

# RESUMMATION OF THRESHOLD CORRECTIONS FOR SINGLE-PARTICLE INCLUSIVE CROSS SECTIONS

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## Abstract

We derive threshold resummations for single-particle and single-jet inclusive cross sections, thus generalizing previous results at fixed invariant mass to a wider class of cross sections with phenomenological interest. We confirm the structure of our resummed expressions by comparison with explicit one-loop calculations for direct photons and heavy quarks.

# 1 Introduction

A central success of quantum chromodynamics (QCD) is the computation of inclusive, short-distance hadron-hadron cross sections. This program combines perturbative coefficient functions for partonic hard scattering with nonperturbative parton distributions and fragmentation functions. In another language, the nonperturbative matrix elements represent effective theories, associated with incoming or observed hadrons, while the perturbative hard-scattering functions match these theories to full QCD at an adjustable scale, conventionally called the factorization scale.

The separation of short- and long-distance dynamics in factorized cross sections is not absolute, however, and soft gluon effects persist in hard scattering functions. Although infrared divergences cancel in perturbative coefficient functions for hard scattering cross sections, finite remainders can give substantial corrections at higher orders of perturbation theory. The universality of soft gluon cancellation makes it possible to resum these remainders to all orders.

These effects are potentially important for single-particle inclusive cross sections for direct photons [1] and heavy quarks [2] and for very high- $p_T$  jet production [3]. For the latter especially, deviations from perturbative predictions may be a signal of new physics. Another outstanding question is the influence of higher orders on global fits to parton distributions [4]. For these applications, as well as for their intrinsic interest, it is important to examine higher orders in  $\alpha_s$  and, where possible, to develop an extended quantitative formalism to estimate their influence. In this paper, we present some of the basic results necessary for this formalism.

The finite remainders of infrared cancellations may be identified with the regions in momentum space where the infrared singularities arise. Singularities associated with “partonic threshold”, at which the partons have just enough energy to produce the final state that defines the cross section, can influence the normalization and shape of the observed cross section indirectly, through a buildup of logarithmically enhanced singular distributions in higher order corrections to hard-scattering functions. Threshold enhancements of this sort have been resummed to all orders for the inclusive Drell-Yan cross section [5, 6]  $d\sigma/dQ^2$ , at fixed pair invariant mass (PIM)  $Q$ , and more recently, for pairs of heavy quarks [7, 8] and jets [9, 10] at fixed invariant mass. However, many cross sections of phenomenological interest, both for the detection of new physics and for the determination of parton distributions, involve the detection of single particles rather than pairs. The purpose of this paper is to extend threshold resummation to cross sections with single-particle inclusive (1PI) kinematics, including direct photon, heavy quark and jet cross sections. The formalism and resummed 1PI cross sections are discussed in the following section, and checked against existing one-loop calculations in Sec. 3.

## 2 Threshold Resummation

A factorized single-particle (denoted collectively as  $c$ ) inclusive cross section at measured momentum  $\ell$  may be written as

$$E_\ell \frac{d\sigma_{AB \rightarrow c(\ell)+X}}{d^3\ell} = \frac{1}{S^2} \sum_{ab} \int dx dy \phi_{a/A}(x, \mu^2) \phi_{b/B}(y, \mu^2) \times \omega_{ab \rightarrow c(\ell)+X} \left( \frac{s_4}{\mu^2}, \frac{t}{\mu^2}, \frac{u}{\mu^2}, \alpha_s(\mu^2) \right), \quad (1)$$

where for  $c$  a photon or jet we introduce the kinematic invariant  $s_4$  by

$$s_4 \equiv s + t + u, \quad (2)$$

in terms of partonic invariants  $s = (p_a + p_b)^2$ ,  $t = (p_a - \ell)^2$ ,  $u = (p_b - \ell)^2$ . With this definition,  $s_4 = M_X^2 + \ell^2$  is the invariant mass squared of the QCD radiation recoiling against the observed particle or jet plus the mass squared of the observed particle or jet. For the production of a pair of heavy particles of mass  $M$ , the corresponding threshold quantity is found from  $s + t_1 + u_1$ , with  $t_1 = t - M^2$  and  $u_1 = u - M^2$ . In Eq. (1) we absorb into  $\omega_{ab}$  “fragmentation” logarithms of  $\ell^2/\mu^2$ . We shall not treat these important corrections, or those associated with photon isolation, here. For jets, we can define  $E_\ell = |\vec{\ell}|$ , and integrate over  $\ell^2$  as part of the sum over final states.

Values of  $x$  and  $y$  for which  $s_4$  vanishes define “partonic threshold”, at which the Born process for direct photon production, or jet production at fixed three-momentum  $\vec{\ell}$ , uses all the available energy. Integration down to  $s_4 = 0$  leads to finite corrections in the cross sections, due to singular distributions of the general form  $\alpha_s^n [\ln^m(s_4/s)/s_4]_+$ , with  $m \leq 2n - 1$ . It is the effects of such distributions that threshold resummation organizes, to all orders in perturbation theory. It is easy to check that the steeper the slopes of the parton distributions at values of  $x$  and  $y$  where  $s_4 = 0$ , the larger are these effects. In essence, threshold resummation summarizes the interplay of parton luminosity and the soft QCD bremsstrahlung associated with the hard scattering. The corresponding recoil of the hard scattering is ignored for this purpose.

