

RADIATIVE CORRECTIONS TO $e\gamma$ SCATTERINGERIC LAENEN^a * AND GERHARD A. SCHULER^{a,b} †

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Abstract

We investigate the effects of photon radiation on deep-inelastic $e\gamma$ scattering. Depending on the set of variables chosen, we find appreciable effects in the kinematic region accessible at LEP2. Convenient analytic results for $O(\alpha)$ corrected differential cross sections are presented.

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1 Introduction

In the period leading up to the start of the HERA program a substantial amount of work was done on radiative corrections to observables in deep-inelastic scattering (DIS) of electrons off protons (see e.g. [1] for an overview). These corrections are substantial enough for both HERA experiments to correct for them. Part of the upcoming LEP2 physics programme involves the measurement of the photon structure function $F_2^\gamma(x, Q^2)$. This structure function is extracted from deep-inelastic electron–photon scattering in the reaction $e^+e^- \rightarrow e^+e^-X$, where one of the leptons escapes undetected down the beam pipe, while the other is measured at rather large angle. It is therefore important to study radiative corrections to deep-inelastic electron–photon scattering. This is our purpose in this paper.

Radiative corrections to $e^+e^- \rightarrow e^+e^-X$ have been calculated for X a (pseudo) scalar particle [2, 3, 4] and $X = \mu^+\mu^-$ [4, 5]. It was found that they are very small (on the percent level) in the no-tag case, when neither the electron nor the positron is being measured. In such a kinematic configuration, when the momentum transfer between the incident and outgoing electron (or positron) is small, the vertex- and bremsstrahlung contributions effectively cancel each other, leaving a small correction dominated by vacuum polarization [3, 4]. This implies that for the equivalent photon spectrum, which is essential to relate $e^\pm\gamma$ with e^+e^- reactions, corrections are small. In contrast, radiative corrections can be sizeable in the case where one of the leptons scatters at a large angle - single tag - and one studies differential cross sections which depend strongly on the energy and angle of the tagged electron (or positron).

Surprisingly, no calculations of the size of radiative corrections for inclusive deep-inelastic electron–photon scattering, i.e. $e\gamma \rightarrow eX$ with both large Q^2 (the absolute value of the transferred momentum squared) and W (the mass of state X), have been performed yet. Correspondingly, in experimental analyses of the (hadronic) photon structure function $F_2^\gamma(x, Q^2)$ radiative corrections have so far not been included (as usual, $x = Q^2/(Q^2 + W^2)$). They are usually assumed to be negligible. Recently, the AMY collaboration [6] estimated the size of radiative corrections by comparing a Monte-Carlo event generator based on the full cross section formula for $e^+e^- \rightarrow e^+e^-\gamma\mu^+\mu^-$ [5] (with the muon mass and electric charge changed to correspond with quark-antiquark pair production) with a generator for $e^+e^- \rightarrow e^+e^-q\bar{q}$ where the cross section for $e\gamma \rightarrow eq\bar{q}$ with a real photon was convoluted with the equivalent photon spectrum. A (positive) correction of order 10% for the visible x distribution was found, which, however, cancelled effectively against the correction due to a non-vanishing target-photon mass. Hence no net correction was applied.

Nevertheless, it is important to understand both corrections separately, and their behavior as a function of the kinematic variables chosen and phase space. Moreover, the hadronic structure of the photon must not be neglected. Here we therefore estimate the size of the radiative corrections to inclusive deep-inelastic

$e\gamma$ scattering, using the full photon structure function.

In the next section we describe the relevant kinematics and formalism and in section 3 we present results.

2 Formalism

We consider the $O(\alpha)$ corrections to deep-inelastic scattering (DIS) of electrons on (quasi-real) photons:

$$e(l) + \gamma(p) \rightarrow e(l') + \gamma(k) + X(p_X) . \quad (1)$$

This process is depicted in Fig.1, in which we indicate all momentum labels.

The target photon $\gamma(p)$ is part of the flux of equivalent photons around the non-tagged lepton. We assume that this flux has a momentum density given by the Weizsäcker-Williams expression

$$f_{\gamma/e}(z) = \frac{\alpha}{2\pi} \left\{ \frac{1 + (1-z)^2}{z} \ln \frac{P_{max}^2}{P_{min}^2} - 2m_e^2 z \left(\frac{1}{P_{min}^2} - \frac{1}{P_{max}^2} \right) \right\} \quad (2)$$

where $P_{min}^2 = (z^2 m_e^2)/(1-z)$ and $P_{max}^2 = (1-z)(E_b \theta_{max})^2$. Here z is the longitudinal momentum fraction of the target photon with respect to its parent lepton, $E_b = \sqrt{s}/2$ is the lepton beam energy, θ_{max} is the anti-tag ¹ angle and $P^2 = -p^2$. In the following we put $P^2 = 0$ and neglect electron masses everywhere except in (2). Moreover we substitute P_{max}^2 by $P_{max}^2 + P_{min}^2$ so that we can easily extend the z range to 1, see [7].

