AL-BĪRŪNĪ ON THE COMPUTATION OF PRIMARY PROGRESSION (TASYĪR*)*

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Introduction

The *Masudic Canon* (al-Qānūn al-Mas'ūdī) of al-Bīrūnī in eleven 'Books' (or large chapters) is one of the most important works of Islamic astronomy. The work is comparable in size and structure to the *Almagest* of Ptolemy (ca. 150 CE). Al-Bīrūnī completed this work between 1030 and 1040 for sultan Mas'ūd of Ghazna, now Ghazni in Afghanistan. An overview of the contents of the *Masudic Canon* can be found in Kennedy's *Studies in the Exact Islamic Sciences*.[1] Unlike Ptolemy in the *Almagest*, al-Bīrūnī added a Book (11) on mathematical astrology, of which Chapter 5 deals with the doctrine of primary progressions. The Arabic text of the *Masudic Canon* was published in an uncritical edition in India[2] which was reprinted in Beirut.[3] The work was also translated into Russian under communist rule; one of the translators, Boris Rozenfeld, once remarked to me that it was the first book on astrology that was printed in Russia after the revolution of 1917.[4] An English translation of the *Masudic Canon* would be important for our knowledge of Islamic astronomy but would require many years of highly specialized work. Al-Bīrūnī wrote in Arabic in a peculiar literary style which is more difficult to understand than the rather straightforward language used by most Islamic mathematicians and astronomers. The appendix to this paper contains a literal translation of Sections 1–4 of Chapter 5 of Book 11,

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[1] Edward Stuart Kennedy, 'Al-Bīrūnī's Masudic Canon', *Al-Abḥāth* 24 (1971), pp. 59–81, repr. in E.S. Kennedy, Colleagues and Former Students, *Studies in the Islamic Exact Sciences* (Beirut: American University of Beirut, 1983), pp. 573–95.

[2] Al-Bīrūnī, *al-Qānūn al-Mas'ūdī*, 3 vols (Hyderabad: Osmaniya Oriental Publications Bureau, 1954–56).

[3] Al-Bīrūnī, *al-Qānūn al-Mas'ūdī, qaddama lahu wa-ḍabaṭahu wa-ṣaḥḥahu 'Abd al-Karīm Sāmī al-Jundī*, 3 vols (Beirut: Dār al-Kutub al ilmiyya, 1422 H./2002).

[4] Abu Raikhan Beruni (973–1048), Izbrannye Proizvedeniya V, part 2, *Kanon Mas'uda, knigi VI–XI*, trans. Boris Abramovich Rozenfeld and Ashraf Akhmedovich Akhmedov (Tashkent: Fan, 1976).

which are related to astrological doctrines of ultimately Greek origin, although some of the mathematical methodology was developed in the Islamic middle ages. I have omitted the final Section 5 of Chapter 5 of Book 11 because it concerns Indian astrology and does not involve advanced mathematics.

I begin the appendix with a translation of al-Bīrūnī's cynical introduction to astrology in his preface to Book 11 of the *Masudic Canon*. Al-Bīrūnī also shows a sceptical attitude towards astrology in his more elementary *Introduction to the Art of Astrology* (*kitāb al-tafhīm li-awā'il ṣinā'at al-tanjīm*), which he wrote in the form of approximately 500 questions and answers for the daughter of a high dignitary, Rayḥāna bint Ḥasan. In that work, al-Bīrūnī begins the final section on astrology as follows:

> We now mention the subjects in the art of the judgement of the stars [i.e., astrology], because its aim is the solution of the question of a person who asks [something about his future], and because it [astrology] is for the majority of people the fruit of the mathematical sciences, although our opinion about this fruit and this art is similar to the opinion of the minority (translation mine).[5]

Commentary on Sections 1–4 of Chapter 5 of Book 11 of the Masudic Canon

The following commentary is introductory; mathematical details will be discussed in a subsequent section of this paper on worked examples. In this and the following section I will assume some familiarity with the concept of the celestial sphere, the coordinate systems on the sphere (ecliptical, equatorial, azimuthal), and the basics of spherical trigonometry, as explained, e.g., in Chapter 1 of Smart's *Textbook on Spherical Astronomy* or Chapter 1 of volume 1 of Chauvenet's *A Manual of Practical and Spherical Astronomy*, (freely accessible on the internet).[6]

Al-Bīrūnī begins section 1 with a brief description of the doctrine of *tasyīr* in general. This doctrine was introduced in Book 3, Chapter 10 of the *Tetrabiblos* of

[5] Al-Bīrūnī, *The Book of Instruction in the Elements of the Art of Astrology* reproduced from Brit. Mus. Ms. Or. 8439, tr. Robert Ramsay Wright (London: Luzac, 1934; repr. in *Islamic Mathematics and Astronomy*, ed. Fuat Sezgin, vol. 29 [Frankfurt: Institute for History of Arabic-Islamic Sciences, 1998]), p. 210.

[6] W.M. Smart, *Textbook on Spherical Astronomy* (Cambridge: Cambridge University Press, 1977); William Chauvenet, *A Manual of Spherical and Practical Astronomy*, 2 vols (London: Trübner and Philadelphia: J. B. Lippincott and Co., 1868), *Vol. 1: Spherical Astronomy*, http://books.google.nl/books?id=KlhRAAAAYAAJ

Ptolemy (ca. 150 CE);[7] it is also known under its Greek name *aphesis* and in the West as the theory of *primary progression(s)* or *direction(s)*.[8]

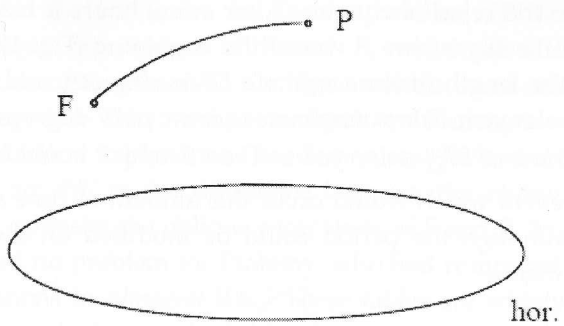

Fig. 1: F and P on same parallel circle.

In the celestial constellation at the moment of birth of a person, two planets or other relevant places are selected using astrological arguments which do not concern us here.[9] For the sake of explanation, I will assume that both places are planets above the horizon. In this case, al-Bīrūnī calls the planet which is more to the West the 'preceding' planet *P*, and the planet which is more to the East the 'following' planet *F*. For the computation of the duration of the life of the person, point *P* was called in Greek the 'aphetic' body, in Arabic the *haylāj*, from Persian *haylij*, 'lord of a building' and in Latin the *hyleg* or *significator*.[10] Point *F* was called the *anairetic* body in Greek, *qāṭi'* ('cutting-off') in Arabic, and *promissor* in Latin.[11]

We now consider the apparent motion of the sky, which was called in ancient

[7] Ptolemy, *Tetrabiblos*, trans. Frank Egleston Robbins, Loeb Classical Library 435 (Cambridge, MA: Harvard University Press, 1940; repr. 1980), pp. 270–307.

[8] Oskar Schirmer, 'Tasyīr', in *Encyclopaedia of Islam*, first ed. (Leiden: Brill, 1934), vol. 4: pp. 751–55.

[9] Auguste Bouché-Leclercq, *L'Astrologie grecque* (Paris: Leroux, 1899), pp. 411–22.

[10] C.L. Nallino, *Al-Battānī sive Albatenii Opus Astronomicum*, editum, Latine versum, adnotationibus instructum, 3 vols (Milano: Pubblicazioni del reale osservatorio di Brera, 1903, 1907, 1899; repr. ed., Frankfurt: Minerva, 1969), 2, p. 355.

