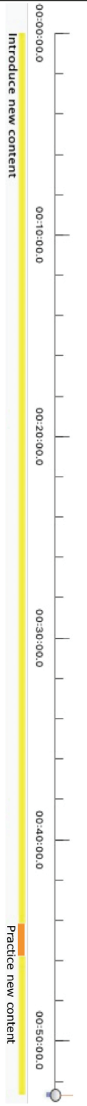
The area of a square with a perimeter of 24 is__________

The area of a regular triangle with a perimeter of 24 is__________

What can be learnt from the content above (what can be concluded from your research findings)?

Fig. 5.7 The task of honeycombs in the original form and in the new version by the teacher

Table 5.9 Timeline and content of the selected usual lesson and the required IBL lesson of T2

Usual lesson		IBL lesson	
	Review • Students draw the bisector of a given angle with ruler and compass individually; A student draws on the blackboard		Set up the task; T2 introduces the context (honeycombs) and the task: *inquire reasons for the shape of honeycombs*
	Set up the task • T2 asks students to draw the shape individually and poses the question: *What is the relationship between the two distances?*		Students explore in groups (P3L3, P4L3, P5L3) • The task (see picture on the right in Fig.7) includes two subtasks and is shown on worksheets • Students work and discuss in groups • T2 walks around to support students in need
	Discuss solutions • Students are guided to consider two types of relationships (positional and quantitative) • They think the distances equal and are guided to clarify the hypothesis (P2L2); Individual students explain mathematical proof (P6L2) • They try but cannot hypothesize for the positional relationship • They are guided to describe what they find in words and compare it with the theorem in the textbook		Whole-class discussion • A student reports a difficulty in subtask one: How to draw a standard regular octagon • Two individual students explain their ideas to help (P6L2)
	Highlight and summarize; Students reflect and extract the theorem (P7L1)		Whole-class discussion • The solution to subtask one by a group is • presented to the class • An individual student describes her reasoning (P6L2) and T2 helps to explain it
	Set up; Students are guided to pose a new question: *Is the converse theorem true?*(P1L2)		Highlight and summarize; T2 directs students to think about other examples in real life
	• Discuss solutions: organized in the same way as "discuss solutions" above • Highlight and summarize; Students reflect on the two theorems (P7L1)		Move on to discuss subtask two • Students continue to work in groups for a while (P3L3, P4L3, P5L3) • Whole-class discussion to check answers and analyze solution procedures • Practice new content; Students apply and calculate individually • Whole class discussion; Students generate a conclusion from the results
	Practice new content • Students work on four exercises individually; T2 walks around to support students in need • T2 asks students that have difficulties with the first exercise to come to the front together and discusses it with them • Students continue to work individually • T2 organizes the second exercise in the same way as in the first exercise until the bell rings		Highlight and summarize • T2 directs students to think about reasons for the shape (regular hexagon) of honeycombs • Students are guided to reflect on the solving process and their collaboration (P7L2)

T2 indicated in the interview that she used to organize small IBL activities in lessons and this was her first time to conduct an IBL lesson. When talking about what makes it an IBL lesson, she emphasized the feature of the task in use: "The problem is open and it comes from real life. It is not a standard task…Students need to interpret it using prior knowledge by themselves". She pointed out that students' collaboration hindered more involvement of IBL because they just started to have group work and were not yet cooperating well. T2 also expressed that if there were more time, she would allow students to share and communicate their solving process. Students indicated in the survey that what they liked most in this lesson were the hands-on activities to draw and try out by themselves, which was reported by 56% of the students.

3.4.3 The Case of T5

The usual lesson of T5 focused on quadratic radical. The lesson showed balanced sections of reviewing, introducing new content and practicing. There were a few occasions for individual students to talk about their solutions to the whole class and show Level 2 in P6 (Communication). T5 asked students what can be analyzed then after they analyzed the result of $\sqrt{a^2}$. It presented Level 2 in P1 (Question) when students were guided to pose the question that what would be the result of $(\sqrt{a})^2$. Students need to investigate the two results, generalize new rules as well as reflect on the similarities and differences between them, which showed Level 3 in P3 (Mathematization) and P4 (Procedure) as well as Level 1 in P7 (Reflection).

The IBL lesson of T5 is a practical lesson (at least once each semester) and the content was not assigned. Teachers could select the content and design the lesson by themselves. This lesson was a demonstration lesson observed by a few teachers and conducted in a specific classroom, and students' seats were changed to be in groups. It lasted for two lesson periods (about 90 minutes) with enough time for students to think and discuss. The lesson was almost all about introducing new content. It focused on a task with a real-life context, i.e., International Horticultural Exposition at Beijing. The exposition has a wide area for visiting, thus it is necessary to offer visitors a route guide to reach the nearest exit among the five gates (see a map of the zone in Fig. 5.8). The main question is how to design this route guide.

The task itself contains many opportunities (Level 3) for students to investigate procedures for solving (P4) and to organize mathematically (P3). For P3, the task needs to be transferred from a real-life context (zone of the exposition) to a mathematical problem (zone as a polygon, each place and exit as a dot). The means for visualizing solution procedures also need to be designed and the solutions are required to be generalized into a rule.

Students got enough time to explore the task by themselves during the lesson. It is interesting that T5 required students to first explore individually for ten minutes and emphasized that they should not communicate with each other. Students got the chance to work in groups later, then they presented and discussed different solutions during whole-class discussion, which involve Level 3 in P5 (Collaboration) and P6 (Communication). It is a pity that students did not have the opportunity to reflect by themselves.

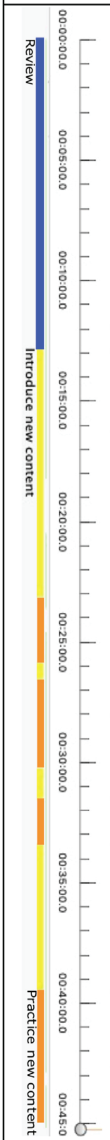
The task might be too challenging for students that T5 chose to guide and support them during the lesson. In the beginning stage, T5 guided students to clarify the requirement of the task that the aim is to find the nearest exit for all the places within the zone. She also helped students with the horizontal mathematization to transfer the real-life context to a mathematical problem. There were several rounds of whole-class discussion to help students to make progress, during which students had the chance to express their ideas and to get support when necessary.

Fig. 5.8 Map used in the task in the worksheet of T5's IBL lesson

T5 regarded the IBL lesson as "considerably" an IBL lesson in the interview. She explained her understanding of IBL and its elements shown in the lesson, "Students actively thought and researched by themselves…It (the task) is not a clear mathematical problem and requires transition, which needs to be inquired…When students came to the mathematical problem, at first they did not know what knowledge to use, then they still cannot solve it because the lines they drew were not enough or too many. They had to find the way to solve it step by step…Maybe IBL is represented in this process with their mathematical thinking being promoted". She acknowledged that for difficult parts she directed students and provided much information. She thought support and guidance still necessary for students, "For example Pythagorean Theorem, so many mathematicians spent so much time to get it, how could students inquire it during one lesson? Maybe it is enough for them to experience it under teacher guidance. The so-called inquiry can be like this".

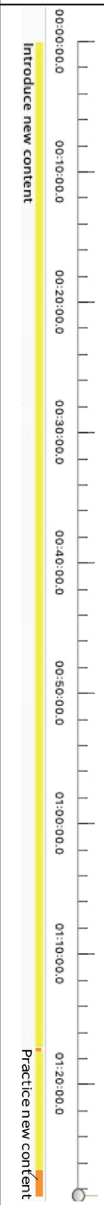
Table 5.10 Timeline and content of the selected usual lesson and the required IBL lesson of T5

Usual lesson



Review • Structure and content of prior knowledge
Review: Square root
• Set up the task: What does *√-3 mean?* • Discuss and summarize the rule, *for √a a≥0*
• Task: *from √a to ³√a and ¹⁰√a* • Discuss solutions
• Set up the task: *What to learn about quadratic radical?* • Students solve individually • Whole-class discussion
• Exercise: *identify examples of quadratic radical* • Individual work; Students talk about solutions (P6L2) during whole-class check; T5 summarizes this type of problem
• Exercise: *What makes √X-2 exist?* • Students explain solutions (P6L2) • Students reflect (P7L1) and T5 summarizes a rule: *a≥0 & √a≥0*
• Exercise: *What makes $\sqrt{a^2}$ exist?* • Whole-class check
• Set up the task: $\sqrt{a^2}=$ • Students explore (P3L3, P4L3) and talk about solutions (P6L2) during whole-class discussion • Students are guided to pose another task (P1L2): $(\sqrt{a})^2=$ • Discuss solutions, reflect and compare the two rules (P7L1)
• Exercise: *What makes √(X+3)/X-1 exist?* • Students talk about solutions (P6L2) during whole-class check
Exercise: *What makes $\sqrt{(-X^2+2X-1)}$, $\sqrt{(-X^2-2X-3)}$ exist?* Individual work
Assign homework

IBL lesson



Set up the task • Introduce the context: Share experience of visiting EXPO 2019 (International Horticultural Exposition) and watch a video • Give the problem: *How to design a route guide for visitors to reach the nearest exit?* • Clarify the task
Horizontal mathematization: From zone of the exposition to zone as polygon & exits as dots
Students explore individually (P3L3, P4L3)
Whole-class discussion to point out the direction: Find all exists and divide the zone
Students continue to explore in groups (P3L3, P4L3, P5L3)
Whole-class discussion • Group presentation, students discuss and justify different solutions procedures (P6L3) • Knowledge involved: Perpendicular bisector; Construct with ruler & compass
Students continue to explore individually to work on a better plan, and then to work in groups (P3L3, P4L3, P5L3)
Whole-class discussion • Group presentation • Discuss the rule: How many lines to consider with 2, 3…n dots (exits)
Highlight and summarize the rule: Consider n(n-1)/2 lines with n dots (exits)
T5 directs students to clarify solution procedures
Students solve individually: Find the zone to reach Gate1
Discuss solutions (how to find the zone that is nearest to each exit) and summarize solution procedures
Assign part of the homework
• Set up a subtask: *Assign exits with functions* • Discuss solutions of the subtask
Assign homework

T5 mentioned that the design of the task was inspired by an experience in teacher training, during which she was provided with a problem to design routes to the nearest exist and she found it really interesting. T5 thought that the lack of time and factors related to students' knowledge and motivation hindered more involvement of IBL in the lesson. She also reflected on her usual lessons that she seldom provided students opportunities to express their ideas or organized group work. Students indicated in the survey that what they liked most in the IBL lesson were the hands-on activities to draw routes by themselves, which was reported by 28% of the students.

