A Light Robust Optimization Approach for Uncertainty-based Day-ahead Electricity Markets

Lina Silva-Rodriguez*†‡, Anibal Sanjab*†, Elena Fumagalli‡, Ana Virag*† and Madeleine Gibescu‡
* Flemish Institute for Technological Research (VITO), Boeretang 200, 2400 Mol, Belgium
{lina.silvarodriguez, anibal.sanjab, ana.virag}@vito.be
† EnergyVille, Thor Park 8310-8320, 3600 Genk, Belgium
‡ Copernicus Institute of Sustainable Development, Utrecht University, Princetonlaan 8a, 3584 CB Utrecht, Netherlands
{l.m.silvarodriguez, e.m.fumagalli, m.gibescu}@uu.nl

***Abstract*—The traditional deterministic day-ahead (DA) market clearing does not accommodate the uncertainty from variable renewable energy sources, resulting in an increasing activation of expensive reserves and curtailment events. Robust optimization (RO) has been proposed to mitigate this uncertainty. However, as RO considers worst-case scenarios, it results in highly conservative solutions. This paper proposes a light robust (LR) DA market clearing mechanism to address these shortcomings, controlling the trade-off between robustness and economic efficiency. This mechanism integrates the uncertainty from renewables in its formulation and allows the derivation of coherent market prices. The optimal bidding strategy of the stochastic participants is mathematically derived, while considering the expectation on the system imbalance. A comparison with the deterministic formulation proves that stochastic producers can economically benefit from the proposed mechanism, encouraging their participation. The derived analytical results are corroborated by numerical results from a case study based on the IEEE 24-node test system.**

***Index Terms*—Day-ahead markets, electricity markets, light robust optimization, renewable energy integration, uncertainty-based market clearing.**

I. INTRODUCTION

Initially designed for dispatchable power plants and a largely inelastic demand, electricity markets are being challenged by an increasing presence of new market participants with different technical characteristics. Among these participants, variable renewable energy sources (VRES) are taking up a key role, in line with the EU decarbonization policy. However, the deterministic market design currently in place does not consider the uncertain nature of VRES, which, in case of forecast errors, augments the likelihood of activation of expensive reserves or curtailment events.

In order to efficiently accommodate VRES, robust optimization (RO), among other approaches, has been adopted in a number of works in the literature [1]–[5]. In RO, the uncertain parameters are assumed to vary in a given uncertainty set. Under this approach, any solution is feasible for any realization within the uncertainty set, and optimal for the worst-case realization [6]. This approach improves upon the deterministic market clearing process by capturing the existing uncertainty. However, disadvantages have prevented its implementation in real-life markets [7]. For example, the high conservativeness of the solution results in a considerably more expensive market clearing result.

This paper addresses this latter shortcoming, and introduces a day-ahead (DA) uncertainty-based market clearing based on the principles of light robust (LR) optimization. The proposed LR-based DA market clearing integrates the uncertainty of VRES in its mathematical formulation using a new bid format that includes an uncertainty range for the stochastic producers (i.e., VRES). Moreover, it explicitly captures the trade-off between the deterministic and the most robust market outcomes via a tunable 'conservativeness' parameter. Finally, it can be efficiently solved owing to its linearity.

This work contributes to the state-of-the-art by (1) proposing an uncertainty-based DA market clearing – using the principles of LR optimization [8] – which allows the market operator to choose the level of robustness and the level of economic efficiency of the DA dispatch (a feature currently missing in deterministic DA markets); (2) presenting a pricing mechanism coherently derived from the proposed formulation; (3) deriving the optimal bidding behaviour of the stochastic market participants with respect to their uncertainty ranges while taking into consideration the expected system imbalance and associated settlement, and (4) showing, based on a comparison with the traditional deterministic DA formulation, that VRES can economically benefit from the proposed LR-based market clearing, encouraging their participation in this market setting.

These results are corroborated by a numerical case study based on the IEEE 24-bus test system. The generated results illustrate the trade-off between robustness and economic efficiency, and the effect of the conservativeness parameter on the dispatched quantities of the producers. Furthermore, the results show the dependence of the submitted uncertainty ranges on the expectation of the total system imbalance. Finally, the comparison with the traditional DA market shows that the profits obtained under the LR formulation are greater than or equal to the profits obtained under the deterministic setting.

The rest of the paper is organized as follows. The LR market

Submitted to the 22nd Power Systems Computation Conference (PSCC 2022).
This work is supported by the energy transition funds project 'EPOC 2030-2050' organised by the Belgian FPS economy, S.M.E.s, Self-employed and Energy.

formulation is introduced in Section II. Section III presents the derivation of coherent prices from the proposed formulation. The bidding strategy analysis is presented in Section IV, while a comparison of profits with the deterministic DA market is presented in Section V. The numerical results are presented in Section VI, while Section VII provides directions for future work and concludes the paper.

II. Light robust day-ahead market clearing

This section first introduces the deterministic DA market formulation, followed by the proposed LR market model.

A. Deterministic day-ahead market

The traditional DA market set-up (as considered in most European electricity markets) relies on price-quantity bids submitted by market participants to a power exchange[1]. As a zonal setting is considered, intra-area network constraints are not explicitly considered. In its most simple form, the DA market clearing (hereinafter the "nominal problem") can be formulated as follows:

$$\max_{\boldsymbol{q^{DD}},\boldsymbol{q^{DS}},\boldsymbol{q^{DG}}} \sum_{d\in\mathcal{D}}(q_d^{DD}p_d^D)-\sum_{s\in\mathcal{S}}(q_s^{DS}p_s^S)-\sum_{g\in\mathcal{G}}(q_g^{DG}p_g^G), \quad (1)$$

subject to:

$$\sum_{d\in D} q_d^{DD} - \sum_{s\in S} q_s^{DS} - \sum_{g\in G} q_g^{DG} = 0, \quad (2)$$

$$q_d^{DD} \leq m_d^D \ \forall d \in \mathcal{D}, \quad (3)$$

$$q_s^{DS} \leq m_s^S \ \forall s \in \mathcal{S}, \quad (4)$$

$$q_g^{DG} \leq m_g^G \ \forall g \in \mathcal{G}, \quad (5)$$

$$q_d^{DD}, q_s^{DS}, q_g^{DG} \geq 0 \ \forall d \in \mathcal{D}, \forall s \in \mathcal{S}, \forall g \in \mathcal{G}, \quad (6)$$

where $q_d^{DD}, q_s^{DS}, q_g^{DG}$ are the decision variables corresponding to the quantities of the accepted demand (d), and dispatched generation from stochastic (s) and conventional (g) producers lying in the sets $\mathcal{D}, \mathcal{S}$, and $\mathcal{G}$, respectively. The problem is seen from the perspective of the market operator, which aims to maximize the social economic welfare (SEW) of the system, as expressed in (1). Under this set-up, market participants submit price(p_d^D, p_s^S, p_g^G)-quantity(m_d^D, m_s^S, m_g^G) bids expressing their willingness to buy or sell. Regarding the constraints, (2) corresponds to the energy balance constraint, while (3)–(5) correspond to the maximum demand and generation bid boundary constraints. Lastly, (6) represents the non-negativity constraints of the decision variables.

