Embodied approaches to functional thinking using digital technology: A bibliometrics-guided review

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Digital technology offers many opportunities for embodied approaches to mathematics education. To investigate what is known from literature about such approaches for the case of Functional Thinking, we carried out a systematic literature review, followed by a bibliometric and an expert content analysis. We included 36 peer-reviewed articles from 1986 to 2020 in the study. As a result, we identified five research themes in the field, which are further merged into three categories labelled Embodiment not central, Pseudo embodiment and Embodiment.

Keywords: Bibliometric analysis, digital technology, embodied cognition, functional thinking.

Introduction

In cognitive science, it is emphasised that cognition originates or is grounded in bodily motions and perceptual experience (Barsalou, 1999; Barsalou, 2008; Lakoff & Johnson, 1999; Lakoff & Núñez, 2001). In this context, recently developed digital technology, including motion detectors and augmented reality, seems to offer opportunities for an embodied approach to mathematics education (Bos et al., 2021; Drijvers, 2019; Nemirovsky et al., 2013). A bibliometric approach, which is an objective method that provides an overview of the knowledge structure of the domain (Li et al., 2019), was applied to explore these opportunities for the case of Functional Thinking (FT), a fundamental learning goal in mathematics education (Thompson, 1994; Vollrath,1989). The research question addressed is as follows: *What is known about the use of digital technology for an embodied approach to the teaching and learning of FT?*

Theoretical underpinnings

We draw on theoretical notions from embodied cognition and design research, digital technology research, and research on functional thinking.

Embodied cognition and embodied design

Several theories concern the role of the body in cognition and learning. Based on Conceptual Metaphor Theory in cognitive linguistics (Lakoff & Johnson, 1980), Lakoff and Núñez (2001) analysed the cognitive structure of mathematics and argued that the kinds of everyday conceptual mechanisms, image schemas, aspectual schemas, conceptual metaphor, and conceptual blends are central to mathematics. Some studies carry a similar idea about mathematics cognition concerning embodied design in function learning (e.g., Font et al., 2010; Oehrtman et al., 2019; Paz & Leron, 2009). From a perceptual perspective, Barsalou frames embodiment through grounding experiences, which is also advocated by Schwartz (1999) and Abrahamson et al. (2016) and employed in their own research. In addition, Shvarts et al. (2021) emphasise that knowledge emerges as part of a complex dynamic behavioural system that is constituted through multiple perception-action loops.

For educational materials, Abrahamson (2009) defined embodied design, which was first proposed by Rompay and Hekkert (2001), as a systematic and procedural design method, helpful in guiding the student's construction of meaning. At first, embodied design was classified into two categories: perception-based design and action-based design (Abrahamson, 2009, 2014; Abrahamson & Lindgren, 2014). Action-based designs aim to ground mathematical concepts in students' natural capacity to adaptively solve sensorimotor problems. Perception-based designs aim to ground mathematical concepts in students' natural perceptual ability in their naive views relating to a situation. Similar to the action-based genre, it is followed by a phase of reflection in which these views are developed. Concerning the role of artefact in learning design, Bos et al. (2021) propose a new type of embodied design, incorporation-based design, which is in a sense the opposite of outsourcing a task to an artefact instead of a person.

Digital technology in mathematics education

A major consideration in designing and using technology in mathematics classrooms is how to identify and use the different didactical functionalities. According to task-based interviews, Günster and Weigand (2020) set up a category system. We followed some of the categories for our study: (1) Feedback through the learning arrangement, (2) Use of sliders, (3) Creating objects, and (4) Adjusting existing objects to analyse the digital technology dimension. These four usages are related to doing mathematics and developing conceptual understanding with possible embodied elements. What's more, an embodied instrumentation approach, which can offer a design heuristic for ICT activities, was proposed by Drijvers (2019). This integrated approach, in which digital technology, mathematical cognition and sensorimotor schemes co-emerge, helps us better understand the relationship between embodied approaches, digital technology, and FT.

Functional thinking

Since the beginning of the twentieth century, functional thinking has been a central aspect of mathematical education throughout primary, secondary, and tertiary education (Vollrath, 1986). Although there is no widely adopted definition of FT, we propose that FT encompasses the process of building, describing, and reasoning with and about functions (Pittalis et al., 2020; Stephens et al., 2017). In a broader interpretation, FT connects to the four main aspects of function distinguished in literature (Confrey & Smith,1995; Doorman et al., 2012; Thompson, 1994; Vinner and Dreyfus, 1989; Vollrath, 1989): a) Function as an input-output assignment; b) Function as a dynamic process of covariation; c) Function as a correspondence relation; d) Function as a mathematical object.