Because the functions  $\omega_{ab}$  are independent of the external hadrons, they are computed in infrared-regulated perturbation theory with  $A$  and  $B$  replaced by partons. To organize singular distributions in  $\omega_{ab}$  at threshold, we rely on further factorization properties of the partonic cross section near  $S_4 = S + T + U = 0$ , where we use capital letters for invariants defined with respect to the overall process,  $T_1 = (p_A - \ell)^2$ , etc. Fig. 1 represents the factorization of the purely partonic cross section  $a + b \rightarrow c + X$  near threshold, for direct photon production. For heavy quark and jet production the corresponding factorizations were discussed in Refs. [7, 8, 9, 10]. The  $h_{ab}$  absorb virtual parton propagators that are off-shell by the order of the momentum transfer. For convenience of notation, we define  $H_{ab} \equiv h_{ab}^* h_{ab}$ . In direct photon production, the two lowest-order reactions in  $H_{ab}$  are  $q + \bar{q} \rightarrow \gamma + q$  and  $g + q \rightarrow \gamma + q$ . The functions  $\psi$  incorporate the dynamics of partons collinear to the incoming partons  $a$  and  $b$  ( $q\bar{q}$  or  $gq$ ). Up to corrections that are finite at threshold, the  $\psi$ 's are flavor-diagonal. The momenta of final state particles associated with  $\psi_{a/a}$  and  $\psi_{b/b}$  may be

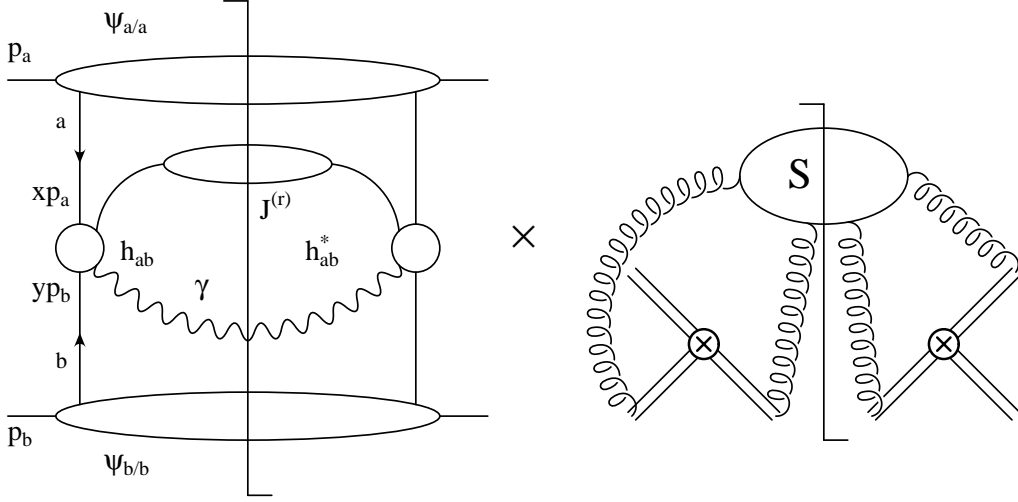


Figure 1: Factorization of direct photon production near threshold.

approximated by  $(1-x)p_a$  and  $(1-y)p_b$ , respectively. The function  $J^{(r)}$  represents partons recoiling against the photon, with total momentum  $p_R$ . Their dynamics is summarized by a two-point function for the field of flavor  $r$  ( $g$  or  $q$ ). Finally, the function  $S(k_S)$  summarizes the dynamics of soft gluons, of total momentum  $k_S$ . As indicated by the double lines in the figure, partons involved in the hard scattering are treated in the eikonal approximation in  $S$  [9, 10]. For jet production, there is an additional jet function associated with the collinear particles that carry the observed momentum  $\ell$ . For jets and heavy quarks,  $H \times S(k_S)$  is a product in the space of color exchange [7, 9, 10], but for direct photon production  $H$  and  $S(k_S)$  are simply functions. At threshold, the dynamics of each of the classes of partons become independent, and the partonic cross section reduces to a convolution [9]. We now turn to the kinematics of QCD radiation near threshold for 1PI processes, which plays a central role in resummation.

