

Relevance of Wrong-Way Risk in Funding Valuation Adjustments

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Abstract

March 2020, the world was thrown into a period of financial distress. This manifested through increased uncertainty in the financial markets. Many interest rates collapsed and funding spreads surged significantly, which increased due to the market turmoil. In light of these events, it is key to understand and model Wrong-Way Risk (WWR) in a Funding Valuation Adjustment (FVA) context. WWR might currently not be incorporated in FVA calculations in banks' Valuation Adjustment (xVA) engines. However, we demonstrate that WWR effects are non-negligible in FVA modeling from a risk-management perspective. We look at the impact of various modeling choices such as including the default times of the relevant parties and we consider different choices of funding spread. A case study is presented for interest rate derivatives.

Keywords: Wrong-Way Risk (WWR), Funding Valuation Adjustment (FVA), computational finance, risk management

JEL: C63, G01, G13, G32

1. Introduction

During the period of financial distress following March 2020, significant market moves took place. Specifically, interest rates (IR) dropped and funding spreads increased drastically, causing banks to experience higher funding costs, which translated into significant losses [2]. These costs associated with the funding, which we consider as losses, depended on the portfolio composition of a dealer, potentially dominated by uncollateralised derivatives (no CSA ¹). These unbalanced portfolios may have been the result of a bank's (many) corporate clients, hedging the IR risk on their loans. The bank receives the fixed rate of the swap (i.e., a receiver swap). When IR drops (e.g., due to central bank interventions), these swaps move deeply into the money (ITM). ² The banks do not receive collateral from the corporates, while having to post collateral themselves when hedging the swap's interest rate risk in the interbank/cleared market, see Figure 1. The funding requirement on this collateral, combined with exploding funding spreads, are likely to have caused the aforementioned funding losses. The loss size depends on the valuation methods, counterparty creditworthiness and the bank's funding risk.

Funding costs of unsecured transactions are incorporated in derivatives pricing through a so-called Funding Valuation Adjustment (FVA), part of the xVA family. ³ FVA represents the funding cost of eliminating market risk on non-perfectly collateralized deals. Together with Credit Valuation Adjustment (CVA), it can be interpreted as the cost of imperfect collateralization. For further intuition on CVA and the hedging thereof, see [30]. The market turmoil and corresponding losses had a significant impact on the derivatives business, see Figure 2.

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¹The Credit Support Annex (CSA) is a legal document that prescribes the collateral part of transactions.

²ATM (at-the-money) means the current value of the swap is zero. ITM (in-the-money) and OTM (out-of-the-money) mean that the current value of the swap is positive and negative, respectively.

³See [13, 14] for further background on xVAs.

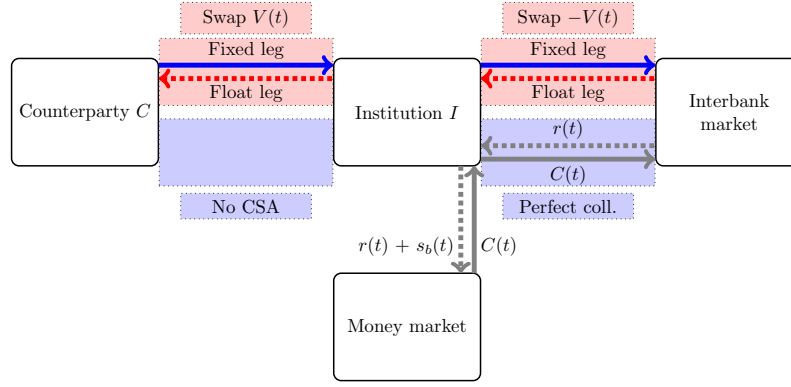


Figure 1: An uncollateralized ITM receiver swap (no CSA) with value $V(t) > 0$, between counterparty C (a corporate) and institution I (a bank). The swap consists of a fixed and a floating leg with fixed and variable cash-flows, respectively. The opposite hedge with value $-V(t) < 0$, in the interbank market, with perfect collateralization (coll.). All values are denoted from the institution's perspective. I needs to post collateral $C(t)$, for the hedge, to the interbank market counterparty. The collateral accrues at the risk-free rate $r(t)$. I needs to fund itself in the money market at the cost of a funding spread $s_b(t)$ over $r(t)$. Dotted lines refer to variable flows, while solid lines refer to fixed flows.

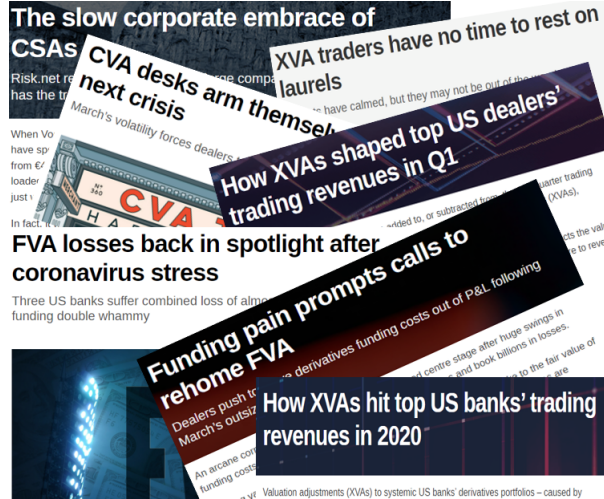


Figure 2: Risk.net headlines [2, 3, 27, 28, 29, 31, 32].

According to ISDA ⁴, Wrong-Way Risk (WWR) occurs when “exposure to a counterparty is adversely correlated with the credit quality of that counterparty” [?]. This is generic WWR, as opposed to specific, which involves the specifics of a deal structure. Right-Way Risk (RWR) is the opposite of WWR, where there is a favourable rather than adverse correlation. ⁵

FVA WWR can be understood as increased funding risk due to increased market risk. For an unbalanced portfolio of receiver swaps, WWR is caused by a negative correlation between interest rates and funding spreads: if IR goes down, exposure goes up, FVA goes up, increasing the funding spread sensitivity, and vice versa. In addition, the funding spread will go up, due to the adverse relationship between IR and funding spread.

To demonstrate the relevance of incorporating WWR in FVA modeling, FVA is plotted through time in Figure 3. These results indicate that FVA can increase significantly under unfavourable market moves. Even without WWR, this increase is significant.

Nevertheless, WWR is non-negligible in FVA modeling. FVA WWR models cross-gamma ⁶

⁴International Swaps and Derivatives Association.

⁵We will use the term WWR to indicate both WWR and RWR, as the difference is only in sign.

⁶Cross-gamma risks are second order partial derivatives with respect to two different linear market data

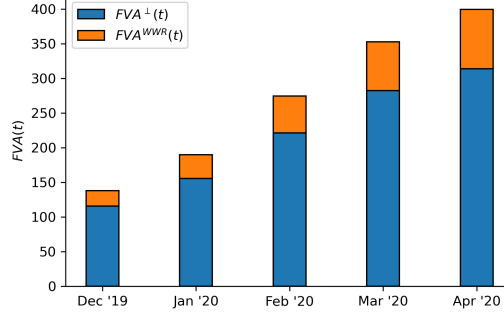


Figure 3: FVA through time for a receiver swap which is ATM in December 2019, with partially synthetic market data. There is a split between the independent part, FVA^{\perp} , and the WWR part, FVA^{WWR} . IR becomes increasingly negative through time, resulting in an ITM swap. The credit spreads are increasing through time. The other parameters are handpicked, such that the implied IR and credit volatilities are increasing through time. The correlation parameters are kept constant. The FVA results are for a credit-based spread, and both party’s default times are excluded from the FVA definition. These concepts will be introduced in Section 2.

risks between funding spreads and market exposure. These cross-gammas are difficult to hedge directly using vanilla derivatives. Alternatively, the frequency of rebalancing delta hedging positions can be increased. Yet, in stressed markets these hedges become increasingly expensive, due to low liquidity. Hence, the WWR premium can be interpreted as a compensation for increased hedging costs. Furthermore, the cross-gammas help an xVA desk to assess their sensitivity to WWR market scenarios. This helps in risk-management, where changing sensitivities can be anticipated when other market factors are changing. Rather than waiting for the overnight xVA run to finish and see how sensitivities are affected, the cross-gamma modeling allows the desk to start look for a suitable hedge on the day the market moves. The next day, banks with similar books will look for similar hedges, and market liquidity might quickly disappear. Hence, the WWR modeling will help the desk to stay within the risk limits. So the effect of adding WWR to the modeling is two-fold: the WWR premium is a compensation for re-hedging at expensive moments, and recognizing earlier when to re-hedge to have lower hedging costs. Furthermore, the cross-gammas will help the P&L explain process [30], to lower the amount of unexplained P&L.

Our contribution is to understand FVA WWR, and how it is driven by various modeling choices. We focus on the inclusion of the default times of a trade’s parties, and the choice of funding spread. These choices have a significant impact on both the FVA levels and the dependency structure, which is important for hedging delta, vega and cross-gamma risks. The FVA equation is split into an independent part and a WWR part to assess the WWR effects in isolation. Including the default times reduces FVA through a credit adjustment factor. This factor results in a more involved dependency structure, resulting in a RWR effect. Furthermore, we consider both a stochastic credit-based and a deterministic IR-based funding spread. The former yields WWR through the stochastic funding spread, possibly dampened by the RWR effects from credit adjustment factors. The latter results solely in a RWR effect.

2. FVA and Wrong-Way Risk

There has been a debate in literature on the legitimacy of incorporating FVA in pricing, initiated by Hull and White [15]. Responses came from both academia and practitioners [11, 12, 21], who do not necessarily agree with this. Hull and White continuously brought new arguments and clarifications to support their statement [16, 18, 17, 19]. For an overview of the arguments made in this debate, see Appendix A. Despite the discussion, market practice has

inputs.

been to include FVA in pricing. Yet, the recent funding losses and the difficulty of hedging have some entities advocating to move FVA out of P&L statements [28].

FVA can be split into a funding benefit (FBA) and cost (FCA). We assume that no profit can be made on potential funding benefits, i.e., an asymmetric funding assumption. In particular, a spread over the risk-free rate is paid when borrowing funds, but when lending out, we earn the risk-free rate. Consequently, $\text{FBA}(t) = 0$, such that $\text{FVA}(t) = \text{FCA}(t)$. In the case of multiple trades, funding benefits can be present implicitly by reduced funding costs.