Methods

To address the research question, we carried out a systematic literature search, followed by a bibliometric clustering (BC) and expert content analysis (Drijvers, Grauwin, & Trouche, 2020). The first step was part of the FunThink Erasmus+ project, a European research project.

Systematic literature search

The literature search was conducted in four databases: ERIC, PsycINFO, Scopus, and Web of Science. We searched for relevant studies published in peer-reviewed journals and written in English without restricting the publication date. Qualitative studies, quantitative studies and mixed-method studies were included. The query focused on Functional Thinking × (Embodiment OR Digital Technology). Our initial search yielded 278 journal articles. After deduplication, 257 unique publications remained. Next, we carried out two rounds of screening. The first round concerned a scan of title, abstract and keywords, to judge each article's relevance to each of the three aspects: Functional Thinking (FT), Embodiment (EM), and Digital Technology (DT). This led to 93 papers – empirical as well as theoretical papers - being selected with the help of ten coders from FunThink project. In the second round, eleven coders participated in the literature appraisal round, during which each coder read full texts and filled in a spreadsheet with the core ideas of each article. We removed the articles coded 0 to 2 as they are perceived as less helpful to our project. As a result, thirty-six articles were included in the final corpus.

Bibliometric clustering and expert content analysis

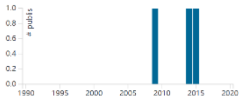
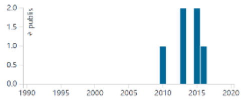
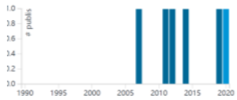
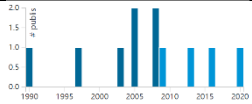
The studies in the final selection were classified with the help of BC techniques (Drijvers, Grauwin, & Trouche, 2020), which provides a sense-making sketch of the 'landscape' of our topic. We did not regard the bibliometric results as strict, exclusive categories; rather, we saw them as analytic tools that help us make sense of the rich diversity in this research field and to locate the main areas of embodied elements. Triangulating the bibliometric findings with expert content analysis helped us to find out a new taxonomy of the studies in relation to the different embodied approaches / embodied elements, which formed a basis for the study's results.

Results

Results from BC techniques include clusters that gather thematically close (based on the references) publications of the studied corpus; overall descriptions of the clusters, including an analysis of publication year, numbers involved, reference, and global meaning (see Table 1); and categories of embodied approaches to the use of digital technology for FT (see Table 2).

The bibliometric clustering leads to five clusters, containing 31 thematically close publications. The quality measure Q=0.429 suggests a meaningful partition. In each cluster, the most frequent references, the most frequent subjects, and the most cited authors are analysed. Table 1 presents some of these features.

Table 1 Description of the five clusters

	Cluster 1 (n=3)	Cluster 2 (n=5)	Cluster 3 (n=6)	Cluster 4 (n=6)	Cluster 5 (n=11)
Time distribution	 between 2009 and 2015	 mainly written after 2020	 mainly after 2013	 since 2007	 since 1990
Top three cited	O' Callaghan (1998) Kennewell(2001) Keong (2005)	Drijvers(2015) Dubinsky (2013) Ellis (2011)	Artigue (2002) Zbiek (2007) Lagrange (2008)	Falcade (2007) Tall (1996) Vollrath (1989)	Nemirovsky (1998) Lakoff (2000) Mariotti (2002)

Our global analysis of tasks	based on computer algebra systems (CAS) but without explicit embodiment	based on animation and dynamic geometry software (DGS) but without explicit embodiment	based on TI-Nspire software/ calculator but without explicit embodiment	involving mouse movements (dragging, sliders, etc.)	based on embodied design

In light of the research question, our main goal is to explore how an embodied approach in learning design can affect developing functional thinking. We merged Cluster 1, Cluster 2 and Cluster 3 into one category labelled as *Embodiment not central*. This category focuses on digital technology-enhanced function learning and teaching without elaborate embodied designs, but has different types of technology. Next, we labelled Cluster 4 *Pseudo embodiment* category, which describes technology-enhanced designs with sort of embodied elements (like dragging). Finally, we labelled Cluster 5 *Embodiment* category, including eleven articles focus on embodied designs for function learning. Figure 1 depicts the results of the BC method, where the node size is proportional to the number of publications contained therein and the line thickness is proportional to the average similarity between the publications of the two linked clusters (in terms of shared references).