Momentum conservation implies that  $xp_a + yp_b = \ell + p_R + k_S$ . Near threshold, and neglecting corrections of order  $S_4^2$ , we readily find that  $S_4$  is a sum of terms, each of which may be associated with one of the functions identified above, through the relations

$$\begin{aligned}
S_4 &= (1-x)2p_a \cdot \hat{p}_R + (1-y)2p_b \cdot \hat{p}_R + 2k_S \cdot \hat{p}_R + p_R^2 + \ell^2 \\
&\equiv \left[ w_a \left( \frac{u}{t+u} \right) + w_b \left( \frac{t}{t+u} \right) + w_S + w_R + w_\ell \right] S \\
&= \left[ (1-x) \left( \frac{u}{t+u} \right) + (1-y) \left( \frac{t}{t+u} \right) + \frac{s_4}{S} \right] S.
\end{aligned} \tag{3}$$

The vector  $\hat{p}_R$  in the first line of Eq. (3) is the momentum of the recoiling jet (or heavy quark) at threshold. In the center of mass frame,  $\hat{p}_R^\mu = (\ell_0, -\vec{\ell}) \equiv \sqrt{S}\zeta^\mu$ . In the second line, we introduce a set of dimensionless weights  $w_i$ , that measure the contribution of each function in Fig. 1 to  $S_4$ . At threshold, each of these weights vanishes. The third line of Eq. (3) relates the overall  $S_4$  to the corresponding quantity,

$s_4$ , defined in Eq. (2), in the standard factorization, Eq. (1). Note that  $w_a \neq 1 - x$  and  $w_b \neq 1 - y$ , because these variables refer to different choices of distributions:  $w_a$  and  $w_b$  to the functions  $\psi$  of Fig. 1, and  $x$  and  $y$  to the distributions  $\phi$  in Eq. (1). In all ratios of kinematic factors, we use the quantities  $t$  and  $u$  characteristic of the partonic hard scattering, which is a good approximation at true threshold, when  $x$  and  $y$  approach unity.

In these terms, the infrared-regulated, perturbative, and factorized cross section,  $a + b \rightarrow c + X$  at fixed  $S_4$ , may be written as an alternate convolution form [11], which directly reflects the organization of Fig. 1,

$$\begin{aligned}
E_\ell \frac{d\sigma_{ab \rightarrow c(\ell)+X}}{d^3\ell} &= H_{ab}(t, u) \int dw_a dw_b dw_S dw_R dw_\ell \\
&\times \delta \left( \frac{S_4}{S} - w_a \left( \frac{u}{t+u} \right) - w_b \left( \frac{t}{t+u} \right) - \sum_{i=S,R,\ell} w_i \right) \\
&\times \psi_{a/a}(w_a, p_a, \zeta, n) \psi_{b/b}(w_b, p_b, \zeta, n) \\
&\times J^{(c)}(w_\ell, \ell, \zeta, n) J^{(r)}(w_R, p_R, \zeta, n) S \left( \frac{w_S S}{\mu^2}, \beta_i, \zeta, n \right). \quad (4)
\end{aligned}$$

When the observed particle  $c$  is a photon,  $J^{(c)}(\ell^2, \ell \cdot \zeta)$  may be replaced by unity (neglecting fragmentation contributions), and for  $c$  a heavy quark both  $J^{(c)}$  and  $J^{(r)}$  may be absorbed into the hard-scattering function  $H$ . All the factors in Eq. (4) are evaluated in  $n \cdot A = 0$  gauge. Essentially the same arguments for the factorized form Eq. (1) apply as well to Eq. (4).

In particular, the functions  $\psi_{i/i}(w_i, p_i, \zeta, n)$  are the distributions of partons  $i$  in parton  $i$  with fixed values of momentum component  $p_i \cdot \zeta$ . They are constructed by direct analogy to ‘‘center-of-mass’’ distributions [5, 7, 9], defined with  $\zeta^\mu = n^\mu = \delta_{\mu 0}$ . For example, as a matrix element in  $n \cdot A = 0$  gauge,  $\psi_{q/q}$  is given by

$$\psi_{q/q}(w, p, \zeta, n) = \frac{1}{2N_c} \frac{p \cdot \zeta}{2\pi p \cdot v} \int_{-\infty}^{\infty} d\lambda e^{-i(1-w)\lambda p \cdot \zeta} \langle q(p) | \bar{q}(\lambda \zeta) \frac{1}{2} v \cdot \gamma q(0) | q(p) \rangle \quad (5)$$

for an external quark of momentum  $p$ . Here  $v^\mu$  is the lightlike vector in the direction opposite to  $p^\mu$ ; for example, when  $\vec{p}$  is in the 3-direction,  $p \cdot v = p^+$ . The factor  $(1/2N_c)$  reflects an average over spin and color.

Given any factorization of the form of Eq. (4), it was shown in Ref. [11] that each of the factorized functions satisfy evolution equations, whose solutions organize all logarithmic  $S_4$ -dependence at leading power. In addition, it is straightforward to verify that if we choose the gauge vector  $n^\mu$  such that  $p_i \cdot n = p_i \cdot \zeta$  for both  $i = a, b$ , the densities  $\psi_{i/i}(w, p, \zeta, n)$  are equal to the center-of-mass densities ( $n^\mu = \zeta^\mu = \delta_{\mu 0}$ ), at leading and next-to-leading logarithm. For direct photon, heavy quark and jet cross sections, such choices are

$$\begin{aligned}
n^\mu &= \frac{p_b \cdot \zeta}{p_a \cdot p_b} p_a^\mu + \frac{p_a \cdot \zeta}{p_a \cdot p_b} p_b^\mu, & \text{photon or jet,} \\
n^\mu &= \zeta^\mu, & \text{heavy quark.}
\end{aligned} \quad (6)$$

For the heavy quark  $\hat{p}_R$  is the momentum of the recoiling (unobserved) member of the pair. In each case,  $\psi_{a/a}$  is a function of  $w_a$  and  $p_a \cdot \zeta = -u/\sqrt{s}$  only, and  $\psi_{b/b}$  a function of  $w_b$  and  $p_b \cdot \zeta = -t/\sqrt{s}$  only.