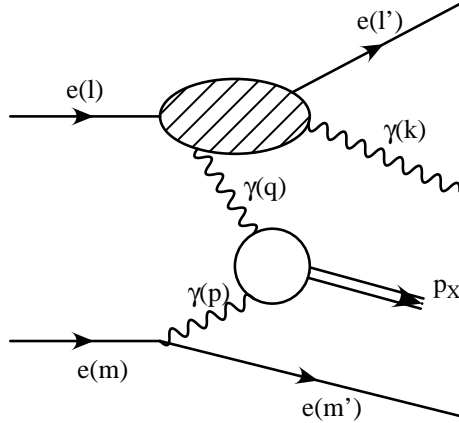


Figure 1: Photon bremsstrahlung from the tagged lepton line in deep-inelastic scattering off an equivalent photon.

The DIS variables can be defined either from the leptonic or the hadronic

¹i.e. all events in which the parent lepton scatters at an angle larger than θ_{max} are rejected.

momenta:

$$\begin{aligned}
q_l &= l - l' & q_h &= p_X - p = q_l - k \\
W_l^2 &= (p + q_l)^2 & W_h^2 &= (p + q_h)^2 = p_X^2 \\
Q_l^2 &= -q_l^2 & Q_h^2 &= -q_h^2 \\
x_l &= Q_l^2/2p \cdot q_l & x_h &= Q_h^2/2p \cdot q_h \\
y_l &= p \cdot q_l/p \cdot l & y_h &= p \cdot q_h/p \cdot l
\end{aligned} \tag{3}$$

Note that both $Q_l^2 = x_l y_l s_{e\gamma}$ and $Q_h^2 = x_h y_h s_{e\gamma}$, where $s_{e\gamma} = (p + l)^2$, but that leptonic and hadronic variables agree only for nonradiative events, i.e. if $k = 0$. We will see that the size of the corrections strongly depends on which set of variables are used in the measurement. In practice one determines Q^2 from the tagged lepton, and x from the (visible) hadronic energy W_h .

The Born cross section (i.e. no $\gamma(k)$ in (1)) is given by

$$\frac{d^2\sigma^B}{dx dQ^2} = f^B(x, Q^2, s), \tag{4}$$

where

$$\begin{aligned}
f^B(x, Q^2, s) &= \frac{2\pi\alpha^2}{xQ^4} F_2(x, Q^2) \\
&\times \int_{z_{min}(x, Q^2, s)}^1 dz f_{\gamma/e}(z) Y_+ \left(\frac{Q^2}{xzs} \right) \left\{ 1 + R \left(x, Q^2, \frac{Q^2}{xzs} \right) \right\}. \tag{6}
\end{aligned}$$

Here we have defined $z_{min}(x, Q^2, s) = Q^2/xs$, $Y_+(y) = 1 + (1 - y)^2$ and

$$R(x, Q^2, y) = \frac{-y^2}{1 + (1 - y)^2} \frac{F_L(x, Q^2)}{F_2(x, Q^2)}. \tag{7}$$

$F_{2,L}$ are the photon structure functions (we have dropped the superscript γ). The above form eq. (6) is useful because F_2 can be factored out of the z integration. For comparison of $e\gamma$ with ep scattering we will also give the cross section in terms of x and y

$$\frac{d^2\sigma^B}{dx dy} = g^B(x, y, s), \tag{8}$$

where

$$g^B(x, y, s) = \frac{2\pi\alpha^2 Y_+(y)}{x^2 y^2 s} \int_{\epsilon(x, y, s)}^1 \frac{dz}{z} f_{\gamma/e}(z) F_2(x, xyzs) \{1 + R(x, xyzs, y)\}. \tag{9}$$

The lower limit $\epsilon = W_{min}^2/(1 - x)ys$ on the z -integration in eq. (9) arises if a lower cut is applied to the invariant hadronic mass W . At the Born level expressions (4) and (8) are equivalent and valid for both sets of variables in (3). For the ep case the full electroweak corrections have been calculated for both neutral current reactions [8, 9], including elastic nucleon scattering [10],

and charged-current reactions [11]. It is well-known that the leading logarithmic approximation (LLA) (in $\ln(Q^2/m_e^2)$) reproduces the exact results to within a few percent however [12, 13]. Here we will therefore work within this approximation and neglect furthermore Z exchange.

