METAPHOR-BASED ALGEBRA ANIMATION

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We investigated whether dynamical animations with visualizations based on metaphorical linking are more effective for grasping algebraic manipulations than static animations. The question is whether and how ideas from conceptual metaphor theory, in particular, embodied cognition, can be drawn to positively contribute to the design of effective animations for visualizing algebraic manipulations. In classroom tests, grade-7 students watched an animated video on algebraic manipulation, either with dynamic visualization, a visualized person dynamically performing the manipulations, or a more static video not based on those ideas. For higher-level students with some pre-knowledge of algebra, we found a small positive effect for dynamic and dynamic embodied videos. For lower-level students with no pre-knowledge of algebra, the embodied animation turned out to be adverse effective.

Keywords: Dynamic visualization, embodied simulation, object collection metaphor.

INTRODUCTION

In the last few years, a revolution in informal mathematics education took place, to our best knowledge, ignored by the mathematics education community. Through his YouTube channel 3Blue1Brown, mathematics educator Grant Sanderson reached tens of millions of students worldwide with dynamically animated videos covering a wide range of topics, including linear algebra, calculus, probability, and machine learning (3Blue1Brown, n.d.). The responses to those videos are jubilant – students claiming to "finally understand the topic". A natural question is whether viewers are simply overawed by the smooth animations, or do animated effects really contribute to better learning outcome. In this study, we focus on one specific feature of these videos: Dynamically animated algebraic manipulations. If one encounters algebraic manipulations in a video, is it beneficial for these to be dynamically animated? We propose a theoretical basis for such animations and study the effect.

Wittmann and collaborators (2013) observed students discussing algebraic manipulations as if the terms were physical objects moving in a landscape. As they explain using Conceptual Metaphor Theory, one might argue such metaphorical language is grounded in spatial embodied experiences (Lakoff & Núñez, 2000): Reasoning about moving physical objects is linked to reasoning about algebra through metaphors. Nicaud and Maffei (2013) explore how this theory leads to an interactive algebra environment where terms can be dragged across the screen. We study the same phenomenon in the non-interactive flat environment of animated video.

No literature has brought such an approach in mathematics education, but recent inquiries by educational psychology researchers have explored the role of embodiment in educational videos (Castro-Alonso et al., 2018; De Koning & Tabbers, 2013; Pouw et al., 2016). More generally, the issue is whether dynamic animations are more effective than static animations, possibly using embodiment (Berney & Bétrancourt, 2016). Dynamic animations use continuous movement and deformation, whereas static animations are more like a traditional slide show portraying a discrete set of images.

In this study, we compare in a quantitative way the effect of a short instruction video on the topic of elementary algebra with respect to three conditions: static animation, dynamic animation, and embodied dynamic animation. The aim is to gain insight in whether these conditions have any quantitatively measurable difference in effect on the learning outcome.

THEORETICAL BACKGROUND

Abrahamson and Lindgren wonder whether "learning environments (can) be designed to foster grounded learning, in which students sustain a tacit sense of meaning from corporeal activity even as they are guided to re-think this activity formally" (Abrahamson & Lindgren, 2014, p.3). They state that "manipulating symbolic notation is cognitively quite similar to physically moving objects in space" (p.3). We consider an instruction video as such a learning environment in which moving the algebraic terms in the plane might support being able to follow and understand algebraic manipulations.

Lakoff and Núñez's theory states that mathematical cognition is organized through certain linking processes (Lakoff & Núñez, 2000). These linking processes, called metaphors, consist of mappings from one conceptual domain – the source domain – to another conceptual domain – the target domain, usually more concrete. For this study, the source-path-goal metaphor, the object collection arithmetic grounding metaphor, and the arithmetic-algebra linking metaphor are important. The source-path-goal metaphor is in play when the changes of position of a term are mapped onto a path of the term as an object in the plane. Such a term is interpreted to have a source—the original position—to move along a path, and a goal—the final position in the equation. The arithmetic grounding metaphor interprets arithmetical operations as manipulations on a collection of objects. The cognitive schemas supporting object collection are image and motor schemas, in particular the hypothesized containment schema, that deals with the metaphorical use of "container" as an object that envelops a collection (Lakoff & Núñez, 2000). For example, addition is then mapped onto the experience of joining the content of two containers. A final metaphor of importance to this study is the arithmetic-algebra linking metaphor: a metaphor that links algebra to arithmetic. Algebraic rules map onto essential characteristics of arithmetic. The hypothesis central to Lakoff and Núñez's theory is that a metaphor transfers the inferential structure of the source domain to the target domain. Applied to our case: the (aspiring) mathematician reasons about arithmetic by mapping reasoning in object collection onto the arithmetic domain; and likewise, they reason about algebraic manipulation by mapping reasoning and experiences in arithmetic computations onto the algebra domain, and about the repositioning of terms as those terms traveling along paths. These metaphors and the way they transfer reasoning inspired a dynamical visual "language" for animating the algebraic manipulation in the dynamic and embodied-dynamic condition.

