Teaching Mathematics and its Applications: An International Journal of the IMA (2022) **41**, 87–91
https://doi.org/10.1093/teamat/hrac008 Advance Access publication 30 May 2022

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Editorial to the special issue on tools to support meaning-making in calculus and pre-calculus education

Rogier Bos[†,*], Anatoli Kouropatov[‡] and Osama Swidan[§]
[†]*Freudenthal Institute, Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands*
[‡]*Levinsky College of Education, Tel Aviv, Shoshana Persiz st., 16, 6937808, Israel*
[§]*Ben Gurion University of the Negev, P.O.B. 653 Beer-Sheva 8410501, Israel*

[*]*Corresponding author. Email:* r.d.bos@uu.nl

[Received February 2022; accepted March 2022]

1. Introduction

Calculus is learned and taught across the globe in secondary schools and higher education institutions. Ongoing research indicates that many students struggle to learn calculus (Bressoud *et al.*, 2016; Tallman *et al.*, 2016); and, equally, teachers struggle to teach calculus in a meaningful way. In order to address this situation, the research community continues to search for new ways and tools to improve the teaching and learning of calculus and pre-calculus and, in particular, support meaning-making in this subject.

In recent decades, various technologies for learning and teaching calculus have been developed. Technologies involving, e.g., augmented reality and virtual reality, GeoGebra or MATLAB software are used to teach pre-calculus concepts at secondary school and more advanced calculus concepts at the university level. Besides these newer tools, traditional tools (like the scientific calculator) are still extensively used.

Mathematics education researchers have emphasized the notion of ‘meaning-making’ for mathematical concepts in general and the construction of meaning in calculus in particular (Kilpatrick *et al.*, 2005; Thompson, 2013, 2016). Many meetings, conferences and other international forums were dedicated to this subject, including the recent *Calculus in Upper Secondary and Beginning University Mathematics Conference* (Monaghan *et al.*, 2020). First, this conference led to a special issue of ZDM, with the theme ‘calculus in high school and college around the world’. Next, there is this special issue for *Teaching Mathematics and Its Applications: International Journal of the IMA*. This issue thematizes the role of tools to support meaning-making in calculus and pre-calculus education.

New technologies give rise to new questions and a fresh look on old questions, in particular on the relation between meaning-making and tool-use in calculus education. This special issue focusses on the following:

- What roles do tools have in students’ meaning-making in pre-calculus and calculus teaching and learning?
- How do students construct meanings of calculus concepts when they use various kinds of tools?

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- Are there successful combinations of digital and non-digital tools that help students in meaning construction?

While some of these questions have been explored before in the literature (Robert & Speer, 2001; Artigue, 2016; Bressoud *et al.*, 2016; Thompson, 2016), this special issue sheds light on some of the most recent approaches that researchers engaged in.

2. Meaning-making in mathematics education

A central issue for educators is how to make concepts in calculus meaningful to students. Meaning-making is complicated, multifaced and widely discussed in the literature, e.g., see important early work by Piaget (Johnckheere *et al.*, 1958). The approaches range from using 'meaning' as a synonym for 'comprehension' to establishing a whole taxonomy for meanings of meaning (Thompson *et al.*, 2014).

Unfortunately, the notion of meaning-making has several meanings in mathematics education literature, and the term 'meaning' differs from one perspective to another. From a cognitivist perspective, one may say that meaning is accomplished through mental organization or reorganization. Socio-culturalists may say that meaning-making takes place through discussion, argumentation and turn-taking. From this point of view, meaning-making is based on public and shared participation in discourse (Seeger, 2011). From an additional perspective, semioticists believe that meaning-making relies on a sign–sign relations, sign–people and even on people–people relations. Some studies address sign–sign relations as a means to reconstruct the system of signs creating mathematical themes in a rationalist spirit, through a lens of an epistemological or historical issue (see, e.g., Rotman, 2000; Duval, 2008; Otte, 2008). While a study on the sign–people relations has been done as a social–emotional reconstruction of what makes mathematics hard or easy for people—inside and outside the classroom (see, e.g., Arzarello & Sabena, 2011; Radford & Roth, 2011). Lastly, from the perspective of embodied cognition, the meaning of mathematical concepts is grounded in corporeal activity and perceptions (Abrahamson & Lindgren, 2014). Meaning can be evaluated with respect to the individual, as well as with respect to groups or institutions. We may say that activity has meaning as part of the curriculum, while students might feel that the same activity lacks meaning. Hence, the question that may arise is what we mean when we articulate making meaning: for who, for what and in which context? Given this complexity, it is appropriate that this special issue encompasses several perspectives on the topic of meaning-making for calculus concepts with tools and includes a discussion of many different perspectives.