We examine FVA WWR for a single ⁷ uncollateralized IR derivative V , between counterparty C and institution I , maturing at time T . All values are denoted from the perspective of party I . The FVA is based on borrowing spread $s_b > 0$ over risk-free rate r .

In Section 2.1, we look at the default processes, affine short-rate dynamics used in this work, and go into the correlation structure of the processes. Then, in Section 2.2, we derive the FVA equation and discuss the impact of including or excluding default times τ_I and τ_C in the FVA integral. Furthermore, the FVA equation and corresponding exposure are split into two parts: an independent part and a WWR part. In Section 2.3, the choice of funding spread is discussed. This choice is then applied to the FVA equation in Section 2.4, and we end up with an FVA exposure including WWR.

2.1. Default processes, model dynamics and correlations

We model default times τ_z , $z \in \{I, C\}$, as the first jumps of a Cox process ⁸ with hazard rate (intensity) λ_z . We impose affine short-rate models [24] for interest rate r and hazard rates λ_I and λ_C . The dynamics can be written as:

$$\bar{z}(u) = x_z(u) + b_z(u), \quad x_z(u) = \mu_z(t, u) + y_z(t, u),$$

where $\bar{z} \in \{r, \lambda_I, \lambda_C\}$ and $z \in \{r, I, C\}$. Furthermore, $y_z(t, u)$ is a stochastic processes, where $\mathbb{E}_t[y_z(t, u)] = 0$. The affine dynamics imply that the corresponding Zero Coupon Bond, $P_z(t, T)$, can be computed analytically.

Dependency between the processes can be introduced by correlating the Brownian motions [23] or using a copula [5, 9]. We choose the former, with independent defaults of counterparties I and C , which is justifiable as this is not the main driver in WWR modelling. Since we look at WWR impacts for IR derivatives, the main driver will be the dependency between the funding spread and the IR exposure. ⁹ Furthermore, independence of defaults is not an uncommon assumption in xVA literature, see for example [8]. The independence of defaults allows for factorization of survival probabilities, which turns out to be helpful in the FVA derivation in Section 2.2.

In terms of the Brownian motions $W(t)$, the correlation assumptions read

$$W_r(t)W_I(t) = \rho_{r,I} \cdot t, \quad W_r(t)W_C(t) = \rho_{r,C} \cdot t, \quad W_I(t)W_C(t) = 0,$$

where the IR-credit correlations $\rho_{r,I}$ and $\rho_{r,C}$ can be estimated historically. If credit data is unavailable, e.g., for illiquid counterparties, techniques exist to map these counterparties to liquid counterparties and the corresponding credit contracts [13].

2.2. FVA equation

Starting from the FVA definition [1], we derive the following expression for the FVA of financial derivative V in Appendix B. We assume that no defaults take place before today (t).

$$\text{FVA}(t) = \mathbb{E} \left[\int_t^{T \wedge \tau_I \wedge \tau_C} e^{-\int_t^u r(v)dv} s_b(u) (V(u))^+ du \middle| \mathcal{G}(t) \right] \quad (2.1)$$

$$\begin{aligned} &= \int_t^T \mathbb{E} \left[e^{-\int_t^u \lambda_I(v) + \lambda_C(v)dv} e^{-\int_t^u r(v)dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(t) \right] du \\ &=: \int_t^T \text{EPE}_{\text{FVA}}(t; u) du. \end{aligned} \quad (2.2)$$

⁷Typically FVA is a quantity of multiple trades in a portfolio or funding set, but for simplicity we only consider single trades.

⁸A Cox process is Poisson process where both the magnitude and the probability of a jump are stochastic [7].

⁹When dealing with credit derivatives, this dependency between defaults should definitely be present.

Here, $(x)^+ = \max\{x, 0\}$, $\mathcal{F}(t)$ is the ‘standard’ default-free filtration and $\mathcal{G}(t)$ is the enriched filtration with all available market information, including defaults. Going forward we write $\mathbb{E}[\cdot | \mathcal{F}(t)] = \mathbb{E}_t[\cdot]$.

As seen from Equation (2.2), FVA represents the cost to fund positive exposure over a period of time.¹⁰ Hence, the FVA integral is over the expected valuation profile with the funding spread, representing the total additional costs to fund at a different rate than the risk-free rate. This is a different type of integral than the CVA integral [13, 14], which is an integral over all possible default times.

In the FVA definition (2.1), the integration range is $[t, T \wedge \tau_I \wedge \tau_C]$.¹¹ If a party defaults before maturity, we need to fund for a shorter period of time than maturity T . This results in a credit adjustment factor $e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} < 1$ for the potential default of I and C . This factor is effectively the survival probability of the relevant parties. It can significantly decrease the overall FVA amount, depending on the credit quality of the parties. Hence, the assumption of including τ_I and/or τ_C in the FVA integral is important.

For example, excluding τ_I and τ_C can be regarded as a conservative assumption.¹² This claim is only valid in the asymmetric funding case where no funding benefits are present.

Depending on the correlations, the credit adjustment factors introduce an extra dependency. This is particularly interesting if the funding spread is driven by the same underlying source of randomness as a credit adjustment factor, i.e., a party’s credit process. In this situation, the WWR effects can become non-intuitive.

Furthermore, the assumption of including τ_I and/or τ_C is important for hedging FVA. Hence, the modeling assumptions may depend on the way an xVA desk hedges its FVA risks. The credit adjustment factor translates into an adjustment of the FVA sensitivities, and introduces new risk factors to which the FVA is sensitive. This impacts first-order delta and vega risks, and introduces new cross-gamma risks with the existing risk factors.

Remark (Additive FVA assumption). *When FVA is seen as the funding extension of the CVA-DVA¹³ paradigm, this will intuitively lead to an extra additive term in the set of xVAs. However, a deal’s future cash-flows will depend on future funding requirements, hence today’s valuation of those cash-flows requires the future funding choices to be modelled [25]. Hence, FVA is not an additive xVA, but the pricing equation is recursive and highly non-linear. Rather than using the FVA definition from Equation (2.1), in absence of CVA and DVA terms, FVA can be defined as “the difference between the price of the deal when all funding costs are set to zero and the price of the deal when funding costs are included” [25]. These recursive systems can be solved using least-squares Monte Carlo regression techniques, see for example the Longstaff-Schwartz method [22]. Under symmetric funding (equal funding and borrowing rates), the recursive nature of the problem disappears, as the funding requirement does not depend on the sign of the exposure [26]. In this case, FVA can be considered as additive, yet this assumption typically does not hold in practice. In [6], the recursive FVA is compared to the additive version through a Non-linearity Valuation Adjustment. The authors state that this framework is not suitable for implementation within banks, but allows to analyze the impact of linearization. Financial institutions often make the simplifying assumption of additive FVA, at the risk of some double counting between the various xVAs. This is also the approach we take, in line with current industry practice.*

$\text{EPE}_{\text{FVA}}(t; u)$ from Equation (2.2) can be written as the sum of the independent exposure $\text{EPE}_{\text{FVA}}^\perp(t; u)$ and a WWR exposure $\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u)$, such that Equation (2.2) changes:

$$\text{FVA}(t) = \int_t^T \text{EPE}_{\text{FVA}}^\perp(t; u) du + \int_t^T \text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u) du =: \text{FVA}^\perp(t) + \text{FVA}^{\text{WWR}}(t). \quad (2.3)$$

Hence, FVA can be split into an independent part and a part that captures the cross-dependencies.

¹⁰Typically, the FVA formula is given in terms of a forward funding spread. However, that is only possible if the funding spread is independent of the exposure, which is currently not the case.

¹¹This means that we integrate to maturity T , or to one of the default times τ_I or τ_C , whichever comes first.

¹²Assume for a moment that both τ_I and τ_C are excluded from the FVA definition, such that in Equation (2.1) integration is over the interval $[t, T]$ regardless of potential defaults. As a result, the credit adjustment factor $e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv}$ will disappear from Equation (2.2). Due to the absence of the credit adjustment factor, the FVA will be substantially larger in absolute terms compared to the previous formulation.

¹³Debit Valuation Adjustment.

2.3. Funding spread

The funding rate should reflect an institution's funding abilities in the market. We mainly focus on a credit-based funding spread, where we include the institution's credit, $\lambda_I(t)$, and a possible liquidity adjustment term $\ell(t)$. For example, $\lambda_I(t)$ can be CDS-based, and $\ell(t)$ can be the bond-CDS basis.¹⁴ Alternatively, $\lambda_I(t)$ can be bond-based or derived from asset swaps. We assume that it is CDS-based. Loss given default LGD_I is assumed to be constant, based on market information. We define borrowing spread $s_b(t)$ as [13]:

$$s_b(t) = \text{LGD}_I \lambda_I(t) + \ell(t).$$

WWR can be introduced through $s_b(t)$, if $\lambda_I(t)$ is stochastic and correlated with the other risk factors. Using the model dynamics from Section 2.1, we write:

$$\begin{aligned} s_b(u) &= \text{LGD}_I [x_I(u) + b_I(u)] + \ell(u) \\ &= \text{LGD}_I [\mu_I(t, u) + b_I(u)] + \ell(u) + \text{LGD}_I y_I(t, u) \\ &=: \mu_S(t, u) + \text{LGD}_I y_I(t, u). \end{aligned} \quad (2.4)$$

Remark (IR-based funding spread). *Another funding spread is based on a borrowing rate $r_b(t)$ which is a constant spread s over an n -month (nM) term rate, e.g., the forward-looking Libor/Euribor or backward-looking SOFR/ESTR. We assume the term rate is a deterministic spread term structure $s_{nM}(t)$ over OIS rate $r(t)$. This yields a deterministic borrowing spread:*

$$s_b(t) = r_b(t) - r(t) = s_{nM}(t) + s. \quad (2.5)$$

Hence, no WWR is introduced through the borrowing spread, but through the credit adjustment factors and exposure only. The IR-based spread is a special case of the credit-based spread: when removing the stochasticity from the latter, i.e., setting $y_I(t, u) = 0$, and choosing the drift $\mu_S(t, u)$ appropriately, the two cases collapse.

2.4. FVA exposure under funding spread assumptions

The funding spread assumptions of Section 2.3 can now be applied to the independent and WWR exposures from Equation (2.3). Derivations of the exposures presented here can be found in Appendix C.