4. Discussion

Findings of this study show that the five teachers adjusted lesson structures when designing and implementing a required lesson that involved students in inquiry-based learning. Compared with the observed usual lessons, every teacher assigned more time to introduce new content in the IBL lesson, while they had various choices considering time spent on the four specific activities of setting up, exploring, discussing and summarizing. The selected usual lessons mainly showed a pattern of "review → introduce new content → practice new content", which was changed to "introduce new content → practice new content" in some of the IBL lessons. Whole-class activity was the dominating form of classroom interaction in both types of lessons.

Inquiry practices were shown in the selected usual lessons for students to organize mathematically, inquire into solution procedures, communicate in whole-class discussion and reflect. Generally these opportunities for IBL were kept or extended a bit in the required IBL lessons, except those related to reflection, and collaboration was conducted in three IBL lessons. A higher level of IBL was relatively present in mathematizing and inquiring into procedures. These results might reflect teachers' understandings about IBL that they seemed to connect it to group work and open problems that could challenge students. They did not have a complete picture of the full IBL cycle, thus they focused more on the problem solving process while ignoring the other phases, especially phases before the solving process, i.e., questioning and hypothesizing.

Compared with the usual lessons that we analyzed, the teachers tended to make changes in the lesson structures of the required IBL lessons, while they did not adjust much with respect to IBL practices. Teachers might feel insecure to design and implement IBL lessons on behalf of an external researcher. Moreover, they might struggle or not be familiar with some elements of IBL that are part of our framework, such as collaborative work in small groups.

The findings are in line with our previous study that analyzed opportunities for IBL provided by textbook tasks. Both studies identified relatively more opportunities in organizing mathematically and inquiring into solution procedures, and the need for attention on questioning and hypothesizing. The main difference lies in that the element of communication is more explicit to be captured in classroom activities than in textbook tasks.

In addition, the findings tend to be in line with existing research that found teachers often not include the whole IBL cycle (Capps & Crawford, 2013), also with research that identified the low level of IBL practices in phases like formulating questions (Lucero et al., 2013). Another previous study of Chinese mathematics lessons found a lack of student collaboration (Cao et al., 2018), which was also shown in the usual lessons in our study, and our case teachers acknowledged in interviews that students usually did not get enough opportunities to have group work.

A general pattern we noticed is that the teachers usually conducted inquiry practices during introducing new content and in the form of whole-class interaction. This finding suggests that the teachers were either quite hesitant in students' capacities to perform individual inquiry, or they did not have sufficient resources at hand for organizing individual inquiry. An inspiring finding is that even in a usual lesson that looked quite rote learning, opportunities for IBL could still be involved in occasions for individual students to explain solutions to the whole class and for students to reflect on their choices in the solving process. Also, main IBL practices appeared at all stages (beginning, middle, end) of the lessons. Opportunities for IBL will be enhanced and more explicitly exploited when teachers are aware of the full IBL cycle, how students' inquiry skills can be developed, and organize their lessons accordingly.

If we had calculated the average percentages of purposes and forms in the observed usual lessons, the numbers would be very close to the situation of Hongkong lessons in TIMSS 1999 (Hiebert et al., 2003), but not match the TALIS finding that 70% of the Shanghai teachers reported frequent group work. Results in this study show that variations of lesson structures exist even among the five teachers, which is in line with findings of previous research (e.g., Clarke et al., 2007). Practices and choices of teachers can be different, thus specific features might be ignored in the report of average percentages. That is why we chose not to present average percentages of purposes and forms, and it is necessary to be careful when interpreting average results in large-scale international projects, also the so-called national patterns generated from them.

The data were obtained from different sources. Most of the results based on our coding of lessons, teachers' perceptions and students' perceptions tend to be congruent. However, some findings seem contradictory. Further research is needed to find out whether these differences reflect general teaching and learning patterns and to shed light on possible reasons.

Findings of this study are based on the 24 lessons of five teachers that we observed for one week and the ten lessons selected for deeper analysis. Further study could testify whether same results would be found in more lessons, other subjects or different cultural contexts. We were aware that we did not intend to generate a universal pattern of lesson structures and IBL practices of Beijing teachers. Nevertheless, the study captured some features that might characterize inquiry-based practices and showed potential starting points to expand mathematics teachers' repertoires for including IBL in regular lessons. The framework to evaluate opportunities for IBL in lessons and examples of classroom practices can be used in teacher professional development programs of IBL. Teachers need to construct a complete picture of the IBL cycle, especially about phases before problem solving processes, and conduct more practices of a higher level of IBL in their classroom teaching.

Chapter 6

Conclusion and Discussion

1. Introduction

This thesis aims to investigate the current situations of inquiry-based learning (IBL) in lower-secondary mathematics education in Beijing and the Netherlands. These two areas are taken as examples of teaching cultures in East Asia and the West. The study also aims to compare the situations in the two areas to indicate what can be learnt from each other.

In this chapter, we begin with going back to the challenging question at the start of Chapter 1, "could you predict where these lessons (see Fig. 6.1) might happen, for example, more possible to appear in East Asia or in the West based on your impressions?" Have you changed your mind after going through this thesis? What is your answer now?

Lesson A (the lesson on the left) employs a common structure of "review→ introduce new content → practice new content". The tasks are not challenging and can be solved by students applying prior knowledge and with teacher support, while opportunities for IBL can still be identified when students are guided by the teacher to pose questions, make hypotheses and reflect on the learning content. Lesson B (the lesson on the right) is almost all about "introduce new content". Students get opportunities to investigate the challenging task in groups before the whole-class discussion. They need to figure out representations and solution procedures by themselves. At the end of the lesson, students are guided to reflect on the solving process. In both lessons, individual students have chances to explain their ideas to the class. Opportunities for IBL are provided in some phases in the two lessons, while Level 3 as a higher level of IBL is only shown in Lesson B.

Differences of lesson structures and opportunities for IBL are obvious in the two lessons. It might be guessed that Lesson A with a well-designed structure and no group work happened in East Asia, and Lesson B with more student exploration and collaboration happened in the West. However, the two lessons were actually conducted by the same Chinese teacher (see the lessons of T2 in Fig. 5.4 & Table 5.9 for details). This is a case to show variations in a teacher's classroom practices, which can be very different and seem like practices happened in two different cultural contexts. In addition, it leads to questions about stereotypes related to teaching cultures in mathematics education in East Asia and the West. To what extent could they represent the situation when variations already exist in an individual teacher's practices? What are the current situations of IBL in lower-secondary mathematics education in Beijing and the Netherlands as examples of the two groups of teaching cultures? Now we try to get an answer through a closer look at findings of this study.

Fig. 6.1 Two mathematics lessons in geometry for grade 8 students

2. Research Overview and Main Results

We explored the current situations of IBL in Beijing and the Netherlands from multiple perspectives of students, teachers, textbooks and classroom practices in the four sub studies (Chapter 2 to Chapter 5). Data from these perspectives are connected. Chapter 2 and Chapter 3 came from one investigation, in which we interviewed teachers and surveyed one class of each teacher. The Beijing (BJ) teachers and most of the Dutch (NL) teachers are using the textbooks included in Chapter 4. The teachers in Chapter 5 were among participants in Chapter 3. Chapter 5 does not include data in the Netherlands because the plan to observe lessons at Dutch schools was hindered by the Covid-19 pandemic. An overview of this thesis with research questions of the sub studies, ways of data collection, and summarized results is shown in Table 6.1.

We return to the main questions of the study and connect findings among the four sub studies.

2.1 What Are the Current Situations of IBL in Lower-secondary Mathematics Education in Beijing and the Netherlands?

The Beijing teachers and students participated in this study indicated that IBL practices were often present in their mathematics lessons: students reported experiencing IBL activities in most lessons; half of the teachers mentioned that they frequently (every lesson or weekly) used IBL in practice. In our analyses of actual practices of five Beijing teachers, they involved some IBL practices in the selected usual lessons and the required IBL lessons. The teachers involved students to mathematize, find procedures, communicate and reflect, while a higher level of IBL was usually not shown in the latter two phases. The other phases (questioning, hypothesizing, collaborating) were ignored. These findings tend to be quite in line with features shown in mathematics textbooks used in Beijing that they provide some opportunities for students to mathematize and figure out procedures while a higher level of IBL is rarely achieved.

In general, the Beijing teachers include elements of IBL in their mathematics lessons, often for students to communicate and reflect and in the problem solving processes, but not in the other phases. These IBL practices are possibly under a higher level of teacher support described by “guidance” and do not involve a higher level of IBL.

