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Action-property duality in embodied design

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For those working on embodied design it is a challenge to create tasks that enable students to develop abstract mathematical concepts. We approach this issue from the perspective of Sfard's notions of saming, encapsulation, and reification. We discuss a duality of properties and actions, and use this duality to review saming, encapsulation and reification from an action- and perception-based perspective. To illustrate the power of this theoretical contribution we discuss one new embodied task design and two from literature: MIT-P for proportion and a design for the gradient of a plane using the Augmented Reality Sandbox.

Keywords: Embodied design, Operational-structural duality, Action-property duality.

Introduction

In her 1991 paper on operational and structural conceptions in the formation of mathematical concepts Sfard writes that "we have good reasons to expect that in the process of concept formation, operational conceptions would precede the structural" (1991, p.10). Research into action-based embodied design seems to support and exploit this view that operations – in the form of goal-oriented actions that develop in the context of motor problems – form a ground for developing mathematical concepts (Abrahamson et al., 2020). Operational-structural theory and Abrahamson's embodied design theories share a central role for the transitions from process to object, and the aim of this paper is to study in more depth how a further application of ideas of operational-structural theory can inform embodied design. In particular, we are interested to see how the terminology of object formation—saming, encapsulation, and reification—as it evolved in Sfard's later work (2008), apply to the context of embodied design. To this purpose we write about *action-property duality*, a duality we believe to be at the heart of students' discovery and development of new mathematics in embodied designs. Whereas in Sfard's later work emphasis lies on how the development of saming, encapsulation, and reification take place in communication—in the introduction of new discourse, through signifiers, like nouns—we would like to draw attention to how these developments take place and are observable in students' non-communicative *actions* in embodied learning environments, in particular their interaction with artifacts (Shvarts et al., 2021).

In the next section we present a theoretical perspective on operational-structural theory and embodied design, immediately adding our view on action-property duality. In this section these theoretical ideas are illustrated a new embodied design for studying quadrilaterals. The next section illustrates how the theory applies to embodied designs in two studies, a well-known example from design for the concept of *proportion*, the MIT-P (Abrahamson & Bakker, 2016), and a more recent embodied design, using augmented reality, for the equation of a plane in a three-dimensional coordinate system and its relation to the gradient vector (Bos et al., 2022).

Theoretical contribution and background

Action-property duality

Let us postulate what we mean by a property in relation to actions:

> A property is an invariant under constrained actions

The invariance means a property is perceived to be the same before and after (and possibly during) the constrained action. Trivial but crucial to us, a property is only invariant under actions that maintain the invariance. For example, an angle is only invariant under angle-preserving actions. There is an interesting duality in this formulation: properties and invariant-preserving actions cannot exist without each other. A naïve perception of sameness underlies the ability to discern a property. Our main claim is that such a naïve sense of sameness co-develops with an ability to maintain the property. To be able to compare the sameness across instances of the property in different objects, transformations of one object to the other need to be performed or imagined. In short, action-property duality refers to the phenomenon that the sameness with respect to a property across objects can only be perceived by transforming those objects into each other while maintaining the property either physically or in the mind's eye.

To make sense of this, let us look at the example of the angles of polygons. Suppose a student is invited to manipulate a polygon by dragging the corners. A naïve perception of *angle* depends on a naïve perception of the *sameness* of angles, since different instances of, e.g., straight angles need to be recognized as the same. As a consequence, a naïve perception of the sameness of angles must co-develop with a naïve ability to maintain an angle while transforming one polygon into another.

Operational-structural development

A theory of operational-structural development within mathematics education developed in the 90s within the framework of traditional cognitive psychology. Sfard argues that operational understanding precedes structural understanding of mathematical concepts (1991). She describes three stages in object/concept development: interiorization, condensation, and reification. Later, Sfard elaborated her perspective on operational-structural issues within the commognitive framework (2008). In the first two columns of Table 1 we present this later view on object formation (cf. Sfard, 2008, p.170). We do not intend to embrace or study the whole theory of commognition here, but find its description of saming, encapsulation and reification most suited to apply in the embodied design context.