$\text{EPE}_{\text{FVA}}^\perp(t; u)$ is written as:

$$\begin{aligned} \text{EPE}_{\text{FVA}}^\perp(t; u) &= P_I(t, u) P_C(t, u) \mu_S(t, u) \mathbb{E}_t \left[e^{-\int_t^u r(v) dv} (V(u))^+ \right] \\ &\quad + \text{LGD}_I \mathbb{E}_t \left[e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} y_I(t, u) \right] \mathbb{E}_t \left[e^{-\int_t^u r(v) dv} (V(u))^+ \right]. \end{aligned} \quad (2.6)$$

Here, survival probabilities $P_I(t, u)$ and $P_C(t, u)$ are independent, resulting from the correlation assumptions in Section 2.1. Furthermore, the second term captures the dependency between the borrowing spread and the credit adjustment factors.

Using the correlation assumptions from Section 2.1, WWR will be present:

$$\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u) = \mathbb{E}_t \left[\left(e^{-\int_t^u r(v) dv} (V(u))^+ - \mathbb{E}_t \left[e^{-\int_t^u r(v) dv} (V(u))^+ \right] \right) e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} s_b(u) \right], \quad (2.7)$$

where $\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u) = 0$ if IR and credit are independent.

Remark (Alternative FVA definition). *If τ_I and τ_C are excluded from the FVA definition, Equations (2.6-2.7) change into*

$$\text{EPE}_{\text{FVA}}^\perp(t; u) = \mu_S(t, u) \mathbb{E}_t \left[e^{-\int_t^u r(v) dv} (V(u))^+ \right], \quad (2.8)$$

$$\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u) = \mathbb{E}_t \left[\left(e^{-\int_t^u r(v) dv} (V(u))^+ - \mathbb{E}_t \left[e^{-\int_t^u r(v) dv} (V(u))^+ \right] \right) s_b(u) \right]. \quad (2.9)$$

Here, the credit adjustment factors have disappeared and a term has vanished, resulting in a more simplistic model.

¹⁴The basis is negative when the CDSs spreads are lower than the bond spread for the same maturity. In March 2020 this spread was negative due to low liquidity.

Remark (IR-based funding spread). *For the IR-based funding spread, Equations (2.6-2.7) change into:*

$$\text{EPE}_{\text{FVA}}^{\perp}(t; u) = P_I(t, u) P_C(t, u) \mu_S(t, u) \mathbb{E}_t \left[e^{-\int_t^u r(v) dv} (V(u))^+ \right], \quad (2.10)$$

$$\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u) = s_b(u) \mathbb{E}_t \left[\left(e^{-\int_t^u r(v) dv} (V(u))^+ - \mathbb{E}_t \left[e^{-\int_t^u r(v) dv} (V(u))^+ \right] \right) e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} \right]. \quad (2.11)$$

So for $\text{EPE}_{\text{FVA}}^{\perp}(t; u)$ from Equation (2.6), the second term drops out. In Equation (2.7), $s_b(u)$ can be moved outside the expectation, and only the correlation between the exposure and the credit adjustment factors remains.

3. FVA Wrong-Way Risk relevance

We illustrate the relevance of including WWR in FVA modeling using numerical examples. Furthermore, we give insights on the inclusion of τ_I and/or τ_C in the FVA definition. Finally, we consider the credit-based and IR-based funding spreads, and show the different WWR/RWR effects in both cases. We assess the correlation impact on FVA through the ratio $\frac{\text{FVA}(t)}{\text{FVA}^{\perp}(t)}$. When this ratio is larger or smaller than 1, there is WWR or RWR, respectively.

We use the FVA definition from Equation (2.3). Using the funding spread assumption of Section 2.3, independent exposures $\text{EPE}_{\text{FVA}}^{\perp}(t; u)$ and WWR exposures $\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u)$ are further specified in respectively Equations (2.6) and (2.7). We consider a 30 year receiver swap with 10000 notional.¹⁵ These results can easily be extended to other financial derivatives. FVA is computed using a Monte Carlo simulation with 10^5 paths and 10 dates per year.

In Section 3.1, we give the affine IR and credit dynamics that fit into the framework introduced in Section 2.1, and we discuss the model calibration stage. In Section 3.2, the choice of market data is motivated. Then we assess FVA WWR for different modeling assumptions: we consider both the credit-based and IR-based funding spreads in Sections 3.3 and 3.4, respectively, and look at including/excluding τ_I and/or τ_C the FVA definition. Furthermore, in Section 3.5, the impact of different market conditions and combinations of model parameters is assessed. There, we also look at payer swaps and the three types of moneyness.

In Appendix D, additional results are presented for the various modeling assumptions, where we also present several exposure profiles. These extra results support the conclusions from the experiments in this section.

3.1. Model dynamics and calibration

For IR process $r(t)$, the Hull-White dynamics with constant volatility are used:

$$r(t) = x_r(t) + b_r(t), \quad dx_r(t) = -a_r x_r(t) dt + \sigma_r dW_r(t). \quad (3.1)$$

The model can be calibrated to a yield curve and a strip of co-terminal swaptions quoted in the market. For sake of simplicity, we focus on a single swaption implied volatility $\sigma_{\text{imp}, r}$ to which the IR model volatility σ_r can be calibrated. Hence, the calibration of the IR process is fully market implied.

A CIR type of model is commonly used in WWR modelling for BCVA purposes [5, 8, 9]. Hence, for the credit processes λ_z , $z \in \{I, C\}$, we choose a CIR credit dynamics with constant volatility:

$$\lambda_z(t) = x_z(t) + b_z(t), \quad dx_z(t) = a_z (\theta_z - x_z(t)) dt + \sigma_z \sqrt{x_z(t)} dW_z(t). \quad (3.2)$$

For these dynamics, potential issues around the origin might arise. When the Feller condition $2a_z\theta_z > \sigma_z^2$ is satisfied, we are not affected by this issue, see [24, p275] for more information.

The CIR credit dynamics calibration should be partially market implied and partially historical. The choice of historical calibration is made due to insufficient liquidity in the CDS

¹⁵Such that all results can be interpreted as basis point effects.

option markets. During the credit calibration, IR and credit are assumed to be independent, such that separability between discount factors and survival probabilities is possible [7]. For each institution $z \in \{I, C\}$, the mean reversion a_z and volatility σ_z parameters can be calibrated to a time series of credit spreads. For a credit process, the intensity should be positive, i.e., $\lambda_z(t) = x_z(t) + b_z(t) > 0$. Then, $x_z(0) > 0$ and θ_z can be chosen such that the Feller condition is satisfied, as well as the condition $b_z(t) = f_z^{Market}(0, t) - f_z^{CIR}(0, t) > 0$ for all t . In other words, the model implied term structure f_z^{CIR} must fit the market term structure f_z^{Market} from the credit curve as closely as possible, while keeping $b_z(t) > 0$. Given $x_z(0)$, θ_z can be explicitly computed as a function of the other parameters.¹⁶

3.2. Market data

The market data used for the experiments in this section are those of April 2020 when markets were stressed, following financial distress in March 2020. In the beginning of 2020, interest rates were negative and continued to drop; the credit curves were relatively stable. Starting March 2020, credit quality across the spectrum started to deteriorate, resulting in higher market implied default probabilities. By April 2020, this lower credit quality was still visible, while interest rates were dropping even further. Clearly, these market conditions match the WWR scenario with an adverse relationship between IR and credit.

Next to market implied data, we use synthetic market data and model parameters. For the yield curves, we use either a market implied EUR 1D curve or a synthetic 5% flat ZC curve, see Appendix E.1. For the credit curves, we consider market implied curves comparable to AAA, BBB or B credit ratings, see Appendix E.2. Correlation parameters are also synthetic in our experiments, but can also be calibrated historically.

We consider low and high values for mean reversion and model volatility for all dynamics, as well as multiple correlation magnitudes and signs. Furthermore, we assume that C equal or lower credit quality than I : the survival probabilities of C are equal to or lower than those of I , and the credit volatility for C is equal to or higher than for I . Without loss of generality, we set liquidity adjustment $\ell(t) = 0$. In Appendix E.3, scenarios are given on which the experiments are based.

3.3. Credit-based funding spread

The results in this section are based on market data scenario 11 in Appendix E.3. The credit adjustment effect of including τ_I and/or τ_C is visible from the $FVA^\perp(t)$ values in Table 1. This effect is the strongest for τ_C , as C has a lower credit quality than I . When including both τ_I and τ_C , the combined effects result in the lowest $FVA^\perp(t)$. The reduction in $FVA^\perp(t)$ can be substantial, illustrated by the 74 basis point reduction in this example, which is approximately a 70% decrease.

	τ_I excl.	τ_I incl.
τ_C excl.	107.64	95.31
τ_C incl.	36.10	33.63

Table 1: $FVA^\perp(t)$ for the various choices of including/excluding τ_I and/or τ_C .

In Figure 4 the correlation effects are illustrated for τ_I included. The inclusion of τ_C is varied. When excluding τ_I , similar results are obtained, apart from a scaling factor due to increased WWR. When looking at correlation effects, we vary correlations $\rho_{r,I}$ and $\rho_{r,C}$ over the interval $[-0.7, 0.7]$.¹⁷ The WWR/RWR effects are non-negligible, as ratio $\frac{FVA(t)}{FVA^\perp(t)}$ is significantly different from 1 for non-zero correlations.

¹⁶In particular, we choose the minimum of the implied θ_z values for all spine dates, where the implied θ_z is computed based on $b_z(t) = 0$ for each pillar date t . The positivity constraints leads to $x_z(0) \leq f_z^M(0, 0)$. Ideally we want to minimize $\int_0^t b_z(u)du$ for all t , such that the stochastic part of the intensity captures the maximum of the market quotes.

¹⁷Correlation parameters $\rho_{r,I}$ and $\rho_{r,C}$ cannot be varied over the interval $[-1.0, 1.0]$ as we have assumed $\rho_{I,C} = 0$. Choosing correlations outside the interval $[-0.7, 0.7]$ will result in a non-SPD correlation matrix.