The Dutch teachers and students indicated that IBL practices were sometimes present in their mathematics lessons: students reported experiencing IBL activities in some lessons; 42% of the teachers mentioned that they sometimes (monthly or occasionally) used IBL in practice. They did not indicate a more frequent use of IBL

Table 6.1 Results of the study at a glance

Chapter	Research questions (RQ)	Data collection	Results
2. Inquiry-based learning practices in lower-secondary mathematics education reported by students from China and the Netherlands	*RQ 2.1*: What do students in Beijing and the Netherlands report about their experience and preference with respect to IBL in lower-secondary mathematics education?	Survey (858 Beijing and 441 Dutch students)	The BJ sample reported experiencing IBL activities in most mathematics lessons while the NL sample in some lessons, and both preferred the same amount of IBL activities as they experienced; Classes with more IBL experience are likely to show a higher preference for IBL activities.
	RQ 2.2: What are the shared and particular features on this issue between the two areas?		The two groups of students reported similar patterns on certain activities. They both experienced more in explaining own ideas, explaining solution strategies, asking questions during investigations and getting extra teacher help. They experienced less in being presented complex mathematical problems, having group work and influencing how the lesson is organized. The BJ sample experienced less on discussing mathematical problems than the NL sample and the NL sample experienced little on posing questions to tackle and influencing how the lesson is organized.
	RQ 2.3: To what extent can the particular features be explained by the stereotypes about the two teaching cultures?		Parts of the results are not in line with the stereotypes about the teacher-centered and rote learning in East Asia, and the student-centered and process-oriented in the West.
3. Beliefs and practices related to inquiry-based learning: Chinese and Dutch lower-secondary mathematics teachers' perspectives	*RQ 3.1*: What do lower-secondary mathematics teachers in Beijing and the Netherlands highlight when they describe their IBL beliefs and IBL practices?	Semi-structured interview (30 Beijing and 19 Dutch teachers)	*Both groups* consistently highlighted students taking responsibility, teacher support and student communication. They also both mentioned student collaboration, IBL tasks as with less given information or small steps, same types of reasons and difficulties to do IBL, and using open problems in practices. *In particular*, the BJ group highlighted support through guidance, lesson design and organization, open problems as IBL tasks, student cognition, collaboration and motivation as well as learning content. The NL group highlighted support through hints/stimulation, added value especially on general skills, student motivation and more time as input, not too open tasks, lack of suitable tasks and the less predictable of IBL as difficulties as well as group discussion.

Chapter	Research questions (RQ)	Data collection	Results
	RQ 3.2: In what ways are the IBL beliefs and IBL practices mentioned in the two areas in line with expectations based on the stereotypes about the two teaching cultures?		The NL group seemed to describe a lower level of teacher support but not indicate a more frequent use of IBL than the BJ group. Many shared IBL beliefs and IBL practices were identified. Some particular elements mentioned in each area are not in line with stereotypes about the whole-class, teacher-centered, rote learning and externally motivated in East Asia, and the student-centered in the West. Most expectations based on the stereotypes are not confirmed.
4. Opportunities for inquiry-based learning in Chinese and Dutch lower-secondary school mathematics textbook tasks	*RQ 4.1*: To what extent are opportunities for IBL provided by tasks in lower-secondary mathematics textbooks in Beijing and the Netherlands?	Textbook tasks (404 Beijing and 244 Dutch tasks)	*The textbooks share* that they allow students to make some choices about mathematizations and solution procedures while they rarely achieve a higher level of IBL. Other phases of the IBL cycle are seldom involved. More IBL opportunities to organize mathematically are present in geometry than in algebra. In general, tasks with contexts show many more opportunities to inquire into solution procedures than tasks without contexts. *In particular*, the NL textbook shows relatively fewer opportunities to organize mathematically and to explore solution procedures than the BJ textbooks, while tasks with real-life contexts in the NL textbook show more opportunities in the two phases than this type of tasks in the BJ textbooks.
	RQ 4.2: In what ways are IBL-related features of these textbooks in line with expectations based on the stereotypes about the two teaching cultures?		The expectation that the NL textbook would show more tasks with a higher level of IBL in various phases is not confirmed. Many shared IBL features between the BJ and NL textbooks were identified. Most expectations based on the stereotypes are not confirmed in this study.
5. Inquiry-based learning practices in lower-secondary mathematics education: An analysis of Chinese teachers' lessons	*RQ 5.1*: To what extent are structures of the required IBL lessons different from those of the selected usual lessons?	Observation (24 lessons of five Beijing teachers); Teacher interview and student survey as additional data	The teachers distributed more time in the IBL lessons to introduce new content, while they had various choices with respect to time spent on specific activities of setting up, exploring, discussing and summarizing. Whole-class activity was the dominating form of classroom interaction in both types of lessons.
	RQ 5.2: To what extent are opportunities for IBL provided in the two types of lessons, and what are the differences?		Opportunities for IBL were provided in the selected usual lessons for students to organize mathematically, inquire into solution procedures, communicate in whole-class discussion and reflect. Generally these opportunities were kept or only extended a little in the required IBL lessons, and collaboration was involved in some of the lessons. A higher level of IBL was present in mathematizing and inquiring into solution procedures. The teachers seemed to focus on problem solving processes while ignoring phases like questioning and hypothesizing.

than the Beijing teachers as expected, while their IBL practices are possibly under a lower level of teacher support described by "hints/stimulation". It is a pity that there was no observational data about the actual practices of Dutch teachers in this study.

The situations of IBL in the Beijing sample and the Dutch sample share quite a lot. Students reported similar patterns of experiences of IBL activities. When describing IBL practices, teachers both emphasized that they allowed students to explore solution procedures by themselves and to express ideas to the whole class, used open problems and provided support. Textbooks in both areas show some opportunities for students to organize mathematically and inquire solution procedures while rarely at a higher level of IBL. The textbooks do not provide enough opportunities for students to question, hypothesize, collaborate, communicate or reflect mathematically.

2.2 What Can Beijing and the Netherlands Learn from Each Other Considering IBL in Lower-secondary Mathematics Education?

The study compared and identified some particular IBL-related features of the Beijing sample and the Dutch sample. Beijing and the Netherlands could reflect on these features and consider what to learn with respect to promote their own situation of IBL.

As for the situation in Beijing, teachers could consider to provide more opportunities for students to discuss mathematical problems by themselves, especially to discuss in groups. They could provide a lower level of teacher support and allow students to initiate activities more. Our Beijing teachers particularly mentioned difficulties such as IBL is hard to design and organize in lessons and students in lack of motivation to do IBL, and ignored the added value of IBL on general skills that would benefit students in future life, and seemed not to have a complete picture of the full IBL cycle, which can be touched in teacher professional development programs. The designers for mathematics textbooks in Beijing could consider to provide more opportunities for students to explore solution procedures in algebra and make better use of real-life contexts to provide opportunities for IBL in tasks.

As for the situation in the Netherlands, teachers could consider to include more practices of IBL in mathematics lessons, for example, by inviting students to pose questions to tackle and design their own procedures to solve complex problems. Students also need to have greater influence on what activities to do and how they are organized in lessons. Some of our Dutch teachers thought IBL tasks to be not too open, and mentioned lack of suitable tasks at hand and the less predictable feature of IBL (may not lead to good results) as difficulties. These perceived difficulties can be touched in teacher professional development programs. What can also be included is to check to what extent teachers' IBL practices are related to their understandings about IBL, and whether teachers are aware of the full IBL cycle. The designers for Dutch mathematics textbooks could think about the way to provide more opportunities for students to organize mathematically and explore solution procedures. Teachers could try to adjust textbook tasks for use and provide more space for students to inquire before possible teacher explanations.

3. Discussion of the Results

3.1 About Results Related to Current Situations of IBL in Beijing and the Netherlands

This study explored the current situations of IBL in the two areas through different perspectives in Chapter 2 to Chapter 5. Some of these findings can be connected to provide more insights.

Connecting results in Chapter 2 and Chapter 3, we have a conjecture that the more frequent IBL practices in Beijing are possibly under a higher level of teacher support and involve a lower level of IBL while the less frequent IBL practices in the Netherlands are possibly involve a higher level of IBL. However, this conjecture is at least not supported by features in textbooks analyzed in Chapter 4 that the Dutch textbooks do not show more opportunities for IBL in more phases than the Beijing textbooks. The conjecture cannot be testified in Chapter 5 because the lack of data about the actual practices of Dutch teachers.

The actual practices of Beijing teachers (Chapter 5) seem to be related to part of their understandings about IBL (Chapter 3). They connect IBL to group work and open problems that could challenge students. Features of their actual practices (Chapter 5) and features of the Beijing textbook tasks (Chapter 4) with respect to IBL are quite in line that students are allowed to make some choices about mathematizations and solution procedures while a higher level of IBL is usually not involved. These suggest the need for attention on questioning and hypothesizing and to achieve higher levels of IBL both in textbook tasks and classroom practices, which might also apply for the Netherlands.

For Beijing, it is shown in Chapter 2, 3 and 5 that there are more involvement of activities related to student communication and teacher support. Chapter 2 and Chapter 5 indicate less involvement of group work. Students in the Dutch sample reported less experience of activity of designing their own procedures than students in the Beijing sample (Chapter 2), which tends to match another finding that fewer opportunities to explore solution procedures are shown in the Dutch textbook than in the Beijing textbooks (Chapter 4).

Findings based on different perspectives can also be not in line with each other. Our Beijing and Dutch teachers mentioned the use of open problems in practices and students taking responsibility by themselves (Chapter 3), while students in both samples reported less experience of being presented problems without obvious solution procedures and influencing how the lesson is organized (Chapter 2), which could be looked into in further research.

The rubric in Chapter 4 and Chapter 5 was adapted from a framework (Capps & Crawford, 2013) built upon a common definition of IBL (National Research Council [NRC], 2000) that evaluated levels of IBL in phases of IBL processes. The original framework by Capps & Crawford (2013) is based on science education and needs to be adjusted to suit the discipline of mathematics. We added a phase of “Mathematization” and set up the IBL processes according to literature (Chapman & Heater, 2010; Pedaste et al., 2015) and our interpretations of IBL. Thus we got a rubric with four levels in seven phases of IBL to evaluate IBL opportunities in textbook tasks and classroom practices. Findings in the two chapters indicate that the Beijing and Dutch textbooks and the Beijing teachers do not provide abundant opportunities for IBL in all the phases. This reflects that understandings about IBL can be different among researchers, textbook designers and teachers. This is not unexpected because interpretations of IBL are various (Chan, 2010; Ibrahim et al., 2017; Turner, Keiffer, & Salamo, 2018). Another way to explain it is that the designers focus more on structuring content knowledge than including elements of IBL in the textbooks, and teachers are hindered by difficulties to incorporate IBL in their teaching practices. As indicated in existing literature (e.g., Turner et al., 2018; Wallace & Kang, 2004), difficulties are related to internal factors such as teachers’ knowledge about IBL and skills to use IBL, and external factors such as school/classroom culture, time, curriculum/materials and students’ conditions. Implementing IBL is a complex task involving many dimensions, our teachers mentioned these external factors as difficulties in Chapter 3, while they also showed some practices of IBL in Chapter 2 and Chapter 5.