Table 1: Operational-structural development from a commognitive and an embodied/action-based perspective

	Commognitive (Sfard, 2008)	Embodied: Perception-Action
Saming	Creating a subsuming discourse on hitherto unrelated objects with the help of a single signifier. Example: "This is a square and	Hitherto unrelated objects are perceived as similar and acted upon in similar ways. Example: two different squares are manipulated

	that is a square too".	with similar dragging schemes.
Encapsulation	Assigning a signifier to a set of objects and using this signifier in the singular when talking about a property of all objects together. Example: "A square has right angles."	A set of objects is perceived and/or acted upon as instances of a more abstract object. Objects in the set can be transformed into each other, if one perceives the defining properties to stay invariant. Example: dragging one square top of a congruent one, while maintaining equal sides and right angles.
Reification	Introducing a noun or pronoun with the help of which narratives about processes on objects can now be told as 'timeless' stories about relations between objects. Example: "These squares are similar through *rotation* and *translation.*"	A series of actions on objects is perceived and performed as part of a single process. Example: rotating a square 90 degrees clockwise and then rotating it back to the original position.

A role of the operational-structural perspective in embodied design

Over the last 20 years, ideas from the psychological theory of embodied cognition have gained currency in mathematics education research, see (Abrahamson & Lindgren, 2014) and references therein. Embodied designs allow students to develop mathematical concepts from naïve perceptions (perception-based designs) or actions (action-based designs) in embodied learning environments (Abrahamson et al., 2020). In perception-based design students are challenged to use their innate perceptive qualities to observe certain events with potential mathematical meaning. Similarly, in action-based designs students are challenged to solve a problem of motor control with potential mathematical meaning. These naïve perceptions and actions are then developed into more robust mathematical concepts with the guidance of a tutor, thus grounding the meaning of the concepts in embodied (perceptive and motoric) experiences (Flood et al., 2020).

Returning to operational-structural development, as presented in the second column of Table 1, we argue, firstly, that saming, encapsulation, and reification are not exclusively revealed through communicative acts, but additionally through non-communicative actions: the way artifacts are handled in an embodied learning environment. This point of view is elaborated in the third column in Table 1. Below we present support for the idea that students' actions evidence stages of saming, encapsulation, and reification, before those stages are communicated through speech or gesture.

Secondly, from an action-property duality perspective, we argue that saming, encapsulation, and reification can be interwoven in embodied design. Motoric fluency in action-based design indicates that the series of necessary actions to solve the motor problem is perceived as part of a single process. These transformations contribute to the discovery of a property (of a new object) and hence contribute to a process of saming and encapsulation. As a consequence, development towards saming/encapsulation and reification are made simultaneously; this rephrases the idea of action-property duality within operational-structural perspective. This way action-based design offers an opportunity for the simultaneous development of a new object and the associated constrained actions (transformations).

To illustrate this let us look again at quadrilaterals. We developed a task series in which a student is invited to move similar quadrilaterals on a multi-touch screen on top of each other by dragging the corners with their fingers (see Figure 2). The corners must be moved independently but simultaneously by four fingers. Moreover, the similarity of the quadrilaterals must be maintained while moving – this is supported by color feedback as in MIT-P. Recognizing similarity of types of quadrilaterals relies on recognizing the similarity of angles and proportions of side lengths. This, in turn, co-develops with the ability to mentally or physically transform one quadrilateral onto/into the other while maintaining those properties. Movements that maintain similarity are turning, dragging, and mirroring. While students try to “same” similar quadrilaterals, they inevitably stumble upon those transformations as naïve actions. Naturally adaptive motor control might lead students to develop distinguishable fluent transformations that could be developed into more rigorously mathematical concepts of rotation, translation, and reflection. This illustrates the main point of how new objects and the associated transformations potentially codevelop in a saming-task.