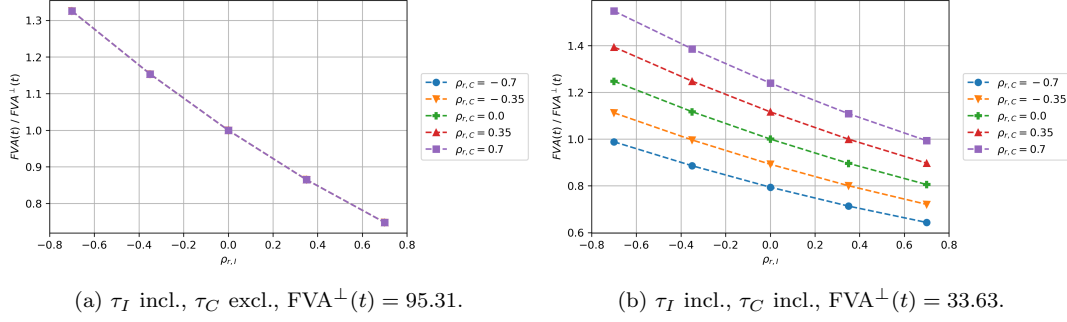


Figure 4: WWR effects for a credit-based spread and an ATM receiver swap.

In Figure 4a, there is net WWR for $\rho_{r,I} < 0$. All lines overlap, which is expected, as τ_C is excluded. The WWR effect comes from the relationship between the funding spread and the discounted exposure. This matches the March 2020 market moves: for negative IR-credit correlation, we expect to see WWR for receiver swaps, i.e., the FVA goes up. Symmetrically, when the correlation sign flips, i.e., $\rho_{r,I} > 0$, WWR changes into RWR. Going forward, we focus on negative correlations, as the symmetry remains: a change in correlation sign will result in a change of WWR/RWR effect. Furthermore, when including τ_I , the credit adjustment effect results in a slight RWR effect for $\rho_{r,I} < 0$: $\frac{FVA(t)}{FVA^\perp(t)}$ is lower than when excluding τ_I .

Comparing Figures 4a and 4b, for $\rho_{r,C} < 0$ there is a RWR effect coming from the credit adjustment factor for C : $\frac{FVA(t)}{FVA^\perp(t)}$ is lower for decreasing $\rho_{r,C}$. Figure 4b illustrates the effects when including all model components. Whether there is net WWR or RWR depends on the magnitude of WWR from the funding spread and the magnitude of the RWR effects from the credit adjustment factors. In turn, this is driven by the correlation parameters, credit parameters, IR parameters and product type. In some cases, the RWR effect from the credit adjustment factors outweighs the WWR effect from the funding spread. For example, when $\rho_{r,C} = -0.7$, for all values of $\rho_{r,I}$ there is overall RWR as $\frac{FVA(t)}{FVA^\perp(t)} < 1$.

Furthermore, the correlation magnitude effect is roughly linear in WWR/RWR, when setting one of the two correlations parameters to zero. When both correlations are non-zero, the correlation effects can be non-trivial due to the mixing of effects.

3.4. IR-based funding spread

The results in this section are based on market data scenario 11 in Appendix E.3. The credit adjustment effect on $FVA^\perp(t)$ is similar to the credit-based spread case from Table 1, as $FVA^\perp(t)$ does not depend on the WWR assumptions.¹⁸

Similarly to the credit-based funding spread results from Section 3.3, now consider the IR-based funding spread results in Figure 5. Now, only RWR effects coming from the credit adjustment factors are expected. In Figure 5a, when both τ_I and τ_C are excluded, there is neither WWR nor RWR present.

The expected RWR for I 's credit adjustment factor is clearly observed in Figure 5b, and RWR is roughly linear in $\rho_{r,I}$. The RWR is limited with a maximum effect of around 3%, explained by the relatively high credit quality of I in this example.

When only including τ_C , see Figure 5c, there is again a roughly linear RWR effect, but much more pronounced, resulting from the lower credit quality for C . Unlike the credit-spread case, the various values of $\rho_{r,I}$ have no impact on the FVA value.

When combining both credit adjustment factors in Figure 5d, as expected, the RWR effect from including τ_C dominates the results. Though there is only RWR, this is non-negligible.

Looking at the correlation magnitude, the same conclusions can be drawn as for the credit-based spread: when turning off one of the two correlations, the effect is roughly linear; when both correlations are non-zero, the effects are non-trivial.

¹⁸Recall that in the IR-based spread case we choose $s_b(u) = \mu_S(t, u)$ such that on average the funding spreads are equivalent for IR-based and credit-based funding spreads.

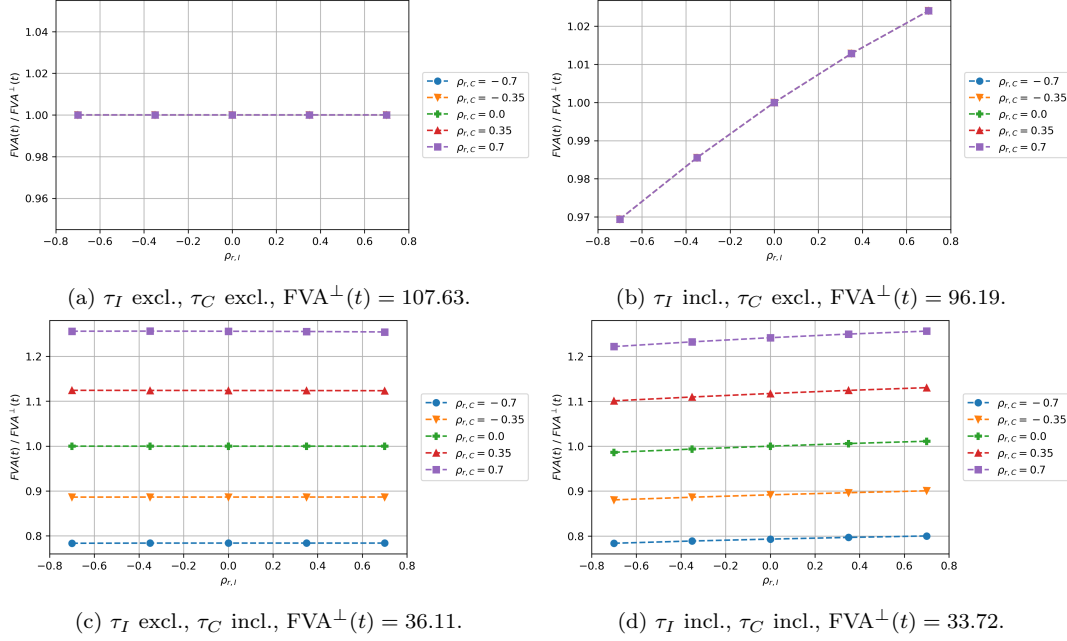


Figure 5: WWR effects for an IR-based spread and an ATM receiver swap.

3.5. Market conditions

Next, the impact of different market conditions and combinations of model parameters is assessed. We look at the effects of IR level (yield curve), IR volatility, credit level (credit curve), credit volatility and correlation parameters on the WWR for an ATM receiver swap. The starting point is scenario 2 in Appendix E.3, and in scenarios 3-21 we assess all effects in isolation. We also look at payer swaps and the three types of moneyness¹⁹ by calculating the par swap rate and applying up and downward shocks of 50 basis points to the par rate.

The IR parameter impacts are summarized in Table 2. When doubling the IR implied volatility (scenario 3), the FVA^\perp increases with roughly a factor 2 as well. For an asymmetric funding curve, an increased IR volatility results in a wider exposure profile. When IR volatilities are correlated with the credit level, this means WWR in the asymmetric funding case.

Change in param.	Scenario	Generic			WWR/RWR for credit-based spread			RWR for IR-based spread	
		FVA^\perp	I disc	C disc	s_b	I disc	C disc	I disc	C disc
$\sigma_r \uparrow$	3	+	o	o	+	+	+	+	+
$a_r \uparrow$	4	-	o	o	-	-	-	-	-
$a_r, \sigma_r \uparrow$	5	+	o	o	+	-	-	+	+
EUR1D curve	6	-	+	+	-	-	-	+	+

Table 2: WWR/RWR effects under different IR scenarios, for both τ_I and τ_C included in the FVA definition. Signs +/o/- indicates that the quantity has gone up / stayed roughly the same / has gone down.

An increased IR mean reversion, a_r , results in a decrease in FVA^\perp , and slightly less WWR (scenario 4). The effects of changing σ_r and a_r are multiplicative (scenario 5). In this case, overall the volatility effect dominates and FVA^\perp increases.

Changing to the EUR1D curve with negative interest rates (scenario 6) implies a change in IR level. In this case, the credit adjustment factor and WWR/RWR effects of including τ_I and/or τ_C are affected. These results are case-specific, non-trivial and no generic conclusion can be drawn.

¹⁹ATM (at-the-money), ITM (in-the-money) and OTM (out-of-the-money).

The credit parameter impacts are summarized in Table 3. A lower credit level (lower rating credit curve) results in a stronger credit adjustment effect of including τ_I and/or τ_C . This is due to a lower credit quality which translates into lower survival probabilities (scenario 7). For a lower credit level of I (scenario 7), the average funding spread goes up, as this is based on I's credit. As a result, the FVA^\perp and WWR/RWR effects go up significantly. FVA^\perp going up in this case can be understood as follows: as credit goes down, the hazard rate goes up, so the funding spread goes up. For a lower credit level of C (scenarios 8 and 10), there is a larger credit adjustment effect of including τ_C .

Change in param.	Scenario	Generic			WWR/RWR for credit-based spread			RWR for IR-based spread	
		FVA^\perp	I disc	C disc	s_b	I disc	C disc	I disc	C disc
BBB curve for I and C with $\sigma_z \downarrow$	7	+	+	+	+	+	+	+	+
BBB curve for C with $\sigma_C \downarrow$	8	o	o	+	o	o	+	o	+
BBB curve for C with similar σ_C	9	o	o	o	o	o	+	o	+
B curve for C with similar σ_C	10	o	o	+	o	o	+	o	+
B curve for C with $\sigma_C \uparrow$	11	o	o	+	o	o	+	o	+
$a_I \uparrow$	12	o	o	o	-	-	-	-	-
$\sigma_I \uparrow$	13	+	o	o	+	+	o	+	o
$a_I, \sigma_I \uparrow$	14	+	o	o	+	+	o	+	o
$a_C \uparrow$	15	o	o	o	o	o	-	o	-
$\sigma_C \uparrow$	16	o	o	o	o	o	+	o	+
$a_C, \sigma_C \uparrow$	17	o	o	o	o	o	+	o	+

Table 3: WWR/RWR effects under different credit scenarios, for both τ_I and τ_C included in the FVA definition. Signs +/o/- indicates that the quantity has gone up / stayed roughly the same / has gone down.