We consider first the radiative corrections to the differential cross section in (8) in terms of x_l and y_l . The $O(\alpha)$ corrections in LLA arise from collinear and soft bremsstrahlung from the initial and final electron that couple to the “probing photon” and from the Compton process². The latter corresponds to the case in Fig.1 where the exchanged photon $\gamma(q)$ is quite soft, but $\gamma(k)$ is radiated at a wide angle causing the lepton $e(l')$ to be tagged. The bremsstrahlung terms are given by

$$\frac{d^2\sigma^{\text{Br}}}{dx dy} = \int_0^1 dx_i D_{e/e}(x_i, Q^2) \left\{ \Theta(x_i - x_i^0) J(x_1, x_2) g^{\text{B}}(\hat{x}, \hat{y}, \hat{s}) - g^{\text{B}}(x, y, s) \right\} \quad (10)$$

where $\hat{s} = x_1 s$, $\hat{x} = x x_1 y / (x_1 x_2 + y - 1)$, $\hat{y} = y x / x_2 \hat{x}$, $J(x_1, x_2) = y / x_1 x_2^2 \hat{y}$, $x_1^0 = (1 - y) / (1 - xy)$ and $x_2^0 = xy + 1 - y$, and

$$D_{e/e}(x, Q^2) = \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m_e^2}\right) \frac{1 + x^2}{1 - x} . \quad (11)$$

In eq. (10) we have suppressed the subscript l on x and y for clarity. Initial-state radiation corresponds to $x_i = x_1$ and $x_2 = 1$ in eq. (10) and vice versa for final-state radiation. The Compton contribution is given by

$$\frac{d^2\sigma^{\text{C}}}{dx_l dy_l} = \int_\epsilon^1 dz f_{\gamma/e}(z) h^{\text{C}}(x_l, y_l, x_l y_l z s, z s) , \quad (12)$$

where

$$h^{\text{C}}(x, y, Q^2, s) = \frac{\alpha^3}{x^2(1-y)s} Y_+(y) \ln\frac{Q^2}{M^2} \int_x^1 \frac{dv}{v} \left[1 + \left(1 - \frac{x}{v}\right)^2 \right] \times F_2(v, Q^2)(1 + R(v, Q^2, y)) . \quad (13)$$

The logarithm $\ln(Q^2/M^2)$ is a result of the absorption of the collinear singularity from the quark to photon splitting by renormalizing the photon density in the quark at scale M . We take M here to be the proton mass [8].

Next we discuss the radiative corrections to the differential cross section expressed in hadronic variables. Now there are, in accordance with the KLN theorem [14], in LLA approximation neither corrections from final state radiation, nor from the Compton process because these process do not affect the kinematic

²As stated above, radiative effects to the Weizsäcker-Williams spectrum are small [4] and we therefore neglect them.

variables. We find that the correction due to initial state radiation can simply be expressed as

$$\begin{aligned} \frac{d^2\sigma^{\text{corr}}}{dx_h dQ_h^2} &= \frac{2\pi\alpha^2}{x_h Q_h^4} \left\{ F_2(x_h, Q_h^2) \int_{z_{\min}(x_h, Q_h^2, s)}^1 dz f_{\gamma/e}(z) g_h \left(x_h, Q_h^2, \frac{Q_h^2}{zx_h s} \right) \right. \\ &\quad \left. + F_L(x_h, Q_h^2) \int_{z_{\min}(x_h, Q_h^2, s)}^1 dz f_{\gamma/e}(z) h_h \left(x_h, Q_h^2, \frac{Q_h^2}{zx_h s} \right) \right\} \quad (14) \end{aligned}$$

where

$$g_h(x, Q^2, y) = \frac{\alpha}{\pi} \ln \frac{Q^2}{m_e^2} \left\{ Y_+(y) \ln(1-y) + y \left(1 - \frac{y}{2}\right) \ln y + y \left(1 - \frac{y}{4}\right) \right\} \quad (15)$$

and

$$h_h(x, Q^2, y) = \frac{\alpha}{\pi} \ln \frac{Q^2}{m_e^2} \left\{ -y^2 \ln(1-y) + \frac{y^2}{2} \ln y - \frac{y}{2} \left(1 - \frac{y}{2}\right) \right\} \quad (16)$$

