

Coupling between splay deformations and density modulations in splay-bend phases of bent colloidal rods

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Using a grand-canonical Landau–de Gennes theory for colloidal suspensions of bent (banana-shaped) rods, we investigate how spatial deformations in the nematic director field affect the local density of twist-bend and splay-bend nematic phases. The grand-canonical character of the theory naturally relates the local density to the local nematic order parameter S . In the splay-bend phase, we find S and hence the local density to modulate periodically along one spatial direction. As a consequence the splay-bend phase has the key symmetries of a smectic rather than a nematic phase. By contrast we find that S and hence the local density do not vary in space in the twist-bend phase, which is therefore a proper nematic phase. The theoretically predicted one-dimensional density modulations in splay-bend phases are in agreement with recent simulations.

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Dispersions of rodlike colloidal particles can spontaneously order into nematic liquid-crystal phases upon compressing a dilute isotropic fluid phase [1,2]. The nematic bulk phase is a homogeneous fluid that exhibits long-range orientational order of the long axes of the rods, which on average align along a so-called nematic director $\hat{\mathbf{n}}$ that is uniform in space. An important characteristic of the nematic phase is its up-down symmetry, which identifies $\hat{\mathbf{n}}$ with $-\hat{\mathbf{n}}$, to be contrasted with polar order of for instance a magnet. The simplest nematic phase does not only feature up-down symmetry but also azimuthal symmetry about $\hat{\mathbf{n}}$. This uniaxial symmetry is broken in the case of a biaxial nematic phase, which features not only ordering of the long axes of the rods along $\hat{\mathbf{n}}$ but also of their short axes along a direction $\hat{\mathbf{m}} \perp \hat{\mathbf{n}}$, with $\pm\hat{\mathbf{m}}$ equivalent [1]. This Letter, however, concerns the interesting and intricate case in which the azimuthal symmetry about $\hat{\mathbf{n}}$ is broken by a *polar* ordering of the short axes of the rods in a preferred direction $\mathbf{P} \perp \hat{\mathbf{n}}$, with nonequivalent \mathbf{P} and $-\mathbf{P}$. This type of ordering is well known to be strongly coupled, via the so-called bend flexoelectric effect [3], to spontaneous bend deformations of the nematic director field, such that $\hat{\mathbf{n}}(\mathbf{r})$ is no longer a spatial constant if $\mathbf{P} \neq \mathbf{0}$ [4]. As a pure bend deformation cannot uniformly fill three-dimensional (3D) space, Meyer [5] and later independently Dozov [6] argued that these local bend deformations should be accompanied by either a twist or a splay deformation. In the former case they theoretically postulated the stabilization of a spatially modulated twist-bend nematic (N_{TB}) phase, that displays a heliconical structure of $\hat{\mathbf{n}}(\mathbf{r})$ with bend and twist deformations in the molecular orientation. In

the latter case, they predicted a spatially modulated splay-bend nematic (N_{SB}) phase that is characterized by alternating layers of splay and bend.

After its theoretical prediction, the N_{TB} phase has actually been observed experimentally in several molecular systems [7–14], and has meanwhile become well established as a new type of orientationally ordered fluid phase. By contrast, the N_{SB} phase has never been observed in the absence of a strong external constraint, such as an electric field [15–17], a planar surface anchoring [18], or a topological constraint [19]. Rather, a huge variety of smectic phases was found either in systems of thermotropic bent-core mesogens [4,20,21] and in systems of bent silica rods [22,23]. Therefore, many doubts subsist on the existence and true nature of the N_{SB} phase. Recently, however, simulations of systems of hard bent spherocylinders revealed that N_{TB} and N_{SB} phases can be thermodynamically stable in bulk, provided the smectic (S_m) phase is destabilized either by polydispersity in particle length or by curvature in the particle shape [24]. Inspired by these simulation results, a stable N_{SB} phase has experimentally been sought for and actually observed, for the first time in a lyotropic system of smoothly curved colloidal rods [25] and later in polydisperse bent silica rods [26]. Recently, simulations and a Maier-Saupe theory showed that the (alleged) N_{SB} phases of monodisperse curved rods display (weak) density modulations [27], which cannot be described by current Oseen-Frank and Landau–de Gennes type theories. The existing theoretical descriptions [5,6,28] are direct extensions of Oseen-Frank theory for the elastic deformations of $\hat{\mathbf{n}}(\mathbf{r})$; they only account for the *direction* of the ordering and ignore the *degree* of ordering that is characterised by the scalar nematic order parameter S . Landau–de Gennes (LdG) theories for thermotropic liquid crystals do account for S but lack any coupling with density ρ [29–33].

In this Letter we investigate the nematic nature of the N_{SB} phase displayed in colloidal systems, by employing a

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recently introduced grand-canonical LdG theory [34] applied to lyotropic curved rods [35], that does involve a coupling between S and ρ . We show that S and therefore also ρ varies periodically along one spatial direction in the N_{SB} phase, such that the N_{SB} phase has actually the symmetries of a smectic rather than a nematic phase. This is not the case, however, for the N_{TB} phase, for which we find that S does not vary in space, and is therefore a proper nematic phase.