Generally, results of this study seem to match what has been found in previous research that teachers may not have a complete understanding of IBL (Chan, 2010) and the full IBL cycle, thus they often do not include all phases of the IBL cycle or involve high-level IBL (Capps & Crawford, 2013; Lucero et al., 2013). This is possibly also impacted by the lack of opportunities for IBL present in textbooks (Aldahmash et al., 2016; X. Li et al., 2018; Ma et al., 2021; Park & Lavonen, 2013).

3.2 About Results Related to Stereotypes about Teaching Cultures in East Asia and the West

As for the current situations of IBL in Beijing and the Netherlands, findings from different perspectives (students, teachers, textbooks) show that the two areas share quite a lot. Besides, some expectations based on the stereotypes about the two teaching cultures are not confirmed by our samples, and particular features in each area cannot be fully explained by the stereotypes. Generally these results challenge the stereotypes about teaching cultures in East Asia and the West.

Findings of this study tend to match studies showing that countries/areas in the two groups of teaching cultures could also share some elements (Hiebert et al., 2003; LeTendre et al., 2001; Ma, 2020; OECD, 2014), and match studies that emphasize diversities within a teaching culture (Andrews, 2016; Clarke et al., 2010; Clarke & Xu, 2008; Felbrich et al., 2012; Kim, 2018; Shimizu & Williams, 2013).

Teaching cultures in East Asia and the West described by features of contrasting dichotomies are at the risk of oversimplification (Clarke, 2006). National teaching patterns indicated by studies like TIMSS 1999 have also been considered to be simplistic and ignore the existing diversity within countries (LeTendre et al., 2001; Leung, 2018). The so-called national patterns generated from average results in large-scale international projects should be interpreted with care. Teaching can be varied due to the domain and topic of the learning content, and individual teachers differ in their instructional practices (Leung, 2018; Seidel & Prenzel, 2006).

We encourage to consider local contexts to acknowledge the variety of practices. Just as what is shown in this study, particular IBL-related features that do not match the stereotypes might be explained by factors within specific context of Beijing and the Netherlands. For example, the relatively more frequent IBL practices in Beijing (Chapter 2) corresponds to what has been pointed out that Chinese education has borrowed some theories and practices from the West during the trend of globalization, and the education reform in China included some elements of IBL (Dai et al., 2011; Liu & Feng, 2015; Tan, 2015). This finding might be in contrast to the label on Chinese education as “teacher-centered, rote learning and passive learners” (Zhao et al., 2016). Dutch mathematics teaching is considered to be impacted by the textbook-oriented culture that teachers may highly rely on textbooks and students’ learning can be limited by textbook worked examples.

4. Implications for Educational Practice

4.1 For Mathematics Teachers and Classroom Teaching

Teachers need to learn to get the shared understanding about what is IBL, what phases are involved, what levels are achieved and how IBL is related to mathematics teaching. Providing complex mathematical problems, letting students pose questions and make hypotheses, organizing group work, encouraging students to have an influence on the lesson have potential for implementing IBL in mathematics. Specifically, mathematics teachers in Beijing might need to allow students to discuss mathematical problems and to participate in IBL activities at their own pace, while Dutch teachers need to encourage students to pose questions to tackle and become less dependent on their textbooks.

Teachers and students might feel uncertain and not be adjusted to the ambiguity in a teaching approach like IBL. The study indicates that classes with more experience of IBL are likely to show a higher preference for IBL activities. This finding should encourage teachers towards more trial of IBL. Teachers need to provide opportunities for students to become familiar with this approach gradually. At start, teachers could try to include IBL for students to communicate and reflect on mathematical issues, and conduct small activities with elements of IBL. After that, teachers can pay attention to cover a complete learning trajectory of IBL and achieve a higher level of IBL. Practically, IBL could happen at more stages, not only during introducing new content.

IBL does not mean just to leave students alone to discover by themselves, teacher support is still necessary. Teachers are responsible to create problem situations, organize activities for student to collaborate and communicate, and provide support when necessary (Artigue & Blomhøj, 2013). Specifically, Dutch teachers could learn from experienced colleagues and example lessons and design their own IBL lessons carefully, for instance with Lesson Study (e.g., Jessen et al., 2022).

4.2 For Teacher Educators and Teacher Professional Development Programs

Teacher educators need to be aware that teachers' interpretations of IBL might be different from those in literature and in curriculum. The rubrics in Chapter 4 and Chapter 5 can be used in teacher professional development programs focusing on IBL as practical frameworks to help teachers to have a better understanding of IBL. With the rubric, they can become familiar with the full IBL cycle and levels of IBL and reflect on their lesson materials and teaching practices accordingly.

In addition, teacher educators need to pay attention to the challenges teachers have when implementing IBL in teaching practices. In addition to difficulties identified in literature (e.g., Turner et al., 2018; Wallace & Kang, 2004), this study found possible reasons why mathematics teachers do not incorporate IBL in usual lessons: 1) they do not consider IBL to be part of the required learning content and learning goals, but something additional; 2) they think IBL is a complete process/cycle to go through and ignore small activities with elements IBL; 3) they think students can only be able to do IBL after being fully prepared for it, such as in cognition and motivation; 4) they emphasize short-term results and think students do not benefit at all if fail to solve an IBL-oriented task. Teacher educators could design activities to deal with these possible misunderstandings about IBL and the external difficulties indicated by teachers in Chapter 3. IBL is helpful for enriching teachers' instruction practices while it is not necessarily considered as a replacement of the current way of teaching.

4.3 For Designers of Mathematics Textbooks and Curriculum

IBL is considered to be a way to foster 21st century skills like creativity, communication and collaboration (Barron & Darling-Hammond, 2010) for students to meet the demand for the quickly changing society. Designers need to be aware of the important role of IBL, move beyond the focus on procedures and algorithm, and intend to include IBL in mathematics textbooks and curriculum.

Our teachers mentioned IBL-oriented tasks as with less given information or small steps and talked about using open problems in practices (Chapter 3), which could be considered by textbook and curriculum designers.

As indicated in Chapter 4, mathematics textbook and curriculum designers need to consider to provide more opportunities for IBL in phases of questioning, hypothesizing, collaborating, communicating and reflecting, and achieve a higher level of IBL. Designers need to provide problems that students do not immediately know what to do for them to learn to inquire mathematically. Designers could also consider to provide more opportunities for students to organize mathematically in algebra chapters, and include various real-life contexts that have potential for IBL in tasks.

5. Limitations and Future Research

Although findings of the successive sub studies help to formulate implications for practice, we shouldn't neglect some of the limitations and be careful in generalizing our results.

First of all, there might be concerns about Beijing and the Netherlands as representatives of the East Asian and Western teaching cultures. We were aware of this issue and made it explicit in the thesis. The two areas were taken as examples other than representatives of these two teaching cultures. Although they have their own regional characteristics, they are also part of and share characteristics of their overarching teaching culture.

Second, we reflect on the issue of sampling in this study. In Chapter 2 and Chapter 3, the participants resulted from convenient sampling. Chapter 4 is based on two chapters selected from each of the three textbooks that are commonly used in the two areas, which involves kind of purpose sampling. Chapter 5 is based on practices of five Chinese teachers. Although we tried to engage participants that are possibly helpful to represent the situation in each area, it has been pointed out in the thesis that the results are limited to these samples within their local contexts and cannot be generalized.

Finally, the study lacks data of actual practices of Dutch teachers in Chapter 5. As a result, comparison cannot be conducted between Beijing and the Netherlands on this issue, nor could these practices be connected to findings based on perspectives of the Dutch students, teachers and textbooks.

However, this study still provides some implications for future research. To begin with, to further testify the stereotypes about teaching cultures, researchers could design to investigate whether differences within countries in East Asia or the West would be bigger than between countries. Besides, researchers could discuss whether random sampling is necessary and possible in small-scale empirical comparative studies like this research.

In addition, further research is needed to look into actual IBL practices of Dutch mathematics teachers and to connect the results to findings of this study. Conjectures such as those about the level of teacher support in Dutch mathematics lessons could be testified. What can also be analyzed are findings based on students' and teachers' report that are contradictory, i.e., the situation of students being presented complex mathematical problems and having an influence on how the lesson is organized in actual practices. Researchers can look into the current situation of IBL in mathematics education in other age levels of students from primary schools and upper-secondary schools as well.

Moreover, in relation to the frameworks, further research could consider to analyze the situation of IBL through different perspectives but based on the same framework, which will be easier to connect findings and help to provide more insights. When thinking about frameworks of IBL, researchers need to continue to focus on the nature of mathematics and the specific features of IBL in mathematics and its pedagogies. More studies are needed to elaborate and improve the frameworks of IBL for mathematics education in this study and the original rubric for science education by Capps and Crawford (2013).

Nevertheless, this study avoids the limitation of investigating the situation of IBL solely from one perspective or only based on self-report. It shows a detailed picture through perspectives of students, teachers, textbooks and classroom practices, and builds upon tools from large international projects like PISA, PRIMAS, TIMSS and TALIS. It is an empirical comparative study that reveals particular features impacted by each of the two teaching cultures and the shared features across cultural boundaries, which leads to a better understanding of the current situation of IBL. It presents a rubric to evaluate IBL in mathematics with four levels in seven phases, which could be used in further research and in practice. This study can be a starting point for teacher professional development projects considering IBL in mathematics education. More research is needed to elaborate frameworks provided in the study and apply them to improve practices of IBL in mathematics education around the world.