[11] Schirmer, 'Tasyīr'.

and medieval terms the daily rotation of the universe (and which is, as we now know, caused by the rotation of the Earth). By means of this daily rotation, the planet *F* will appear to move in a western direction, on a path along the celestial sphere parallel to the celestial equator. After a few hours it may end up at the same position in the sky where *P* was at the beginning (Fig. 1). The astrologer then computed the length of the *tasyīr* arc *FP* in degrees, and converted each degree into one solar year. For example, a *tasyīr* arc of 57 degrees and 11 minutes would be converted to $57\frac{11}{60}$ solar years. Then *F* and *P* could be astrologically connected to an event which would occur this amount of time after the birth of the individual, although the period could be modified on the basis of other astrological arguments.

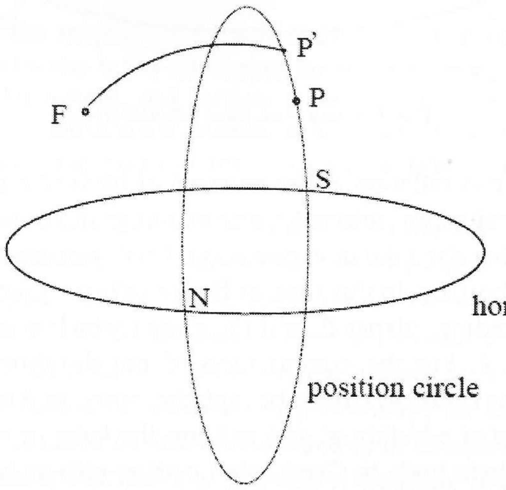

Fig. 2: F and P on different parallel circles.

Usually, the planet *F* will not pass through the initial position of *P* in the sky but will appear to move lower or higher, as in Figure 2. Then the astrologers considered what I will call the *position circle* of *P* – that is, the great circle of the celestial sphere through *P* and the north point *N* and south point *S* of the horizon. I will also use the term *position semicircle* of *P* for the arc *NPS* of the position circle with endpoints *N* and *S*. Now the *tasyīr* arc is arc *FP′* where *P′* is the intersection of the position semicircle of *P* and the path of *F*. I note that the whole process is only concerned with the (apparent) daily rotation of the

universe caused by the rotation of the earth, and not with real motions of *P* and *F* with respect to the fixed stars.

If *P* is on the meridian, arc *FP′* is the difference between the (modern) right ascensions of *P* and *F*. If *P* is on the Eastern horizon, *FP′* is the difference between the 'oblique ascensions' of *F* and *P*, which also depend on the geographical latitude of the observer. Oblique ascensions are no longer used in modern astronomy but they were standard in Ptolemaic and medieval astronomy; for didactical explanations see Evans and Pederson.[12] For *P* on the Western horizon, arc *FP′* is the difference between the oblique descensions, which are directly opposite the oblique ascensions of F and P. In these cases the computation caused no problem for Ptolemy, who had computed tables of right and oblique ascensions in *Almagest* II:8.[13] These tables are widely available and will be used in the worked examples in the subsequent section of this paper. For *P* not in the meridian or horizon plane, Ptolemy presents an approximate computation[14] which al-Bīrūnī discusses in Section 1 and which was often used in the Islamic world. The method only requires tables of right ascension and of oblique ascension for one's locality. Before al-Bīrūnī the same method had also been presented by al-Battānī (ca. 900 CE),[15] and by Kūshyār ibn Labbān (tenth century).[16] Below I will discuss the numerical examples given by Ptolemy, using al-Bīrūnī's methodology. Ptolemy only discusses the case where *P* and *F* are on the ecliptic but, at the end of Section 1, al-Bīrūnī slightly adapts the procedure so it can also be used for planets with non-zero latitude.

Al-Bīrūnī makes the following remark on terminology. Although *F* is carried

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[12] James Evans, *History and Practice of Ancient Astronomy* (New York: Oxford University Press, 1998), pp. 109–21; Olaf Pedersen, *A Survey of the Almagest* (Odense: Odense University Press, 1974; repr. ed. New York: Springer, 2010), pp. 98–101, 110–15.

[13] Gerald J. Toomer, *Ptolemy's Almagest* (London: Duckworth, 1984; repr. ed. Princeton: Princeton University Press, 1998), pp. 18, 100–01.

[14] Ptolemy, *Tetrabiblos*, pp. 292–95.

[15] Carlo Alfonso Nallino, *Al-Battānī sive Albatenii Opus Astronomicum*, editum, Latine versum, adnotationibus instructum (Milano: Pubblicazioni del reale osservatorio di Brera, 1903, 1907, 1899; repr. ed., Frankfurt: Minerva, 1969), 1: pp. 131–34, 3: pp. 200–02.

[16] Michio Yano and Merce Viladrich, 'Tasyīr computation of Kūshyār ibn Labbān', *Historia Scientiarum* 41 (1991): pp. 4–7; Kūshyār Ibn Labbān, *Introduction to Astrology*, ed. and trans. Michio Yano (Tokyo: Institute for the Study of Languages and Cultures of Asia and Africa, 1997), pp. 160–67.

to P by the daily rotation of the universe, the astrologers say that P 'progresses' to F, and al-Bīrūnī also uses this standard terminology. This 'progression' is visualized in Figure 3. For point P on the ecliptic, we can define a series of points F_1, F_2, . . . F_n on the ecliptic such that the distance of F_n (along its circle parallel to the equator) to the position semicircle NPS is exactly n degrees in the direction of the daily rotation of the universe.

Then we can say that in the *tasyīr* doctrine, P after one year progresses to F_1, after two years to F_2, and so on. I note that this 'progression' is in the usual order of the signs of the ecliptic, and that the ecliptical longitude increases if P 'progresses' to F_1, then to F_2, etc.

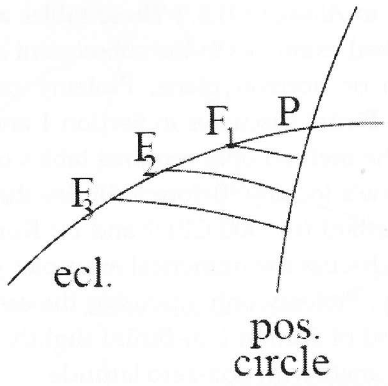

Fig. 3: Progression of P towards F.

Al-Bīrūnī's Section 2 has the somewhat mysterious title 'on mixing degrees with ascensions'. The word 'degree' (*daraja*) in the title must refer to ecliptical degrees; time-degrees on the equator are called *azmān*, 'times'. Section 2 begins with a general introduction to variable quantities and the process of interpolation. Al-Bīrūnī goes on to discuss another computation preliminary to the determination of *tasyīr* for planets with non-zero latitude, as an alternative to his own computation at the end of Section 1. By 'mixing degrees with ascensions' the astrologers wanted to compute for such a planet P, which is outside the ecliptic, the intersection T of its position semicircle with the ecliptic. The ecliptical longitude λ_T of this point of intersection may differ by a few degrees from the planet's ecliptical longitude λ_P obtained from astronomical handbooks (unless the position circle of the planet passes through the pole of the ecliptic and is perpendicular to the ecliptic). The computation of λ_T from λ_P is complicated and al-Bīrūnī only sketches an approximate method which resembles the computation in Section 1. The astrologers apparently continued the *tasyīr* computation with λ_T instead of λ_P.

Remarkably, λ_T emerged again in the twentieth century in the Dutch astrological school headed by Theo J.J. Ram (1884–1961) as the 'true zodiacal longitude' of a planet. The concept was introduced by the Dutch astrologer Leo Knegt (1882–1957).[17]

Al-Bīrūnī was aware of the fact that the computations in Section 1 and 2 are based on a geometrically approximate method. In Section 3 he describes a geometrically exact method of computation. He says that this method requires tables of oblique ascensions, not only for one's own geographical latitude but also for sufficiently many latitudes between one's own latitude and zero. Al-Bīrūnī says that the method can be used if one's mathematical competence is limited, but this is rather optimistic; the crux is a complicated trigonometrical computation of a quantity that he calls the 'latitude' of the position circle through the planet P. Al-Bīrūnī considers this position circle as a horizon for a new locality and then proceeds to compute the 'latitude' of that locality, which latitude is the length of the great circle arc from the celestial pole perpendicular to the new 'horizon'. Once this has been done, al-Bīrūnī prescribes that the computation of *tasyīr* should be based on oblique ascensions for the latitude of this new horizon. His point of view is abstract because if P is in the Western hemisphere, he considers the necessary oblique 'descensions' as oblique ascensions for localities of southern latitude. If a celestial object sets on the Western horizon at a locality on the Northern hemisphere of the Earth, it rises on the diametrically opposite locality on earth, in the Southern hemisphere. Thus al- Bīrūnī considers oblique ascensions of localities that were believed to be uninhabited in his time.