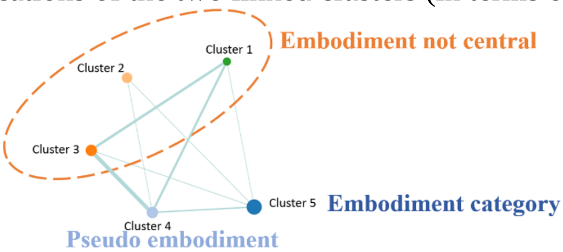
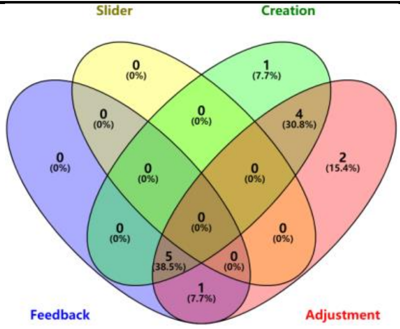
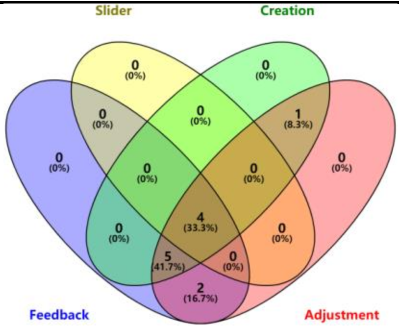
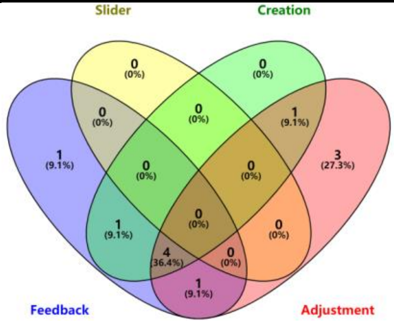


Figure 1 The network of clusters

Following the labelled categories, Table 2 illustrates how the possible embodied elements (the use of slider, create object, feedback through the learning arrangement and adjust object) are involved in the three types of embodiments. First, the use of sliders only appears in the Pseudo embodiment, that is, most designs only allow students to control sliders in the digital environment by mouse. Second, students are given many opportunities to create and adjust functional objects, mainly digital geometric objects, in the designs from Embodiment not central and Pseudo embodiment. But in the Embodiment category, about half of the designs offer the existing, elaborate objects that students only need to adjust. Finally, feedback from the digital environment appears more frequently in the Pseudo embodiment category.

Table 2 Categories of Articles Based on the Use of Digital Technology and Embodiment

Embodiment not central (n=13)	Pseudo embodiment (n=12)	Embodiment (n=11)
		

As a final result, given the theoretical underpinnings and bibliometric results described above, our content analysis led to the identification of three categories of embodied approaches: Embodiment not central, Pseudo embodiment and Embodiment.

Embodiment not central

In the Embodiment not central category, the most common configuration of the designs is creating and adjusting objects with/without feedback. And in these designs, students are allowed to adjust objects by inputting different values or pressing buttons on the calculators. Considering the mathematical object aspect of FT, especially the aspect graphing, GeoGebra, Graphmatica and TI-Nspire software/calculator are used to help students detect the effects of changing parameters in function on its graphical representation through supporting the modelling of different scenarios that allow students to study the effect of changes in the value of one variable on the other (Duijzer et al., 2019; Jon, 2013; Ogbonnaya, 2010). Along with the adjusting functionality, feedback from digital technology was also emphasised in the studies. For example, Asli Özgün-Koca (2016) pointed out that the feedback from the representations on the screen might help students recognise their misconceptions and overcome them through additional interactions with the digital tool. In addition, digital technology has the potential to motivate students and instil a curiosity that enables them to learn more when receiving real-time feedback from the tool (Ogbonnaya, 2010).