Let us briefly summarize those perspectives on meaning-making that we encounter in this special issue and what role tools play in this process. In his contribution, Swidan (2022) considers meaning-making from a socio-cultural approach by focusing on the role of social interaction between students and digital tools, emphasizing the role of argumentation in the process of meaning-making. For him, meaning-making is disclosing objects through layers: layers of meaning. Wangberg *et al.*'s (2022) paper treats meaning-making from the socio-cultural perspective. They report how students collaboratively make sense of the derivative concept while learning in a multivariable calculus course. For this purpose, the students used non-digital tools—plastic surface models, inclinometer and accompanying contour maps—to endow with meaning context-rich problems involving the function of two variables. Bos *et al.* (2022) consider meaning-making from an embodied cognition point of view. For them, meaning-making happens through embodied interaction with artefacts, enriched by an augmented reality technology designed for this purpose. In their paper, Kouropatov and Ovodenko (2022) consider meaning-making as a cognitive process of Abstraction in Context (Dreyfus *et al.*, 2015). They discussed the learning of the inflection point concept and considered the conceptual knowledge of the concept as a productive meaning

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of it. Kouropatov and Ovodenko document the meaning-making process of the concept as it is learned in a dedicated digital learning environment designed for this purpose. Elias and Dreyfus's (2022) paper treats the meaning-making concept like Kouropatov and Ovodenko's. Both consider meaning-making as a process of abstraction in context. While Kouropatov and Ovodenko focus on making sense of inflection points through digital tools, Elias and Dreyfus report how students make sense of the convergence concept using digital and non-digital tools.

These approaches show how meaning-making undoubtably requires artefacts, including language. The discussion about the place and roles of tools in mathematics education is prevalent and well represented in the literature (Monaghan *et al.*, 2016). The studies in this special issue use digital and non-digital artefacts (Trouche, 2003) in tasks designed to foster the construction of particular meanings. The reported tools' interfaces (ibid.) are very diverse: from a sandbox to interactive digital teaching units.

3. Conclusions and further developments

Let us sketch some of the results of the paper in this issue. In four studies the task design involves interactive computer visualization. In two of those individual meaning-making is analysed. From this several conclusions are drawn. Kouropatov and Ovodenko (2022) find support for the claim that the tool they use facilitated students' meaning-making process in case of inflection points by offering two central functionalities: zooming and dragging. However, meaning-making depended on the flexibility of students' existing knowledge and representations. Elias and Dreyfus's (2022) analysis of their interviews reveals that manual plotting supported students' development of intuition about convergence and the technological tool supported students in constructing a notion of 'as close as one pleases', thus making a step in the direction of the formal definition of limit.

Swidan (2022) focusses on shared meaning-making by students. His study reveals that the utterances of the students while engaging with the digital tool were predominantly transactive. In other words, their questions and statements to one another tended to be clearly connected to the utterances that came before them, suggesting that using the digital tool and working on the task in pairs, using the same screen, kept the students' discussion focused and coherent.

Finally, two studies have a physical artefact as central feature of the task design. Both studies are concerned with teaching the slope of graphs of functions of two variables for which they provide real-world models: a hilly landscape made of sand and made of plastic, respectively. Bos' *et al.* (2022) case study demonstrates the feasibility of the augmented reality sandbox as embodied learning environment for this calculus subject. In action- and perception-based tasks the student uses the height lines feedback to manipulate a plastic plane into positions of varying slope. Additionally, rolling a marble and using sticks, the students can mathematize the notion of steepness into the notion of gradient and relate this to the plane equation. A new type of embodied task is introduced, incorporation, where a critical affordance is deliberately removed and the student reproduces its functionality without technology. Wangberg *et al.*'s (2022) show that the features of the plastic surface models productively supported the students' discussions and learning by simultaneously introducing ambiguity while providing ways for students to share their ideas. As in Swidan's study, students' small group discussions are highlighted as the arena where meaning-making takes place.

From what we discussed above, one can conclude that there are significant differences and similarities among the papers in this special issue. The aspects of similarity are as follows: all articles relate to the practical aspect the meaning-making; all articles evolve around tools purposefully designed to support meaning-making of specific calculus concepts; and all articles claim that researchers got evidence for the tool's importance in this process of meaning-making. The aspects of difference are as follows: all authors

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treated 'meaning' differently; and all authors use different theoretical frameworks for tools' design and data analysis. This reflects the fascinating reality of research in mathematics education. Indeed, even though all authors have similar ambitions concerning calculus education, it seems like they are working on different 'pieces of reality'. Can we integrate these pieces into a holistic picture? Should we?

These considerations lead us to several questions for future research.

1. Do we need to make an effort to consolidate the meaning of meaning? Is there a uniform role for tools in this consolidation?
2. What is the role of previous knowledge in new meaning-making? This question is strongly interwoven with the question of Thompson & Harel (2021, p. 516): 'What prior-to-calculus mathematical meanings and reasoning abilities might students develop in elementary, middle, and secondary grades to enhance their transition to calculus?'
3. How can we share and generalize our insights regarding tools' design?

These questions require collaboration. We hope that this special issue constitutes one more step towards it.

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