The deterministic market clearing approach does not consider the stochastic nature of market participants (e.g., VRES). Therefore, the forecast of the next day's operating conditions is used internally by VRES owners to determine their bid's quantities m_s^S. Given the uncertain nature of these participants, forecast errors can occur, resulting in a possible mismatch between the quantities committed in the DA dispatch and the actual delivery. This imbalance represents an additional cost not only for the participant responsible for the imbalance, but also to the entire system due to the activation of reserves to restore the system balance.

[1] This work focuses on European electricity markets, which differ from other settings that are in place in, for example, North-American markets, where unit-based nodal market approaches are used.

B. Light robust formulation

Since VRES producers are influenced by weather conditions (e.g., wind speed and solar irradiance), their highly variable electricity production is considered an uncertain parameter. To capture this uncertainty, this work proposes to clear the DA market using a new LR problem formulation. LR optimization is a flexible modelling technique first proposed in [8] for linear programming as a compromise between the quality and the robustness of a derived solution. Quality refers to the optimality of the solution for the nominal case (e.g., the most probable case), while robustness refers to the feasibility of the solution when uncertain input parameters are considered[2]. The proposed LR formulation for uncertainty-based clearing of the DA market (hereinafter referred to as the LR DA market clearing) is presented next.

The LR DA market clearing accommodates a new bid format, namely, uncertainty bids, to be used by the stochastic producers. The bid quantity $\tilde{m}_s^S$ is the uncertain parameter that lies in the interval-based uncertainty set $U : [m_s^S - \hat{m}_s^S, m_s^S + \hat{m}_s^S]$. Given the fact that an individual negative imbalance (i.e., actual productions lower than the DA commitments) might require the activation of expensive upward reserves or in extreme cases load shedding, this work focuses on deviations towards the lower bound of the uncertainty set. In this regard, a stochastic participant s, would be allowed to submit an uncertainty bid in the form of $(m_s^S, \hat{m}_s^S, p_s^S)$, where m_s^S is their most probable production, $\hat{m}_s^S$ is the maximum anticipated negative deviation from m_s^S, and p_s^S is the associated price. Fig. 1 illustrates the uncertainty range in the bid format.

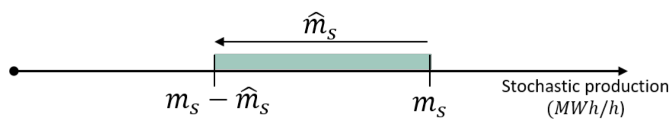

Fig. 1. Uncertainty bid representation.

Considering the LR optimization principles [8], the deterministic formulation and the proposed uncertainty bid format, the uncertain DA market is formulated as an LR DA market, as follows:

$$\min_{\boldsymbol{q^D},\boldsymbol{q^S},\boldsymbol{q^G},\gamma_s} \sum_{s\in\mathcal{S}} \gamma_s, \quad (7)$$

subject to:

$$\sum_{s\in\mathcal{S}}(q_s^S p_s^S)+\sum_{g\in\mathcal{G}}(q_g^G p_g^G)-\sum_{d\in\mathcal{D}}(q_d^D p_d^D)\leq -z^*(1-\rho); \mu_1, \quad (8)$$

[2] A generalized light robustness concept is proposed in [9]. The concept has been used to solve problems on train timetable scheduling [8], [10], [11], cyclic master surgery scheduling [12], airport runway scheduling [13], and optimization of Markov decision processes [14].

$$\sum_{d\in\mathcal{D}} q_d^D - \sum_{s\in\mathcal{S}} q_s^S - \sum_{g\in\mathcal{G}} q_g^G = 0 \; ; \lambda, \tag{9}$$

$$q_d^D \leq m_d^D \; ; \mu_{2_d} \; \forall d \in \mathcal{D}, \tag{10}$$

$$q_s^S \leq m_s^S \; ; \mu_{3_s} \; \forall s \in \mathcal{S}, \tag{11}$$

$$q_g^G \leq m_g^G \; ; \mu_{4_g} \; \forall g \in \mathcal{G}, \tag{12}$$

$$q_s^S - \gamma_s \leq m_s^S - \hat{m}_s^S \; ; \mu_{5s} \; \forall s \in \mathcal{S}, \tag{13}$$

$$q_d^D, q_s^S, q_g^G, \gamma_s \geq 0 \; ; \mu_{6d}, \mu_{7s}, \mu_{8g}, \mu_{9s}, \; \forall d, \forall s, \forall g, \tag{14}$$

where z^* is the optimal solution for the nominal problem (i.e, the deterministic market clearing (1)-(5)) and ρ is a conservativeness parameter which can be used by a market or system operator to balance the quality (i.e, economic efficiency) and the robustness of the solution. In this respect, $\rho = 0$ corresponds to the nominal problem, while $\rho = \rho_{max}$ results in the most robust solution[3]. As evident, (8) defines the maximum deterioration from the nominal case (z^*) using the conservativeness parameter ρ. Constraints (9)–(12) ensure the optimal solution of the LR problem is feasible for the nominal case. Furthermore, the robustness goal is represented by the right-most part of constraint (13) whose level of compliance is defined by the slack variable γ_s, which is minimized by the auxiliary objective function (7). Hence, in addition to the uncertainty bid submitted, the slack variable γ_s contributes directly to defining the dispatch space of q_s^S as captured in (13). The resulting dispatch limits of q_s^S with respect to the submitted uncertainty bid and γ_s are depicted in Fig. 2. In other words, γ_s takes positive values when the robustness goal is relaxed (reaching its maximum value when $\rho = 0$) and equals zero when the reached solution corresponds to the most robust case. As such, when ρ is increased, this allows deviating from the nominal solution, z^*, as captured by (8). This, in turn, enables γ_s for all $s \in \mathcal{S}$ to decrease, constraining the dispatch of stochastic units' outcomes (as shown in (13)) in a sense that renders the solution less sensitive to downward deviations in stochastic units' outputs, and hence, more robust. In this LR DA formulation, the most robust dispatch is chosen among all the solutions satisfying the maximum objective function deterioration captured in (8).