The corrections are large at large y (soft region), and can in fact be resummed by simple exponentiation, with the result

$$\begin{aligned} g_h(x, Q^2, y) &= Y_+(y) \left\{ \exp \left[\frac{\alpha}{\pi} \left(\ln \frac{Q^2}{m_e^2} - 1 \right) \ln(1-y) \right] - 1 \right\} \\ &\quad + \frac{\alpha}{\pi} \ln \frac{Q^2}{m_e^2} \left\{ y \left(1 - \frac{y}{2}\right) \ln y + y \left(1 - \frac{y}{4}\right) \right\} \quad (17) \end{aligned}$$

and

$$\begin{aligned} h_h(x, Q^2, y) &= -y^2 \left\{ \exp \left[\frac{\alpha}{\pi} \left(\ln \frac{Q^2}{m_e^2} - 1 \right) \ln(1-y) \right] - 1 \right\} \\ &\quad + \frac{\alpha}{\pi} \ln \frac{Q^2}{m_e^2} \left\{ \left(\frac{y^2}{2} \right) \ln y - \frac{y}{2} \left(1 - \frac{y}{2}\right) \right\} \quad (18) \end{aligned}$$

In the next section we use the formulae listed in the above to estimate the size of the corrections.

3 Results

Here we study the radiative correction to deep-inelastic electron-photon scattering numerically. For the results presented below we use $\sqrt{s} = 175$ GeV and $W_{\min} = 2$ GeV. For the parton densities in the photon we use set 1 of [15]. This set has an already low minimum Q^2 of $Q_0^2 = 0.36$ GeV². However in radiative events even lower values of Q^2 contribute. We therefore extrapolate below this value by

$$\begin{aligned} F_2^\gamma(x, Q^2 < Q_0^2) &= F_2^\gamma(x, Q_0^2) \left(\frac{Q^2}{Q_0^2} \right)^2 + \left(1 - \frac{Q^2}{Q_0^2} \right) \frac{Q^2(1-x)}{112 \text{ GeV}^2} \\ &\quad \times \left(0.211 \left(\frac{W^2}{\text{GeV}^2} \right)^{0.08} + 0.297 \left(\frac{W^2}{\text{GeV}^2} \right)^{-0.45} \right), \quad (19) \end{aligned}$$

(this expression correctly approaches the $\gamma\gamma$ total cross section in the small Q^2 limit) which vanishes as $Q^2 \rightarrow 0$, as required by gauge invariance. Furthermore we have neglected F_L^γ , i.e. we put R in (6) and (9) to zero. We found that its inclusion has a negligible effect.

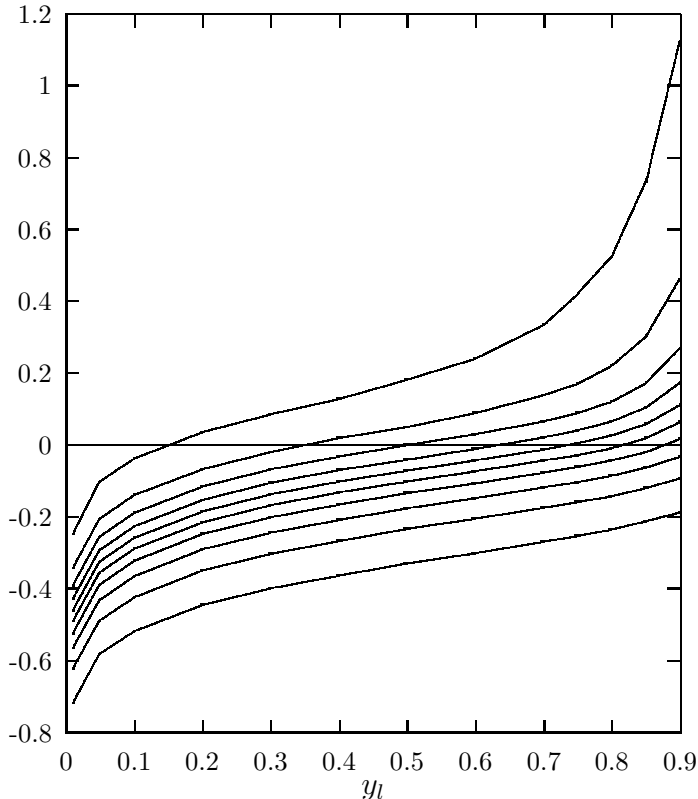


Figure 2: The ratio $\delta(x_l, y_l)$ (20) vs. y_l for various x_l values. Top curve is for $x_l = 0.01$, the next for $x_l = 0.1$. Each subsequent curve represents an increase of x_l by 0.1.