We organize singular behavior at threshold in terms of a Laplace (or Mellin) transform,

$$\tilde{\omega}_{ab} \left( N, \frac{t}{\mu^2}, \frac{u}{\mu^2}, \alpha_s(\mu^2) \right) = \int_0^S \frac{ds_4}{S} e^{-N(s_4/S)} \omega_{ab} \left( \frac{s_4}{\mu^2}, \frac{t}{\mu^2}, \frac{u}{\mu^2}, \alpha_s(\mu^2) \right), \quad (7)$$

where  $s_4$  is defined in Eq. (2) above. In the transform, a singular distribution  $[\ln^m(s_4/s)/s_4]_+$  produces  $\ln^{m+1} N$ , plus lower powers of  $\ln N$ . By comparing the moments with respect to  $S_4$  of Eq. (1) for initial state *partons*,  $A = a$  and  $B = b$  with moments of Eq. (4), and using the relation between  $S_4$  and  $s_4$  in Eq. (3), we derive [9]

$$\begin{aligned} \tilde{\omega}_{ab} \left( N, \frac{t}{\mu^2}, \frac{u}{\mu^2}, \alpha_s(\mu^2) \right) &= H_{ab}(t, u) \left[ \frac{\tilde{\psi}_{a/a}(N \frac{u}{t+u}, p_a \cdot \zeta) \tilde{\psi}_{b/b}(N \frac{t}{t+u}, p_b \cdot \zeta)}{\tilde{\phi}_{a/a}(N \frac{u}{t+u}, \mu^2) \tilde{\phi}_{b/b}(N \frac{t}{t+u}, \mu^2)} \right] \\ &\times \tilde{J}^{(e)}(N, \ell \cdot \zeta) \tilde{J}^{(r)}(N, p_R \cdot n) \tilde{S} \left( \frac{S}{N\mu^2}, \beta_i, \zeta, n \right) + \mathcal{O} \left( \frac{1}{N} \right). \quad (8) \end{aligned}$$

As in the case of  $\omega_{ab}$ , for each function  $f$  the moment is  $\tilde{f}(N) \equiv \int_0^1 e^{-Nw} f(w)$ . For large  $N$ , the precise upper limit is unimportant. The factors  $t/(t+u)$  and  $u/(t+u)$  are characteristic of resummation for the single-particle cross section. Solving the evolution equations for each of the functions in (8) [9, 11], we derive an explicit expression for  $\tilde{\omega}_{ab}(N)$ , whose inverse transform [12, 13, 14, 15] is the fully-resummed hard scattering function for cross sections with single-particle inclusive kinematics,

$$\begin{aligned} \tilde{\omega}_{ab} \left( N, \frac{t}{\mu^2}, \frac{u}{\mu^2}, \alpha_s(\mu^2) \right) &= \exp \left\{ \sum_{i=a,b} E_{(i)}(N_i, p_i \cdot \zeta) \right. \\ &\quad \left. - \int_{\mu}^{p_i \cdot \zeta} \frac{d\mu'}{\mu'} [\gamma_f(\mu') - \gamma_{ff}(N, \mu')] \right\} \exp \left\{ \sum_{j=c,r} E'_{(j)}(N, p_j \cdot n) \right\} \\ &\times H_{ab}(t, u) \tilde{S}(1, \beta_i, \zeta \cdot n) \exp \left\{ \int_{\mu}^{\sqrt{S/N}} \frac{d\mu'}{\mu'} 2\text{Re}\Gamma_S^{(ab)}(\mu') \right\}. \quad (9) \end{aligned}$$

The first exponential, which gives the  $N$ -dependence of the ratios of wave functions  $\tilde{\psi}$  and  $\tilde{\phi}$  in the  $\overline{\text{MS}}$  scheme, is precisely the same as for heavy quark and dijet production,

$$\begin{aligned} E_{(f)}(N_i, M) &= - \int_0^1 dz \frac{z^{N_i-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} A_{(f)}[\alpha_s(\lambda M^2)] \right. \\ &\quad \left. + \frac{1}{2} \nu^{(f)}[\alpha_s((1-z)^2 M^2)] \right\}, \quad (10) \end{aligned}$$

where again the extra factors in  $N_a \equiv N(-u/s)$  and  $N_b \equiv N(-t/s)$  reflect the kinematics of single particle inclusive cross section.  $A_{(f)}$  is given by the standard