In a recent meta-analysis of 140 pair-wise comparisons, Berney and Bétrancourt (2016) found a significant advantage for dynamic animations over static in learning outcome. However, in only 31% of the studies dynamic animation was superior, compared to 10% where static was superior, and 59% with no significant difference. For the eleven studies on mathematics videos in their meta-analysis, they found a small negative effect for dynamic animation, but this did not include the topic of algebra. Ayres and Paas find advantages and disadvantages for learning through dynamic animation (2007). An advantage of dynamic animation is that movement and changes can be visualized in a life-like way. Being able to see instead of having to infer the motion and changes helps students follow reasoning steps. Another advantage is that dynamic animation facilitates cueing: movement can be used to direct students' attention. A disadvantage is that information tends to be transient: Information comes and goes, and remembering and integrating it all is a challenge.

Our research question is what the effect is of dynamic videos—where animations are based on the discussed metaphors—on learning outcome, compared to static visualization, showing the algebraic manipulations line by line. The next question is what the effect on learning outcome is of showing a part of the body performing the dynamically animated manipulations.

METHOD AND MATERIALS

We performed a pilot study, a first experiment, and a second experiment. The dynamics and embodiment might influence both the retention and the understanding of the information in the video (Pouw et al., 2016). The retention of information influences the performance on reproductive tasks, whereas processing and understanding the information influences performance on more challenging tasks. In the second experiment, we measured performance on reproductive and on more challenging tasks separately. Pouw et al. (2016) show that embodied animation can be more effective for students with a lower level of achievement in the subject of the video. Therefore, we performed the experiment on two populations: Higher-level mathematics students with little prior knowledge of algebra in the first experiment; lower-level mathematics students with no prior knowledge of algebra in the second experiment. The level was assessed by the teachers of the students.

For the experiments, there were two experimental groups and a control group. We designed three versions of the same video (see https://tinyurl.com/anialg). For each 92 seconds video, we used the same audio, as well as the same algebraic expressions, fonts and font size, colours, timing, outlay and design. The video shows five worked examples about basic manipulations in algebra: Scalar multiplication by a positive integer as repeated addition: $a + a + a + a + a = 5a$; addition of similar terms: $5a + 3a = 8a$; commutativity: $5a + 3b = 3b + 5a$; a combination of those: $5a + 3b + 3a = 8a + 3b$; distributivity: $3(5a + 3b) = 15a + 9b$. In Table 1, we present screenshots from the three versions of the intervention video. In the *Static* video, the algebraic expressions appear line by line at the moment they are addressed in the audio. The *Dynamic* video and the *Embodied dynamic* video use dynamic animations based on the discussed metaphors for algebra. The difference between *Dynamic* and *Embodied dynamic* is that, in the embodied dynamic version, a person is visualized performing the manipulations (upper body in examples 1, 2 and 3 and only the hands in examples 4 and 5).