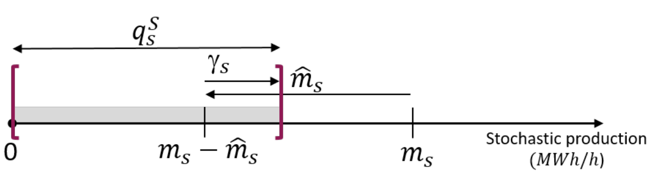

Fig. 2. Dispatch limits of q_s^S.

[3]Note that the most robust solution corresponds to the solution for the worst-case realization of the interval-based uncertainty sets defined by the uncertainty bids submitted by the stochastic participants.

III. Pricing scheme

Given the structure of the LR formulation, it is not possible to derive the market price from the dual variable λ of the energy balance constraint (9), as in the traditional DA market formulation[4], as the objective function (7) does not capture the social economic welfare. Therefore, we propose to derive the market price from a modified DA formulation. This modified formulation is made of problem (1)–(6) in which (4) is replaced with (13) for $\gamma_s = \gamma_s^*$. The modified DA formulation is as follows[5].

$$\max_{\boldsymbol{q^{MD}}, \boldsymbol{q^{MS}}, \boldsymbol{q^{MG}}} \sum_{d\in\mathcal{D}} q_d^{MD} p_d^D - \sum_{s\in\mathcal{S}} q_s^{MS} p_s^S - \sum_{g\in\mathcal{G}} q_g^{MG} p_g^G, \tag{15}$$

subject to:

$$\sum_{d\in D} q_d^{MD} - \sum_{s\in S} q_s^{MS} - \sum_{g\in G} q_g^{MG} = 0 \; ; \lambda_m, \tag{16}$$

$$q_d^{MD} \leq m_d^D ; \mu_{m1d} \; \forall d \in \mathcal{D}, \tag{17}$$

$$q_s^{MS} - \gamma_s^* \leq m_s^S - \hat{m}_s^S \; ; \mu_{m2s} \; \forall s \in \mathcal{S}, \tag{18}$$

$$q_g^{MG} \leq m_g^G \; ; \mu_{m3g} \; \forall g \in \mathcal{G}, \tag{19}$$

$$q_d^{MD}, q_s^{MS}, q_g^{MG} \geq 0 \; ; \mu_{m4d}, \mu_{m5s}, \mu_{m6g} \forall d, \forall s, \forall g. \tag{20}$$

This formulation allows the market operator to derive the prices from the dual variable λ_m of the energy balance constraint (16), as further shown next. Indeed, by transferring the optimal slack variable γ^* from the LR DA to the modified problem, as shown in (18), the dispatched quantities of the stochastic producers in the two formulations (q_s^{S*} and q_s^{MS*} for all $s \in \mathcal{S}$) are the same, enabling the derivation of the prices from the modified formulation[6]. This statement is proven as follows using the Karush-Kuhn-Tucker conditions (37)–(60) included in the Appendix.

Theorem 1: If the optimal objective function of the LR DA problem is strictly positive ($\sum_{s\in S} \gamma_s > 0$) in a competitive market, such that at least one conventional producer is dispatched, then $q_s^{MS*} = q_s^{S*}$, for all $s \in \mathcal{S}$.

Proof: Without loss of generality, let us consider stochastic producer i (i.e. $s \triangleq i$) to be the cheapest stochastic producer. As such, when $\sum_{s\in S} \gamma_s > 0$, then γ_i is positive. For the derivation of the proof of Theorem 1, we first begin by proving the following two claims.

Claim 1: If $\sum_{s\in S} \gamma_s > 0$, then (13) is binding and at least one stochastic producer (i.e., i) is dispatched ($q_i^S > 0$).

Proof: If $\sum_{s\in S} \gamma_s > 0$, then at least one stochastic producer has an associated $\gamma_s > 0$, namely, $\gamma_i > 0$. By complementary slackness, if $\gamma_i > 0$, then $\mu_{9i} = 0$, see (50). If

[4]This pricing mechanism corresponds to a uniform pay-as-cleared pricing used in European DA markets.

[5]Note that superscript M is added to the nomenclature of the decision variables to explicitly highlight that they belong to the modified DA formulation.

[6]Note that γ_s defines the level of robustness of the dispatched quantity q_s^S with respect to the uncertainty bids submitted by each stochastic producer.

$\mu_{9_i} = 0$, then $\mu_{5_i} = 1$, see (40). By complementary slackness (46), if $\mu_{5_i} = 1$, then (13) is binding:

$$q_s^S - \gamma_s = m_s^S - \hat{m}_s^S. \tag{21}$$

By definition, we know that $m_i^S - \hat{m}_i^S > 0$. Therefore, if $\gamma_i > 0$, then $q_i^S > 0$. This captures the case of stochastic producers as their bids are typically cheaper than the bids of conventional generators. □

Claim 2: If $\sum_{s \in S} \gamma_s^* > 0$ and $q_s^{MS} > 0$, then (18) is binding.

Proof: Again, let i be the stochastic producer being dispatched ($q_i^{MS} > 0$) whose γ^* is positive ($\gamma_i^* > 0$). By complementary slackness, if $q_i^{MS} > 0$, then $\mu_{m5_i} = 0$, see (60). If $\mu_{m5_i} = 0$, then (53) can be written as:

$$-\lambda_m + p_i^S + \mu_{m2_i} = 0. \tag{22}$$

Since the market is competitive, there are other market players (either conventional or stochastic) that are dispatched by the market operator. Let these players be more expensive than stochastic player i. Let j be a conventional producer (i.e., $g \triangleq j$) dispatched by the market ($q_j^{MG} > 0$) whose price p_j^G is greater than p_i^S. By complementary slackness, if $q_j^{MG} > 0$, then $\mu_{m6_j} = 0$, see (58). If $\mu_{m6_j} = 0$, then equation (51) can be written as:

$$-\lambda_m + p_j^G + \mu_{m3_j} = 0. \tag{23}$$

By definition $\mu_{m3_j} \geq 0$, then, (23) can be converted from equality to inequality:

$$p_j^G \leq \lambda_m. \tag{24}$$

Substituting (22) in (24): $\mu_{m2_i} \geq p_j^G - p_i^S$. Since $p_j^G > p_i^S$, then $\mu_{m2_i} > 0$. By complementarity slackness (57), if $\mu_{m2_i} > 0$, then (18) is binding, which proves claim 2 and returns:

$$q_s^{MS} - \gamma_s^* = m_s^S - \hat{m}_s^S. \tag{25}$$

□

Given the results of Claim 1 and Claim 2, the equivalence between the dispatched quantities in the two formulations can be readily shown. In fact, solving (21), from Claim 1, for γ_s and substituting it in (25), from Claim 2, it follows that $q_s^{S*} = q_s^{MS*}$ for all $s \in \mathcal{S}$. ■

As such, the dispatched quantities and the market price can be obtained from the modified formulation. Focusing on the binding constraints (21) and (25), it is relevant to highlight that the DA quantities of the stochastic players q_s^S are defined by the bid quantities $m_s, \hat{m}_s$ and the slack variables γ_s. Fig. 3 illustrates the relation between the variables and the parameters.