For comparison with the ep case we now show in Fig.2 the correction $\delta(x_l, y_l)$, using (8), (10) and (12), defined by

$$\frac{d^2\sigma^{\text{Br}}}{dx_l dy_l} + \frac{d^2\sigma^{\text{C}}}{dx_l dy_l} = \frac{d^2\sigma^{\text{B}}}{dx_l dy_l} \delta(x_l, y_l) . \quad (20)$$

Note that we use here the leptonic variables to conform with the ep case. A closer examination reveals that initial and final state radiation are similar in order of magnitude throughout most of the x, y region, whereas the Compton contributions is appreciable only in the small and medium x , large y region. Also we note that if one freezes F_2 at $F_2(Q_0^2)$ for $Q^2 < Q_0^2$, instead of extrapolating as in (19), we find that only the $x_l = 0.01$ curve changes significantly. It decreases at small and medium y_l by up to 50%. Note that inclusion of final state radiation implies a perfect measurement of the energy and momentum of the tagged lepton, even in the presence of collinear radiation.

We see in Fig.2 that radiative effects can in principle be large, around 40% for medium x and small y .

Next we show in Fig.3 the correction factor $\delta(x_h, Q_h^2)$, defined by

$$\frac{d^2\sigma^{\text{corr}}}{dx_h dQ_h^2} = \frac{d^2\sigma^{\text{B}}}{dx_h dQ_h^2} \delta(x_h, Q_h^2) \quad ., \quad (21)$$

in terms of hadronic variables, as a function of x_h for various choices of Q_h^2 , cf. (14). Here only initial state lepton bremsstrahlung is taken into account, because the scattered lepton is not used in constructing the kinematic variables.

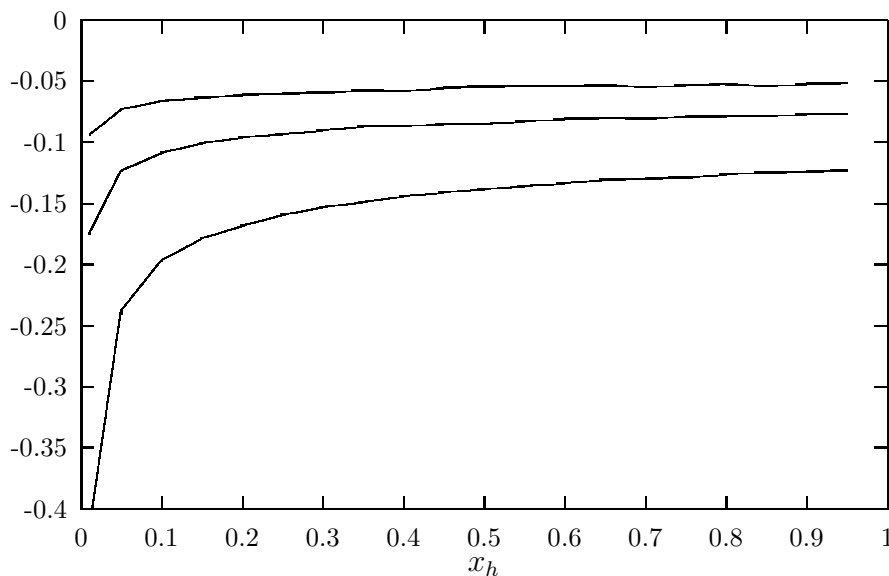


Figure 3: The ratio $\delta(x_h, Q_h^2)$ vs. x_h for three Q_h^2 values. Top curve: $Q_h^2 = 1 \text{ GeV}^2$. Middle curve: $Q_h^2 = 10 \text{ GeV}^2$. Lower curve: $Q_h^2 = 100 \text{ GeV}^2$.

We see that the corrections are sizable only for large Q^2 . Using however the resummed version of (17) and (18), we find that the corrections are reduced by about an order of magnitude.

Thus we conclude that the size of the radiative corrections depends significantly on the set of variables chosen. These corrections can in principle be quite large. As noted before, in practice mixed variables are used. In view of the results obtained in this paper, we think a more careful study, involving Monte Carlo simulation of the full final state, is warranted.

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