Table 1. Comparing visualization in intervention videos

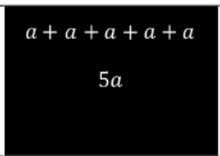
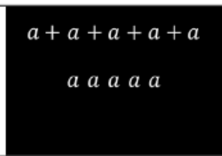
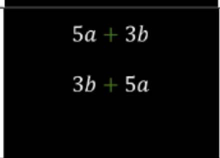
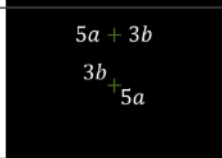
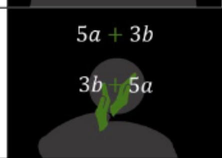
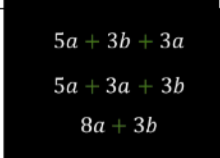
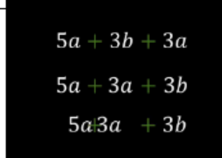
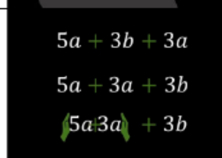
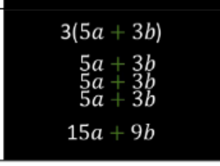
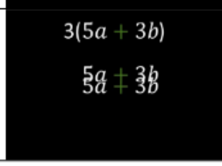
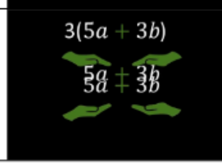
Time	Static	Dynamic	Embodied dynamic
0:17			
0:45			
1:05			
1:27			

Table 2 explains for each algebraic manipulation how the metaphorical mappings lead to a choice of dynamical visualization. The first column contains the algebraic manipulation rule, the second column the linking metaphor of the algebraic rule in arithmetic, the third column the two grounding metaphors, and the fourth the choice of visualization based on these grounding metaphors. We discuss here the reasoning behind the visualization of *commutativity*: The rule states that the algebraic terms in a sum can be interchanged, e.g., $5a + 2b = 2b + 5a$. This rule is an abstraction of the experience in arithmetic that numbers in a sum can be exchanged, e.g., $7 + 8 = 8 + 7$. These experiences and the rule are related by a linking metaphor, where the algebraic rule represents an *essential* property of arithmetic (so-called metonymy). The arithmetic can metaphorically be linked to the physical model of heaps of objects. The containment scheme allows one to interpret a heap of objects as one unit, a container, with an individual position. This can also be seen as a form of metonymy: the heap itself is not contained in an object, but one has a natural notion of *in the heap* and *not in the heap*. Inspired on the source-path-goal schema, commutativity is hence linked to moving heaps of objects as a whole: Interchanging the positions of the heaps does not influence the result of joining them. Therefore, in the dynamic visualization of commutativity, the terms $5a$ and $2b$ are treated as containers with a position that can interchange position by moving. To refer to these schemes more directly, the embodied dynamic animation presents a person moving the terms as if they were physical containers.

Table 2. How algebraic rules are visualized based on metaphorical mappings

Algebra over positive integers	Arithmetic	Object collection and source-path-goal metaphor	Dynamic visualization
Scalar multiplication	Multiplication as repeated addition	Swiping the similar objects on a heap Or the other way around: spreading the similar object on a heap out over space.	In a swiping gesture, the same variables in the summation together join into a number-times-variable expression In a spreading gesture away from the number-times-variable expression, the separate occurrences of the variable appear
Adding similar terms	Addition (distributivity: $3a + 2a = (2 + 3)a$))	Swiping heaps of similar objects together on a new heap	In a swiping gesture, the similar number-times-variable terms are joined together and there appears a new number-times-variable expression where the number is the sum of the previous numbers
Commutativity	Commutativity of number addition	Moving heaps of objects (in a container) around to make them interchange position	Hands move the number-times-variable terms around the plus sign to make them interchange position.
Distributivity	Distributivity in arithmetic	Reorganizing heaps of objects. More precisely: one has a number of heaps of two different object types and joins all objects of same type together on heaps	In a spreading gesture, the scalar multiplication of the terms between the brackets separates into occurrences of these terms (as in scalar multiplication) Then a swiping gesture joins the similar number-times-variable terms (as in adding similar terms)

The design included a pilot study in two classes, the first experiment in five classes and the second experiment in four classes. For each class, the teacher divided the students into three levels, based on their own assessment: below average, average, above average. The division of students over the conditions was such that each condition has about the same number of students of each math level. The pilot study took place in two 7^{th} grade classes of a Dutch pre-university level secondary school to test the set-up and have indicative results. In one of the pilot classes—like the classes in our first experiment of pre-university level—we found an effect of $d = 0.81$ (Cohen's d) of the embodied dynamical condition. A power analysis suggested the size for each group in the first experiment to be 38 for a power of $1 - \beta = 0.8$. After that, we performed the first experiment, involving 132 seventh graders (boys and girls, age 12 - 13) of a Dutch pre-university level secondary school. The students had prior knowledge of what a variable is and had had a very limited primer on algebraic manipulation. The second experiment involved 42 seventh graders (age 12 -13) of one Dutch pre-vocational secondary school (*VMBO*) and 49 seventh graders (age 12 -13) of another Dutch pre-vocational secondary school (*VMBO*). These students had no prior knowledge of algebra.