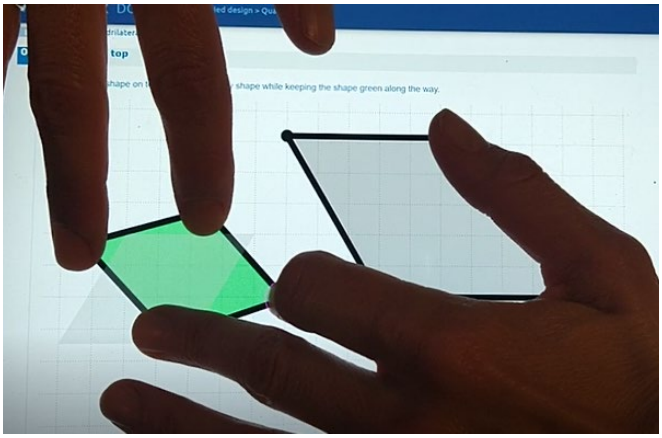

Figure 2. Action-based task: saming flexible quadrilateral by dragging four corners

Examples of embodied designs: the role of transformations

In this section we present two examples of the role of the action-property duality and the operational-structural development in embodied design. We emphasize how fluent motor-action could be reified into mathematical transformations (seen as objects).

Example 1: embodied design for proportion

A well-studied example of embodied design is the action-based task for proportion based on dragging two vertical bars (Abrahamson & Bakker, 2016). The student is encouraged to find positions where the heights of the bars are in a fixed proportion (e.g. $2:3$) by receiving green feedback when such a position is achieved, changing to red if the heights are not according to this proportion. Once a position has been achieved the student is invited to move the bars in a way that maintains the green feedback. In our interpretation from a procedural-structural perspective the student is hence invited to perform constrained transformations on the system of two vertical bars. The outcome is not only a naïve conception of proportion, but also a naïve conception of those transformations that leave a proportion invariant. Flood et al. (2020) report on a student, Ben, who arrived at a solution where the bars move with constant, but different speeds. We infer that Ben not only explored the invariant proportion, but also the actions that maintain the invariant. The latter could be reified into the notion of geometric vertical multiplication, i.e. the transformation that leaves the proportion of height invariant (see Figure 3). In particular, Ben established how (what we call) vertical multiplication has properties different from vertical translation, a transformation that does not leave the proportion of heights invariant. This transformation alludes to the property of proportional variables that, increasing one variable with a factor, the other must increase with the same factor. In general, this again illustrates how the process of saming situations of two bars' heights, the defining property of proportion codevelops with an ability to perform fluent vertical transformation, which could be reified into a mathematical notion of vertical multiplication.

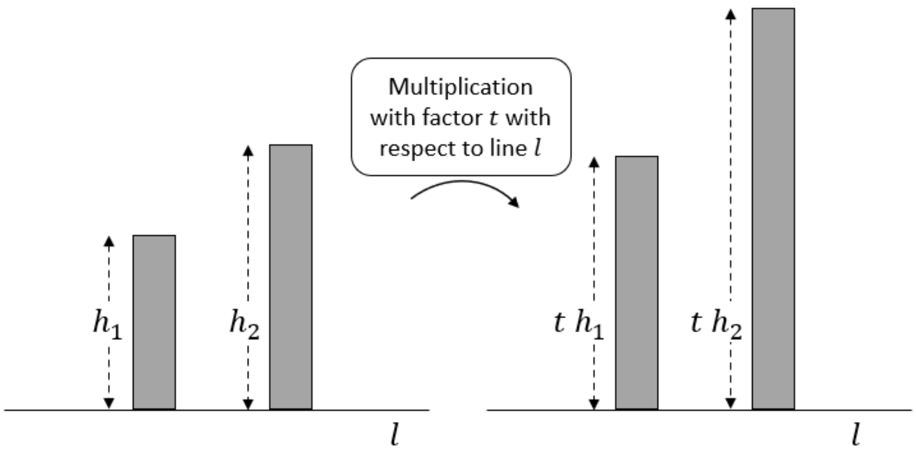

Figure 3. Vertical multiplication as a transformation that leaves the proportion of heights invariant