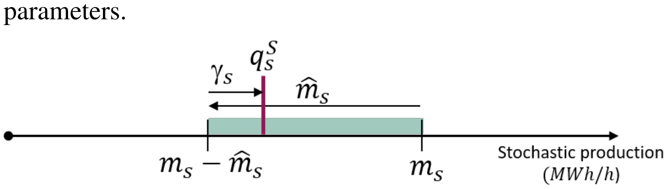

Fig. 3. LR DA dispatched quantities according to binding constraints.

In this respect, the entire market clearing process is composed of three main steps. First, the nominal problem is solved to obtain the optimal solution z^*. Second, considering the uncertainty bids, the conservativeness parameter[7] ρ and the optimal solution z^*, the LR problem is solved, and the resulting optimal slack variable γ^* is sent to the last step. Lastly, the modified DA market is cleared to obtain the dispatched quantities and the market price. The process is illustrated as a flow diagram in Fig. 4.

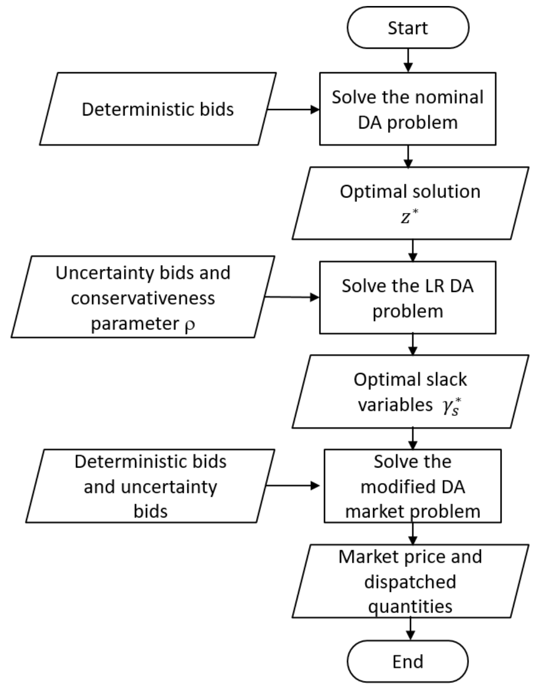

Fig. 4. LR DA market clearing process.

IV. Optimal Bidding Strategy

As the LR DA formulation introduces a novel bidding format and market clearing method, investigating the bidding behaviour of stochastic producers under this new setting is paramount to anticipate the performance of the market and its implications on the different market participants.

Under the LR DA approach, stochastic participants bid a maximum negative deviation $\hat{m}_s$. Hence, the player can either decide to bid its true negative deviation $\hat{m}_s^T$ or an untruthful deviation that considers a strategic deviation χ_s (representing a shorter or larger uncertainty range), expressed as follows:

$$\hat{m}_s = \hat{m}_s^T - \chi_s. \tag{26}$$

Therefore, when considering a bidding strategy, a player's decision variable corresponds to the choice of χ_s, as the most probable generation m_s is a relatively known quantity that can be inferred from the unit technical characteristics and weather conditions. Fig. 5 illustrates the relation between the aforementioned parameters and variables.

[7] ρ can be freely specified by the market or system operators to achieve the intended balance between robustness and optimality.

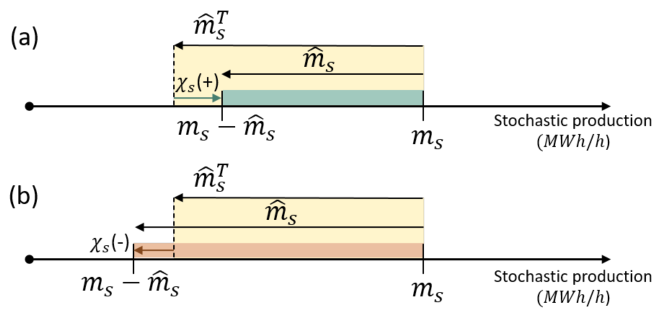

Fig. 5. Positive (a) and negative (b) strategic deviation.

As the profits achieved by a player do not only depend on its DA dispatch but also on its production in real time, the imbalance settlement stage is key in the derivation of the optimal bidding strategies. In European electricity markets, imbalance settlement is performed by the transmission system operators (TSOs), who pay or charge balancing responsible parties (BRP) for their imbalances [15]. The single price imbalance settlement mechanism is currently in place in most European markets and is being further adopted as part of the harmonization process [15]. Under the single imbalance pricing scheme [16], BRPs are settled according to their individual imbalance (i) and the total system imbalance (I). Table I illustrates this imbalance pricing mechanism, where p^I is the imbalance price and p^{DA} is the DA market price.

TABLE I
SINGLE PRICE IMBALANCE SETTLEMENT MECHANISM

	$I > 0$	$I < 0$
$i > 0$	TSO pays: $p^I < p^{DA}$	TSO pays: $p^I > p^{DA}$
$i < 0$	BRP pays: $p^I < p^{DA}$	BRP pays: $p^I > p^{DA}$

When bidding strategically, market participants choose the optimal strategic deviation χ_s that maximizes their expected profit π_s^{LR}. The profit π_s^{LR} is made up of the revenues from selling electricity in the DA market and the revenues or costs from the imbalance settlement stage:

$$\pi_s^{LR} = \ q_s^S \ p^{DA} + i_s \ p^I, \tag{27}$$

where i_s is the player's imbalance given by:

$$i_s = q_s^{RT} - q_s^S, \tag{28}$$

where q_s^{RT} is the actual realization of VRES production (which is considered exogenous), and q_s^S is the dispatched quantity from the LR DA market (21).

