HEAVY-QUARK CORRELATIONS IN DIRECT PHOTON-PHOTON COLLISIONS

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Abstract

In two-photon collisions at LEP2 and a future $e^+e^-$ linear collider heavy quarks (mainly charm) will be pair-produced rather copiously. The production via direct and resolved photons can be distinguished experimentally via a remnant-jet tag. We study correlations of the heavy quarks at next-to-leading order in QCD in the direct channel, which is free from phenomenological parton densities in the photon. These correlations are therefore directly calculable in perturbative QCD and provide a stringent test of the production mechanism.
1. Introduction

The production of heavy quarks in two-photon collisions has interesting aspects. Each of the photons can behave as either a pointlike or a hadronic particle [1]. Consequently one distinguishes in such collisions direct- (both photons are pointlike), single resolved- (one photon is pointlike, the other hadronlike), and double resolved (both are hadronlike) production channels. The resolved channels require the use of parton densities in the photon, whereas the production via the direct channel is free of such phenomenological inputs and depends only on the QCD coupling and the heavy quark mass. The heavy mass provides the hard scale for the perturbative analysis and ensures that the separation into direct and resolved production channels is unambiguous even at the next-to-leading order (NLO) level. Hence production via the direct channel is directly calculable in perturbative QCD (pQCD) and in principle the best way for examining the validity of such an analysis and for confronting the pQCD prediction with experiment.

Two-photon collisions can be investigated at $e^+e^-$ colliders, where a large number of equivalent photons is generated. Charm quark production in two-photon collisions has been analysed in many experiments. One has mainly studied the reaction $e^+e^- \rightarrow e^+e^- D^{*\pm}X$ with neither outgoing lepton tagged (“no-tag”), because it proceeds predominantly via the fusion of two equivalent photons to produce open charm ($\gamma\gamma \rightarrow c\bar{c}$). The existence of the $D^{*\pm}$ has been inferred either from direct reconstruction [2] or from unfolding the distribution of soft pions [3] resulting from its decay. There have in addition been studies that use soft leptons [4] and kaons [5] to tag charm quarks.

Due to the low experimental acceptance of heavy quark production in two-photon collisions this reaction has been studied also theoretically at next-to-leading order in QCD only in the single-particle inclusive case. Ref. [6] concentrated on the no-tag case, and ref. [7] on the case where one of the outgoing leptons is tagged. At LEP2 and a future $e^+e^-$ linear collider (NLC) the higher cms energy and large luminosity will lead to fairly copious production of charm quark pairs. Thus it will become possible to measure both heavy quarks and analyse their correlations. The study of these correlations constitutes a more comprehensive test of the theory and is our purpose in this letter. Heavy-quark correlations have been investigated theoretically also in hadroproduction [8], photoproduction [9] and electroproduction [10], and experimentally in [11]. We concentrate here on the no-tag case and, to eliminate the uncertainties related to the parton densities in the photon, on the direct channel only. Note that the TOPAZ collaboration [5] has recently shown that the direct channel may be isolated experimentally from the resolved ones by detecting the photon-remnant jet, present in the resolved channels only.

The paper is organized as follows: In section 2 we describe our method of calculation and in section 3 we show heavy quark correlations for LEP2, and a NLC at a center of mass energy of 500 GeV. We conclude in section 4.
2. Method

In this section we describe the method we used to calculate the QCD corrections to the process
\[ \gamma(k_1) + \gamma(k_2) \rightarrow Q(p_1) + \overline{Q}(p_2), \]  
where \( Q(\overline{Q}) \) is a heavy (anti)-quark. We want to have full exclusive information about the final state. Our method is a special case of a more general method for performing exclusive higher order QCD calculations [12].