The worked examples below illustrate the computation of the 'latitude' of the position circle, and also the resulting exact determination of the *tasyīr* arc. Our worked examples for al-Bīrūnī's sections 1 and 3 are the same and will illustrate

[17] See Leo Knegt, *Astrologie, Wetenschappelijke Techniek, een studiewerk voor meergevorderden* (Amsterdam: J.F. Duwaer en zonen, 1928), pp. 76–79 for an introduction, computations, and a list of λ_T and λ_P for the horoscope of the Dutch Queen Wilhelmina (1880–1962). See also the website, www.wva-astrologie.nl.

the difference between the approximate and exact computation. In these examples, the differences correspond to 9 months and 2.5 years for the astrological prediction. We may conclude that, for a serious astrological prediction, the standard approximation method in Section 1 is pretty useless. This is probably the reason why computations on the basis of Section 3 are often met with in later Islamic astrology – for example in the *Zīj* of Ulugh Beg (ca. 1420).[18]

In Section 4, al-Bīrūnī describes the solution of an inverse problem: if the position of *P* on the ecliptic and the *tasyīr* arc is given, find the position of *F* on the ecliptic. In Figure 3, we have to find the ecliptical longitude of point *F* to which *P* 'progresses' in a given amount of time. Al-Bīrūnī presents an approximate solution which seems to agree well with the approximation in his Section 1 and therefore must have been rather useless for the exact solution of the problem, at least if the given number of years is sufficiently large.

In order to facilitate the *tasyīr* computation for a fractional number of solar years, al-Bīrūnī then presents a table in which any number of days of a solar year can be converted to minutes and seconds of arc (assuming that a complete solar year is equivalent to one degree of *tasyīr* arc). The manuscripts and printed edition contain this table in alphanumerical (*abjad*) notation. The table has been transcribed in the Appendix and some obvious scribal errors have been corrected in the process.

Worked examples, and some additional commentary

The following numerical examples were treated by Ptolemy.[19] I use the same latitude that was used by Ptolemy and also his tables of right and oblique ascensions in *Almagest* I:15, II:8.[20] Al-Bīrūnī computed slightly different tables of oblique ascensions which are not easily available to the reader who does not know Arabic.

Example 1: The locality is such that the longest day is 14 hours (Lower Egypt, latitude $\phi = 30°22'$); *P* and *F* are on the ecliptic at 0° Aries (*P*) and at 0° Gemini = 30° Taurus (*F*). One is required to compute the *tasyīr* in the following four cases:

[18] Louis Pierre Eugene Amelie Sédillot, *Prolégomènes des Tables astronomiques d'Oloug-Beg* (Paris: Typographie de Firmin Didot Frères, 1853), pp. 208–09, 211–12.
[19] Ptolemy, *Tetrabiblos*, III:10, pp. 294–305.
[20] Toomer, *Ptolemy's Almagest*, pp. 100–03.

(a) *P* is on the meridian, above the horizon (at midheaven) or below the horizon (imum coeli). Solution: the *tasyīr* arc is the right ascension of the signs Aries plus Taurus, that is 57°44′.

(b) P is at the Eastern horizon. Solution: the *tasyīr* arc is the oblique ascension of the signs Aries plus Taurus, taken from the oblique ascension table for Lower Egypt ($\phi = 30°22'$), namely 45°5′.

(c) *P* is on the Western horizon. Solution: the *tasyīr* arc is the oblique descension of the signs Aries and Taurus, which is equal to the oblique ascension of the two diametrically opposite signs of Libra and Scorpio, taken from the table for Lower Egypt: 250°23′ – 180°0′ = 70°23′.

(d) *P* is in the Western quadrant above the horizon, with distance 45° to the meridian plane (measured on the celestial equator). In modern terms, the hour angle of *P* is 3^h. Solution: Since *P* is on the celestial equator, its half-day arc is 90°, and Ptolemy's seasonal ('ordinary') hours coincide with his equinoctial hours which are still used today. In this case, al-Bīrūnī's first arc is the right ascension arc 57°44′ determined in (a), and his second arc is the oblique descension arc 70°23′ determined in (c). His and Ptolemy's approximate computation boils down to the following.

If the distance of *P* to the meridian plane is $a°$, and the half-day arc of *P* is *D*, the *tasyīr* arc is the weighted average $57°44' + \frac{a}{D}(70°23' - 57°44')$. In Ptolemy's case, $D = 90°$ and $a = 45°$ so the *tasyīr* arc is 64°4′. In the *Tetrabiblos* III:10 Ptolemy rounds all *tasyīr* arcs to integer degrees.

I finish by one more working example for the same locality, but with $D \neq 90°$:

(e) Take *P* and *F* on the ecliptic such that *P* = 0° Taurus, *F* = 0° Cancer, and assume that *P* is in the Eastern quadrant above the horizon, at a distance of a = 20° from the meridian plane measured along a parallel circle. (In modern terms, the hour angle of *P* is 22^h40^m.) Solution: The day arc 2*D* of *P* is the part of this parallel circle above the horizon, and its length can be computed as the oblique ascension of *P* + 180° minus the oblique ascension of *P*; so $2D = 214°47' - 20°53' = 193°54'$, whence $D = 96°57'$.

Al-Bīrūnī's first and second arcs are the right and oblique ascensions (ϕ = 30°22′) of arc *PF*, that is 90°0′ – 27°50′ = 62°10′ and 75°0′ – 20°53′ = 54°7′.

Thus the *tasyīr* arc is $62°10' - \frac{20}{96°57'}(62°10' - 54°7') = 60°30'$.

In the more refined method for planets *P* of non-zero latitude, the ecliptical coordinates longitude and (non-zero) latitude of the planet are first transformed into the equatorial coordinates declination δ, and right ascension α. Using the geographical latitude ϕ, one can then compute the point on the equator which rises or sets together with *P* as $\alpha \pm \arcsin(\tan\delta \tan\phi)$, without reference to ecliptical coordinates. Using tables of right and oblique ascensions, one can then compute al-Bīrūnī's 'degree of the rising point', the point on the ecliptic that rises at the same moment as *P*, and the 'degree of transit', which is the point on the ecliptic which passes through the meridian plane at the same time as *P*.

Section 1 of Chapter 5 of Book 5 of the *Masudic Canon* agrees with al-Bīrūnī's non-technical explanation of *tasyīr* in the *Introduction to Astrology*:

> . . . the meaning of *tasyīr* here, concerning births, is not by equal (ecliptical) degrees, but by degrees of ascension. As for the degree of the ascendent and the stars which are in it, its *tasyīr* is by oblique ascension of the locality, for every degree one year. And as for the degree of the descendent and the planets which are in it, it is by the descensions of the locality, and they are the ascensions of the degree opposite the [setting, that is the] rising (degree) and the signs which follow it, because the descensions of every sign in the locality are equal to the ascensions of the (sign) opposite it. For the degrees of the midheaven and the imum coeli, and the planets which are in it, will have *tasyīr* in all localities in right ascension. If a planet which is 'progressing' is not in these four degrees, but if it is between two degrees in the cardinal planes, its *tasyīr* is by ascensions which are mixed between the ascensions of the two cardinal degrees, by a long procedure and a difficult computation.[21]

The computation by Ptolemy and by al-Bīrūnī in his Section 1 is based on the idea that position circles are approximately the same as hour lines for seasonal hours. We can see this in the following way. If the sun is in *P* in the Western or Eastern quadrant above the horizon such that $\frac{\mathrm{a}}{\mathrm{D}} = \frac{\mathrm{n}}{6}$ for constant $n = 1,\ldots 5$, the time of day is the beginning of a constant seasonal hour, namely ($7-n$) if *P* is in the Eastern quadrant and ($7+n$) if *P* is in the Western quadrant.