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Summary

Mathematics is considered to be a human activity instead of a closed ready-made system, and students should actively participate in the process of constructing mathematical knowledge (Freudenthal, 1973; Van den Heuvel-Panhuizen & Drijvers, 2020). These features of mathematics are fostered in inquiry-based learning (IBL), which is interpreted as a teaching approach that invites students to learn in similar ways as how mathematicians work (Maaß & Doorman, 2013; Siegel & Borasi, 2003). The understandings and practices related to IBL might be impacted by teaching cultures, of which those in East Asia and the West are considered to be remarkably different and have led to stereotypes. The study tried to move beyond these stereotypes and explored the current situations of IBL with open views. China, specifically Beijing, and the Netherlands were taken as examples of the two teaching cultures. Perspectives of students, teachers, textbooks and classroom practices were explored, with main questions as follows: *1) What are the current situations of IBL in lower-secondary mathematics education in Beijing and the Netherlands? 2) What can Beijing and the Netherlands learn from each other considering IBL in lower-secondary mathematics education?*

Chapter 2 explores the questions from a students' perspective. It focuses on students' experience of IBL-related activities in lower-secondary mathematics lessons and their preference. 858 Beijing students from 30 classes and 441 Dutch students from 19 classes participated in the survey. The questionnaire was built upon items from international projects of PISA (Programme for international student assessment) and PRIMAS (Promoting IBL in mathematics and science education across Europe). It includes two scales. The first part is an IBL experience scale with 15 items that represent two categories of IBL, i.e., students take responsibility in inquiry processes and the teacher guides the inquiry processes. The second part is an IBL preference scale consisting of five items selected from the IBL experience scale.

Based on data from the survey, average scores on each item and on scales were calculated for the Beijing sample and the Dutch sample and ranked within each scale. Independent samples *t* test was conducted based on "mathematics grade" (low-achievers, high-achievers), and correlation analysis was performed between IBL experience and IBL preference. Moreover, results in the two samples were compared and connected with expectations based on the stereotypes.

Results show that generally the Beijing sample reported experiencing IBL activities in most mathematics lessons and the Dutch sample in some lessons. Students' reports show similar patterns. They both experienced more in explaining own

ideas, explaining solution strategies, asking questions during investigations and getting extra teacher help. They both experienced less in being presented complex mathematical problems, having group work and influencing how the lesson is organized. Students in both samples preferred the same amount of IBL activities as they experienced. They both preferred most for group work to happen more, and least for being presented complex problems. Compared with high-achievers in that sample, low-achievers in the Beijing sample reported less IBL preference, while low-achievers in the Dutch sample reported less IBL experience. A positive correlation between IBL experience and IBL preference of each class is also suggested. Part of the results are not in line with the stereotypes about aspects of teacher-centered and rote learning in the East Asian teaching culture, and aspects of student-centered and process-oriented in the Western teaching culture. Expectations based on the stereotypes are not confirmed in the chapter.

Chapter 3 explores the main issue of this thesis from a teachers' perspective. It focuses on beliefs and practices related to IBL described by lower-secondary mathematics teachers. 30 Beijing and 19 Dutch teachers participated in the semi-structured interviews. They were mathematics teachers of the students in Chapter 2. The teachers were not provided with a pre-set definition of IBL, while two mathematical tasks with features of IBL were included in the interviews to provide contexts. The tasks provoked teachers to express their views about factors to consider before doing IBL, activities to do in IBL lessons and outcomes of IBL. In addition, the interviews included other questions about beliefs (general views, attitude, reasons for, difficulties, strategies) and practices (frequency of using IBL , a recent IBL lesson, the role as teacher in the lesson) related to IBL.

These topics were inspired by information from the PRIMAS project and were used as main codes to code the interviews. Possible sub codes emerged from the interviews were also used. High-ranking statements within each main code that most represent what teachers mentioned were extracted. Quotations and two cases of teachers were presented as well. Moreover, we identified possible misunderstandings that might hinder teachers' use of IBL, connected the IBL beliefs with reported IBL practices, and compared the results with expectations based on the stereotypes.

Results show that 50% of the Beijing teachers and 37% of the Dutch teachers reported that they used IBL frequently in mathematics lessons. Compared with the Beijing teachers, the Dutch teachers did not indicate a more frequent use of IBL as expected, and they even showed a relatively less positive attitude towards IBL. The two groups of teachers mentioned many shared IBL beliefs and practices.

They consistently mentioned students taking responsibility by themselves, teacher support and student communication in regard to both their beliefs and practices of IBL. They also both considered IBL as one of the ways to teach and learn mathematics, mentioned IBL tasks as without much information or intermediate steps, paid attention to student collaboration in classroom activities, took the four positive outcomes of IBL as reasons to do it, listed factors related to contexts and perceived features of IBL as difficulties, and used open problems in their practices. Our Beijing teachers and Dutch teachers also paid particular attention to some aspects, part of which do not match the stereotypes about aspects of whole-class instruction, teacher-centered teaching, rote-like learning and external motivation in the East Asian teaching culture, and aspects of student-centered learning in the Western teaching culture. Both consistencies and inconsistencies were found between these teachers' beliefs and reported practices related to IBL. Although the level of teacher support indicated by the Dutch teachers (described by "hints/stimulation") seemed to be lower than that of the Beijing teachers (described by "guidance"), most expectations based on the stereotypes are not confirmed in this chapter.

Chapter 4 takes a textbook perspective. It focuses on to what extent IBL practices in lower-secondary mathematics education are supported by opportunities in textbooks through analyzing tasks in two Beijing textbooks and one Dutch textbook. These mathematics textbooks are in use by the teachers in Chapter 3 and still commonly used in the two areas. Chapters with similar topics and matching numbers of pages were selected, resulting in the analysis of an algebra chapter about quadratic equations including factorization and a geometry chapter about similarity. 404 Beijing and 244 Dutch algebra and geometry tasks were coded. The analytical framework was built upon the rubric of Capps and Crawford (2013), with IBL processes from literature and the nature of mathematics taken into consideration. The framework evaluates four IBL levels in seven phases of IBL.

In the analysis, percentages of IBL levels in the seven phases were calculated for each textbook. The phases with some IBL opportunities were taken for further analysis with tasks separated for different content (algebra, geometry) and context types (none, mathematical, camouflage, real-life). Examples of tasks with and without opportunities for IBL were shown for illustration. Shared and particular IBL-related features of the textbooks were identified and compared with expectations based on the stereotypes.

Results show that many shared IBL features between the Beijing and Dutch textbooks were identified. The textbooks allow students to make some choices to organize mathematically and explore solution procedures while higher levels of IBL are rarely achieved. They seem not involve students to question, hypothesize, collaborate, communicate or reflect. The textbooks provide more IBL opportunities to organize mathematically in the selected geometry chapters than in the selected algebra chapters. Generally more IBL opportunities to organize mathematically and to explore solution procedures are shown in tasks with contexts than in tasks without contexts. Particular IBL-related features of the textbooks in each area were identified as well, part of which do not match the stereotypes about teaching cultures. The Dutch textbook involves relatively fewer opportunities to organize mathematically and to explore solution procedures than the Beijing textbooks. For some categories, each textbook presents a different pattern, or differences exist between the two Beijing textbooks and tend to be more prominent than those between the Beijing and Dutch textbooks. Most expectations based on the stereotypes are not confirmed in the chapter.

Chapter 5 looks into the perspective of classroom practices. It focuses on to what extent IBL practices are involved in Chinese lower-secondary mathematics lessons. This chapter is based on 24 lessons of five Beijing teachers that are among participants in Chapter 3, including 19 usual lessons and 5 required IBL lessons. As for the required IBL lessons, each teacher was asked to design and implement a mathematics lesson with elements of IBL based on his or her understandings towards it. Additional data from post-lesson teacher interviews and student surveys about the IBL lessons was also included.

The videos or audios of the lessons were coded on structure and on opportunities for IBL. The framework to code lesson structures consists of two parts. The first one focuses on purposes of classroom activities with categories (review, introduce new content, practice new content) from TIMSS project (Trends in international mathematics and science study). The second one focuses on forms of classroom interaction with categories (whole-class, group, individual) built upon TALIS project (Teaching and learning international survey). The framework to evaluate opportunities for IBL was based on the rubric used in Chapter 4 and it was adjusted to suit the coding of lessons other than textbooks.

In the analysis, the percentages of categories in purposes and forms were calculated for the 24 lessons. Five usual lessons were selected, together with the five required IBL lessons, were analyzed further to code opportunities for IBL. Features were identified and compared among these teachers, and compared between the two types of lessons. Results were also compared with results from teacher interviews and student surveys. Moreover, graphical representations and cases of three teachers were shown in this chapter.

Compared with the usual lessons, the teachers adjusted lesson structures in the IBL lessons to distribute more time to introduce new content, while they showed different choices regarding the change of time on the four specific activities (set up, explore, discuss and summarize). As for the purposes, the selected five usual lessons mainly showed a pattern of "review → introduce new content → practice new content", while in some of the IBL lessons it was changed to "introduce new content → practice new content". Whole-class activity was the dominating form in both types of lessons. IBL practices were shown in the selected usual lessons for students to organize mathematically, explore solution procedures, communicate and reflect. A higher level of IBL was relatively present in the first two phases. When the teachers designed and implemented the IBL lessons, they did not adjust much with respect to IBL practices. They seemed to connect IBL to group work and open problems that could challenge students, and they seemed not to have a complete picture of the full IBL cycle. Students need to be provided with opportunities to think about questions to tackle by themselves, and more hypotheses should be involved. Most of the results based on the coding of lessons, teachers' perceptions and students' perceptions tend to be congruent, while some findings seem contradictory and could be testified in further research.

Chapter 6 shows an overview of findings in the four chapters above and connects these findings to answer the main questions of this study. Results related to the current situations of IBL in the two areas were discussed, also results related to the stereotypes about teaching cultures in East Asia and the West. The stereotypes are challenged by our findings. Countries or areas in the two groups of teaching cultures could also share some elements, and variations within a teaching culture cannot be ignored. Based on these results, implications were provided for mathematics teachers, teacher educators as well as textbook and curriculum designers. We also reflected on the study and provided insights for further research. For example, researchers can continue to investigate the current situation of IBL in mathematics education in primary schools and upper-secondary schools. More research is needed to elaborate and improve the frameworks of IBL for mathematics education provided in this study.

Generally, results of this thesis seem to match findings in literature that mathematics teachers may not have a complete understanding of IBL and the full IBL cycle, thus they often do not include all phases of the IBL cycle or involve high-level IBL in their teaching. This is possibly also related to the lack of abundant opportunities for IBL

present in textbooks. Finally, the study suggests to consider specific local contexts when interpreting educational practices other than regarding them solely from features of broader teaching cultures.