Increasing the mean reversion a_I (scenario 12) results in a smaller FVA^\perp . This comes from the choice of funding spread, which is based on λ_I . There is smaller RWR from the inclusion of τ_I and/or τ_C . Increasing the credit volatility σ_I (scenario 13) results in an increased FVA^\perp and increased WWR effect. The effects of changing σ_I and a_I are multiplicative (scenario 14). In this case, overall the volatility effect dominates and FVA^\perp increases.

When increasing the mean reversion a_C (scenario 15), only the RWR effect from including τ_C is affected: the amount of RWR goes down. Increased credit volatility σ_C for the same credit curve (scenarios 9, 11 and 16) does not seem to change the credit adjustment effect of including τ_C . Hence, the credit adjustment effects are purely driven by the credit curve itself. Similarly to institution I , the effects of the changes in a_C and σ_C are multiplicative (scenario 17). In this example, the effect of the change in σ_C dominates.

The correlation parameter impacts are summarized in Table 4. Changes in correlation magnitude or sign do not affect the level of FVA^\perp or the credit adjustment effects of including τ_I and/or τ_C .

Change in param.	Scenario	Generic			WWR/RWR for credit-based spread			RWR for IR-based spread	
		FVA^\perp	I disc	C disc	s_b	I disc	C disc	I disc	C disc
$\rho_{r,I} \times 2$	18	o	o	o	+	o	-	+	o
$\rho_{r,C} \times 2$	19	o	o	o	o	o	+	o	+
$\rho_{r,I}, \rho_{r,C} \times 2$	20	o	o	o	+	o	o	+	+
$\rho_{r,I}, \rho_{r,C} \times -1$	21	o	o	o	- (-)	- (-)	- (-)	- (-)	- (-)

Table 4: WWR/RWR effects under different correlation scenarios, for both τ_I and τ_C included in the FVA definition. Signs +/o/- indicates that the quantity has gone up / stayed roughly the same / has gone down. '(-)' refers to a change in sign.

Doubling the magnitude of $\rho_{r,I}$ (scenario 18) results in around twice as much WWR from the funding spread. When excluding τ_I and τ_C , the WWR effect from the funding spread is approximately linear in correlation magnitude. On the other hand, doubling the magnitude of $\rho_{r,C}$ (scenario 19) results in a larger RWR effect from including τ_C . When doubling the magnitude of both $\rho_{r,I}$ and $\rho_{r,C}$ (scenario 20), there is approximately twice as much WWR from the funding spread. Relatively speaking, the RWR effects are the same as before the correlation magnitude increases. If either $\rho_{r,I} = 0$ or $\rho_{r,C} = 0$, an approximate linear effect in correlation can be observed. Furthermore, if $\rho_{r,I}, \rho_{r,C} \neq 0$, the correlation effects are non-trivial due to the mixing of effects.

Similar to the observations from Section 3, changing the signs of $\rho_{r,I}$ and $\rho_{r,C}$ to positive correlations (scenario 21), results in a change in WWR sign, i.e., WWR changes into RWR and vice versa. All the effects are relatively of a lower magnitude than for negative correlations. Furthermore, the WWR/RWR results are non-symmetric in correlation sign.

For the various moneyiness types compared to the ATM case we find that ITM (OTM) makes FVA^\perp go up (down) and the percentage of WWR/RWR go down (up). Also, ITM (OTM) results in a lower (higher) credit adjustment effect compared to the ATM case. Furthermore, ITM (OTM) causes the WWR from the funding spread to go down (up) in relative sense w.r.t. the ATM case.

Changing to a payer swap causes RWR to change into WWR and vice versa. Furthermore, all WWR/RWR effects are lower in magnitude for payer swaps compared to receiver swaps in this particular example. Hence, the WWR/RWR effects are non-symmetric in swap-type.

For payer swaps, the WWR from the funding spread is much closer to being symmetric in correlation sign than for receiver swaps. Hence, the non-linearities have less impact for payer swaps. All other correlation and moneyiness effects described above are invariant in swap type.

4. Conclusion

We have shown that WWR effects are non-negligible and play an important role in FVA modeling, especially from a risk-management perspective when dealing with cross-gamma risks. Hence, we wanted to understand FVA WWR and how it is impacted by different modeling choices. The model reproduced the WWR effects observed in the March 2020 market moves. The modeling choices turn out have a significant impact on the FVA levels and the dependency structure. We split the FVA equation into two parts, an independent and a WWR part, to examine the WWR effects in isolation. There is a substantial credit adjustment effect from adding τ_I and/or τ_C in the FVA model, where we have seen examples of up to 70% reduction in FVA. This effect increases for lower credit quality. In practice, the choice of including τ_I and/or τ_C depends on an institution's preferences. Yet, portfolio FVA computation and trade level attribution should always be possible under these modeling choices. This can be challenging, and requires further research.

The credit-based funding spread results in WWR, while the credit adjustment effects translate into RWR (for a receiver swap and $\rho_{r,I}, \rho_{r,C} < 0$). Depending on correlations, credit parameters, IR parameters and product type, the net result is WWR or RWR. Changing the correlation sign causes WWR to change into RWR and vice versa. For an IR-based funding spread, there is only RWR coming from the credit adjustment factors. The credit adjustment effect is similar for all funding spreads. That the model can only generate RWR in this case means that the WWR as observed in the March 2020 market turmoil is not captured under this specific set of assumptions. Rather than exhibiting WWR, the effect is completely opposite.

Furthermore, we analyzed the different impacts the market and model parameters can have on the various components of the modeling. For IR and credit we see that the effects of σ_z and a_z are multiplicative, but in these examples the volatility effects dominate. Furthermore, the credit adjustment effects are unaffected by σ_z and a_z , but driven by the IR and credit levels. In particular, a lower credit level results in a stronger credit adjustment effect.

Correlations do not affect FVA^\perp and the credit adjustment effects, but only the WWR/RWR results. For both funding spread assumptions, the correlation effects in isolation are roughly linear, but when combined, non-linearities are introduced. In addition, the WWR/RWR is non-symmetric in correlation sign.

This absence of symmetry also holds for different swap types. Furthermore, like for a change in correlation sign, a change in swap type results WWR to change sign. The swap's moneyness affects everything: FVA^\perp , credit adjustment effects and WWR/RWR effects. This is because moneyness affects the exposures, which is naturally present in all the aforementioned items.

The conclusions for the single IR derivative naturally extend to an ITM portfolio of FVA sensitive trades. Due to the long-lasting low IR environment, such a portfolio becomes slowly less ITM. As the new trades continue to stay at-market, the ITM effect slowly fades away, though this might take a long time, especially when there are many long-dated ITM trades in the portfolio. If interest rates would go up in the future, this would mean a completely opposite situation where the portfolio is OTM. WWR effects will always strongly depend on the portfolio composition, see Figure 6. Actively making the portfolio less ITM results in less FVA variability, but this may be costly and not always feasible.

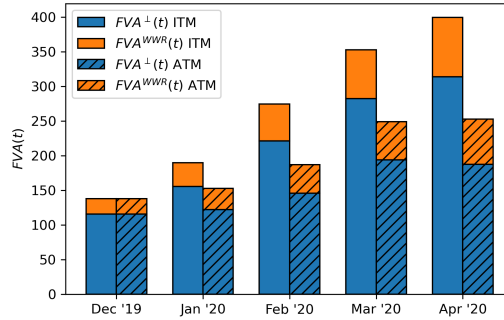


Figure 6: The same situation as in Figure 3, but compared with a similar ATM trade at all dates. If at each point in time a portfolio is rebalanced such that it is ATM rather than ITM, the increase through time of overall FVA is significantly less, but relatively the FVA^{WWR} becomes more important.

We have focused on FVA WWR in a qualitative sense. Yet, it is unclear how to compute this quantity efficiently, as the current Monte Carlo approach is not feasible in practice. There, additional sample paths are simulated for each of the credit processes, on top of the existing paths for the market exposure. As the number of risk-factors is typically already large, adding even more risk-factors in the simulation phase is undesired, especially regarding the large amount of counterparties a bank may have. This leaves room for our further research, to develop efficient calculation methods for WWR in FVA.

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Appendix A. The FCA debate

There has been a debate in literature on the legitimacy of incorporating FCA in pricing, initiated by Hull and White [15]. They argue that FCA (they call this FVA) should not be considered when valuing a derivative nor when determining the prices charged by the dealer. According to the authors, the risk-free rate should be used for discounting when valuing derivatives as this is required by the risk-neutral valuation paradigm. They counter the argument that funding hedging positions would be a natural cause of including FCA, postulating that hedges are zero NPV instruments, and that hence the hedging decision should not impact valuation. Furthermore, they argue that FCA is in conflict with the corporate finance theory principle that pricing and funding should be kept segregated: discounting when valuing a project should reflect the risk of the project rather than the risk of the firm. In conclusion, they state that FCA is an adjustment that moves away from computing the economic value, and should thus not be included in pricing.

In their original argument, Hull and White [15] claim that Burgard and Kjaer [10] make the same point of not including FCA in valuing or pricing derivatives, but with a different set of arguments. Burgard and Kjaer state that funding costs are indeed present in practice, as the funding party needs to be compensated for the possibility that the issuer might default on the received funding, and include them in their pricing equations through an FCA term. Then, using a simple balance sheet and funding model, they show that when adding the derivative asset and the funding liability to the balance sheet,

funding costs can be mitigated and that the FCA term is reduced to 0 as a result. As a result, prices are now symmetric between the issuer and the counterparty. However, from a practical point of view, they argue the operational challenges associated to this theoretical argument, as it requires the ability to completely hedge all risks related to the issuer and counterparty default. This is only possible when the issuer is able to buy and sell arbitrary quantities of their own bonds, such that the spread earned compensates for the funding costs. Furthermore, they state some other special cases where the funding adjustment drops out from the valuation.

In a clarifying note [11], Burgard and Kjaer shed some further light on their original arguments and relate them to those of Hull and White [15]. They see the FCA as the expected value of the windfall to the issuer's bondholders at the issuer's default, when the derivative is ITM for the issuer. The authors argue in which three cases the FCA term should not be included:

1. When the issuer is able to buy and sell arbitrary quantities of their own bonds, such that the spread earned compensates for the funding costs.
2. When the ITM derivative can be used as collateral to obtain cheaper funding than the otherwise unsecured funding.
3. When adding the derivative asset and the funding liability to the balance sheet, the issuer's recovery changes and thus the funding rate, resulting in a marginal funding rate equal to the risk-free rate.

All of these cases are only valid from a theoretical perspective though according to the authors, in a practical setting they all have their shortcomings/issues.