The imbalance price depends on the total system imbalance I. When I is positive, p^I is lower than p^{DA}. On the contrary, when I is negative, p^I is greater than p^{DA}. These characteristics are aligned with the single imbalance pricing method, where the balancing marginal incremental price and marginal decremental price follow the described trend. p^I can then be captured using the downward-sloping function:

$$p^I = -bI + p^{DA}. \tag{29}$$

The total system imbalance, I, is the sum of the imbalance of the player acting strategically (i_s) and the imbalance caused by other stochastic players (i_{-s}). Without loss of generality, it is assumed that only stochastic producers contribute to the total imbalance, as other resources are dispatched deterministically[8]:

$$I = i_s + i_{-s}. \tag{30}$$

The optimal bidding strategy of a stochastic producer can, then, be derived as follows.

Proposition 1: The optimal strategic deviation is given by:

$$\chi_s^* = q_s^{RT} - m_s + \hat{m}_s^T - \gamma_s + \ \frac{i_{-s}}{2}. \tag{31}$$

Proof: After replacing (21), (26), and (28)–(30) in (27), the first order optimality condition of the expected profit in (27) is derived to obtain the optimal strategic deviation χ_s^*.[9] ■

Equation (31) provides relevant information about the bidding behaviour of player s. Since $\frac{d\chi_s}{di_{-s}} > 0$, this implies that the economic incentives to bid a shorter uncertainty range $\hat{m}_s$ (positive strategic deviation χ_s) increases with the expected aggregate imbalance from the other stochastic producers. On the other hand, if the aggregated imbalance is expected to be negative, the player is incentivized to bid a larger uncertainty range $\hat{m}_s$ (negative strategic deviation χ_s). The obtained result is linked to the imbalance settlement scheme considered in this analysis. In this regard, the work in [17] demonstrates that, in the traditional DA market, the economic incentives to intentionally be in an imbalanced position are lower under the dual pricing scheme than under the single pricing scheme.

V. MARKET PARTICIPANTS PROFITS UNDER LIGHT ROBUSTS AND DETERMINISTIC DAY-AHEAD MARKET CLEARING

To anticipate their incentive to participate in the proposed LR market framework, an investigation of whether stochastic players are in a more advantageous position under the LR approach than under the traditional DA market is provided next.

In the LR market clearing, under the optimal bidding strategy in (31), the player's profit in (27) is expressed as:

$$\pi_s^{LR} = \ q_s^{RT} \ p^{DA} + b \ \frac{i_{-s}^2}{4}. \tag{32}$$

As the true most probable forecast m_s^S is considered to be known, no strategic behaviour is considered in the

[8] Any participant having non-controllable outputs (including, e.g., due to unplanned outages) can be readily considered in the proposed formulation as stochastic, if needed.

[9] Since $\nabla^2 \pi_s^{LR}(\chi_s) < 0$, then, χ_s^* is a local maximum point.

deterministic DA formulation[10]. Similarly to the LR case, the expected profit under the traditional deterministic DA formulation is calculated considering the revenues and costs from the DA market and the imbalance settlement. Since the stochastic producers are assumed to be the cheapest bidders, they are guaranteed to be entirely dispatched, based on their bid quantity ($q_s^S = m_s^S$). As such, the individual imbalance of the stochastic player under the deterministic DA market is calculated as:

$$i_s^D = q_s^{RT} - m_s^S. \tag{33}$$

Considering the DA dispatch and the imbalance settlement stage, the expected profit of a stochastic player participating in the deterministic DA market is expressed as:

$$\pi_s^D = q_s^{RT} p^{DA} + b(-(q_s^{RT} - m_s^S)^2 - i_{-s}(q_s^{RT} - m_s^S)). \tag{34}$$

The following results show that it is always advantageous for stochastic producers to participate in the LR DA market, as $\pi_s^{LR} \geq \pi_s^D$.

Proposition 2: If $i_{-s} \neq 2(-q_s^{RT} + m_s^S)$, then $\pi_s^{LR} > \pi_s^D$. Otherwise, $\pi_s^{LR} = \pi_s^D$

Proof: Comparing the profits in (32) and (34), it is observed that the expression $q_s^{RT} p^{DA} + b$ is common to both equations. Therefore, the following residual components of the LR and deterministic profits are compared:

$$\pi_s^{rLR} = \frac{i_{-s}^2}{4}, \tag{35}$$

$$\pi_s^{rD} = -(q_s^{RT} - m_s^S)^2 - i_{-s}(q_s^{RT} - m_s^S). \tag{36}$$

Subsequently, if $\pi_s^{rLR} = \pi_s^{rD}$, then $\pi_s^{LR} = \pi_s^D$. The root of the quadratic equation $\frac{i_{-s}^2}{4} + i_{-s}(q_s^{RT} - m_s^S) + (q_s^{RT} - m_s^S)^2 = 0$ is unique and corresponds to $i_{-s} = 2(-q_s^{RT} + m_s^S)$, which is the only point in which π_s^{rD} and π_s^{rLR} are equal. Since there is only one interception point, and the linear function (36) always intercepts the vertical axis at a negative value, the power function (35) is greater than (36) when $i_{-s} \neq 2(-q_s^{RT} + m_s^S)$. ■

This comparison with the traditional deterministic DA formulation demonstrates that VRES can economically benefit from the proposed LR-based market clearing, encouraging their participation in this market setting.

VI. Case Study

A. Input data

The case study considers an adaptation of the IEEE RTS 24-bus test system, whose original data is available in [18]. In this case study, a market with 12 conventional producers, 17 consumers, and 6 wind farms is cleared for 1 hour to test the proposed LR formulation[11]. The price-quantity bids of the conventional generators and the demand levels are obtained directly from the test system data, while the demand bid prices were chosen from a uniform distribution in the range [0,50]. Regarding the stochastic producers, historical data of the wind power production in Belgium from the Belgian TSO, Elia [20], was used to define the bid quantities. Since the information corresponds to the total aggregated production, different data sets were used to obtain the capacity factor for each wind farm considered in the case study. In this regard, m_s^S and $\hat{m}_s^S$ were calculated as, respectively, the mean and two standard deviations of each dataset. The obtained uncertainty bids are presented in Table II.