In the first experiment, the written pre-test and post-test consisted of three items in line with the worked examples, e.g., simplify $3a + 2b + 2a$. For the second experiment, the tests had sixteen instead of three items: eight reproductive items and eight more challenging items: an improvement implemented to avoid the ceiling effect that could have played a role in the first experiment. In the first tests, students had as much time as they needed, whereas, in the second, they had precisely five minutes to complete as many of the sixteen items as they could. All tests were hand-marked by the researchers using a complete and strict marking model. In the first experiment, maximally 7 points could be scored. In the second experiment, 2 points per item could be scored, which leads to a maximum of 32 points. These pre- and post-test scores were taken as variables and analyzed statistically using SPSS version 25. For the first experiment we conducted an ANCOVA test to determine a statistically significant difference between the control, dynamical and embodied dynamical conditions on the post-test score controlling for the pre-test score. For the second experiment, we conducted one-way ANOVA tests to compare the post-test results on the reproductive and challenging items between control, dynamical and embodied dynamical conditions

RESULTS

For the first experiment, we carried out Levene's test and normality checks on the post-test data and the assumptions were met. The ANCOVA test revealed no significant effect of video-type on the post-test controlling for the pre-test score ($F(2, 128) = 0.459$, $p = 0.633$). Estimates of the means, standard errors and 95% confidence intervals are presented in Table 3 and Figure 1.

Table 3. Estimates for the post-test scores in the first experiment (maximum score is 7)

Condition	N	Mean	Std. error	Lower bound	Upper bound
Control	43	5.607	0.274	5.065	6.148
Dynamic	45	5.887	0.268	5.357	6.416
Embodied	44	5.955	0.271	5.419	6.491

Note: the covariate pre-test score was evaluated at 1.659

The effect of the dynamic condition (compared to control) was small, $d = 0{,}15$; and for the embodied condition also small, $d = 0{,}20$.

In the second experiment, the pre-test scores were zero for all candidates; not surprisingly, since they had no prior knowledge of algebra. The ANOVA test revealed no statistically significant difference between groups ($F(2, 88) = 0.824, p = 0.442$). The means, standard errors and 95% confidence intervals are presented in Table 4.

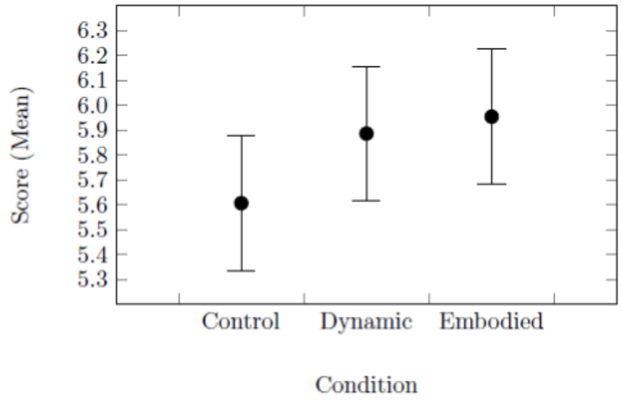


Figure 1. Estimates for the post-test scores in the first experiment (maximum score is 7)

Table 4. Post-test scores on the reproductive items in the second experiment (maximum score is 16)

Condition	N	Mean	Std. error	Lower bound	Upper bound
Control	31	6.419	0.772	4.843	7.996
Dynamic	29	6.828	0.878	5.030	8.626
Embodied	31	5.371	0.834	3.668	7.074

We conducted a one-way ANOVA test to compare the post-test result on the challenging (transfer) items between control, dynamical and embodied dynamical conditions. There was no statistically significant difference between groups ($F(2, 88) = 0.265, p = 0.768$). The means, standard errors and 95% confidence intervals are presented in Table 5.

Table 5. Post-test scores on the challenging items in the second experiment (maximum score is 16)

Condition	N	Mean	Std. error	Lower bound	Upper bound
Control	31	1.790	0.423	0.927	2.654
Dynamic	29	1.414	0.354	0.688	2.140
Embodied	31	1.855	0.559	0.713	2.996

Figure 2 presents the total scores for the post-test. There was no effect of the dynamic condition, $d = 0.01$; and for the embodied condition, there was a small adverse effect, $d = -0.14$.