The Born process (1) is described by the differential cross section
\[ d\sigma^{(0)} = \frac{4\alpha_e^2 e_Q^4 N_c}{s} \frac{t_1 + u_1 + 4m^2 s}{t_1 u_1} \left( 1 - \frac{m^2 s}{t_1 u_1} \right) \times \frac{d^3 p_1}{2\sqrt{|p_1|^2 + m^2}} \frac{d^3 p_2}{2\sqrt{|p_2|^2 + m^2}} \delta^{(4)}(k_1 + k_2 - p_1 - p_2). \]  
(2)

Here \( e_Q \) is the charge of the heavy quark in units of \( e \), \( N_c = 3 \) the number of colors, and \( m \) the mass of the heavy quark. The kinematic invariants are defined by
\[ s = (k_1 + k_2)^2, \quad t_1 = (k_1 - p_1)^2 - m^2, \quad u_1 = (k_1 - p_2)^2 - m^2. \]  
(3)

The virtual QCD corrections to the Born process consist of the interference between the Born amplitude (depicted e.g. in Fig. A1 in [6]) and its one-loop corrections. Explicit results have already been presented in [13] and we will not repeat the details of the calculation here. We merely note that we regularized the ultraviolet (UV) and infrared (IR) singularities that occur in the virtual corrections by working in \( d = 4 - 2\epsilon \) dimensions, and absorbed the UV singularities via mass renormalization in the on-shell scheme. We are then left with only IR singularities, which appear as \( 1/\epsilon \) poles and factorize into a universal factor multiplying the Born differential cross section, eq.(2).

The bremsstrahlung corrections at NLO are due to the radiation of a gluon from one of the heavy quarks
\[ \gamma(k_1) + \gamma(k_2) \rightarrow Q(p_1) + \overline{Q}(p_2) + g(k_3). \]  
(4)

Since our method here is a little different from what was done previously in the literature, we give a few more details. Note first of all that when a gluon is radiated from the heavy quark, no collinear singularity occurs, because it is shielded by the heavy quark mass. We divide up the phase space into a “soft” region and a “hard” region. The soft region is defined by the condition
\[ 0 \leq s_{13}, s_{23} \leq s_{\text{min}} \]  
(5)
where \( s_{13} = 2p_i \cdot k_3 \) (\( i = 1, 2 \)) and \( s_{\text{min}} \) is an arbitrary cut-off, to be chosen small. The hard region is the complementary one.

In the hard phase space region, one can work in 4 dimensions and perform the phase space integrations numerically, allowing for easy implementation of experimental cuts. As is well known, in the soft region both the phase space and the matrix element factorize in
the limit of small $s_{\text{min}}$. In both cases, one of the factors contains the quantum numbers of the gluon, and the other is only related to the lower order process. As a consequence, one may perform the integral of the momentum of the gluon in this region analytically in $d$-dimensions. Specifically one must do the integral

$$
4\pi\alpha_s C_F \int d\text{PS(soft)} K(\text{soft}).
$$

with the color factor $C_F = (N^2 - 1)/(2N)$. Here the soft gluon phase space factor is

$$
d\text{PS(soft)} = \frac{(4\pi)^\epsilon}{16\pi^2 \Gamma(1 - \epsilon)} ds_{13} ds_{23} (s_\beta)^{2\epsilon - 1} \left[ s_{12}s_{13}s_{23} - m^2(s_{13}^2 + s_{23}^2) \right]^{-\epsilon}
$$

where $\beta = \sqrt{1 - 4m^2/s}$ and $s_{12} = 2p_1 \cdot p_2$. Note that the expression in square brackets must be positive. The soft gluon matrix element factor can easily be found in the eikonal approximation, and is

$$
K(\text{soft}) = 4 \left( \frac{s_{12}s_{13}s_{23} - m^2(s_{13}^2 + s_{23}^2)}{s_{13}^2 s_{23}^2} \right).
$$

Thus, upon combining both factors and integrating over the range (3), one obtains a universal factor multiplying the differential Born cross section (2). This factor contains $1/\epsilon$ poles which cancel against those originating in the virtual corrections. The soft contribution to the fully differential cross section can then finally be written as