TABLE II
UNCERTAINTY BIDS FOR STOCHASTIC PRODUCERS

s	p_s^S (€/MWh)	m_s^s (MWh)	$\hat{m}_s^S$ (MWh)	s	p_s^S (€/MWh)	m_s^s (MWh)	$\hat{m}_s^S$ (MWh)
1	1	111.08	66.38	4	3	105.32	59.98
2	1.5	102.42	67.88	5	2.5	112.38	72.26
3	2	93.72	82.18	6	3.5	66.24	42.14

B. Light robust day-ahead market clearing

To conduct a sensitivity analysis, the DA market was cleared using the LR formulation (7)–(14) for different values of the conservativeness parameter ρ. It is relevant to highlight that $\rho = 0$ corresponds to the deterministic case, in which no uncertainty is considered, while $\rho = \rho_{max} = 0.0625$ corresponds to the most robust case.

One of the effects of ρ on the market dispatch is illustrated in Fig. 6, where the dispatched quantities of each stochastic producer are shown. It is observed that, if the market operator fixes $\rho = 0$, all the stochastic producers are dispatched at their most probable production m_s^S. As greater values of ρ are chosen, more uncertainty is included, reducing the dispatched generation towards the lower bound $\hat{m}_s^S$. This reaction is observed first in the dispatch of the most expensive producer (i.e., stochastic producer 6), while the cheapest unit (i.e., stochastic producer 1) is the last to experience the dispatch reduction. When the market operator aims to have a more robust dispatch, additional generation from conventional producers is required to cover the reduction of the dispatched stochastic production. This outcome is illustrated in Fig. 7, where the total demand, is included in addition to the total dispatched quantities from conventional and stochastic producers. The SEW of the system is affected by the conservativeness parameter as well. As expected, a greater SEW is obtained at the nominal case ($\rho = 0$), when also the maximum total γ is obtained. As ρ increases (and total γ decreases), greater deviations from the optimal solution of the nominal case are allowed, losing optimality but gaining robustness. The effect of ρ on the SEW is shown in Fig. 8, showing the decrease in SEW with an increase in ρ, as more traditional generation is dispatched to replace VRES (this latter result is shown in Fig. 7).

C. Bidding behaviour

To illustrate the bidding behaviour of the stochastic producers, the profit maximization problem (27) is solved. Here,

[10] Note that the bid format in the deterministic DA market is made of the most probable production m_s^S and the price p_s^S.

[11] The case study was modelled in Julia using JuMP [19] and solved using GLPK and Ipopt.

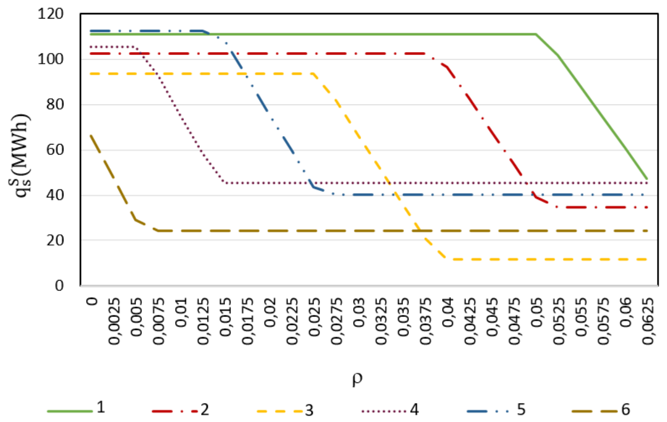

Fig. 6. Dispatched quantities of stochastic producers for different conservativeness levels.

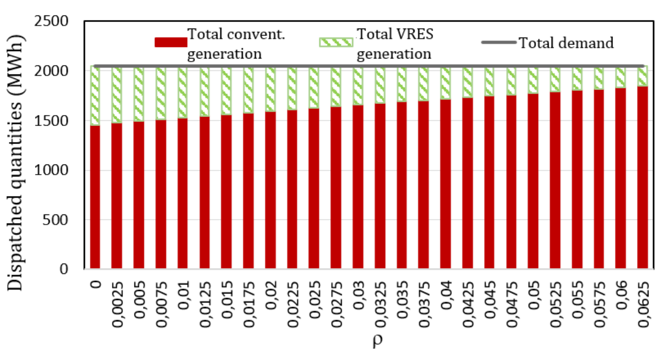

Fig. 7. Dispatched quantities for different values of conservativeness parameter.

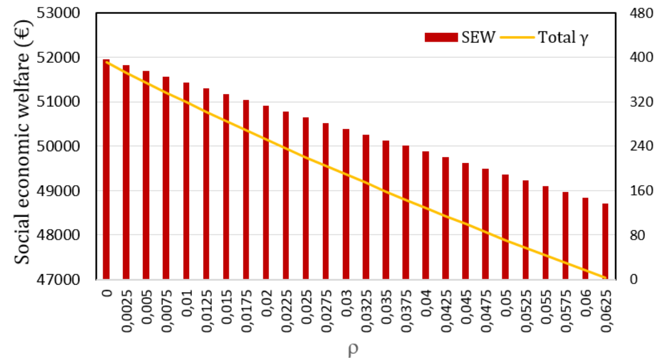

Fig. 8. Social economic welfare for different conservativeness levels.

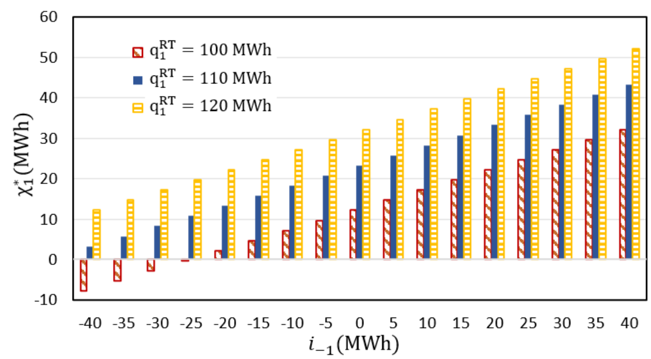

Fig. 9. Strategic deviation χ_1^* for stochastic producer 1.

the results from the perspective of the stochastic player 1 are presented for $\rho = 0.055$. A sensitivity analysis with respect to q_1^{RT} and i_{-1} is performed to assess the impact of these parameters on the decision variable χ_1. Fig. 9 illustrates how χ_1 varies under three different expected real-time realizations for the chosen stochastic producer (i.e., 100MWh, 111MWh and 120MWh) and various imbalances caused by the other stochastic producers. These results show that player 1 is incentivized to increase its strategic deviation χ_1 (i.e., bidding a shorter uncertainty range) if it expects the other participants to be in an aggregated positive imbalance. This is due to the fact that by bidding a shorter uncertainty range, a stochastic producer increases the likelihood of its actual realization being below its DA dispatch, thus being in a negative imbalance. This allows the stochastic producer to benefit from the expected positive total system imbalance (and, as a result, help balancing the system). This reaction is more prominent when a greater actual realization is expected ($q_1^{RT} = 120$). The contrary takes place if the aggregated imbalance of the other participants is expected to be negative, in which case, player 1 is incentivized to reduce its strategic deviation χ_1 and bid a broader uncertainty range. These results are consistent with the analysis and conclusions drawn in Section IV.