In Section 3 (see Figure 4 below), al-Bīrūnī assumes that the preceding planet is *K* and that its altitude *KZ* and azimuth *ZE* (reckoned from the East or West point of the local horizon) have been computed from the declination and the hour angle. His idea is to first find arcs *KH* and *HZ* from arcs *KZ* and *ZE*; mathematically speaking he performs a transformation of coordinates of *K* to a

[21] Al-Bīrūnī, *The Book of Instruction*, pp. 326–27.

new system, in which the basic circle of reference is the prime vertical. The computation can be described in more modern notation as follows. Make $R = 60$ equal to the radius of the sphere and write Sin and Cos for the sine and cosine functions used by al-Bīrūnī; so $\mathrm{Sin}\, x = R\sin x$ and $\mathrm{Cos}\, x = R\cos x$, where sin and cos are the modern sine and cosine.

Step 1: We have $\mathrm{Sin}\, HK = \mathrm{Sin}\, EZ \cdot \mathrm{Cos}\, KZ / R = \mathrm{Sin}\, EZ \cdot \mathrm{Sin}\, KS / \mathrm{Sin}\, ZS$ and hence we find arc *HK*.

Step 2: We have $\mathrm{Sin}\, EH = \mathrm{Sin}\, KZ / [\mathrm{Cos}\, HK / R]$ and hence we find arc *EH*.

Step 3: We have $\mathrm{Sin}\, TM = \mathrm{Cos}\, EH\ \mathrm{Sin}\, \phi / R$ where $\phi = TD$ is the geographical latitude. Then *TM* is the 'latitude' of the 'horizon' *HKMD*. Thus, from the intersection of the ecliptic with the position circle *HKMD* can find the intersection with the celestial equator using the table of oblique ascensions for 'latitude' *TM*. Al-Bīrūnī's figure (Figure 4 below) is not realistically drawn; because *K* is a planet, it will not be too far away from the celestial equator.

For worked example (e) in Section 1, this computation is as follows. For $P = K$ standard computations produce a zenith distance $z = 26°20'$, and azimuth $a = 42°18'$ reckoned from the East point of the horizon, or 47°42′ reckoned from the South point. In modern terms $\sin(HK) = \sin z \sin a$, so arc $HK = 17°22'$ (where *K* should be left of the prime vertical); $\sin EH = \cos z / \cos HK$ so $EH = 69°54'$ (note that *E* is also left of the prime vertical); $\sin TM = \sin EH \sin\phi$ so $TM = 10°0'$. For this latitude, the oblique ascension for the arc *FP* is the oblique ascension of 30 Gemini (85°32′) minus that of 30 Aries (25°45′), so the correct *tasyīr* arc is 59°47′. The standard approximation method in Section 1 produced 60°30′. For the astrological interpretation, the difference between correct and approximate computation corresponds to a difference of nine months in the life of the person.

In worked example (d) we find in the same or easier ways, $HE = 40°47'$ and $TM = 22°30'$. Because $P = K$ is in the Western quadrant above the horizon, we need to compute for this latitude the oblique descension of Taurus and Gemini, which is equal to the oblique ascension for Libra and Scorpio. This can be found, either by linear interpolation between Ptolemy's two tables of oblique ascensions for $\phi = 16°27'$ and $\phi = 23°51'$ or by direct trigonometrical computation, as 66°39′. This accurate *tasyīr* arc differs from the approximation 64°6′ of Section 1 by 2°33′, corresponding to a difference of more than 2.5 years in the astrological predictions for the person who was born at the time when the celestial configuration was as in example (d).

I illustrate the computation in Section 4 by a worked example similar to (e).

Assume that the locality is again the same (ϕ = 30°22′), that P is again the point 0° Taurus on the ecliptic, and that the *tasyīr* arc is 60°30′, and that P is in the Eastern quadrant above the horizon with a distance 20° to the meridian (measured along a circle parallel to the equator).

We find al-Bīrūnī's first arc as follows. Since the right ascension of 0° Taurus is 27°50′, we look for an ecliptical degree whose right ascension is 27°50′ + 60°30′ = 88°20′ and we find (by linear interpolation in Ptolemy's table for right ascensions) 28°28′ Gemini, which is the endpoint of the first arc; hence the first arc is 58°28′.

We then find al-Bīrūnī's second arc. Since the oblique ascension of 0° Taurus is 20°53′, we look for an ecliptical degree whose oblique ascension is 20°53′ + 60°30′ = 81°23′. We find the endpoint of the second arc as 5°42′ Cancer, so the second arc is 65°42′. As above we have a = 20, D = 96°57′, so the correction is $\frac{20}{96°57'}$(65°42′ − 58°28′) = 1°30′. Thus arc PF = 58°28′ + 1°30′ = 59°28′, so F is in 29°58′ Gemini, very close to 30° Gemini in worked example (e) in Section 1. If al-Bīrūnī had believed in the value of astrology, he probably would have given a more correct solution such as the one in Section 3.

Appendix: Translations from the eleventh Book of the Masudic Canon

The following abbreviations will be used:
H: the Arabic edition in vol. 3 of Al-Bīrūnī, *al-Qānūn al-Mas'ūdī*, 3 vols (Hyderabad: Osmaniya Oriental Publications Bureau, 1954–56),
J: the Arabic edition in vol. 3 of Al-Bīrūnī, *al-Qānūn al-Mas'ūdī, qaddama lahu wa-ḍabaṭahu wa-ṣaḥḥahu 'Abd al-Karīm Sāmī al-Jundī*, 3 vols (Beirut: Dār al-Kutub al ilmiyya, 1422 H./2002), which seems to be largely a reprint of H,
R: the Russian translation in Abu Raikhan Beruni (973–1048), Izbrannye Proizvedeniya V, part 2, *Kanon Mas'uda, knigi VI–XI*, trans. Boris Abramovich Rozenfeld and Ashraf Akhmedovich Akhmedov (Tashkent: Fan, 1976).
All explanatory additions by me appear in square brackets.

The Eleventh Book of the Masudic Canon

[H 1354, J 317, R 449]

This art [of astronomy] to which this book is devoted, is sufficient in itself, because of its great intrinsic value. Thus it is not very attractive to the hearts [of students], who cannot imagine any delight except in preparations for sensual pleasures, and who cannot see any use except in worldly matters. Since they found nothing desirable in it, they disliked and loathed it, so they turned their back on it and on its people. For this reason, the ancients versified the events of this world in terms of the rulings [of the stars], and they found somewhat satisfactory methods for obtaining knowledge of them [the events] by means of the influences of it [astronomy]. They founded the art of the judgements [of the stars, i.e., astrology] on these [methods], all the time presenting it to them [i.e., the 'hearts' of the students] as absolutely the fruit of that [i.e., astronomy], so they would pursue [it]. They [the ancients] knew that the greed of the general public for knowledge on how to increase the good and how to avoid harm would take severe blame and harsh disasters away from them [the students of astronomy and astrology].[22]

Some of the principles of the art of the judgements of the stars are related to computation. The astrologers find it sufficient to slavishly follow the relevant rules without critical investigation. But since that [i.e., such rules] cannot be reduced to [logical] necessity, it is possible to have differences [i.e., different opinions] in it, so the methods in it have become manifold. This [eleventh] book describes most of them, so it is distinguished from the earlier [works on the subject].[23]

The fifth chapter on the procedures of *tasyīr*, in five sections [H 1393, J 343, R 474]