A transformation can also be associated with the eye movements from one bar to the other. For the task to make any sense the bars need to be considered 'the same' by the student: There must be reason to compare the heights. Some students tend to focus on an imaginary diagonal line between the tips of their two hand dragging the tops of the bars, a so-called attentional anchor (Abrahamson & Bakker, 2016). We argue that this diagonal line is a naïve conception that could (or even should) be reified into a more formal mathematical notion: the transformation of one bar into the other through enlargement with respect to a point (see Figure 4). The diagonal line is the essential ingredient for this transformation. This transformation alludes to the property of proportional variables $H_1 \sim H_2$ that one is a multiple of the other: $H_1 = c\,H_2$. So, yet another process of saming leads to a potential transformation to be reified.

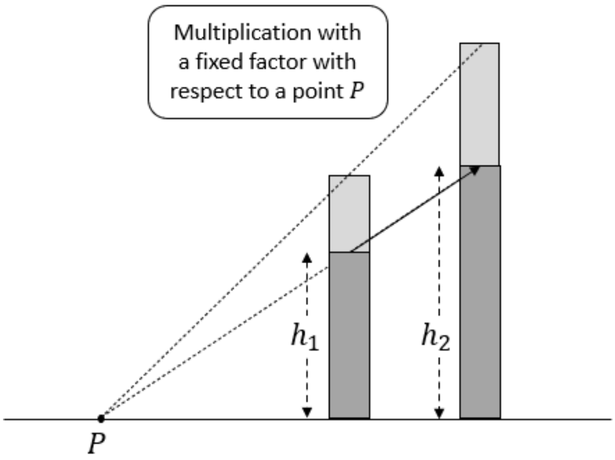

Figure 4. The diagonal line (attentional anchor) can be extended to form an essential ingredient for multiplication with respect to point P: a transformation from bar to the other (ignoring the width).

Example 2: embodied design for the gradient of a plane

In a recent study the author and collaborators investigated embodied design tasks for plane equations ($z = a\,x + b\,y + c$) and the relation to the gradient vector (a, b) (Bos et al., 2022). The embodied learning environment consisted of an Augmented Reality Sandbox (ARSB) – cf. https://web.cs.ucdavis.edu/~okreylos/ResDev/SARndbox/ – together with some plastic planes and bamboo sticks (see Figure 5). The ARSB consists of a box of sand of area roughly 0,6m^2, a stereo camera and a projector. The stereo camera captures images of the height of the sand and objects in the box, and the projector projects height lines and color feedback onto the sand and objects accordingly.

The first few tasks of the teaching sequence aim for students to discover the relations between the parameters a, b and c in the equation $z = a\,x + b\,y + c$ and the transformations rotation parallel to the y-axis, rotation parallel to the x-axis, and translations. In this paper we would like to highlight the next task in the sequence. This task aims to support development of the meaning of the gradient vector in a qualitative way. A vector is defined by its two properties: direction and length. In the case of the gradient vector these properties correspond to direction of the steepest ascent, and to the steepness, respectively. Applying the action-property perspective, each property has associated transformations that leave the property invariant. In the case of direction these are translations and rotations around an axis perpendicular to the direction; in case of steepness these are translations and rotations around a vertical axis. The task is divided into two sub-tasks accordingly: (1) move a circular plane in a way that the direction of the height lines stays the same, but the distance varies; meanwhile, roll a marble down the plane in several positions and try to explain the rolling direction; (2) do the same, but keep the distance between the height lines the same, and explain the speed of the marble. Observing the marble adds a perception-based element to these action-based tasks. Below is an excerpt from a dialogue between tutor Rogier and student Tiago, the case student in the study presented in (Bos et al., in press). The fragment begins while Tiago is working on sub-task 2.