expression [16],  $A_{(f)}(\alpha_s) = C_f \left( \alpha_s/\pi + (1/2)K(\alpha_s/\pi)^2 \right) + \dots$ , where  $C_f = C_F$  ( $C_A$ ) for an incoming quark (gluon), and  $K = C_A (67/18 - \pi^2/6) - 5/9n_f$ , with  $n_f$  the number of quark flavors. Finally,  $\nu^{(f)} = 2C_f (\alpha_s/\pi) + \dots$ . In the same exponential, the integral of the difference  $\gamma_f - \gamma_{ff}$  gives the scale evolution of the ratio  $\tilde{\psi}/\tilde{\phi}$  for flavor  $f$  [9].

The second exponential in Eq. (9) is associated with the final state jets. For heavy quarks it is absent. Adopting the notation of [9], we have

$$E'_{(f)}(N, M) = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{(1-z)^2}^{(1-z)} \frac{d\lambda}{\lambda} A_{(f)}[\alpha_s(\lambda M^2)] + B'_{(f)}[\alpha_s((1-z)M^2)] \right\}, \quad (11)$$

where  $A_{(f)}$  is the same as in Eq. (10), while the flavor-dependent  $B'$  is identified by comparing the one-loop expansion of  $E'$  to the one-loop two-point function of the relevant parton. For quarks and gluons the one-loop results are

$$B'_{(g)} = \frac{\alpha_s}{\pi} \left\{ C_A \left[ \frac{1}{2} \frac{n_f}{3C_A} - \frac{11}{12} - 1 + \ln(2\nu_g) \right] \right\}, \quad B'_{(q)} = \frac{\alpha_s}{\pi} \left\{ C_F \left[ -\frac{7}{4} + \ln(2\nu_q) \right] \right\}, \quad (12)$$

where we define  $\nu_i \equiv (\beta_i \cdot n)^2/n^2$  for a particle of velocity  $\beta_i$ . Finally, the last exponential in Eq. (9) is associated with soft emission. The “soft anomalous dimension”  $\Gamma_S$  depends on the kinematics of the hard scattering. The one-loop matrix anomalous dimensions for jet production were extensively discussed in Refs. [9, 10], and for heavy quarks in Ref. [7]. In these cases, the exponentials of the matrix soft anomalous dimension are ordered, and occur in traces with matrices of hard scattering functions.

We note that exponents from both the incoming and outgoing jets are double logarithmic. For the initial-state jets they are positive, and enhance the cross section, but for the recoiling jet they are negative, and suppress it. They are already present in the singularities at partonic threshold in the explicit one-loop calculation of direct photon production, as we will see in the next section.

The suppression associated with the recoiling final state jet for direct photon production tends to oppose the enhancement that is found from initial state jets in the production of heavy pairs [5, 6, 13, 14, 15]. As pointed out in Ref. [9], however, this relative suppression depends on the manner in which the cross section is constructed. The distinguishing criterion is whether the cross section is defined in such a way that partonic threshold requires that  $\ell^2 = p_R^2 = 0$ . For a jet or photon at fixed 3-momentum this is indeed the case, as we see in Eq. (3). Even a slight smearing of the jet momentum, however, such as in the cross section  $d^2\sigma_{\text{jet}}/dT dU$ , allows  $p_R^2$  and  $\ell^2$  to vary, and eliminates double-logarithmic suppression due to final state interactions.

### 3 One-loop Expansions

We now verify that the one-loop expansion of Eq. (4), and therefore Eq. (9), indeed reproduces the singular functions for direct photon production given in Ref. [17, 18], and similarly that, in the case of heavy quark production, the expansion of Eq. (9) reproduces the one-loop singular threshold behavior given in [19]. Note that to one

loop the individual contributions simply add up in Eq. (4). In addition, for finite contributions in the  $\overline{\text{MS}}$  scheme, we may take  $x = y = 1$ , because at zeroth order  $\phi_{a/a}(x) = \delta(1 - x)$ , and  $\phi_{b/b}(y) = \delta(1 - y)$ . Therefore, by the third line in Eq. (3),  $S_4 = s_4$ , and we need not distinguish between these two variables at one loop. It is important to keep in mind that, following our comments after Eq. (3),  $x = 1, y = 1$  does *not* imply  $w_a = 0, w_b = 0$ . The value of the calculations below, of course, is not to rederive known results, but to confirm the exponentiated forms that organize singular distributions to all orders.