The first section, on the well-known method in this

The astrologers assume some planet or place in the ecliptic in order to make predictions. They make (1) the [number of] time-degrees between it and another planet or its ray or something similar, correspond to (2) [real] amounts of time, in such a way that the first resembles the second or exemplifies the second. They call the procedure for obtaining these time-degrees *tasyīr* [i.e., 'progression']. By this term they indicate that we make [in our imagination] some planet progress to some [other planet or place], and then it will reach [that planet or place] in a certain amount of time: years, months or days. For easy expression, let us call the first of the two [planets] the 'preceding', since it precedes in the primary [i.e., daily] motion, and the other [planet], at which [the *tasyīr*] ends, the 'following'. The terminology and the procedure of them [the astrologers] may give the impression that the *tasyīr* is directed from the preceding [planet] and ends at the following [planet], but this is not the case. Its [real] meaning is contradictory to that impression: it is the arrival of the following[24] [planet], by

[22] Al-Bīrūnī probably means that because the general public believes that astrology can be useful, a skilled astronomer and astrologer can have a good reputation and make a living.
[23] Al-Bīrūnī means that Book 11 of the Qānūn is the first collection of different, mutually inconsistent astrological computations.
[24] Following R, p. 475 n. 69, p. 607, I read instead of *al-awwal* in H 1393:14 and J 343:12 *al-tālī* to make mathematical sense.

the primary motion [i.e., the daily motion of the sky], at the place of the preceding [planet]. They [the astrologers] agree that if the circle of the preceding [planet] is the meridian above or below [the horizon], the time-degrees of the *tasyīr* are degrees of right ascension of the [arc] between it and the following [planet]; and that if the circle of the preceding [planet] is the horizon, these time-degrees are degrees of oblique ascension of the [arc] between the two [planets] if the [preceding planet] is on the Eastern horizon, and degrees of oblique descension if the [preceding planet] is on the Western horizon. Thus, if [the preceding planet] is between these fundamental [half-]planes, it is necessary that the procedure for them is by means of the ascensions [defined] by the [semi]circle [H 1394] passing through the preceding [planet] and through the two poles of the prime vertical, or by their descensions, in analogy with what has been explained above[25] for the procedure for [computing] the projection of rays by means of ascensions that are mixed between the ascensions for the fundamental semicircles. For that procedure [for the rays] is adapted from the procedure for the *tasyīr*. So in this [procedure for the *tasyīr*] also: the ratio of the difference between the right ascension and the ascension of the circle through the preceding [planet] to the difference between the right ascension and the ascension or descension of the locality is [supposed to be] equal to the ratio of the distance [arc] between the preceding [planet] and the meridian to half its day arc [if it, sc. the preceding planet, is] above the earth or half its night arc [if it is] below the earth.

Its computation: we derive the distance [arc] of the preceding [planet] from the meridian circle above the horizon:[26] if it [the preceding planet] is above the earth, we subtract the right ascension of the tenth [house] from its [i.e., the planet's] right ascension, if it is in the Eastern quadrant; and we do the opposite of that [i.e., we subtract the planet's right ascension from

[25] See Section 1 of Chapter 4 of Book 11 of the *Masudic Canon* [H].
[26] In Chapter 3 of Book 11 of the *Masudic Canon*, Al-Bīrūnī explains that the 'distance' is computed along a circle parallel to the celestial equator.

the right ascension of the tenth house], [if it is] in the Western [J 344]
[quadrant]; or [we derive] its distance from the meridian circle below the horizon, if it [the preceding planet] is below the earth: we subtract the right ascension of the fourth [house] from its right ascension if it is in the Eastern quadrant and we do the opposite [if it is] in the Western [quadrant]. Then we subtract the right ascension of the preceding [planet] from the right ascension of the following [planet], and the remainder is the first [arc of] time-degrees. We do the same [subtraction] with the two oblique ascensions [of the preceding and following planet] for the locality, if the preceding [planet] is in the rising half [of the celestial sphere], and with the two oblique descensions if it is in the setting half. I mean by the oblique descensions the two oblique ascensions of the degrees diametrically opposite to the two degrees of them for the same [locality]. The remainder is the second [arc of] time-degrees. Then we multiply the difference between these two [arcs of] time-degrees by the distance of the preceding [planet to the meridian] and we divide the result by half its day arc if it is above the earth and by half its night arc if it is under
the earth. Then the result is [called] the correction. We add it to the [H 1395]
first [arc of] time-degrees if they are less than the second, and we subtract it from them [the first time-degrees] if they are more than the second. The result after the addition or subtraction is the desired time-degrees of the *tasyīr*.

A more refined method: if we desire a more refined method, just as we desired in the projection of rays for the case where the planet is at a distance of a [non-zero] latitude from the ecliptic, then it is necessary to consider the preceding celestial body by itself, regardless of its ecliptical degree. If it is in the orb of the meridian itself, above or below the horizon,[27] then we use the right ascensions, and[28] we take them between the two [ecliptical] degrees of transit of the preceding and the following [planet]. Then

[27] For *nqshmā* in H 1395:7 I read *nafsihimā*. J 344:14 incorrectly changes the word to *naqsimuhumā*, as in the footnote on H 1395.

[28] I have emended *aw* in H 1395:8 and J 344:15 to *wa-*.

the result is the time-degrees of the *tasyīr*. And if the preceding [planet] is on the Eastern horizon, the time-degrees of the *tasyīr* are the difference between the oblique ascensions for the locality of the two [ecliptical] degrees of their rising points; and if on the Western horizon, then [the *tasyīr* arc is] the difference between the oblique ascensions of the degrees diametrically opposite to the [ecliptical] degrees of their setting points. According to this analogy, the ascensions of [bodies] between these fundamental planes are mixed between these [right and oblique] ascensions. Their procedure is that we compute the distance in right ascension between the [ecliptical] degree of transit of the preceding [planet] and the midheaven or the *imum coeli*, and the first [arc of] time-degrees in them [right ascensions] also, between the [ecliptical] degrees of transit of the preceding and the following [planets], and the second [arc of] time-degrees between the oblique ascensions for the locality of the two [ecliptical] degrees of their rising points, if the preceding [planet] is in the rising hemisphere, or between the oblique ascensions of the two [ecliptical] degrees diametrically opposite to the degrees of their setting points, if the preceding [planet] is in the setting hemisphere. For the correction, and the condition about adding or subtracting [it], we follow what has been explained above. Thus the time degrees of the *tasyīr* are obtained by means of half the day arc of the preceding [planet] itself or half its night arc, but not by means of half the day arc or night arc of its [ecliptical] degree.

[H 1396]

The second section. On mixing the degrees [of a planet] by means of the ascensions and the use of them

Magnitudes which change in the neighbourhood of two consecutive cardinal planes [horizon, meridian] share in that change according to the distance from them, if they happen to be between the two circles which define them

[the cardinal planes]. Ascensions belong to them [i.e., to such [J 345]
magnitudes], as has been explained above in sufficient [detail]. So there is a rule for their similarity which is in two types. One type is for a magnitude which is bounded between its [maximal] value and its absence [i.e., being zero], either at the beginning or at the end, such as the altitude [of a celestial body]. For this begins with its absence [zero] at the horizon and reaches its [maximal] value in the meridian. Another example is the distance in azimuth, taken from the meridian line. It begins at its maximum, at the rising [of the planet] at the horizon, it ends with its absence [being zero] in the meridian circle. The case of the 'equation of day' is similar.

The second type is the [magnitude] which oscillates between the two cardinal planes, between two [maximal and minimal] values: it is greater than the minimum and less than the maximum according to its position with respect to the cardinal plane[s]. Such is the day arc. Just like the distance between rising point and East point, the day arc is different in magnitude for horizons [of localities] with [non-zero] latitude, and fixed at its average magnitude in the meridian.[29] Another example is the ortive amplitude for them [i.e., for these horizons], for the ortive amplitude reduces to the declination [of the planet] when it is at midheaven, and if it is [computed for a position circle] between them [meridian and local horizon] it is less than the ortive amplitude [on the local horizon] and greater than the declination. For it is always [equal to an arc] on the [position] circle which is the horizon of a latitude less than the latitude of the locality.