Given the optimal strategic deviation, the total expected profit, made of the revenues and costs from the LR DA market and the imbalance settlement stage, is calculated. These results are compared with the profits from the traditional deterministic DA formulation to determine which formulation yields a better profit for the stochastic producers. Fig. 10 illustrates the results obtain under different actual realizations. Fig. 10a illustrates the case in which $q_1^{RT} = 110MWh < m_1^S$. Fig. 10b shows the results obtained when $q_1^{RT} = 120MWh > m_1^S$. Finally, Fig. 10c corresponds to the case in which $q_1^{RT} = 111MWh = m_1^S$. These 3 cases represent the possible relations between q^{RT} and m_1^S, which affect the calculation of the profits. The results show that, regardless of the relation between q^{RT} and m_1^S, the profits obtained under the LR formulation are always higher than the profits obtained under the traditional deterministic formulation, except at the interception point where $\pi_1^D = \pi_1^{LR}$. These results are consistent with the analysis included in Section V.

VII. Conclusions and Future Work

This work has proposed a DA market clearing that accommodates the uncertainty from stochastic producers using LR optimization and uncertainty bids. This market clearing allows the market operator to define the level of conservativeness of the system using a conservativeness parameter. In addition, the proposed market clearing mechanism can be readily used to derive coherent electricity prices. Moreover, the optimal

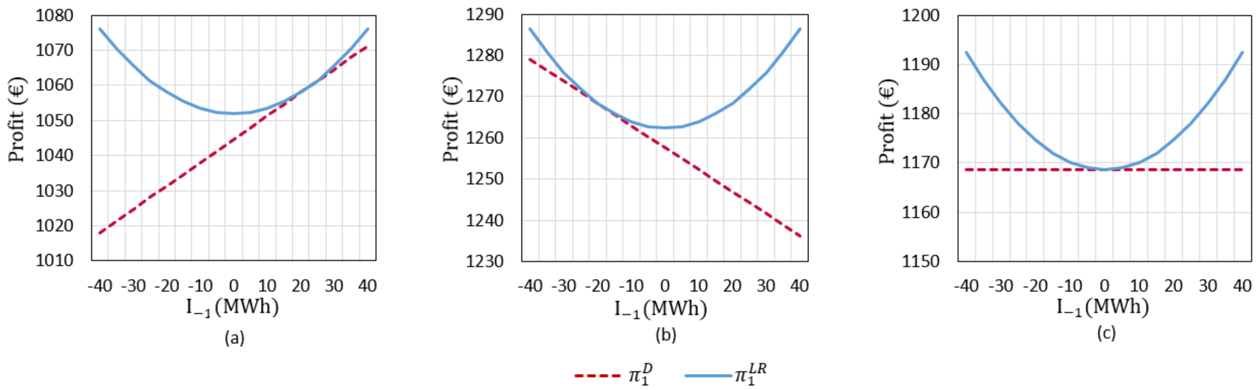

Fig. 10. Comparison of profits under the LR formulation and the traditional deterministic market clearing. (a) $q_1^{RT} < m_1^S$, (b) $q_1^{RT} > m_1^S$, (c) $q_1^{RT} = m_1^S$.

bidding strategy of the stochastic producers was derived. The analysis showed that the economic incentive to bid a shorter uncertainty range increases with the expectation of a positive aggregated imbalance from the other producers. On the other hand, stochastic producers are incentivized to bid a larger uncertainty range if they expect the aggregated imbalance from the other producers to be negative. The analysis showed that stochastic producers benefit from this new market clearing. By comparing the proposed LR DA market with the traditional DA market, it was shown that the profits from the LR formulation are always greater than or equal to the profits from the traditional deterministic DA formulation. The analytical derivations and results were showcased through a case study using an updated IEEE 24-node test system. In general, the proposed LR DA formulation provides a balance between the robustness of the market dispatch against uncertainty and the economic efficiency of the market, a useful feature for both market and system operators.

This work paves the way for future research directions. In this regard, an advanced formulation including co-optimization of energy and reserves is currently under consideration. This would allow incorporating further benefits of the proposed formulation, since additional gains in SEW can be achieved when reserves are considered in the DA clearing. In addition, a game-theoretic analysis is also envisioned to further assess the bidding behaviour of the participants under this setting.

References

[1] M. Z. J. Morales, A. Conejo, H. Madsen, P. Pinson, *Integrating Renewables in Electricity Markets*, 1st ed. Springer US, 2014.

[2] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 52–63, Feb. 2013.

[3] M. Zugno and A. J. Conejo, "A robust optimization approach to energy and reserve dispatch in electricity markets," *European Journal of Operational Research*, vol. 247, no. 2, pp. 659–671, Jun. 2015.

[4] R. Jiang, J. Wang, and Y. Guan, "Robust unit commitment with wind power and pumped storage hydro," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 800–810, May 2012.

[5] R. Jiang, J. Wang, M. Zhang, and Y. Guan, "Two-stage minimax regret robust unit commitment," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2271–2282, Aug. 2013.

[6] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski, *Robust Optimization*. Princeton University Press, Dec. 2009.

[7] L. Silva-Rodriguez, A. Sanjab, E. Fumagalli, A. Virag, and M. Gibescu, "Short term wholesale electricity market designs: A review of identified challenges and promising solutions," *Renewable and Sustainable Energy Reviews*, vol. 160, p. 112228, Feb. 2022.

[8] M. Fischetti and M. Monaci, "Light robustness," in *Robust and Online Large-Scale Optimization. Lecture Notes in Computer Science*. Springer, Berlin, Heidelberg, 2009, vol. 5868 LNCS, pp. 61–84.

[9] A. Schöbel, "Generalized light robustness and the trade-off between robustness and nominal quality," *Mathematical Methods of Operations Research*, vol. 80, no. 2, pp. 161–191, Oct. 2014.

[10] M. Goerigk, M. Schmidt, A. Schöbel, M. Knoth, and M. Müller-Hannemann, "The price of strict and light robustness in timetable information," *Transportation Science*, vol. 48, no. 2, pp. 225–242, May 2014.