DISCUSSION

From the ANCOVA and ANOVA tests, we may conclude that there are no significant effects of the dynamic and embodied dynamic condition on learning outcome as measured by the tests. This result adds to the 59% of no-significant-difference-results of Berney and Bétrancourt (2016). Even though this puts some weight in the scale in favor of “no added value of dynamics”, for us, there is a lesson in the methods used, since we believe there may have been an effect that we failed to pick up to a significant level. The power-analysis based on the pilot seems to have had an outcome too much in favor of the dynamic conditions leading to a rather small n. Moreover, a 91-second video is a very short intervention. Combined with a possible ceiling effect in the test, we feel a more precise method is needed to measure the effect in a significant way.

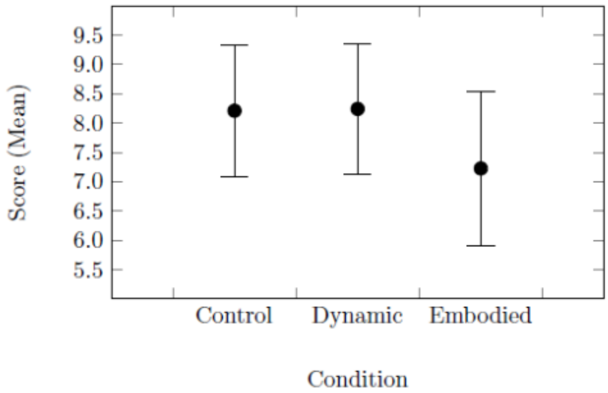


Figure 2. Total post-test scores in the second experiment (maximum score is 32)

Positive learning effects of embodied simulation, as observed in the first experiment, depend on whether the goal of the movement is understood (Van Gog et al., 2009). For the higher-level students of the first experiment (with a bit of prior knowledge of algebra), the goal of the algebraic manipulations, namely simplifying the expressions, was probably clear. On the contrary, the lower-level students of the second experiment may have struggled to grasp this goal from the visualized manipulations. The local goal of the movements themselves must have been clear to all students, though since the voiceover explained, for example, “we now join these terms” or we now “switching these terms”. But possibly this is not enough: the global goal of where the algebraic manipulations are leading (i.e., a simplification) may have to be clear as well. The adverse effect found for the embodied condition in the second experiment (with lower-level students) is in line with the results of Castro-Alonso et al. (2018), who find that the effectiveness of dynamic animations reduces when showing hands.

The animations in this study are based on a chain of metaphors, first, a linking metaphor linking arithmetic to algebra, and next to grounding metaphors: the source-path-goal metaphor and the object collection metaphor. We questioned whether these metaphors could be supportive in animated algebra instruction videos. The measured small effects give rise to some optimism, but in particular for higher-level students with some prior knowledge of algebra. Presumably, the arithmetic grounding metaphorical link needs to be well-understood by students for the fundamental metonymy of commutative algebra—that letters take the role of numbers—to be accepted and “recognized” in the video. There may also be concern about the *transitivity* of the two metaphorical links: the object collection metaphor supports arithmetic reasoning, and arithmetic reasoning may support algebraic reasoning, but does that imply that the object collection metaphor supports algebraic insight? In each metaphorical link, part of the inferential structure is preserved, and part is lost. We believe enough supportive inferential structure is maintained, but the outcome of this research might imply that reality is more complicated.

A limitation of watching a video is that it is based on *motor mirroring* (embodied simulation) and not on *motor execution* (enactment). The motoric or bodily engagement is of low level (Duijzer et al., 2019). Also, the level of immediacy is low: Students watching the dynamic and, in particular, the embodied dynamic video have to rely on embodied simulation of previously acquired sensorimotor experiences. Including enactment in the intervention may increase the effect of the video.

This study focused on the effect of dynamical animation on being able to grasp and follow algebraic manipulations. In many videos—like those of 3Blue1Brown—algebraic manipulative skills themselves do not form the learning goal, rather those skills form a prerequisite, applied in a step to reach an interesting result. To make another step in our understanding of dynamic animation of algebra in such videos, our follow-up study investigates whether, in those cases, animated algebra contributes to learning outcomes.

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