$$
d\sigma^{(1)}(\text{soft}) = S_F(s, m^2, s_{\text{min}}) d\sigma^{(0)}
$$

where

$$
S_F = \left( \frac{\alpha_s C_F}{\pi} \right) \left\{ -2 \left( 1 + \frac{1 - 2m^2}{s} \right) \ln \frac{x}{\beta} \left( \ln x - \ln \left( \frac{s}{s_{\text{min}}} \right) - \ln \beta \right) \\
-2 \left( \ln(1 - x) + \ln(1 + x) - \ln x \right) + 1 - \beta \\
- \frac{1}{\beta} \left( 1 - \frac{2m^2}{s} \right) \ln x \left( 1 + 2 \ln \left( 1 + \frac{x}{1 - x} \right) \right) \\
+ \frac{1}{2\beta} \left( 1 - \frac{2m^2}{s} \right) \left( \text{Li}_2 \left( 1 - \frac{1}{x^2} \right) - \text{Li}_2 \left( 1 - x^2 \right) \right) \\
+ \frac{m^2}{s\beta} \ln x \left( \frac{(1 + x)(1 - x)}{x} \right) + \frac{3s}{2m^2} \left( 1 - \frac{2m^2}{s} \right) \ln x \right\}.
$$

Here $x = (1 - \beta)/(1 + \beta)$ and $\text{Li}_2(z)$ is the dilogarithmic function as defined in [14].

Finally, one is left with a two-to-two particle contribution (consisting of the Born and soft-plus-virtual corrections) and the two-to-three particle contribution in the hard region. Each contribution depends on the theoretical cut-off $s_{\text{min}}$, but as long as $s_{\text{min}}$ is small enough compared to the typical scale of our process, the sum does not. This we checked explicitly.
3. Results

Using the method described in the previous section we have constructed a Monte Carlo program for the reaction $\gamma\gamma \rightarrow Q\overline{Q}$ for direct photons, including the complete $\mathcal{O}(\alpha_s)$ corrections, which is fully exclusive in all final state particles. We checked that we could reproduce the results in \cite{1} for the total cross section and single particle transverse momentum ($p_t$) and rapidity ($y$) distributions for the direct channel. We only present results for charm quark production because the bottom quark production rate is very much reduced in two-photon collisions due to charge and phase space suppression.

We first list the default choices we made for various parameters for producing the results shown in the rest of this section. To compute $\alpha_s$ we used the two loop expression with $\Lambda_{QCD}^{(5)} = 0.215$ GeV and $n_{\text{lf}}$ active flavors, where $n_{\text{lf}}$ is the number of flavors with mass less than the renormalization scale. For the charm quark mass we used 1.5 GeV. The center of mass energy was chosen to be 175 (500) GeV for LEP2 (NLC). For the renormalization scale we took $\mu = \sqrt{m^2 + (p_t^2(Q) + p_t^2(\overline{Q}))/2}$. In the present process the choice of scale only affects the value of $\alpha_s$. We used the Weizsäcker-Williams density of \cite{17} with an anti-tag angle $\theta_{\text{max}}$ of 30 (175) mrad for the case of LEP2 (NLC). At the NLC beamstrahlung is expected to play an important rôle, so we include its effect here by adopting for its spectrum the expression given in \cite{16}, with parameters $\Upsilon_{\text{eff}} = 0.039$ and $\sigma_z = 0.5$ mm \cite{17} corresponding to the TESLA design. For the NLC we will as default coherently superimpose the Weizsäcker-Williams density and the beamstrahlung density, in order to incorporate the case where one photon is of beam- and the other of bremsstrahlung origin.

For most results we have not used charm-to-D meson fragmentation function. For the cases that we do, which we indicate explicitly, we employed the Peterson et al. parametrization \cite{18}

$$D(z) = \frac{N}{z(1-1/z - \epsilon/(1-z))^2}$$

with $\epsilon = 0.06$ the value given in \cite{19} for the case of charm. Our interest when including the fragmentation function lies mainly in how it changes the shapes of distributions, rather than their normalization. Hence we choose $N$ such that $\int_0^1 dz D(z) = 1$.

We will only present one single particle distribution here, since such distributions have already been studied in \cite{20,21}. Fig.1 shows the single particle $p_t$ distribution at LO, NLO, and at NLO with fragmentation. We see that inclusion of NLO corrections decreases the cross section at large $p_t$ and enhances it at small $p_t$, and that the application of the fragmentation function softens it considerably.