The discussion of FCA evolved by responses from others [12, 21], amongst which practitioners. They structurally counter the arguments by Hull and White, claiming that the assumptions of the BSM economy no longer hold as the market is incomplete, and always using the risk-free rate for discounting is inappropriate in that case. Also, the argument of project-specific discounting rather than firm-based discounting is argued against: adding a risk-free hedged portfolio of derivatives does in practice not affect the rate at which the bank can borrow, let alone the fact that existing debt cannot be renegotiated as a result of adding this portfolio.

In turn, Hull and White counter these responses [16], casting doubt on the market incompleteness argument, and challenge the justification of FCA using a hedging argument. One of their key arguments/assumptions is the idea that when an issuer hedges, it reduces risk and therefore will have a lower funding spread as a result.

Consecutively in [17, 19], Hull and White argue that the return on an investment should be driven by the risk of the investment rather than the average funding cost of the company undertaking the investment. This relies on the assumption that debtholders of the issuer understand all the risks taken and continuously reflect this in the pricing of debt. The advocates against including FCA in pricing are financial economists who work with marginal funding costs, as opposed to financial engineers who are in favour of including FCA and use average funding costs. Concerns are expressed about FCA being an asymmetric adjustment that will jeopardize the ability of two parties to agree on a price for a deal. Their view is that inclusion of funding costs lacks a theoretical basis, but they recognize the practical situation of institutions including it in pricing. A possible future scenario according to the authors is FCA being used for internal decision making when taking on a deal or not.

In [18], Hull and White summarize old arguments and give new ones as well. A key new insight is that funding adjustments are driven by derivatives desk performance measurement: they are required to earn a target funding rate.

Appendix B. FVA derivation

The following filtrations are relevant for us: $\mathcal{F}(t)$ is the 'standard' default-free filtration; $\mathcal{H}_I(t) = \sigma(\{\tau_I \leq s\} : s \leq t)$ is the filtration generated by the default time τ_I ; $\mathcal{H}_C(t) = \sigma(\{\tau_C \leq s\} : s \leq t)$ is the filtration generated by the default time τ_C ; $\mathcal{G}(t) := \mathcal{F}(t) \otimes \mathcal{H}_I(t) \otimes \mathcal{H}_C(t)$ is the enriched filtration containing all available market information.

The independence of defaults can be translated into a useful result for later derivations. As we model times to default τ_z , $z \in \{I, C\}$, as the first jumps of a Cox process with hazard rate (intensity) $\lambda_z(t)$, we can write for $s \leq t$ [7]:

$$\mathbb{Q}(t < \tau_z | \mathcal{F}(s)) = \mathbb{E}[\mathbb{1}_{\{t < \tau_z\}} | \mathcal{F}(s)] = \mathbb{E}\left[e^{-\int_0^t \lambda_z(v) dv} \middle| \mathcal{F}(s)\right], \quad (\text{B.1})$$

where $\mathcal{F}(t)$ is the 'standard' default-free filtration. Combining this result with the assumption of independent default times τ_I and τ_C , results in the following assumption.

Assumption 1 (Independent defaults). *We assume that default times τ_I and τ_C are conditionally independent on $\mathcal{F}(t)$, hence $\rho_{I,C} = 0$ and, for $s \leq t$,*

$$\begin{aligned} \mathbb{Q}(t < \tau_I, t < \tau_C | \mathcal{F}(s)) &= \mathbb{Q}(t < \tau_I | \mathcal{F}(s)) \cdot \mathbb{Q}(t < \tau_C | \mathcal{F}(s)) \\ &\stackrel{(B.1)}{=} \mathbb{E} \left[e^{-\int_0^t \lambda_I(v) dv} \middle| \mathcal{F}(s) \right] \cdot \mathbb{E} \left[e^{-\int_0^t \lambda_C(v) dv} \middle| \mathcal{F}(s) \right] \\ &= \mathbb{E} \left[e^{-\int_0^t \lambda_I(v) + \lambda_C(v) dv} \middle| \mathcal{F}(s) \right]. \end{aligned} \quad (B.2)$$

For FVA, we can derive the following expression starting from its definition [1], under the single assumption of conditional independence of defaults:

$$\begin{aligned} \text{FVA}(t) &= \mathbb{E} \left[\int_t^{T \wedge \tau_I \wedge \tau_C} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ du \middle| \mathcal{G}(t) \right] \\ &= \mathbb{E} \left[\int_t^T \mathbb{1}_{\{t < u < \tau_I\}} \mathbb{1}_{\{t < u < \tau_C\}} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ du \middle| \mathcal{G}(t) \right] \\ &\stackrel{\text{Fubini}}{=} \int_t^T \mathbb{E} \left[\mathbb{1}_{\{t < u < \tau_I\}} \mathbb{1}_{\{t < u < \tau_C\}} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{G}(t) \right] du \\ &\stackrel{\text{Lemma [4]}}{=} \mathbb{1}_{\{t < \tau_I\}} \mathbb{1}_{\{t < \tau_C\}} \int_t^T \frac{\mathbb{E} \left[\mathbb{1}_{\{u < \tau_I\}} \mathbb{1}_{\{u < \tau_C\}} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(t) \right]}{\mathbb{Q}(t < \tau_I, t < \tau_C | \mathcal{F}(t))} du \\ &\stackrel{(B.2)}{=} \frac{\mathbb{1}_{\{t < \tau_I\}} \mathbb{1}_{\{t < \tau_C\}}}{e^{-\int_0^t \lambda_I(v) + \lambda_C(v) dv}} \int_t^T \mathbb{E} \left[\mathbb{1}_{\{u < \tau_I\}} \mathbb{1}_{\{u < \tau_C\}} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(t) \right] du, \end{aligned} \quad (B.3)$$

where in the last step we also used that $e^{-\int_0^t \lambda_I(v) + \lambda_C(v) dv}$ is $\mathcal{F}(t)$ -measurable.

For convenience, we re-iterate Lemma 5.1.2 from [4] we have used. See also Section 22.5 from [7] for further context and background.

Lemma 1. *Let $\mathcal{G}(t) := \mathcal{F}(t) \otimes \sigma(\{\tau \leq s\} : s \leq t)$. When X is a $\mathcal{F}(u)$ integrable random variable then for $t < u$:*

$$\mathbb{E} [X \mathbb{1}_{\{\tau > u\}} | \mathcal{G}(t)] = \mathbb{1}_{\{\tau > t\}} \frac{\mathbb{E} [X \mathbb{1}_{\{\tau > u\}} | \mathcal{F}(t)]}{\mathbb{E} [\mathbb{1}_{\{\tau > t\}} | \mathcal{F}(t)]}.$$

By $\mathcal{F}(t) \subset \mathcal{F}(u)$ we can use the tower property to rewrite the following expectation, using that $e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+$ and $e^{-\int_0^u \lambda_I(v) + \lambda_C(v) dv}$ are $\mathcal{F}(u)$ -measurable.

$$\begin{aligned} &\mathbb{E} \left[\mathbb{1}_{\{u < \tau_I\}} \mathbb{1}_{\{u < \tau_C\}} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(t) \right] \\ &\stackrel{\text{Tower prop.}}{=} \mathbb{E} \left[\mathbb{E} \left[\mathbb{1}_{\{u < \tau_I\}} \mathbb{1}_{\{u < \tau_C\}} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(u) \right] \middle| \mathcal{F}(t) \right] \\ &= \mathbb{E} \left[\mathbb{E} \left[\mathbb{1}_{\{u < \tau_I\}} \mathbb{1}_{\{u < \tau_C\}} \middle| \mathcal{F}(u) \right] e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(t) \right] \\ &\stackrel{(B.2)}{=} \mathbb{E} \left[\mathbb{E} \left[e^{-\int_0^u \lambda_I(v) + \lambda_C(v) dv} \middle| \mathcal{F}(u) \right] e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(t) \right] \\ &= \mathbb{E} \left[e^{-\int_0^u \lambda_I(v) + \lambda_C(v) dv} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \middle| \mathcal{F}(t) \right]. \end{aligned} \quad (B.4)$$

We write $\mathbb{E}[\cdot | \mathcal{F}(t)] = \mathbb{E}_t[\cdot]$ for ease of notation. Furthermore, we assume that no defaults take place before t , which in practice will be today. In other words, we have $\mathbb{1}_{\{t < \tau_I\}} \mathbb{1}_{\{t < \tau_C\}} = 1$. Going back to FVA from Equation (B.3) we can now write the following result:

$$\begin{aligned} \text{FVA}(t) &\stackrel{(B.4)}{=} \frac{1}{e^{-\int_0^t \lambda_I(v) + \lambda_C(v) dv}} \int_t^T \mathbb{E}_t \left[e^{-\int_0^u \lambda_I(v) + \lambda_C(v) dv} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \right] du \\ &= \int_t^T \mathbb{E}_t \left[e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} e^{-\int_t^u r(v) dv} s_b(u) (V(u))^+ \right] du \\ &=: \int_t^T \text{EPE}_{\text{FVA}}(t; u) du, \end{aligned} \quad (B.5)$$

where in the second step we again used that $e^{-\int_0^t \lambda_I(v) + \lambda_C(v) dv}$ is $\mathcal{F}(t)$ -measurable.