1 Tiago Like this you get [it] too, and then you can make all those movements again" [Tiago refers to horizontal and vertical translations] but then, in any

case, it will remain just as steep. [Then Tiago begins to rotate the plane round the place where it touches the sand (see Figure 3)]
2 Tiago Whether you hold it like this or this.
3 Rogier Ah.. What movement are you making?
4 Tiago I rotate it. I rotate it round an axis. That is actually what I'm doing.
[Tiago keeps rotating the plane. He chooses a correct position, then waits for the feedback to update, thus clearly using the affordance of the ARSB for establishing a new action.]
5 Rogier Do you pay attention to anything in particular while rotating?
6 Tiago No, nothing special. Yes, that I keep it equally steep. Otherwise, it doesn't matter whether you rotate around the point at the bottom [where it touches the sand] or at the top [where Tiago holds it]

Tiago fluently performs a series of actions he calls rotation (line 4), "saming" the plane positions with equal steepness. In line 1 Tiago mentions that during his actions the plane "remain just as steep". This is the first time he associates this adjective "steep" to the plane as a property: one of the two which define the gradient vector. Moreover, Tiago's explanation in line 6 and remarks earlier in course of the experiment suggest he perceives an imagined triangle as depicted in Figure 4 as an attentional anchor, where the angle between the diagonal on plane and the horizontal line on the sand is a measure of steepness. In the light of Table 1 it is important to stress how the introduction of the noun "steep" (commognition perspective, column 2) is preceded and accompanied by the development of fluent action to solve the motor problem and a perception (attentional anchor) that facilitates this action (action-based perspective, column 3).

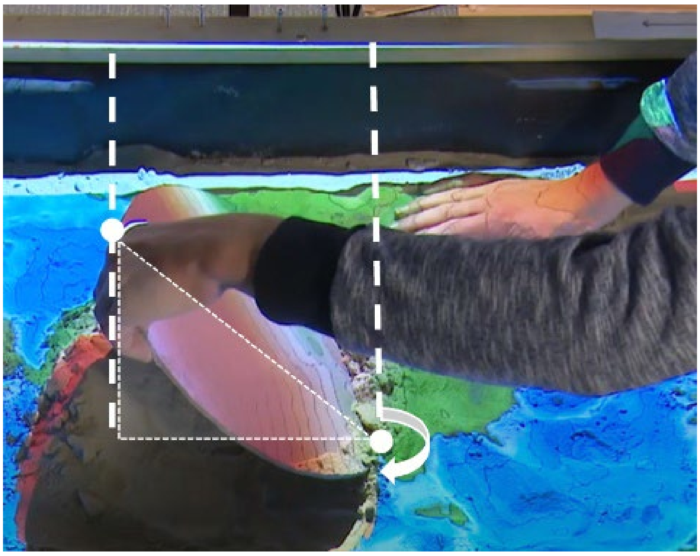

Figure 5. Tiago manipulating a plane while keeping the distance between height lines invariant

Table 2 summarizes the action-property duality for the two properties that constitute the gradient vector. The table highlights how each property is closely connected to an action that can be reified into a transformation as an object. We observed how the actions described in the first column are performed as single process with a clear goal, e.g. in line 6 during rotation "I keep it equally steep". The reification of the object "gradient vector" goes hand in hand with the reification of rotation round a vertical axis as a "keeping equally steep"-action and rotating round a horizontal axis as a "keeping same direction"-action.

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I would like to thank Anna Shvarts for many discussions and the organization of the Embodied Instrumentation Group meetings at the Freudenthal Institute. The ideas in this paper originate from the inspiring conversations on embodiment with all participants.

Table 2. Action-property duality for two properties that constitute the gradient vector qualitatively

Action	Invariant (property)	Property gradient	Changing
Rotation round vertical axis	Steepness, angle of imagined triangle	Length	Direction of steepest ascent, direction of rolling marble
Rotation round horizontal axis	Direction of steepest ascent, direction of rolling marble	Direction	Steepness, angle of imagined triangle, speed of marble

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