### 3.1 Direct Photon Production

The two lowest order partonic subprocesses for direct photon cross sections are the ‘‘Compton’’ process,  $q_a(p_a) + g(p_b) \rightarrow q_r(p_r) + \gamma(\ell)$  and the ‘‘annihilation’’ process,  $q_a(p_a) + \bar{q}_b(p_b) \rightarrow g(p_r) + \gamma(\ell)$ . For ease of comparison we cast our answers in terms of the kinematic variables used by Gordon and Vogelsang (GV) in Ref. [18]:

$$v \equiv \frac{s+t}{s}, \quad z \equiv \frac{-u}{s+t}, \quad (13)$$

so that  $1 - z = s_4/(s+t)$  (to avoid confusion, we use  $z$  rather than GV’s  $w$ ). The Born cross sections for these two subprocesses are given by

$$v(1-v)z \frac{d^2\sigma_{q\bar{q}}^{(0)}}{dv dz} = \frac{2C_F}{N_c} \frac{\pi\alpha\alpha_s e_q^2}{s} T_{q\bar{q}} \delta(1-z), \quad (14)$$

$$v(1-v)z \frac{d^2\sigma_{qg}^{(0)}}{dv dz} = \frac{1}{N_c} \frac{\pi\alpha\alpha_s e_q^2}{s} T_{qg} v \delta(1-z), \quad (15)$$

with

$$T_{qg} = 1 + (1-v)^2, \quad T_{q\bar{q}} = v^2 + (1-v)^2. \quad (16)$$

In the above,  $N_c = 3$  is the number of colors, and  $e_q$  the electric charge of the quark. The contributions from the singular functions in the NLO corrections found by GV may be written as

$$v(1-v)zs \frac{d^2\sigma_{ij}^{(1)}}{dv dz} = \alpha\alpha_s^2 e_q^2 \left\{ c_3^{ij} \left[ \frac{\ln(1-z)}{1-z} \right]_+ + c_2^{ij} \left[ \frac{1}{1-z} \right]_+ + c_b^{ij} \left[ \frac{1}{1-z} \right]_+ \ln \frac{\mu^2}{s} \right\}, \quad (17)$$

where  $ij = qg$  or  $q\bar{q}$  and  $\mu$  is the factorization scale. To derive the  $c^{ij}$  in our formalism we need the one-loop expressions for the exponents in Eq. (9), or, equivalently, the one-loop corrections to the functions in Eq. (4). Both direct photon production subprocesses receive contributions from two incoming and one outgoing jet function, and from the relevant soft function.

As stated in the previous section, one may define all functions in Eq. (4) as operator matrix elements [7, 9, 5]. We assume they are all normalized to  $\delta(w_i)$ , where the weights  $w_i$  are defined in Eq. (3). We have computed the one-loop contributions to these matrix elements in  $n \cdot A = 0$  gauge, with  $n^\mu$  chosen as in Eq. (6).



The one-loop incoming-parton contribution in the  $\overline{\text{MS}}$  scheme is given by

$$\psi_{a/a}^{(1)}(w, n) = 2C_a \left[ \frac{\ln w}{w} \right]_+ - C_a \frac{1}{w_+} + C_a \frac{1}{w_+} \ln(2\nu_a) - C_a \ln\left(\frac{\mu^2}{s}\right) \frac{1}{w_+}, \quad (18)$$

where  $a = q, g$ , with  $C_q = C_F$  and  $C_g = C_A$ , and where we recall that  $\nu_i \equiv (\beta_i \cdot n)^2/n^2$  for a particle of type  $i$  and velocity  $\beta_i$ .

Correspondingly, the correction to this order for the outgoing gluon jet is (see Eqs. (11) and (12))

$$J^{(g),(1)}(w, n) = -C_A \left[ \frac{\ln w}{w} \right]_+ + C_A \left( \frac{1}{2} \frac{n_f}{3C_A} - \frac{11}{12} - 1 + \ln(2\nu_g) \right) \frac{1}{w_+}, \quad (19)$$

and for the outgoing quark jet [5]

$$J^{(q),(1)}(w, n) = -C_F \left[ \frac{\ln w}{w} \right]_+ + C_F \left( -\frac{7}{4} + \ln(2\nu_q) \right) \frac{1}{w_+}. \quad (20)$$

Notice that, indeed, the signs of the double-logarithmic terms in the  $\psi$ 's and the  $J$ 's correspond to Sudakov enhancement and suppression, respectively.

The soft functions  $S$  in Eq. (4) are ‘‘eikonal’’ cross sections constructed from Wilson lines, path-ordered exponentials of the gauge fields, in color representations and along paths that reflect the incoming and outgoing partons at threshold [7, 10]. In the case of direct photon production, there are two Wilson lines in the fundamental representation, representing quarks (antiquarks), and one in the adjoint, representing the gluon, coupled at a vertex  $T_a^{(F)}$ , the generator of  $SU(N_c)$  in the quark representation. At one loop, soft functions are of the form  $(1/w_+)2\text{Re}\Gamma_S^{(ij)}$ , where  $\Gamma_S^{(ij)}$  is the one-loop anomalous dimension of the vertex, labelled by the equivalent initial state partons  $i$  and  $j$ .