Pseudo embodied approach

Compared to the first category, embodied elements in this second category are more visible in the learning design, such as slider using and mouse dragging tasks. Mouse movements play an important role in using DGS (e.g., Cabri and GeoGebra) or in other digital environments (e.g., the Digital Mathematics Environment DME). There are two different settings of sliders: a) continuously slider (free movement on a bar without restriction), b) discrete slider (static selection of particular values). In Liang and Moore's case (2020), students could drag the endpoint (without restriction) to vary the length of the bar, which leads to the dynamic point on the circle moving correspondingly. Students can recognise amounts of change (covariation aspect of FT) when the perceptual material was given, but can not anticipate, represent or regenerate the changes when the perceptual material is absent.

Apart from using sliders, this category includes diverse mouse movements that provide more opportunities for students to create or explore relationships between entities and variables. The Arrow Chain module in DME, for example, is designed to foster conceptual understanding of the notion of function, where the main aspects of function in this design are input-output assignment, dynamic process of covariation, and mathematical object with different representations (Doorman et al., 2012). Students are able to drag and connect machines into function chains. In doing so, the idea of embodying the functional level to compose as well as the input-output assignment is clear. This design could offer educators and researchers some informed directions or ideas for using the technologies to achieve specific learning goals.

In addition to the different mouse movements of embodied elements, the type of feedback from digital technology can also differ among designs. The abovementioned design shows that the movement of connecting embodies the input-output process essential for the function notion, but there is no feedback on the movement itself. Falcade et al. (2007) designed two tasks with real-time feedback on the screen that allows the user to feel functional dependency in the domain of space and time. Students can find the effect of moving one of these points at a time and observe the traces they make through the Trace tool. The traces of points on the screen provide real-time feedback and serve as a cognitive anchor for learning about and understanding abstract concepts (Cox, 1999; Reiner, 2009).

Embodied approach through digital technology

This category of embodied studies in our corpus includes physical motions with the help of digital technology, especially registering movement digitally, processing, and providing feedback. A main similarity between the papers is the presence of (adaptive) motor tasks. In accordance with the three types of embodied task designs, action-based, perception-based, and incorporation-based (Bos et al., 2021), the following analyses provide insight into the possible embodied instrumentation approach (Drijvers, 2019) used in the studies of the third category.

Most studies used function-related tasks from an action-based perspective (8 out of 11). The embodiment of actions can supplement the input received from other modalities (e.g., vision), enabling students to construct richer multimodal representations to support more complex understanding (Drijvers, 2019). Distinctive regarding the understanding of functions, Nemirovsky et al. (2013) designed a mathematical instrument called Drawing in Motion, which is a prototype exhibit that requires physical engagement and collaboration between two people who jointly produce a graph on a displayed Cartesian coordinate plane through a large LCD screen. It did provide a new perspective of understanding function using the embodied instrumentation approach, compared to the conventional ways of thinking about functions (e.g., dynamic/process and static/structural conceptions). The authors claim that, given suitable mathematical instruments and practices, even young learners can engage in the learning of functions with the emphasis on the parameterisation of time.

Studies in the genres of perception-based and incorporation-based designs concerning FT so far are rare. Ferrara & Ferrari (2020) used WiiGraph software to engage pairs of students with functions through graphing motion, and one of their tasks, named *Line option for a+b*, showed the perception features. They even drew a conclusion that aspects of coordination and imagination push the mathematical activity further no matter whether the tool is in use or not. Again, the significance of perceptual experiences in the learning of function has been proved. The graphing motion technology, which allows working with couples of positions over time graphs, provides students with the opportunity to observe in real-time the graph of the sum of two functions on the screen, and then gain perceptual experiences supporting a concrete understanding of function.

Conclusion

With the aim of exploring how an embodied approach in learning design can affect students' FT with the support of digital technology, this study performed an expert content analysis using the BC approach. The literature analysis based on the selected corpus revealed three categories: (1) Embodiment not central; (2) Pseudo embodiment; and (3) Embodiment. All three categories have distinctive features in characterising the embodied elements of technology-enhanced learning designs. In the Embodiment not central category, embodiment remains implicit with keyboard strokes tasks and mouse-clicking tasks occupying the most learning designs. In the Pseudo embodiment category, mouse movements, as a distal movement, play an important role that can be made more proximal through touch screen technology and gestures that more closely correspond to the actual movements intended in using sliders or adjusting geometric objects. In the Embodiment category, digital technology allows for an embodied approach to register movement, process, provide feedback. From a methodological point of view, the bibliometric clustering technique did not offer new insights but did confirm our impression on how embodied approaches are involved in the domain of function.

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