And to this specialism belongs what they do,[30] because they

[29] The day arc of a planet is the arc of its apparent daily orbit above the horizon. This arc is always 180° for an observer on the terrestrial equator and it varies between a minimum and a maximum value with average 180° for observers in the tropical and temperate latitudes. In the context of *tasyīr*, the 'horizon' through the planet is the circle through the planet and the North and South point of the local horizon, so if a planet is at the meridian, its 'horizon' is the meridian itself and the celestial pole is located on it. So mathematically speaking, the 'horizon' of the planet is then the horizon for an observer on the terrestrial equator.

[30] Al-Bīrūnī is a bit sarcastic.

urgently need it, where they use both the degrees of rising and transit. If they use [only] one of these degrees [i.e., one of the two values] for the whole distance between the cardinal planes, the transition to the other [value] is [with a jump] at the [moment of]
reaching the other cardinal plane without any orderly approach [H 1397]
towards it [the new value]. This is unsatisfactory from a theoretical point of view.

The computation of the degree which is a mixture between the two above-mentioned degrees follows the analogy which has been mentioned for the projection of rays and the *tasyīr*. We obtain half the day arc of the body of the planet, not [necessarily] for the planet itself, but for its ecliptical degree. Then we multiply (1) the difference between the degree of transit and the rising degree in the ascending hemisphere, or the difference between the degree of transit and the degree of setting in the descending hemisphere by (2) the distance of the degree of transit from[31] the tenth house [if it is] above the earth, and we divide the product by (3) half the day arc of the planet; or [we multiply (1)] by (2) the distance of the degree of transit from the fourth house [if it is] below the earth, and we divide the product by (3) half the night arc of the planet. Thus we find the correction of the degree. Then we see: if the degree of transit is before the degree of rising or setting, whichever of the two is used, then we add the correction of the degree to the degree of transit. And if it [the degree of transit] is after [the degree of rising or setting] we subtract it [the correction] from it [the degree of transit]. The result is the degree which belongs to the planet in accordance with its position between the two cardinal planes. According to this analogy, the *tasyīr* will be made for the [astrological] powers of the planets, if they are between[32] the two cardinal planes.

[31] I have corrected *fī l-ʿāshir* in H 1397:6 J 345:19 to *ʿani l-ʿāshir*, in analogy with H 1397:7, J 345:21 *ʿani l-rābiʿ*.

[32] Instead of *fī*, 'in' in H 1396:12 and J 345:25 I read *bayna*, 'between' to make mathematical sense.

The third section. On the method which I preferred in tasyīr computation

For someone who can obtain [tables of] ascensions for the [i.e., all] latitudes less than the latitude of his locality, the above-mentioned procedure can be reduced from complexity to simplicity, and one can get rid of the carelessness and approximation which are involved in it, even if one's [mathematical] power is but little. [J 346]

For this, let *ABGD* be the circle of the meridian with pole *E*, and *BED* the horizon with
pole *S*, and *AEG* the equator with pole *T*, [H 1398]

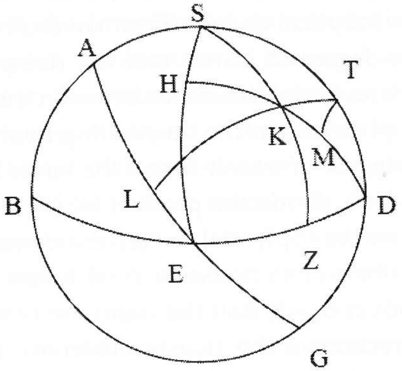

Fig. 4: Al-Bīrūnī's own computation.

and *K* the position of the preceding planet, between two fundamental [half-planes], and *TKL* the [declination] circle of its distance from the equator, and by this circle its position is known at the [given] time. We let pass through it *SKZ*, its altitude circle, to find its azimuth, and we draw *SHE*, the prime vertical. Then it is known that, if the azimuth is known, the ratio of the Sine of *EZ* to the Sine of *ZS*, the quadrant, is equal to the ratio of the Sine of *HK* to the Sine of *KS*, the complement of the altitude of the preceding [planet], so *KH* is known. But the ratio of the Sine of *DK*,[33] the complement of *KH*, to the Sine of *KZ*, is equal to the ratio of the Sine of *DH*, the quadrant, to the Sine of *EH*. So *EH* is known, but it

[33] I have corrected the scribal error *ZK* in H 1398:9 and J 346:7.

is the magnitude of the angle *EDH*, so the angle *SDH* is known, because it is the complement of it. But the ratio of the Sine of it to the Sine of the right angle *M* is equal to the ratio of the Sine of *TM*, which is perpendicular to *DH*, to the Sine of *DT*, the latitude of the locality. But *TM* is the latitude of the locality whose horizon is circle *DKH*. If it [the planet] is in the ascending hemisphere, it [the latitude *TM*] is in the [same] direction of the latitude of the locality [of the observer, i.e., Northern latitude], and therefore ascensions are used for it. But if it [the planet] is in the descending hemisphere, then *TM* is the latitude of that position [i.e., of the position circle through the planet], but in the direction different from the latitude of the locality [of the observer], and therefore the descensions of that locality are used in it, since they are equal to the ascensions there [for a locality of Southern latitude].

So either we compute the ascensions or descensions between the preceding [planet] and the following [planet], by [computing ourselves] the equation of day of the preceding [planet] on the [H 1399]
horizon *DKH*, or we use the [tables of] ascensions which have been issued for the above-mentioned [i.e., all] latitudes; then [the result] is the [arc of] *tasyīr* which we wanted, in time-degrees.

The computation of this is as follows: we compute the altitude of the preceding [planet] and its azimuth from the given moment of time. Then we multiply the Sine of the azimuth by the Cosine of the altitude, and we divide by sixty. The result is a Sine, of which we take the arc. We divide the Sine of the altitude by the Cosine of it [i.e., of the arc] divided by sixty, and we take the arc corresponding to the result [considered as Sine]. We multiply the Cosine of it by the Sine of the latitude of the locality divided by sixty. The quotient is the Sine of the latitude of the circle of the *tasyīr* [i.e., the position circle of the preceding planet], and that is the horizon such that its ascensions or descensions are used for the *tasyīr* of the preceding [planet].

The fourth section, on the determination of the amounts of the tasyīr

If a known [amount of] time is assumed for us, and one wants to know where the preceding [planet] arrives in this [amount of time] as a result of *tasyīr*, we take for each complete solar year of the designated period one time-degree; and [we take] for the remainder in months, days, and smaller intervals, the share[34] of one time-degree which is allotted to it, [one time degree being] sixty minutes – by converting the remaining days of the year in our amount [of time], expressed in their smallest units, into minutes of days and higher sexagesimal fractions [of days], and by multiplying them by the sixty minutes which are in one time-degree, and by dividing the result by the magnitude of the year, and the result is the desired [number of] minutes of the time-degree. They are added to them [the integer number of time-degrees]. [J 437]

It is easier to compute for the remainder which does not complete a solar year, the argument of the sun and its apogee. We multiply the sum of these two [arcs] by ten minutes [= 1/6], and in this way we also extract the minutes of the time-degrees, which are added to the integer [time-degrees].[35] So if our time-degrees have been obtained, we add them to the right ascensions of

[34] I have emended *wa-ḥiṣṣatuhā* in H1399:13 and J 347:2 to *ḥiṣṣatuhā* for mathematical sense.