概要

数学被认为是一项人类活动 (human activity)，而非一个预设好的封闭系统，学生应该积极参与到建构数学知识的过程当中 (Freudenthal, 1973; Van den Heuvel-Panhuizen & Drijvers, 2020)。探究性学习(inquiry-based learning)与数学的这些特点相契合，它可以被解读为一种教学方式，鼓励学生以类似数学家工作方式的方法来学习(Maaß & Doorman, 2013; Siegel & Borasi, 2003)。对探究性学习的理解和运用可能受到教学文化的影响，而东亚和西方的教学文化被认为存在显著差异，这产生了一些刻板印象。本研究尝试超越这些刻板印象，以开放的眼光来探索探究性学习的现状，将中国的北京地区与荷兰作为这两种教学文化的例子，从学生、教师、教材和课堂教学实践四个视角切入，主要问题如下: 1) 北京与荷兰的初中数学教育中探究性学习的现状如何？2) 北京与荷兰在这个主题上可以相互学习什么?

第二章基于学生视角进行探索。它聚焦学生在初中数学课中经历探究性学习相关活动的情况，以及对这些活动的偏好。来自30个班级的858名北京学生以及来自19个班级的441名荷兰学生参与了问卷调查。所用问卷基于两个国际项目中的问卷条目调整形成，即PISA（国际学生评估项目）和PRIMAS（欧洲数学与科学教育中的探究性学习提升项目）。该问卷包括两个量表。其一为探究性学习经历量表，它由15个条目构成，呈现了探究性学习的两大方面：学生在探究过程中承担责任，教师引导探究过程。其二为探究性学习偏好量表，由探究性学习经历量表中选取的5个条目构成。

基于问卷数据，分别计算北京样本和荷兰样本每个条目和两个量表的均值，根据分值在量表内部排序，并对"数学成绩"的两个分组（低分组、高分组）开展独立样本t检验。同时，对探究性学习经历与探究性学习偏好进行相关分析。此外，比较北京、荷兰两个样本的各项结果，并将其与基于刻板印象的预期结果进行对照。

结果显示，北京样本表明他们在大部分数学课上经历了探究性活动，而荷兰样本表示他们在有些数学课上经历了探究性活动。两组学生的问卷结果呈现出相似的模式：他们在解释自己的想法、说明解题策略、在探究过程中提出问题、获得老师的额外帮助这四项活动上经历得更多；他们在遇到复杂的数学问题、开展小组合作、对课堂活动如何开展产生影响这三项活动上经历得更少。在偏好方面，两组学生都希望探究性活动保持目前的数量，他们最希望开展更多的小组合作，最不希望遇到更多复杂的数学问题。与所在样本中的高分组相比，北京样本中的低分组汇报了更低的探究性学习偏好，而荷兰样本中的低分组汇报了更少的探究性学习经历。

结果还显示了学生（以班级为单位）的探究性学习经历与探究性学习偏好之间的正相关关系。部分结果与刻板印象中东亚教学文化的教师中心、机械学习特点并不相符，也不符合刻板印象中西方教学文化的学生中心、过程导向特点。基于刻板印象的预期结果在本章中没有得到证实。

第三章从教师视角进行探索。它聚焦初中数学教师描述的探究性学习观念及实践。30位北京教师和19位荷兰教师参与了半结构化访谈。他们是第二章问卷调查中两地学生的数学老师。访谈中没有给出预设的探究性学习的定义，而是向老师们呈现了两道具有探究性学习特征的数学题。这两道题目提供情境，激发老师们表达看法，关于开展探究性学习需要考量的因素、探究课上开展的活动以及探究性学习的结果。此外，访谈还涉及了与探究性学习的观念和实践相关的其他问题，前者包括总体看法、态度、使用理由、难点、策略，后者包括使用频率、一节近期的探究课、作为教师在该课中的角色。

这些主题受到PRIMAS项目内容的启发而形成，并作为主编码用于访谈数据的编码，被使用的还有从访谈中生成的子编码。之后，基于教师所提及的与各主编码相关的内容，提取出最能代表这些内容的表述。此外，还呈现了一些访谈原句和两个教师案例。再者，识别了可能阻碍教师运用探究性学习的一些误解，将教师的探究性学习观念与其自陈的探究性学习实践联系起来，并将研究结果与基于刻板印象的预期结果进行比较。

结果显示，在参与这一子研究的教师中，50%的北京教师和37%的荷兰教师表示，他们在数学课上经常运用探究性学习。与北京教师相比，荷兰教师没有像预期的那样更为频繁地开展探究性学习，反而对它表现出相对不那么积极的态度。两组教师提及很多共同的探究性学习观念与实践。他们都一再提到学生自己承担责任、教师支持和学生交流，既指向探究性学习观念，也指向相关的实践。他们还都认为探究性学习是数学教学与学习的其中一种方法，谈到探究性题目不会给出太多信息或者中间步骤，关注课堂活动中的学生合作，将探究性学习的四个正向结果作为运用它的理由，将与情境相关和与探究性学习特征相关的因素作为使用难点，并且在实践中采用开放性题目。这些北京教师和荷兰教师也有各自关注的一些方面，其中一部分不符合刻板印象中东亚教学文化的全班教学、教师中心、机械学习、外部激

励特点，也不符合刻板印象中西方教学文化的学生中心特点。在这些教师的探究性学习观念与自陈性实践之间，既发现了一致性，也发现了矛盾性。尽管荷兰教师表明的教师支持水平（描述为"提示/激发"）似乎低于北京教师所表明的（描述为"引导"），大部分基于刻板印象的预期结果在本章中没有得到证实。

第四章采取教材视角，通过分析两本北京教材（人教版、京教版）和一本荷兰教材中的题目，聚焦初中数学教材在多大程度上支持探究性学习实践。第三章中的教师所用的就是这些数学教材，它们在北京、荷兰两地仍被广泛使用。这一子研究筛选三本教材中主题相似、篇幅相符的章节，选出一个代数章节（一元二次方程与/含因式分解）和一个几何章节（相似）进行分析，对书中的404道北京、244道荷兰代数与几何题进行编码。所用分析框架建立在 Capps & Crawford（2013）框架的基础之上，并考虑已有文献中的探究过程以及数学学科的性质而加以调整，它评估七个探究阶段上的四个探究性学习水平。

在数据分析部分，计算每本教材七个阶段上各个探究性学习水平的百分比，对有探究机会的阶段开展进一步分析，将题目根据内容（代数、几何）和情境类型（无、数学情境、虚假情境、现实情境）细分。同时，将有、无探究机会的题目作为例子加以说明。此外，识别了三本教材共同的以及各有的与探究性学习相关的特点，并与基于刻板印象的预期结果进行比较。

研究结果显示，三本北京、荷兰教材之间存在很多共同的探究性学习相关特点。这些教材允许学生在以数学的方式组织、探索解题步骤两个阶段上做出一些决策，但是很少达到较高的探究性学习水平。它们似乎没有让学生参与到提问、假设、合作、交流或反思当中。相比所选代数章节，这些教材在所选几何章节中提供了更多的探究机会。整体而言，相比没有情境的题目，带有情境的题目在以数学的方式组织、探索解题步骤两个阶段上表现出更多的探究机会。同时，识别了北京、荷兰教材各自具有的特点，其中一部分与关于教学文化的刻板印象不符。与北京教材相比，荷兰教材在以数学的方式组织、探索解题步骤两个阶段上涉及的探究机会相对较少。在某些类别上，每本教材表现出不同的模式，或者两本北京教材之间的差异可能比北京、荷兰教材之间的差异更为突出。大部分基于刻板印象的预期结果在本章中没有得到证实。

第五章从课堂教学实践视角切入，聚焦中国的初中数学课在多大程度上涉及探究性学习实践。本章基于5位北京教师（来自第三章的教师被试）的24节课，包括19节日常课和5节探究课。关于探究课，要求每位教师根据自己对探究性学习的理解，设计并教授一节包含探究性学习元素的数学课。另外，以课后教师访谈和探究课后学生问卷调查的数据作为补充。

该子研究基于课堂视频或音频，对课的结构和其中涉及的探究机会进行编码。在结构方面，使用的编码框架由两部分组成。其一，聚焦课堂活动的目的，采用TIMSS项目（国际数学和科学趋势研究）中的三个类别——回顾、新知、练习。其二，聚焦课堂互动的形式，在TALIS项目（教师教学国际调查）类别的基础上调整形成三个类别——全班活动、小组活动、个人活动。此外，在探究机会方面，其评估框架基于第四章使用的框架并有所调整，以从适用于编码教材转变为适用于编码课堂。

在分析中，计算了这24节课在结构方面（目的、形式）各个类别上的百分比，从中选取出5节日常课，连同5节探究课一起，进一步分析课上的探究机会。同时，识别相关特点，并对这些教师之间、两种课型之间加以比较。之后，将课堂分析的结果与教师访谈和学生问卷的结果进行比较。此外，本章还将结果可视化，并呈现三位教师的案例。

结果显示，与该子研究中的日常课相比，老师们调整了探究课的结构，将更多时间分配给新知部分，但是他们在新知部分的四个具体活动（导入、探索、讨论、总结）上的时间分配各有特点。在课堂结构的目的方面，选取的5节日常课主要呈现“回顾→新知→练习”的模式，而在部分探究课中它被调整为“新知→练习”模式。两种课型都以全班活动作为主要的课堂互动形式。5节日常课在以数学的方式组织、探索解题步骤、交流、反思这几个阶段上涉及探究性学习实践，并在前两个阶段上表现出相对较高的探究性学习水平。这些教师设计和开展5节探究课时，在探究性学习实践方面的调整较少，他们似乎将探究性学习与小组活动以及有挑战性的开放性问题联系起来，而且似乎对探究性学习的整个循环缺乏完整的认识，需要为学生提供相应的机会来思考提出以待解答的问题，并且涉及更多假设。大部分从课堂分析、教师访谈和学生问卷中得出的结果是一致的，但有些结果似乎存在矛盾，可以在未来的研究中加以检验。