The rules necessary for the computation of the  $\Gamma_S^{(ij)}$  are given in Refs. [7, 10]. Straightforward calculation shows that

$$\Gamma_S^{(qq)} = \frac{\alpha_s}{2\pi} \left\{ C_A \left[ -\ln\left(\frac{2u}{t}\right) + 1 - i\pi - \ln(\nu_g) \right] + C_F \left[ 2\ln\left(\frac{-u}{2s}\right) + 2 - \ln(\nu_{q_a}\nu_{q_r}) \right] \right\}, \quad (21)$$

for the Compton process, and

$$\Gamma_S^{(q\bar{q})} = \frac{\alpha_s}{2\pi} \left\{ C_A \left[ \ln\left(\frac{tu}{2s^2}\right) + 1 + i\pi - \ln(\nu_g) \right] + C_F \left[ -2\ln(2) + 2 - 2i\pi - \ln(\nu_{q_a}\nu_{\bar{q}_b}) \right] \right\}, \quad (22)$$

for the annihilation process. Substituting the above results into Eq. (4), accounting carefully for the kinematic factors in the delta function that relates all the weights, and multiplying with the Born cross sections, given in Eqs. (14) and (15), gauge

dependence cancels, and we find

$$\begin{aligned}
c_3^{gg} &= T_{qg} \frac{1}{N_c} (C_F + 2N_c)v, & c_b^{gg} &= -T_{qg} \frac{1}{N_c} (C_F + N_c)v, \\
c_2^{gg} &= T_{qg} \left[ -\ln\left(\frac{1-v}{v}\right) - C_F \left( \frac{3}{4N_c} - \frac{1}{N_c} \ln v \right) \right] v, \\
c_3^{q\bar{q}} &= T_{q\bar{q}} \frac{2C_F}{N_c} (4C_F - N_c), & c_b^{q\bar{q}} &= -T_{q\bar{q}} \frac{4C_F^2}{N_c} \\
c_2^{q\bar{q}} &= T_{q\bar{q}} \left[ \frac{C_F n_f}{3N_c} - \frac{11}{6} C_F + 2C_F \ln(1-v) - 4 \frac{C_F^2}{N_c} \ln\left(\frac{1-v}{v}\right) \right]. \tag{23}
\end{aligned}$$

These coefficients agree precisely with those of GV in Ref. [18]. We note that the coefficients of the leading terms exhibit contributions from final state jets, which act to suppress the cross section, as discussed at the end of Sec. 2. From the initial state functions  $\psi$  alone, these coefficients would have been  $2C_F + 2C_A$  for the Compton process, and  $4C_F$  for the annihilation process (the same as in the Drell-Yan cross section).

### 3.2 Heavy Quark Production

As a further illustration, we consider heavy quark production through the partonic subprocess  $q(p_a) + \bar{q}(p_b) \rightarrow \bar{Q}(l) + X$ . The Mandelstam variables  $s, t_1, u_1$  are defined below Eq. (2), where now  $s_4 = s + t_1 + u_1$ . We shall derive the one-loop singular functions in this process, given in Ref. [19], in the  $\overline{\text{MS}}$  scheme. The results may be represented as

$$s^2 \frac{d^2 \sigma_{q\bar{q}}^{(1)}}{dt_1 du_1} = \frac{\alpha_s}{\pi} \sigma^{(0)} \left\{ c_3 \left[ \frac{\ln(s_4/m^2)}{s_4} \right]_+ + c_2 \left[ \frac{1}{s_4} \right]_+ + c_b \left[ \frac{1}{s_4} \right]_+ \ln \frac{\mu^2}{m^2} \right\} \tag{24}$$

with  $\mu$  the factorization scale. The plus distribution in terms of the dimensionful variable  $s_4$  may be represented as

$$\left[ \frac{\ln^i(s_4/m^2)}{s_4} \right]_+ = \lim_{\Delta \rightarrow 0} \left\{ \frac{\ln^i(s_4/m^2)}{s_4} \theta(s_4 - \Delta) + \frac{1}{i+1} \ln^{i+1} \left( \frac{\Delta}{m^2} \right) \delta(s_4) \right\}. \tag{25}$$

The function  $\sigma^{(0)}$  is the lowest order cross section (see Ref. [19]) with the factor  $\delta(s_4)$  removed.

We follow the same methods as in the previous subsection, and use the one-loop results for the  $\psi_{i/i}$  densities of Eq. (18). For the case at hand we find from Eq. (3)

$$w_a = \frac{s_4}{m^2} \left( \frac{m^2}{-u_1} \right), \quad w_b = \frac{s_4}{m^2} \left( \frac{m^2}{-t_1} \right). \tag{26}$$