[35] Since the mean sun moves with constant speed and makes one complete revolution in one solar year, al-Bīrūnī can use the arc of the mean sun from the beginning of Aries in the first column of his table. In Book 6 of the *Masudic Canon*, he adds a table in which he lists for the *n*-th day (n = 1...60) two columns in six sexagesimals, namely the increase of the argument of the mean sun, reckoned from the apogee, from the beginning of the first day to the beginning of the *n*-th day, and the motion of the apogee in the same period, see [1, vol. 2, pp. 697–700]. The first column in the present table is adapted from the table in Book 6 by adding the two columns and truncating the sum. The second column in the present table is obtained from the first by means of division by 360, because 360 degrees of solar motion, one solar year, correspond to a *tasyīr* arc of one degree. The division by 360 is accomplished by multiplication by 1/6 ('ten minutes') and shifting the result by one sexagesimal place. Al-Bīrūnī indicates the sexagesimal shift by his statement that the quotient is in 'minutes of the time degree' rather than 'time-degrees' itself.

the degree of transit of the preceding [planet], and we convert the result into an [ecliptical] arc, and the result is the first arc. [H 1400]

Then we also add to the [oblique] ascensions for the locality of its rising degree, if it is in the rising half, the same [amount] which we have added to the ascensions of its degree of transit, and we convert the result to an [ecliptical] arc by means of the [oblique] ascensions for the locality, and the result is the second arc. And if it is in the setting half, we add this amount to the [oblique] ascensions of the degree diametrically opposite to its setting degree for the locality, and we convert the sum to an arc by means of these [oblique ascensions], and we add to the result one hundred and eighty degrees, and the result is the second arc.

Then we multiply the difference between it and the first [arc] by the distance between the preceding [planet] and the tenth [house] and we divide the product by half the day arc if it is[36] above the earth and by half the night arc if it is under it, then the result is the correction. We add it to the first arc if it is less than the second and we subtract it from it is if it is more. The result after the addition or the subtraction is the place to which the preceding [planet] is transported by the tasyīr, that is to say, the place in the ecliptic to whose [position] circle it is transported by the prime motion. Further, it is clear that the first arc is the desired [arc] if the preceding [planet] is on the meridian, above or under the earth, and then we do not need the second [arc]; and that the desired [arc] is the second arc if it [the preceding planet] is on the horizon, and then we do not need the first [arc].

I have placed in this table opposite the elapsed days the mean [motion] of the sun and its share of one time-degree. From this is also found, by the property of four numbers in proportion, the days and their fractions which correspond to [given] fractions of degrees in the *tasyīr*. [H 1401, J 438]

[36] I have changed *fawqa* in H 1400:8 and J 347:16 to *in kāna fawqa* for mathematical sense.

day	solar year			time deg.			day	solar year			time deg.		
	°	′	″	°	′	″		°	′	″	°	′	″
1	0	59	8	0	0	10	31	30	33	18	0	5	5
2	1	58	16	0	0	20	32	31	32	27	0	5	15
3	2	57	23	0	0	30	33	32	31	35	0	5	25
4	3	56	33	0	0	40	34	33	30	43	0	5	35
5	4	55	41	0	0	49	35	34	29	51	0	5	45
6	5	54	50	0	0	59	36	35	29	0	0	5	55
7	6	53	58	0	1	9	37	36	28	8	0	6	5
8	7	53	6	0	1	19	38	37	27	15	0	6	14
9	8	52	15	0	1	29	39	38	26	24	0	6	24
10	9	51	23	0	1	39	40	39	25	33	0	6	34
11	10	50	31	0	1	48	41	40	24	41	0	6	44
12	11	49	40	0	1	58	42	41	23	50	0	6	54
13	12	48	48	0	2	8	43	42	22	58	0	7	4
14	13	47	56	0	2	18	44	43	22	6	0	7	14
15	14	47	5	0	2	28	45	44	21	15	0	7	24
16	15	46	13	0	2	37	46	45	20	23	0	7	33
17	16	45	21	0	2	47	47	46	19	31	0	7	43
18	17	44	30	0	2	57	48	47	18	40	0	7	53
19	18	43	38	0	3	7	49	48	17	48	0	8	3
20	19	42	47	0	3	17	50	49	16	56	0	8	13
21	20	41	55	0	3	27	51	50	16	5	0	8	23
22	21	41	3	0	3	37	52	51	15	13	0	8	32
23	22	40	11	0	3	46	53	52	14	21	0	8	42
24	23	39	20	0	3	56	54	53	13	30	0	8	52
25	24	38	28	0	4	6	55	54	12	38	0	9	2
26	25	37	36	0	4	16	56	55	11	47	0	9	12
27	26	36	45	0	4	26	57	56	10	55	0	9	22
28	27	35	53	0	4	36	58	57	10	3	0	9	32
29	28	35	2	0	4	46	59	58	9	12	0	9	41
30	29	34	10	0	4	56	60	59	8	20	0	9	51

day	solar year			time deg.			day	solar year			time deg.		
	°	′	″	°	′	″		°	′	″	°	′	″
61	60	7	29	0	10	1	91	89	41	39	0	14	57
62	61	6	37	0	10	11	92	90	40	47	0	15	7
63	62	5	45	0	10	21	93	91	39	56	0	15	17
64	63	4	54	0	10	31	94	92	39	4	0	15	26
65	64	4	2	0	10	41	95	93	38	12	0	15	36
66	65	3	10	0	10	50	96	94	37	21	0	15	46
67	66	2	19	0	11	0	97	95	36	29	0	15	56
68	67	1	27	0	11	10	98	96	35	38	0	16	6
69	68	0	35	0	11	20	99	97	34	46	0	16	16
70	68	59	44	0	11	30	100	98	33	54	0	16	26
71	69	58	52	0	11	40	101	99	33	3	0	16	35
72	70	58	0	0	11	50	102	100	32	11	0	16	45
73	71	57	9	0	11	59	103	101	31	19	0	16	55
74	72	56	17	0	12	9	104	102	30	28	0	17	5
75	73	55	25	0	12	19	105	103	29	36	0	17	15
76	74	54	34	0	12	29	106	104	28	44	0	17	24
77	75	53	42	0	12	39	107	105	27	53	0	17	34
78	76	52	50	0	12	49	108	106	27	1	0	17	44
79	77	51	59	0	12	59	109	107	26	9	0	17	54
80	78	51	7	0	13	9	110	108	25	18	0	18	4
81	79	50	15	0	13	18	111	109	24	27	0	18	14
82	80	49	24	0	13	28	112	110	23	34	0	18	24
83	81	48	32	0	13	38	113	111	22	43	0	18	34
84	82	47	40	0	13	48	114	112	21	51	0	18	43
85	83	46	49	0	13	58	115	113	21	0	0	18	53
86	84	45	57	0	14	8	116	114	20	8	0	19	3
87	85	45	6	0	14	18	117	115	19	17	0	19	13
88	86	44	14	0	14	27	118	116	18	25	0	19	23
89	87	43	22	0	14	37	119	117	17	33	0	19	33
90	88	42	31	0	14	47	120	118	16	41	0	19	43