第六章概括呈现了以上四个章节的结果，回应主要的研究问题。基于研究结果，对北京、荷兰两地的探究性学习现状进行讨论，也讨论了被这些结果所挑战的对东亚、西方教学文化的刻板印象。两组教学文化中的国家或地区之间可能也有一些共同元素，同时，不应忽视每组教学文化的内部差异。在研究结果的基础上，提出可供数学教师、教师教育者、课程与教材设计者参考的建议。此外，对这项研究进行了反思，给出对未来研究的启示。例如，研究者可以继续关注探究性学习在小学和高中数学教育中的现状；需要开展更多研究来进一步阐明和提升本文中的数学探究性学习框架。

本研究的结果整体上与已有文献中的发现一致，即数学教师可能还未形成对探究性学习以及整个探究循环的全面理解，因此他们往往没能把握完整的探究过程，或者没能在教学中涉及高水平的探究性学习。这或许也与教材中缺少充足的探究机会有关。最后，本研究建议，解读教育实践时要考虑当地的特定情境，而不是仅基于所属的宽泛的教学文化的特征来进行考量。

概要

Samenvatting

Wiskunde wordt beschouwd als een menselijke activiteit in plaats van een gesloten kant-en-klaar systeem, en leerlingen moeten actief deelnemen aan het proces van het construeren van wiskundige kennis (Freudenthal, 1973; Van den Heuvel-Panhuizen & Drijvers, 2020). Deze kenmerken van wiskunde worden bevorderd in onderzoekend leren (IBL), dat wordt opgevat als een onderwijsaanpak die leerlingen uitnodigt om te leren op manieren die vergelijkbaar zijn met hoe wiskundigen werken (Maaß & Doorman, 2013; Siegel & Borasi, 2003). De opvattingen en praktijken met betrekking tot onderzoekend leren kunnen worden beïnvloed door onderwijsculturen, waarvan die in Oost-Azië en het Westen als zeer verschillend worden beschouwd en die hebben geleid tot stereotypen. De studie probeerde deze stereotypen te overstijgen en onderzocht de huidige situaties van onderzoekend leren. China, met name Beijing, en Nederland zijn genomen als voorbeelden van de twee onderwijsculturen. Perspectieven van leerlingen, leraren, tekstboeken en lespraktijken werden onderzocht, met de volgende hoofdvragen: *1) Wat zijn de huidige situaties van onderzoekend leren in het lager secundair wiskundeonderwijs in Beijing en Nederland? 2) Wat kunnen Beijing en Nederland van elkaar leren als het gaat om onderzoekend leren in de onderbouw van het voortgezet wiskundeonderwijs?*

Hoofdstuk 2 verkent de vragen vanuit het perspectief van de leerlingen. Het richt zich op de ervaring van leerlingen met IBL-gerelateerde activiteiten in wiskundelessen in de onderbouw van het voortgezet onderwijs en hun voorkeur. 858 leerlingen uit 30 klassen in Beijing en 441 leerlingen uit 19 klassen in Nederland hebben deelgenomen aan het onderzoek. De vragenlijst was gebaseerd op items uit internationale projecten van PISA (Programme for international student assessment) en PRIMAS (Promoting IBL in mathematics and science education across Europe). Hij omvat twee schalen. Het eerste deel is een IBL-ervaringsschaal met 15 items die twee categorieën van onderzoekend leren vertegenwoordigen, d.w.z., leerlingen nemen verantwoordelijkheid in onderzoeksprocessen en de leraar begeleidt de onderzoeksprocessen. Het tweede deel is een IBL-voorkeursschaal bestaande uit vijf items geselecteerd uit de IBL-ervaringsschaal.

Gebaseerd op gegevens uit de enquête werden de gemiddelde scores op elk item en op de schalen berekend voor de Beijing steekproef en de Nederlandse steekproef en gerangschikt binnen elke schaal. Een onafhankelijke steekproeven *t*-toets werd uitgevoerd op basis van "wiskundeniveau" (laag-presteerders, hoog-presteerders), en een correlatie-analyse werd uitgevoerd tussen IBL-ervaring en IBL-voorkeur. Bovendien werden de resultaten in de twee steekproeven vergeleken en in verband gebracht met de verwachtingen op basis van de stereotypen.

De resultaten laten zien dat de Beijing-steekproef rapporteerde IBL-activiteiten te hebben ervaren in de meeste wiskundelessen en de Nederlandse steekproef slechts in sommige lessen. De gegevens van de leerlingen laten ook vergelijkbare patronen zien. Ze ervoeren beiden meer in het uitleggen van eigen ideeën, het uitleggen van oplossingsstrategieën, het stellen van vragen tijdens onderzoeken en het krijgen van extra hulp van de leraar. Ze ervoeren beiden minder in het voorgelegd krijgen van complexe wiskundige problemen, het werken in groepen en het beïnvloeden van hoe de les wordt georganiseerd. Leerlingen in beide steekproeven gaven de voorkeur aan dezelfde hoeveelheid IBL-activiteiten als ze ervaren. Ze hadden beiden de meeste voorkeur voor meer groepswerk, en de minste voorkeur voor het werken aan complexe problemen. Vergeleken met hoogpresteerders in die steekproef hadden laagpresteerders in de Beijing-steekproef minder IBL-voorkeur, terwijl laagpresteerders in de Nederlandse steekproef minder IBL-ervaring rapporteerden. De resultaten suggereren een positieve correlatie tussen IBL-ervaring en IBL-voorkeur binnen deze groepen. Een deel van de resultaten is niet in overeenstemming met de stereotypen over aspecten van leraar- en proceduregerichte praktijken in de Oost-Aziatische onderwijscultuur, en leerling- en procesgerichte praktijken in de westerse onderwijscultuur. Verwachtingen gebaseerd op de stereotypen worden niet bevestigd in dit hoofdstuk.

In **hoofdstuk 3** wordt het belangrijkste onderwerp van dit proefschrift vanuit het perspectief van de docenten belicht. Het richt zich op overtuigingen en lespraktijken met betrekking tot onderzoekend leren, beschreven door wiskundeleraren in de onderbouw van het voortgezet onderwijs. Aan de semigestructureerde interviews namen 30 Beijing en 19 Nederlandse leraren deel. Zij waren de wiskundeleraren van de leerlingen uit hoofdstuk 2. De leraren kregen geen vooraf vastgestelde definitie van onderzoekend leren, terwijl twee wiskundige taken met kenmerken van onderzoekend leren in de interviews werden opgenomen om context aan de vragen te geven. De taken lokten leerkrachten uit om hun mening te geven over factoren die ze moeten overwegen voor de implementatie van onderzoekend leren (IBL), activiteiten te doen in IBL-lessen en de resultaten van IBL. Daarnaast bevatten de interviews andere vragen over overtuigingen (algemene opvattingen, houding, redenen voor, moeilijkheden, strategieën) en lespraktijken (frequentie van het gebruik van IBL, een recente IBL-les, de rol van de docent) met betrekking tot IBL.

Deze onderwerpen waren geïnspireerd op informatie uit het PRIMAS-project en werden gebruikt als hoofdcodes bij het coderen van de interviews. Mogelijke subcodes die uit de interviews naar voren kwamen werden ook gebruikt. Hooggenoteerde uitspraken binnen elke hoofdcode die het meest representatief waren voor wat leraren noemden, werden eruit gehaald. Citaten en twee casussen van leerkrachten werden ook gepresenteerd. Bovendien identificeerden we mogelijke misverstanden die het gebruik van onderzoekend leren door leraren in de weg zouden kunnen staan; we vergeleken de IBL-overtuigingen met de gerapporteerde IBL-lespraktijken, en vergeleken de resultaten met de verwachtingen op basis van de stereotypen.

De resultaten laten zien dat 50% van de leraren in Beijing en 37% van de Nederlandse leraren rapporteerden dat ze IBL frequent gebruikten in wiskundelessen. Vergeleken met de Beijing leraren, gaven de Nederlandse leraren niet, zoals verwacht, vaker een indicatie van gebruik van onderzoekend leren, en ze toonden zelfs een relatief minder positieve houding tegenover onderzoekend leren. De twee groepen leraren noemden veel gedeelde IBL-overtuigingen en lespraktijken. Ze noemden leerlingen die zelf verantwoordelijkheid nemen, ondersteuning door de leraar en communicatie tussen leerlingen met betrekking tot zowel hun overtuigingen als hun lespraktijken van onderzoekend leren. Ze beschouwden ook allebei IBL als een van de manieren om wiskunde te onderwijzen en te leren en beschreven IBL taken als taken zonder veel informatie of tussenstappen. Ze besteedden aandacht aan de samenwerking van leerlingen in klasactiviteiten. Ze noemden als moeilijkheden of uitdagingen het vinden van geschikte contexten en het gebruik van open problemen. Onze Beijing leraren en Nederlandse leraren besteedden ook bijzondere aandacht aan enkele aspecten, waarvan een deel niet overeenkomen met de stereotypen over klassikale instructie, docentgericht onderwijs, uit het hoofd leren en externe motivatie in de Oost-Aziatische onderwijscultuur, en aspecten van leerlinggericht onderwijs in de westerse cultuur. Zowel consistenties als inconsistenties werden gevonden tussen de overtuigingen van deze leraren en de gerapporteerde praktijken met betrekking tot onderzoekend leren. Hoewel de mate van ondersteuning door de leraar die de Nederlandse leraren aangaven (beschreven als “hints/stimulering”) lager leek dan die van de Beijing leraren (beschreven als “begeleiding”), werden de meeste verwachtingen gebaseerd op de stereotypen in dit hoofdstuk niet bevestigd.

Hoofdstuk 4 heeft een tekstboekperspectief. Het richt zich op de vraag in hoeverre IBL-lespraktijken in het lager middelbaar wiskundeonderwijs worden ondersteund door mogelijkheden in tekstboeken door middel van het analyseren van taken in twee Beijing-tekstboeken en een Nederlands tekstboek. Deze wiskundehandboeken zijn in gebruik door de leraren in hoofdstuk 3 en zijn nog steeds algemeen in gebruik in de twee gebieden. Hoofdstukken met vergelijkbare onderwerpen en overeenkomend aantal pagina’s werden geselecteerd, wat resulteerde in de analyse van een algebrahoofdstuk over kwadratische vergelijkingen met inbegrip van factorisatie en een meetkundehoofdstuk over gelijkvormigheid. 404 Beijing- en 244

Nederlandse algebra- en meetkundeopgaven werden gecodeerd. Het analytisch kader was gebaseerd op de rubric van Capps en Crawford (2013), waarbij IBL-processen uit de literatuur en de aard van wiskunde in aanmerking zijn genomen. Het raamwerk evalueert vier IBL-niveaus in zeven fasen van IBL.