Appendix C. Exposure derivation

The exposure $\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u)$ in Equation (2.2) can be considered in the following generic form:

$$\text{EPE}_{\text{FVA}}(t; u) = \mathbb{E}_t [f(t, u; \lambda_I, \lambda_C) g(t, u; r) h(t, u; r, V)]. \quad (\text{C.1})$$

The choice of functions $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ depends on the borrowing spread assumption, as well as on the inclusion/exclusion of τ_I and/or τ_C in the FVA definition. In [20], a model-independent approach is used for this, where the expectation is decomposed into simpler expectations, standard deviations and correlations. Unfortunately, this yields rather complicated expressions that are not trivial to compute. Instead of decomposing into correlations, we decompose into covariances. Using the covariance definition iteratively, we can write

$$\begin{aligned} \text{EPE}_{\text{FVA}}(t; u) &= \mathbb{E}_t [f] \mathbb{E}_t [g] \mathbb{E}_t [h] \\ &\quad + \mathbb{E}_t [h] \text{Cov}_t (f, g) + \mathbb{E}_t [(h - \mathbb{E}_t [h]) (fg - \mathbb{E}_t [f] \mathbb{E}_t [g] - \text{Cov}_t (f, g))] \\ &=: \text{EPE}_{\text{FVA}}^\perp(t; u) + \text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u). \end{aligned} \quad (\text{C.2})$$

The initial expectation can be written as the sum of the independent exposure $\text{EPE}_{\text{FVA}}^\perp(t; u)$ and a term capturing the cross-dependencies, $\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u)$, which is driven by the correlation assumptions. For the credit-based funding spread from Equation (2.4), the functions in Equation (C.2) are: $g(t, u; r) = 1$, $h(t, u; r, V) = e^{-\int_t^u r(v) dv} (V(u))^+$, such that $\mathbb{E}_t [e^{-\int_t^u r(v) dv} (V(u))^+]$ is simply the discounted positive exposure, readily available from an existing xVA engine. Finally, define $f(\cdot)$ as:

$$\begin{aligned} f(t, u; \lambda_I, \lambda_C) &= e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} s_b(u), \\ \mathbb{E}_t [f(t, u; \lambda_I, \lambda_C)] &= P_I(t, u) P_C(t, u) \mu_S(t, u) + \text{LGD}_I \mathbb{E}_t [e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} y_I(t, u)], \end{aligned}$$

where survival probabilities $P_I(t, u)$ and $P_C(t, u)$ are independent due to the assumption of independent defaults of counterparties I and C from Section 2.1.

Applying this to the two exposure types in Equation (C.2) yields:

$$\begin{aligned} \text{EPE}_{\text{FVA}}^\perp(t; u) &= P_I(t, u) P_C(t, u) \mu_S(t, u) \mathbb{E}_t [e^{-\int_t^u r(v) dv} (V(u))^+] \\ &\quad + \text{LGD}_I \mathbb{E}_t [e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} y_I(t, u)] \mathbb{E}_t [e^{-\int_t^u r(v) dv} (V(u))^+], \end{aligned} \quad (\text{C.3})$$

$$\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u) = \mathbb{E}_t \left[\left(e^{-\int_t^u r(v) dv} (V(u))^+ - \mathbb{E}_t [e^{-\int_t^u r(v) dv} (V(u))^+] \right) e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv} s_b(u) \right]. \quad (\text{C.4})$$

Remark (Alternative FVA definition). *The definition of $f(\cdot)$ above is based on the inclusion of both τ_I and τ_C in the FVA definition. If both are excluded, the credit adjustment factor term disappears and we have $f(t, u; \lambda_I, \lambda_C) = s_b(u)$ with $\mathbb{E}_t [f(t, u; \lambda_I, \lambda_C)] = \mu_S(t, u)$, as $\mathbb{E}_t [y_I(t, u)] = 0$.*

Remark (IR-based funding spread). *For the IR-based funding spread from Equation (2.5), we take the same definition of $h(\cdot)$, as before. For $f(\cdot)$ and $g(\cdot)$, we write:*

$$\begin{aligned} f(t, u; \lambda_I, \lambda_C) &= e^{-\int_t^u \lambda_I(v) + \lambda_C(v) dv}, & \mathbb{E}_t [f(t, u; \lambda_I, \lambda_C)] &= P_I(t, u) P_C(t, u), \\ g(t, u; r) &= s_b(u), & \mathbb{E}_t [g(t, u; r)] &= s_b(u). \end{aligned}$$

The IR-based borrowing spread is a special case of the credit-based borrowing spread. If in the credit-based spread case we set $y_I(t, u) = 0$, then, in terms of the formulae, this collapses to the IR-based spread case. The drift term $\mu_S(t, u)$ can be chosen to make these two cases equivalent.

Appendix D. Additional results for modeling assumptions

The first results correspond to the market data and model parameters from scenario 1 in Appendix E.3, for an ITM receiver swap and a credit-based funding spread. We examine the various choices of including τ_I and/or τ_C in the FVA definition.

In Figure D.7, exposure plots for $\text{EPE}_{\text{FVA}}(t; u)$ and $\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u)$ are presented for the case that both τ_I and τ_C are excluded from the FVA definition. From Figure D.7a, it is clear that the funding spread s_b results in WWR, since the analytic exposure without WWR is below all other exposure profiles. The same conclusion is drawn from Figure D.7b where $\text{EPE}_{\text{FVA}}^{\text{WWR}}(t; u) > 0$ for all u . Hence, this WWR effect is clearly non-negligible and plays an important role in FVA modeling.

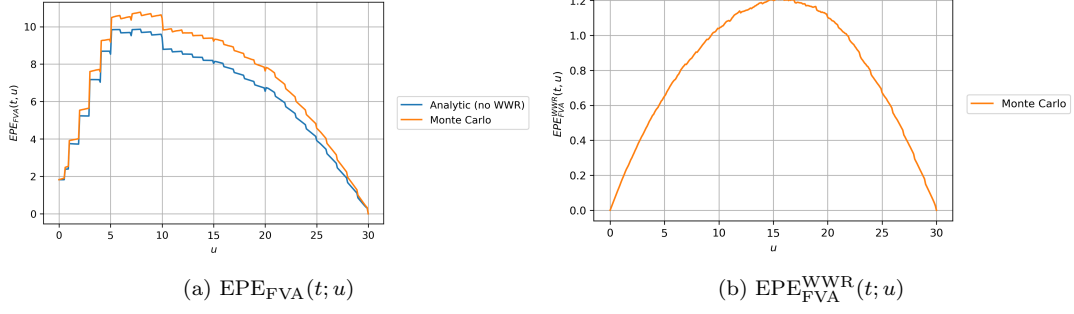


Figure D.7: Credit-based spread, ITM receiver swap, τ_I excl., τ_C excl., $FVA^\perp(t) = 193.3481$. The ‘Analytic’ exposure profile is taken from the existing xVA engine where all quantities are assumed to be independent, and can hence be computed analytically for an IR swap. This exposure profile and corresponding $FVA^\perp(t)$ serve as a point of reference to illustrate the amount of WWR that is present after introducing the dependencies.

τ_I	τ_C	$FVA^\perp(t)$	$FVA^{WWR}(t)$	WWR %
Excl.	Excl.	193.3481	24.0972	12.46
Incl.	Excl.	169.9607	18.2658	10.75
Excl.	Incl.	136.5265	6.6041	4.84
Incl.	Incl.	122.3386	4.7654	3.90

Table D.5: Credit-based spread, ITM receiver swap. ‘WWR %’ refers to the percentage of $FVA^{WWR}(t)$ w.r.t. $FVA^\perp(t)$, i.e., $\frac{FVA^{WWR}(t)}{FVA^\perp(t)} \cdot 100\%$.

In Table D.5, the corresponding FVA and WWR numbers are presented. These results support the conclusions made from Figure D.7, but now in terms of FVA numbers rather than exposure profiles. When including τ_I in the FVA definition, the plots in Figure D.7 do not change significantly in shape, but the exposure profile gets scaled down slightly due to the credit adjustment effect. In Table D.5 this credit adjustment is clearly present when looking at the FVA^\perp and FVA^{WWR} numbers. In the current situation, the inclusion of τ_I translates into RWR. This the credit adjustment effect is not the same as RWR, but it is a separate effect. The credit adjustment affects the FVA level, while the dependency structure with the existing factors results in the RWR effect, which is seen from the lower percentage of WWR in Table D.5.

When τ_I is still excluded but τ_C is included in the FVA definition, both the $EPE_{FVA}(t; u)$ and $EPE_{FVA}^{WWR}(t; u)$ change significantly in shape and magnitude, see Figure D.8. This is the result of the different credit curve for C , which is different in both shape and magnitude due to the lower credit quality than I . In general, the credit adjustment effect from including τ_z , $z \in \{I, C\}$, in the FVA definition increases for worse credit quality. In Table D.5 it is clear that the inclusion of τ_C results in

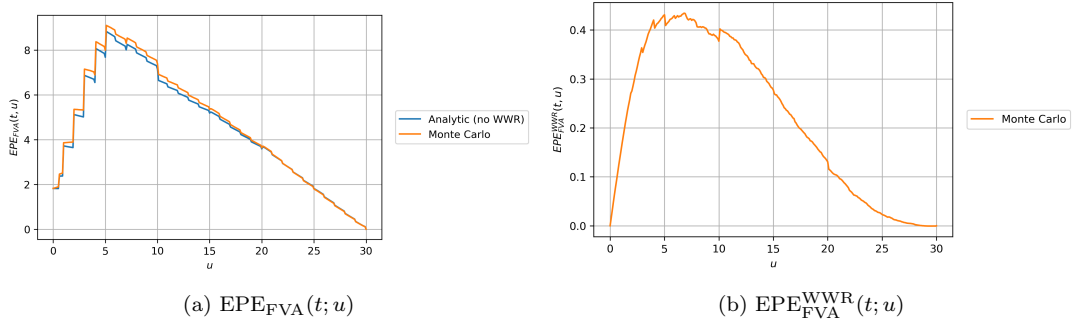


Figure D.8: Credit-based spread, ITM receiver swap, τ_I excl., τ_C incl., $FVA^\perp(t) = 136.5265$.

RWR. Due to the higher credit volatility for C compared to I , this RWR effect is stronger than when only τ_I was included. This is illustrated by the significantly lower WWR percentage in Table D.5. Despite the significant reduction in overall WWR (due to the RWR effect from the inclusion of τ_C),

the WWR effect coming from the funding spread still dominates.

Finally, when including both τ_I and τ_C in the FVA definition, the plots in Figure D.8 do not change significantly in shape, but only get scaled down slightly due to both credit adjustment effects. As expected, the overall credit adjustment effect in Table D.5 is strongest when both τ_I and τ_C are included. This also holds for the RWR that results from this inclusion, which is demonstrated by the $FVA^{WWR}(t)$ and ‘WWR %’ numbers, which are the lowest of all results presented so far.

For an IR-based funding spread, the credit adjustment effects are similar as presented so far. Now, there is no WWR but only RWR coming from the inclusion of τ_I and/or τ_C . In both absolute and relative sense, the RWR numbers are of lower magnitude than those for the credit-based spread. See for example the results in Table D.6, which reports around 6.6% of RWR when both τ_I and τ_C are included. This should be compared to the change in ‘WWR %’ in Table D.5, where there is around 8.6% of RWR in the credit-based spread case due to the inclusion of both τ_I and τ_C .

τ_I	τ_C	$FVA^\perp(t)$	$FVA^{WWR}(t)$	WWR %
Incl.	Incl.	123.1260	-8.1066	-6.58

Table D.6: IR-based spread, ITM receiver swap.