[11] M. Fischetti, D. Salvagnin, and A. Zanette, "Fast approaches to improve the robustness of a railway timetable," *Transportation Science*, vol. 43, no. 3, pp. 321–335, Aug. 2009.

[12] C. Mannino, E. J. Nilssen, and T. E. Nordlander, "A pattern based, robust approach to cyclic master surgery scheduling," *Journal of Scheduling*, vol. 15, no. 5, pp. 553–563, May 2012.

[13] A. Heidt, H. Helmke, M. Kapolke, F. Liers, and A. Martin, "Robust runway scheduling under uncertain conditions," *Journal of Air Transport Management*, vol. 56, no. Part A, pp. 28–37, Sept. 2016.

[14] P. Buchholz and D. Scheftelowitsch, "Light robustness in the optimization of Markov decision processes with uncertain parameters," *Computers and Operations Research*, vol. 108, pp. 69–81, Apr. 2019.

[15] "COMMISSION REGULATION (EU) 2017/ 2195 - of 23 November 2017 - establishing a guideline on electricity balancing," *Official Journal of the European Union*, vol. L 312, pp. 6–53.

[16] Elia, "Tariffs for maintaining and restoring the residual balance of individual access responsible parties. Period 2020-2023," Tech. Rep., 2019. [Online]. Available: https://www.elia.be/en/customers/invoicing-and-tariffs

[17] S. Clò and E. M. Fumagalli, "The effect of price regulation on energy imbalances : a difference in differences design," *Energy economics*, vol. 81, pp. 754–764, Jun. 2019.

[18] C. Ordoudis, P. Pinson, J. M. Morales, and M. Zugno, "An Updated Version of the IEEE RTS 24-Bus System for Electricity Market and Power System Operation Studies," 2016. [Online]. Available: https://backend.orbit.dtu.dk

[19] I. Dunning, J. Huchette, and M. Lubin, "JuMP: A modeling language for mathematical optimization," *SIAM Review*, vol. 59, no. 2, pp. 295–320, 2017.

[20] Elia, "Wind-power generation," 2021. [Online]. Available: https://www.elia.be/en/grid-data/power-generation/wind-power-generation

APPENDIX

The Karush–Kuhn–Tucker conditions of the LR DA formulation introduced in (7)–(14) and used in Section III are as follows:

$$-\lambda + \mu_1 p_g^G \; + \mu_{4g} - \mu_{8g} = 0 \;\forall g \in \mathcal{G}, \tag{37}$$

$$\lambda - \mu_1 p_d^D \; + \mu_{2d} - \mu_{6d} = 0 \;\forall d \in \mathcal{D}, \tag{38}$$

$$-\lambda + \mu_1 p_s^S \; + \mu_{3s} + \mu_{5s} - \mu_{7s} = 0 \;\forall s \in \mathcal{S}, \tag{39}$$

$$1 - \mu_{5s} - \mu_{9s} = 0 \;\forall s \in \mathcal{S}, \tag{40}$$

$$\sum_{d \in \mathcal{D}} q_d^D - \sum_{g \in \mathcal{G}} q_g^G - \sum_{s \in \mathcal{S}} q_s^S = \; 0, \tag{41}$$

$$0 \leq \sum_{d \in \mathcal{D}} q_d^D p_d^D - \sum_{s \in \mathcal{S}} q_s^S p_s^S - \sum_{g \in \mathcal{G}} q_g^G p_g^G - z^*(1-\rho) \perp \mu_1 \geq 0, \tag{42}$$

$$0 \leq -q_g^G + m_g^G \perp \mu_{4g} \;\; \geq 0 \;\forall g \in \mathcal{G}, \tag{43}$$

$$0 \leq -q_d^D + m_d^D \; \perp \mu_{2d} \;\; \geq 0 \;\forall d \in \mathcal{D}, \tag{44}$$

$$0 \leq -q_s^S + m_s^S \perp \mu_{3s} \geq 0 \;\forall\; s \in \mathcal{S}, \tag{45}$$

$$0 \leq -q_s^S + m_s^S - \hat{m}_s^S \; + \gamma_s \perp \mu_{5s} \geq 0 \;\forall\; s \in \mathcal{S}, \tag{46}$$

$$0 \leq \; q_g^G \perp \mu_{8g} \geq 0 \;\forall\; g \in \mathcal{G}, \tag{47}$$

$$0 \leq \; q_d^D \perp \mu_{6d} \geq 0 \;\forall\; d \in \mathcal{D}, \tag{48}$$

$$0 \leq \; q_s^S \perp \mu_{7s} \geq 0 \;\forall\; s \in \mathcal{S}, \tag{49}$$

$$0 \leq \gamma_s \perp \mu_{9s} \geq 0 \;\forall\; s \in \mathcal{S}. \tag{50}$$

The Karush–Kuhn–Tucker conditions of the modified DA formulation (15)–(20) used in Section III are as follows:

$$-\lambda_m + p_g^G \; + \mu_{m3g} - \mu_{m6g} = 0 \;\forall g \in \mathcal{G}, \tag{51}$$

$$\lambda_m - p_d^D \; + \mu_{m1d} - \mu_{m4d} = 0 \;\forall d \in \mathcal{D}, \tag{52}$$

$$-\lambda_m + p_s^S \; + \mu_{m2s} - \mu_{m5s} = 0 \;\forall s \in \mathcal{S}, \tag{53}$$

$$\sum_{d \in \mathcal{D}} q_d^{MD} - \sum_{g \in \mathcal{G}} q_g^{MG} - \sum_{s \in \mathcal{S}} q_s^{MS} = \; 0, \tag{54}$$

$$0 \leq -q_g^{MG} + m_g^G \perp \mu_{m3g} \;\; \geq 0 \;\forall g \in \mathcal{G}, \tag{55}$$

$$0 \leq -q_d^{MD} + m_d^D \; \perp \mu_{m1d} \;\; \geq 0 \;\forall d \in \mathcal{D}, \tag{56}$$

$$0 \leq -q_s^{MS} + m_s^S - \hat{m}_s^S \; + \gamma_s^* \perp \mu_{m2s} \geq 0 \;\forall s \in \mathcal{S}, \tag{57}$$

$$0 \leq \; q_g^{MG} \perp \mu_{m6g} \geq 0 \;\forall g \in \mathcal{G}, \tag{58}$$

$$0 \leq \; q_d^{MD} \perp \mu_{m4d} \geq 0 \;\forall d \in \mathcal{D}, \tag{59}$$

$$0 \leq \; q_s^{Ms} \perp \mu_{m5s} \geq 0 \;\forall d \in \mathcal{D}. \tag{60}$$