day	solar year			time deg.			day	solar year			time deg.		
	°	′	″	°	′	″		°	′	″	°	′	″
121	119	15	50	0	19	52	151	148	50	0	0	24	48
122	120	14	58	0	20	2	152	149	49	8	0	24	58
123	121	14	6	0	20	12	153	150	48	16	0	25	8
124	122	13	15	0	20	22	154	151	47	25	0	25	18
125	123	12	23	0	20	32	155	152	46	33	0	25	28
126	124	11	31	0	20	42	156	153	45	41	0	25	38
127	125	10	40	0	20	52	157	154	44	50	0	25	47
128	126	9	48	0	21	1	158	155	43	58	0	25	57
129	127	8	56	0	21	11	159	156	43	6	0	26	7
130	128	8	5	0	21	21	160	157	42	15	0	26	17
131	129	7	13	0	21	31	161	158	41	23	0	26	27
132	130	6	21	0	21	41	162	159	40	31	0	26	37
133	131	5	30	0	21	51	163	160	39	40	0	26	46
134	132	4	38	0	22	1	164	161	38	48	0	26	56
135	133	3	46	0	22	11	165	162	37	56	0	27	6
136	134	2	55	0	22	20	166	163	37	5	0	27	16
137	135	2	3	0	22	30	167	164	36	13	0	27	26
138	136	1	11	0	22	40	168	165	35	21	0	27	36
139	137	0	20	0	22	50	169	166	34	30	0	27	46
140	137	59	28	0	23	0	170	167	33	38	0	27	56
141	138	58	36	0	23	9	171	168	32	46	0	28	5
142	139	57	45	0	23	19	172	169	31	55	0	28	15
143	140	56	53	0	23	29	173	170	31	3	0	28	25
144	141	56	2	0	23	39	174	171	30	12	0	28	35
145	142	55	10	0	23	49	175	172	29	20	0	28	45
146	143	54	18	0	23	59	176	173	28	29	0	28	55
147	144	53	26	0	24	9	177	174	27	37	0	29	4
148	145	52	35	0	24	18	178	175	26	45	0	29	14
149	146	51	43	0	24	28	179	176	25	54	0	29	24
150	147	50	51	0	24	38	180	177	25	2	0	29	34

day	solar year			time deg.			day	solar year			time deg.		
	°	′	″	°	′	″		°	′	″	°	′	″
181	178	24	10	0	29	44	211	207	58	21	0	34	39
182	179	23	19	0	29	54	212	208	57	29	0	34	49
183	180	22	27	0	30	3	213	209	56	38	0	34	59
184	181	21	35	0	30	13	214	210	55	46	0	35	9
185	182	20	45	0	30	23	215	211	54	54	0	35	19
186	183	19	52	0	30	33	216	212	54	3	0	35	29
187	184	19	0	0	30	43	217	213	53	11	0	35	39
188	185	18	9	0	30	53	218	214	52	19	0	35	48
189	186	17	17	0	31	3	219	215	51	28	0	35	58
190	187	16	25	0	31	13	220	216	50	37	0	36	8
191	188	15	33	0	31	22	221	217	49	44	0	36	18
192	189	14	42	0	31	32	222	218	48	53	0	36	28
193	190	13	50	0	31	42	223	219	48	1	0	36	38
194	191	12	59	0	31	52	224	220	47	9	0	36	48
195	192	12	7	0	32	2	225	221	46	18	0	36	58
196	193	11	15	0	32	12	226	222	45	26	0	37	7
197	194	10	24	0	32	22	227	223	44	34	0	37	17
198	195	9	32	0	32	31	228	224	43	43	0	37	27
199	196	8	40	0	32	41	229	225	42	51	0	37	37
200	197	7	49	0	32	51	230	226	42	0	0	37	47
201	198	6	57	0	33	1	231	227	41	8	0	37	57
202	199	6	6	0	33	11	232	228	40	16	0	38	7
203	200	5	14	0	33	21	233	229	39	25	0	38	16
204	201	4	22	0	33	31	234	230	38	33	0	38	26
205	202	3	31	0	33	41	235	231	37	41	0	38	36
206	203	2	39	0	33	50	236	232	36	50	0	38	46
207	204	1	47	0	34	0	237	233	35	58	0	38	56
208	205	0	56	0	34	10	238	234	35	6	0	39	6
209	206	0	4	0	34	20	239	235	34	15	0	39	16
210	206	59	12	0	34	30	240	236	33	22	0	39	26

day	solar year			time deg.			day	solar year			time deg.		
	°	′	″	°	′	″		°	′	″	°	′	″
241	237	32	31	0	39	35	272	268	5	50	0	44	41
242	238	31	40	0	39	45	273	269	4	58	0	44	51
243	239	30	48	0	39	55	274	270	4	6	0	45	1
244	240	29	56	0	40	5	275	271	3	15	0	45	11
245	241	29	5	0	40	15	276	272	2	23	0	45	20
246	242	28	13	0	40	24	277	273	1	31	0	45	30
247	243	27	21	0	40	34	278	274	0	40	0	45	40
248	244	26	30	0	40	44	279	274	59	48	0	45	50
249	245	25	38	0	40	54	280	275	58	56	0	46	0
250	246	24	46	0	41	4	281	276	58	5	0	46	9
251	247	23	55	0	41	14	282	277	57	13	0	46	19
252	248	23	3	0	41	24	283	278	56	21	0	46	29
253	249	22	11	0	41	33	284	279	55	30	0	46	39
254	250	21	20	0	41	43	285	280	54	38	0	46	49
255	251	20	28	0	41	53	286	281	53	46	0	46	59
256	252	19	36	0	42	3	287	282	52	55	0	47	8
257	253	18	45	0	42	13	288	283	52	3	0	47	18
258	254	17	53	0	42	23	289	284	51	12	0	47	28
259	255	17	2	0	42	33	290	285	50	20	0	47	38
260	256	16	10	0	42	43	291	286	49	29	0	47	48
261	257	15	18	0	42	52	292	287	48	37	0	47	58
262	258	14	26	0	43	2	293	288	47	45	0	48	7
263	259	13	35	0	43	12	294	289	46	54	0	48	17
264	260	12	43	0	43	22	295	290	46	2	0	48	27
265	261	11	51	0	43	32	296	291	45	10	0	48	37
266	262	11	0	0	43	42	297	292	44	19	0	48	47
267	263	10	5?	0	43	52	298	293	43	27	0	48	57
268	264	9	16	0	44	1	299	294	42	35	0	49	7
269	265	8	25	0	44	11	300	295	41	43	0	49	17
270	266	7	33	0	44	21	301	296	40	51	0	49	26
271	267	6	41	0	44	31	302	297	40	0	0	49	36

day	solar year			time deg.			day	solar year			time deg.		
	°	′	″	°	′	″		°	′	″	°	′	″
303	298	39	8	0	49	46	334	329	12	27	0	54	52
304	299	38	16	0	49	56	335	330	11	35	0	55	2
305	300	37	25	0	50	6	336	331	10	44	0	55	12
306	301	36	33	0	50	16	337	332	9	52	0	55	22
307	302	35	41	0	50	26	338	333	9	0	0	55	31
308	303	34	50	0	50	35	339	334	8	9	0	55	41
309	304	33	58	0	50	45	340	335	7	17	0	55	51
310	305	33	6	0	50	55	341	336	6	25	0	56	1
311	306	32	15	0	51	5	342	337	5	34	0	56	11
312	307	31	23	0	51	15	343	338	4	42	0	56	21
313	308	30	31	0	51	25	344	339	3	50	0	56	30
314	309	29	40	0	51	35	345	340	2	59	0	56	40
315	310	28	48	0	51	45	346	341	2	7	0	56	50
316	311	27	56	0	51	54	347	342	1	15	0	57	0
317	312	27	5	0	52	4	348	343	0	24	0	57	10
318	313	26	13	0	52	14	349	343	59	32	0	57	20
319	314	25	21	0	52	24	350	344	58	40	0	57	30
320	315	24	30	0	52	34	351	345	57	49	0	57	39
321	316	23	38	0	52	44	352	346	56	57	0	57	49
322	317	22	46	0	52	54	353	347	56	6	0	57	59
323	318	21	55	0	53	3	354	348	55	14	0	58	9
324	319	21	3	0	53	13	355	349	54	22	0	58	19
325	320	20	12	0	53	23	356	350	53	31	0	58	28
326	321	19	20	0	53	33	357	351	52	39	0	58	38
327	322	18	29	0	53	43	358	352	51	47	0	58	48
328	323	17	37	0	53	53	359	353	50	56	0	58	58
329	324	16	45	0	54	3	360	354	50	4	0	59	8
330	325	15	53	0	54	13	361	355	49	12	0	59	18
331	326	15	2	0	54	23	362	356	48	21	0	59	28
332	327	14	10	0	54	33	363	357	47	29	0	59	37
333	328	13	19	0	54	42	364	358	46	38	0	59	47
							365	359	45	46	0	59	57