In summary, both WWR and RWR effects are clearly non-negligible and thus play an important role in FVA modeling. For the credit-based spread, the stochastic funding spread results in WWR in this example, and the inclusion of τ_z , $z \in \{I, C\}$, generates RWR. It depends on correlations, credit parameters, IR parameters and product type whether the net result is WWR or RWR. In case of an IR-based funding spread, there is only RWR from the inclusion of τ_z . All results presented are in line with the conclusions from Section 3.

Appendix E. Market data and model parameters

Appendix E.1. Yield curves

t	DF(t)	ZC(t)	t	DF(t)	ZC(t)
0.00	1.000000	0.00	0.00	1.000000	0.000000
0.25	0.987578	0.05	0.25	1.001187	-0.004744
0.50	0.975310	0.05	0.50	1.002448	-0.004891
0.75	0.963194	0.05	0.75	1.003773	-0.005021
1.00	0.951229	0.05	1.00	1.005158	-0.005145
1.50	0.927743	0.05	1.50	1.008088	-0.005370
2.00	0.904837	0.05	2.00	1.011132	-0.005535
2.50	0.882497	0.05	2.50	1.014134	-0.005614
3.00	0.860708	0.05	3.00	1.016990	-0.005616
4.00	0.818731	0.05	4.00	1.022401	-0.005538
5.00	0.778801	0.05	5.00	1.026945	-0.005318
6.00	0.740818	0.05	6.00	1.030583	-0.005021
7.00	0.704688	0.05	7.00	1.033099	-0.004652
8.00	0.670320	0.05	8.00	1.034654	-0.004258
9.00	0.637628	0.05	9.00	1.035117	-0.003835
10.00	0.606531	0.05	10.00	1.034622	-0.003404
12.00	0.548812	0.05	12.00	1.031876	-0.002615
15.00	0.472367	0.05	15.00	1.025681	-0.001690
20.00	0.367879	0.05	20.00	1.021923	-0.001084
25.00	0.286505	0.05	25.00	1.032268	-0.001270
30.00	0.223130	0.05	30.00	1.053926	-0.001751

(a) 5% flat ZC curve.

(b) EUR1D curve.

Table E.7: Yield curves. DF denotes discount factor, ZC denotes zero-coupon.

Appendix E.2. Credit curves

t	DF(t)	ZC(t)	t	DF(t)	ZC(t)	t	DF(t)	ZC(t)
0.00	1.000000	0.000000	0.00	1.000000	0.000000	0.00	1.000000	0.000000
0.50	0.998984	0.002034	0.50	0.994676	0.010677	0.50	0.963312	0.074757
1.00	0.997659	0.002343	1.00	0.988348	0.011720	1.00	0.919991	0.083391
2.00	0.993528	0.003247	2.00	0.970999	0.014715	2.00	0.831220	0.092431
3.00	0.987626	0.004151	3.00	0.948562	0.017603	3.00	0.741957	0.099488
4.00	0.979424	0.005198	4.00	0.920897	0.020602	4.00	0.657705	0.104750
5.00	0.969391	0.006217	5.00	0.888371	0.023673	5.00	0.579658	0.109064
7.00	0.946630	0.007835	7.00	0.828067	0.026952	7.00	0.439022	0.117601
10.00	0.912382	0.009170	10.00	0.745380	0.029386	10.00	0.296235	0.121660
15.00	0.861670	0.009926	15.00	0.632957	0.030490	15.00	0.160474	0.121975
20.00	0.813199	0.010339	20.00	0.537460	0.031045	20.00	0.085857	0.122754
30.00	0.721512	0.010880	30.00	0.386538	0.031684	30.00	0.023506	0.125016

(a) AAA-rating curve. (b) BBB-rating curve. (c) B-rating curve.

Table E.8: Credit curves. DF denotes discount factor, ZC denotes zero-coupon.

Appendix E.3. Scenarios

- This is the scenario on which the graphs in Appendix D are based:
 - IR: $x_r(0) = 0.0$, $a_r = 1e - 05$, $\sigma_r = 0.00284$, EUR1D yield curve (see Table E.7b), ATM $\sigma_{\text{imp},r} = 0.1$;
 - Credit for I: $x_I(0) = 0.0016939$, $a_I = 0.05$, $\theta_I = 0.015390$, $\sigma_I = 0.02$, $\text{LGD}_I = 0.6$, AAA-rating credit curve (see Table E.8a), ATM $\sigma_{\text{imp},I} = 0.07351$;
 - Credit for C: $x_C(0) = 0.0063774$, $a_C = 0.2$, $\theta_C = 0.035447$, $\sigma_C = 0.08$, $\text{LGD}_C = 0.6$, BBB-rating credit curve (see Table E.8b), ATM $\sigma_{\text{imp},C} = 0.12090$;
 - Correlation: $\rho_{r,I} = -0.35$, $\rho_{r,C} = -0.5$, $\rho_{I,C} = 0.0$.
- This is the base scenario on which most of the scenarios to follow are based:
 - IR: $x_r(0) = 0.0$, $a_r = 1e - 05$, $\sigma_r = 0.00774$, 5% flat zero-coupon yield curve (see Table E.7a), ATM $\sigma_{\text{imp},r} = 0.1$;
 - Credit for I: $x_I(0) = 0.0016939$, $a_I = 0.05$, $\theta_I = 0.015390$, $\sigma_I = 0.02$, $\text{LGD}_I = 0.6$, AAA-rating credit curve (see Table E.8a), ATM $\sigma_{\text{imp},I} = 0.07395$;
 - Credit for C: $x_C(0) = 0.0016939$, $a_C = 0.05$, $\theta_C = 0.015390$, $\sigma_C = 0.02$, $\text{LGD}_C = 0.6$, AAA-rating credit curve (see Table E.8a), ATM $\sigma_{\text{imp},C} = 0.07395$;
 - Correlation: $\rho_{r,I} = -0.35$, $\rho_{r,C} = -0.35$, $\rho_{I,C} = 0.0$;
- Scenario 2, but with $\sigma_{\text{imp},r} = 0.2$ s.t. $\sigma_r = 0.01556$.
- Scenario 2, but with $a_r = 0.05$ s.t. $\sigma_r = 0.01285$.
- Scenario 2, but with $a_r = 0.05$ and $\sigma_{\text{imp},r} = 0.2$, s.t. $\sigma_r = 0.02578$.
- Scenario 2, but with the EUR1D yield-curve (see Table E.7b), s.t. now $\sigma_r = 0.00284$ to still get $\sigma_{\text{imp},r} = 0.1$.
- Scenario 2, but with BBB-rating credit curves for I and C (see Table E.8b), s.t. $x_z(0) = 0.0098774$, $\theta_z = 0.041033$ and $\sigma_{\text{imp},z} = 0.05224$, for $z \in \{I, C\}$.
- Scenario 2, but with BBB-rating credit curve for C (see Table E.8b), s.t. $x_C(0) = 0.0098774$, $\theta_C = 0.041033$ and $\sigma_{\text{imp},C} = 0.05224$.
- Scenario 8, but with $a_C = 0.2$ and $\sigma_C = 0.045$ s.t. we match $\sigma_{\text{imp},C}$ and $\int_0^t b_C(u)du$ from scenario 8. This results in $x_C(0) = 0.0078774$, $\theta_C = 0.033825$, ATM $\sigma_{\text{imp},C} = 0.07359$.
- Scenario 8, but with B-rating credit curve for C (see Table E.8c), $a_C = 0.06$ and $\sigma_C = 0.045$ s.t. we match $\sigma_{\text{imp},C}$ from scenario 8 and get as close as possible to $\int_0^t b_C(u)du$ from scenario 8. This results in $x_C(0) = 0.071957$, $\theta_C = 0.16435$, ATM $\sigma_{\text{imp},C} = 0.07330$.
- Scenario 10, $a_C = 0.02$ and $\sigma_C = 0.08$ s.t. the implied volatility is higher. In particular, we get $x_C(0) = 0.057657$, $\theta_C = 0.44319$, ATM $\sigma_{\text{imp},C} = 0.13624$.
- Scenario 8, but with $a_I = 0.15$ such that $x_I(0) = 0.0011139$, $\theta_I = 0.012183$, ATM $\sigma_{\text{imp},I} = 0.05871$. When increasing a_I , we get more curvature in $f^{CIR}(0, t)$, s.t. it is closer to $f^M(0, t)$, resulting in a lower $\int_0^t b_I(u)du$. This means that we capture more of the market with the model. Furthermore, when increasing a_I , $x_I(0)$ and θ_I and $\sigma_{\text{imp},I}$ go down.

13. Scenario 8, but with $\sigma_I = 0.04$ s.t. $x_I(0) = 0.0016539$, $\theta_I = 0.016763$, ATM $\sigma_{\text{imp},I} = 0.14155$. When increasing σ_I , there are no significant changes in $f^{CIR}(0, t)$, so $\int_0^t b_I(u)du$ is hardly affected. Also, $\sigma_{\text{imp},I}$ scales like σ_I , so a twice as large σ_I results in a doubling of $\sigma_{\text{imp},I}$.
14. Scenario 8, but with $a_I = 0.15$ and $\sigma_I = 0.04$ s.t. $x_I(0) = 0.00052392$, $\theta_I = 0.012475$, ATM $\sigma_{\text{imp},I} = 0.11033$.
15. Scenario 8, but with $a_C = 0.15$ s.t. $x_C(0) = 0.0088774$, $\theta_C = 0.033506$, ATM $\sigma_{\text{imp},C} = 0.03840$.
16. Scenario 8, but with $\sigma_C = 0.04$ s.t. $x_C(0) = 0.0097774$, $\theta_C = 0.045113$, ATM $\sigma_{\text{imp},C} = 0.10292$.
17. Scenario 8, but with $a_C = 0.15$ and $\sigma_C = 0.04$ s.t. $x_C(0) = 0.0087774$, $\theta_C = 0.034305$, ATM $\sigma_{\text{imp},C} = 0.07579$.
18. Scenario 9, but with $\rho_{r,I} = -0.7$.
19. Scenario 9, but with $\rho_{r,C} = -0.7$.
20. Scenario 9, but with $\rho_{r,I} = \rho_{r,C} = -0.7$.
21. Scenario 9, but with $\rho_{r,I} = \rho_{r,C} = 0.7$ (opposite sign compared to scenario 20).