Because the heavy quark mass prevents collinear singularities, all the final state contributions can be included in the soft function, via its anomalous dimension. Therefore we can replace both final state jet functions in Eq. (4) by unity. (Equivalently, we

absorb their finite corrections into the hard-scattering function.) Although the soft anomalous dimension is a matrix in the space of color tensors, the Born cross section in the  $q\bar{q}$  channel projects out only the octet-octet component. In Ref. [7] this was computed to be<sup>1</sup>

$$\begin{aligned}\Gamma_S^{\mathbf{8}} &= \frac{\alpha_s}{\pi} \left\{ C_F \left[ 4 \ln \left( \frac{u_1}{t_1} \right) - \ln(2\sqrt{v_q v_{\bar{q}}}) - L_\beta - \pi i \right] \right. \\ &\quad \left. + \frac{C_A}{2} \left[ -3 \ln \left( \frac{u_1}{t_1} \right) - \ln \left( \frac{m^2 s}{u_1 t_1} \right) + L_\beta + \pi i \right] \right\},\end{aligned}\quad (27)$$

where

$$L_\beta = \frac{1 - 2m^2/s}{\beta} \left\{ \ln \left( \frac{1 - \beta}{1 + \beta} \right) + i\pi \right\}, \quad \beta = \sqrt{1 - 4m^2/s}.\quad (28)$$

Its one-loop contribution is again  $(1/w_+)2\text{Re}\Gamma_S$ . Combining these results, we find the same  $\overline{\text{MS}}$  scheme singular functions for heavy quark production in the  $q\bar{q}$  channel as in Eq. (28) of Ref. [19]:

$$\begin{aligned}c_3 &= 4C_F \\ c_2 &= C_F \left[ -2 \ln \left( \frac{-u_1}{m^2} \right) - 2 \ln \left( \frac{-t_1}{m^2} \right) - 2 + 2 \ln \left( \frac{s}{m^2} \right) \right. \\ &\quad \left. - 8 \ln \left( \frac{u_1}{t_1} \right) - 2\text{Re}L_\beta \right] + C_A \left[ -3 \ln \left( \frac{u_1}{t_1} \right) - \ln \left( \frac{m^2 s}{u_1 t_1} \right) + \text{Re}L_\beta \right] \\ c_b &= -2C_F.\end{aligned}\quad (29)$$

Note that in order to obtain the resummed heavy quark production cross section in the DIS factorization scheme, one would divide Eq. (8) by  $(\tilde{F}_{2,a}/\tilde{\phi}_{a/a}) \times (\tilde{F}_{2,b}/\tilde{\phi}_{b/b})$  with  $F_{2,a}$  the hard part of the deep-inelastic scattering process [11].

Finally, we would like to make a general remark on the connection of resummed cross section formulas for single-particle inclusive (1PI) and pair-invariant mass (PIM) kinematics. As observed in Sec. 2, to next-to-leading logarithm, for  $p_i \cdot n = p_i \cdot \zeta$ , the densities  $\psi_{i/i}$ ,  $i = q, g$  are all identical, whether they fix the energy or some other component of the incoming parton momentum. The relation of the weight  $w_i$  to the total weight  $s_4$  is in general different for 1PI kinematics (Eq. (3)) and PIM kinematics [5, 9]. Therefore the only extra terms in the contributions from the incoming partons, after transforming  $\zeta^\mu$  (and therefore  $n^\mu$ ) from  $\delta_{\mu 0}$  to  $\hat{p}_R^\mu$ , arise from the weight relation in Eq. (3). Performing the same transformation in the soft function in Eq. (4) again yields extra terms, but here they are due to the gauge dependence of the soft anomalous dimension. Kinematic effects from the weight relation are absent.

The above observations apply when the vector  $\zeta^\mu$  is time-like, for both PIM and 1PI kinematics. In this manner one may transform the PIM resummed cross section into the 1PI resummed cross section (9), and back, the differences being easily computed by changing the gauge in the soft anomalous dimension  $\Gamma_S$ , and by replacing the moment variable  $N$  by  $N_a$ , for incoming parton  $a$ , as in Eq. (10).

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<sup>1</sup>Note that we have not normalized  $\Gamma_S$  to the Drell-Yan soft anomalous dimension, so that the expression (27) has an extra term  $+\alpha_s C_F/\pi$  compared to the result in [7].

## 4 Conclusion

We have already mentioned the relevance of resummed single-particle cross sections to the theory and phenomenology of QCD at high momentum transfer. In the context of single-particle inclusive cross sections, we have explored how threshold resummations provide information on higher-order corrections. As in the cases of heavy quark pair and Drell-Yan cross sections, the resummed exponents  $E$  and  $E'$  for threshold resummation contain infrared renormalons, which must be eliminated by a prescription that depends, in general, on nonperturbative parameters. In certain cross sections at high scales, cross sections may be quite independent of these parameters, but this subject merits further study. We believe that the formalisms outlined here will find useful applications [21].

Before concluding, we should point out that threshold resummation is not the only possible organization of higher-order corrections associated with soft gluon emission. Of particular interest for the kinematic shape of 1PI cross sections is the resummation of enhancements associated with points in phase space at which partonic transverse momenta vanish [22, 23]. This would lead to a “ $k_T$ -resummation” for 1PI cross sections. Such a formalism exists for the Drell-Yan and related processes [24, 25, 26] at measured (small) pair  $Q_T$ , but we know of no fully-developed method for  $k_T$ -resummation in single-particle kinematics, or discussion of its relation to threshold resummation. We believe that the organization of threshold singular distributions will also be a valuable step toward a fuller control over higher-order corrections including transverse momentum effects.

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