Turning to correlations, we begin by showing distributions which allow a comparison between the LO and NLO calculations. In Fig.2 we show the cross section versus the invariant mass $M_{Q\overline{Q}}$ of the heavy quark pair for LEP2 and NLC at both LO and NLO. Notice in Fig.2 the sizable difference that occurs at both small and large invariant masses when including the NLO corrections. This can be understood as follows. Consider first the situation where the two photons collide with all the momentum of their parent leptons. Denoting the invariant mass of the heavy quark pair in this case by $\widetilde{M}_{Q\overline{Q}}$, then at LO
Figure 1: Single charm quark $p_t$ spectrum at LEP2, comparison of LO, NLO and NLO with fragmentation.

Figure 2: $M_{q\bar{q}}$ distribution for charm for LEP2 and NLC at both LO and NLO.
$\hat{M}_{Q\bar{Q}}$ is fixed at $\sqrt{s_{\gamma\gamma}}$. At NLO it may assume smaller values, and there the cross section is positive. For $\hat{M}_{Q\bar{Q}} = \sqrt{s_{\gamma\gamma}}$ one has at NLO also a negative contribution coming from the virtual graphs. To go back to the case of LEP2 and NLC we must fold with the photon spectrum. A given $\hat{M}_{Q\bar{Q}}$ value then contributes to the spectrum for $M_{Q\bar{Q}}$ under the restriction $M_{Q\bar{Q}} < \hat{M}_{Q\bar{Q}}$, so that at large $M_{Q\bar{Q}}$ the LO spectrum is mainly modified by the negative contribution at $\hat{M}_{Q\bar{Q}} = \sqrt{s_{\gamma\gamma}}$, and at small $M_{Q\bar{Q}}$ by the positive contributions at smaller $\hat{M}_{Q\bar{Q}}$.

In Fig.3 we show the $\Delta R$ distribution, defined by $\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2}$, at LO and NLO for both LEP2 and the NLC. Here $\Delta \phi$ is the azimuthal angle between the charm and anticharm in the plane transverse to the beam axis and $\Delta \eta$ is the pseudo-rapidity difference of the two heavy quarks. At LO $\Delta R > \pi$, but at NLO $\Delta R$ may also assume values below that. Note that NLO effects seem to be mostly active for $\Delta R \lesssim 4$.

We now show two distributions which are only non-trivial at NLO (and higher orders). In Fig.4 we present the $p_t$ distribution of the charm-anticharm pair, and in Fig.5 the azimuthal correlation between the two heavy quarks. We also show in Fig.4 the NLC curves with only beamstrahlung photons and with only Weizsäcker-Williams photons for the purpose of comparison. One observes in Fig.4 that at the NLC charm pairs produced by beamstrahlung photons prefer to have a lower $p_t$ than those due to WW equivalent photons. This is a consequence of the TESLA beamstrahlung spectrum, which is enhanced at small $z$ and depleted at large $z$ compared to the WW spectrum ($z$ is the momentum fraction of the photon relative to its parent lepton).
Figure 4: $p_t(c\bar{c})$ distribution for charm and anti-charm quark at LEP2 and NLC.

Figure 5: $\Delta\phi$ distribution for charm and anti-charm quark at LEP2 and NLC.
In Fig.5 we see that the $\Delta \phi$ distributions are all quite uniform. We observe however that at the NLC for the case of charm the beamstrahlung contribution dominates the $WW$ one.

Finally we comment on the consequences of choosing different values of the renormalization scale $\mu$ and the charm mass $m$. To see how Figs.1-3 change when varying $\mu$ one can simply rescale the differences between the LO and NLO curves according the change in $\alpha_s$, whereas in Figs.4 and 5 the whole curve will change by an overall factor. We further remark that choosing a different value for $m$ changes mainly the normalizations of the curves shown in this section, but not their shapes.

4. Conclusions

In this paper we have presented a NLO calculation of heavy quark production in direct two-photon collisions. We have described our calculation method and presented numerical studies of various correlations between the heavy quarks. We observed that the inclusion of the NLO corrections significantly modifies the shapes and normalizations of the distributions we studied. Experimentally such studies will be challenging at LEP2 due to the low charm acceptance, but they are certainly feasible at a future $e^+e^-$ linear collider.

References