In de analyse werden percentages van IBL-niveaus in de zeven fasen berekend voor elk tekstboek. De fasen met enkele mogelijkheden voor onderzoekend leren werden genomen voor verdere analyse met taken gescheiden voor verschillende inhoud (algebra, meetkunde) en contexttypes (geen, wiskundig, camouflage, real-life). Voorbeelden van opgaven met en zonder mogelijkheden voor onderzoekend leren werden ter illustratie getoond. Gedeelde en specifieke IBL-gerelateerde kenmerken van de tekstboeken werden geïdentificeerd en vergeleken met verwachtingen gebaseerd op de stereotypen.

De resultaten laten zien dat er veel gedeelde IBL-kenmerken tussen de Beijing- en Nederlandse tekstboeken zijn geïdentificeerd. De tekstboeken laten leerlingen een aantal keuzes maken om zich wiskundig te organiseren en oplossingsprocedures te verkennen, terwijl hogere niveaus van onderzoekend leren zelden worden bereikt. Ze lijken leerlingen niet te betrekken bij vragen stellen, hypothesen opstellen, samenwerken, communiceren of reflecteren. De leerboeken bieden meer IBL-mogelijkheden om wiskundig te organiseren in de geselecteerde meetkundehoofdstukken dan in de geselecteerde algebrahoofdstukken. Over het algemeen worden er meer IBL-mogelijkheden om wiskundig te organiseren en om oplossingsprocedures te verkennen getoond in opgaven met contexten dan in opgaven zonder. Bijzondere IBL-gerelateerde kenmerken van de tekstboeken in elk gebied werden ook geïdentificeerd, waarvan een deel niet overeenkomen met de stereotypen over onderwijsculturen. Het Nederlandse leerboek bevat relatief minder mogelijkheden om wiskundig te organiseren en om oplossingsprocedures te verkennen dan de Beijing-leerboeken. Voor sommige categorieën vertoont elk leerboek een ander patroon, of er bestaan verschillen tussen de twee Beijing-tekstboeken die opvallender lijken dan die tussen de Beijing- en Nederlandse tekstboeken. De meeste verwachtingen op basis van de stereotypen worden in het hoofdstuk niet bevestigd.

Hoofdstuk 5 gaat in op het perspectief van de lespraktijk in de klas. Het richt zich op de vraag in hoeverre IB- lespraktijken aan de orde zijn in Chinese wiskundelessen in de onderbouw van het voortgezet onderwijs. Dit hoofdstuk is gebaseerd op 24 lessen van vijf leraren uit Beijing die tot de deelnemers van hoofdstuk 3 behoren, waaronder 19 gebruikelijke lessen en 5 verplichte IBL-lessen. Wat betreft de verplichte IBL-lessen werd elke leraar gevraagd om een wiskundeles te geven en een wiskundeles met elementen van onderzoekend leren te ontwerpen en uit te voeren. De IBL-lessen werden opgenomen; leraren werden na de les geïnterviewd en leerlingenenquêtes werden afgenomen.

De opnames van de lessen werden gecodeerd op structuur en op mogelijkheden voor onderzoekend leren. Het raamwerk om lesstructuren te coderen bestaat uit twee delen. Het eerste deel richt zich op de doelen van klassikale activiteiten met categorieën (herziening, introductie van nieuwe inhoud, oefening van nieuwe inhoud) uit het TIMSS project (Trends in International Mathematics and Science Study). Het tweede deel richt zich op vormen van interactie in de klas met categorieën (hele klas, groep, individueel) op basis van het TALIS project (Teaching and learning international survey). Het kader voor de evaluatie van mogelijkheden voor IBL was gebaseerd op de in hoofdstuk 4 gebruikte rubric, en werd aangepast aan de codering van andere lessen dan tekstboeken.

In de analyse werden de percentages van de categorieën in doelen en vormen berekend voor de 24 lessen. Vijf gebruikelijke lessen werden geselecteerd, samen met de vijf verplichte lessen van onderzoekend leren; ze werden verder geanalyseerd om de mogelijkheden voor onderzoekend leren te coderen. Kenmerken werden geïdentificeerd en vergeleken tussen deze leraren, en tussen de twee soorten lessen. Resultaten werden ook vergeleken met resultaten uit interviews met leraren en enquêtes onder leerlingen. Bovendien werden grafische voorstellingen en casussen van drie docenten getoond in dit hoofdstuk.

Vergeleken met de gewone lessen pasten de leraren de lesstructuren in de IBL-lessen aan om meer tijd te verdelen om nieuwe inhoud te introduceren, terwijl ze verschillende keuzes toonden met betrekking tot de verandering van tijd voor de vier specifieke activiteiten (opstellen, verkennen, bespreken en samenvatten). Wat betreft de doelstellingen, vertoonden de vijf geselecteerde gewone lessen hoofdzakelijk een patroon van “herbekijken → nieuwe inhoud introduceren → nieuwe inhoud inoefenen”, terwijl het in sommige van de IBL-lessen veranderd was in “ nieuwe inhoud introduceren→ nieuwe inhoud inoefenen”. Klasactiviteit was de overheersende vorm in beide typen lessen. IBL-lespraktijken werden aangetoond in de geselecteerde gebruikelijke lessen met mogelijkheden voor leerlingen om wiskundig te organiseren, oplossingsprocedures te verkennen, te communiceren en te reflecteren. Een hoger niveau van onderzoekend leren was relatief aanwezig in de eerste twee fasen. Toen de leraren de IBL-lessen ontwierpen en implementeerden , pasten ze niet veel aan met betrekking tot IBL-lespraktijken. Ze leken onderzoekend leren te verbinden met groepswerk en open problemen die leerlingen konden uitdagen, en ze leken geen compleet beeld te hebben van de volledige IBL-cyclus. Leerlingen moeten de kans krijgen om na te denken over vragen om zelf aan te pakken, en er zouden meer hypothesen bij betrokken moeten worden. De meeste van de resultaten op basis van de codering van lessen, de percepties van leerkrachten en de percepties van leerlingen lijken overeen te stemmen, terwijl sommige bevindingen tegenstrijdig lijken en in verder onderzoek zouden kunnen worden getoetst.

Hoofdstuk 6 geeft een overzicht van de bevindingen in de vier bovenstaande hoofdstukken en verbindt deze bevindingen om de hoofdvragen van deze studie te beantwoorden. Resultaten met betrekking tot de huidige situaties van onderzoekend leren in de twee gebieden werden besproken, ook resultaten met betrekking tot de stereotypen over onderwijsculturen in Oost Azië en het Westen. De stereotypen worden door onze bevindingen in twijfel getrokken. Landen of gebieden in de twee groepen van onderwijsculturen zouden ook enkele elementen kunnen delen, en variaties binnen een onderwijscultuur kunnen niet worden genegeerd. Op basis van deze resultaten werden implicaties gepresenteerd voor wiskundeleraren, lerarenopleiders en ontwerpers van leerboeken en leerplannen. We reflecteerden ook op de studie en verschaften inzichten voor verder onderzoek. Onderzoekers kunnen bijvoorbeeld doorgaan met het onderzoeken van de huidige situatie van onderzoekend leren in het wiskundeonderwijs op basisscholen en middelbare scholen. Meer onderzoek is nodig om de kaders van onderzoekend leren voor het wiskundeonderwijs die in deze studie zijn gegeven uit te werken.

In het algemeen lijken de resultaten van deze dissertatie overeen te komen met bevindingen in de literatuur dat wiskundeleraren mogelijk geen volledig begrip hebben van IBL en de volledige IBL-cyclus, waardoor ze vaak niet alle fasen van de IBL-cyclus benutten of IBL op hoog niveau gebruiken in hun onderwijs. Dit is mogelijk ook gerelateerd aan het gebrek aan overvloedige mogelijkheden voor onderzoekend leren in tekstboeken. Tenslotte suggereert de studie om rekening te houden met specifieke lokale contexten bij het interpreteren van onderwijspraktijken, in plaats van deze uitsluitend te beschouwen op basis van kenmerken van bredere onderwijsculturen.

Curriculum Vitae

Luhuan Huang was born on 3 August 1991 in Yongxin, Jiangxi Province, China. After completing her secondary education at Renbishi High School in 2009, she entered Beijing Normal University and studied there for seven years. She obtained her bachelor and master degree in the Faculty of Education, also a dual bachelor degree in the School of Foreign Languages and Literature. In 2016, she received funding from the China Scholarship Council (CSC) for her PhD project at Utrecht University. The project is supervised by Prof. dr. W.R. van Joolingen and Dr. L.M. Doorman. Research findings of the project are presented in this dissertation. Luhuan has returned to China and will continue her future career in the field of education.

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Publications Related to this Dissertation

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Utrecht University

Faculty of Science
Freudenthal Institute

Mathematics is considered to be a human activity and students should actively participate in the learning process. These features are fostered in inquiry-based learning (IBL). IBL is interpreted as a teaching approach that invites students to learn in similar ways as how mathematicians work. The understanding and practices of IBL might be impacted by teaching cultures, of which those in East Asia and the West are considered to be remarkably different and have led to stereotypes. This study tried to move beyond these stereotypes and explored the current situations of IBL in mathematics education. China, specifically Beijing, and the Netherlands were taken as examples of the two teaching cultures. Perspectives of students, teachers, textbooks and classroom practices were explored.

Results seem to show that mathematics teachers may not have a complete understanding of IBL and the IBL cycle, thus they often do not include the full IBL cycle or involve high-level IBL in their teaching. This is possibly also related to the lack of abundant opportunities for IBL present in textbooks. The study also challenges the stereotypes and suggests to include specific local contexts when interpreting educational practices other than considering them solely from features of broader teaching cultures.