We follow Ref. [35] and extend the Oseen-Frank theory of Selinger and co-workers [28] to a mesoscopic LdG theory where the director $\hat{\mathbf{n}}$ is replaced by the standard second-rank, symmetric, and traceless tensor $\mathbf{Q}(\mathbf{r})$ with Cartesian components $Q_{\alpha\beta}(\mathbf{r})$ for $\alpha, \beta = x, y, z$. The eigenvector of \mathbf{Q} , corresponding to the maximum modulus of a nondegenerate eigenvalue, defines the nematic director $\hat{\mathbf{n}}$ of the system [1]. A vector field $\mathbf{P}(\mathbf{r})$ with Cartesian coordinates $P_\gamma(\mathbf{r})$ for $\gamma = x, y, z$ describes the polar order in a direction perpendicular to $\hat{\mathbf{n}}$. In order to describe lyotropic systems that become ordered with increasing density, we follow Ref. [34] and set up a Landau expansion for which we use the chemical-potential-dependent grand potential $\Omega(\mu)$ rather than the temperature-dependent Helmholtz (or Gibbs) free energy $F(T)$. More specifically, for a system of hard bent rods modeled as curved or kinked rods of contour length L and diameter D , at chemical potential μ in a macroscopic volume V , and at fixed temperature T , we write the excess-over-isotropic LdG grand potential as $\Delta\Omega(\mathbf{Q}, \mathbf{P}) = \int_V d\mathbf{r}(\Delta\omega_b + \omega_{eP})$, where $\Delta\omega_b$ denotes a bulk term

$$\beta B_2 \Delta\omega_b = \frac{2}{3} a \beta (\mu^* - \mu) Q_{\alpha\beta} Q_{\beta\alpha} - \frac{4}{3} b Q_{\alpha\beta} Q_{\beta\lambda} Q_{\lambda\alpha} + \frac{4}{9} d Q_{\alpha\beta} Q_{\beta\alpha} Q_{\lambda\rho} Q_{\rho\lambda}, \quad (1)$$

and ω_{eP} an elastic-polar term

$$\beta B_2 \omega_{eP} = \frac{2}{9} l_1 (\partial_\alpha Q_{\beta\lambda}) (\partial_\alpha Q_{\beta\lambda}) + \frac{2}{9} l_2 (\partial_\alpha Q_{\alpha\lambda}) (\partial_\beta Q_{\beta\lambda}) + e_2 P_\alpha \left(\delta_{\alpha\beta} + \frac{2}{S_0} Q_{\alpha\beta} \right) P_\beta + e_4 P_\alpha P_\alpha P_\beta P_\beta - \lambda P_\alpha (\partial_\beta Q_{\alpha\beta}) + \kappa (\partial_\alpha P_\beta) (\partial_\alpha P_\beta). \quad (2)$$

We express $\Delta\omega_b$ and ω_{eP} in units of $\beta^{-1} = k_B T$ with k_B the Boltzmann constant, and in units of the second virial coefficient $B_2 = \pi L^2 D / 4$ of long needles in the isotropic (I) fluid phase, which is a convenient unit of volume for systems of rodlike particles. We use the Einstein summation convention for repeated indices, and indicate the Landau phenomenological parameters by $a, b, d, l_1, l_2, e_2, S_0, e_4, \lambda$, and κ . We note that only the quadratic term $Q_{\alpha\beta} Q_{\beta\alpha}$ has a μ -dependent prefactor that changes sign at the nematic spinodal μ^* and drives the phase transitions. Stability of the grand-potential expansion in the dilute limit requires $a > 0$ and $e_2 > 0$, while stability with respect to an unlimited growth of \mathbf{Q}, \mathbf{P} , and $\partial_\alpha P_\beta$ requires that $d > 0$, $e_4 > 0$, and $\kappa > 0$, respectively. The sign of the b term determines the nature of the I - N transition, rodlike for $b > 0$ and platelike for $b < 0$. The coefficients $2/S_0$ and λ represent the \mathbf{Q} - \mathbf{P} coupling and the bend flexoelectric \mathbf{P} - $\nabla\mathbf{Q}$ coupling, respectively. In order to describe a favored polarization perpendicular to the nematic director, leading to a bend flexoelectric effect, we set $S_0 > 0$. Finally the coefficients l_1 and l_2 are related to

the Oseen-Frank elastic constants [36,37] through the relations $\beta B_2 K_{11} = \beta B_2 K_{33} = (2l_1 + l_2) S^2$ and $\beta B_2 K_{22} = 2l_1 S^2$. As shown in Ref. [35], a complete mapping exists between our LdG theory and the Oseen-Frank theory of Selinger and co-workers [28].

Minimization of the grand potential $\Delta\Omega(\mathbf{Q}, \mathbf{P})$ with respect to \mathbf{Q} at $\mathbf{P} \equiv \mathbf{0}$, $\nabla\mathbf{Q} \equiv \mathbf{0}$, and $\nabla\mathbf{P} \equiv \mathbf{0}$, gives a first-order I - N transition at $\beta\mu_{IN} \equiv \beta\mu^* - b^2/(4ad)$ [34,35]. The nematic order parameter equals $S_I(\mu) = 0$ for $\mu \leq \mu_{IN}$, and

$$S_N(\mu) = \frac{3b}{8d} \left(1 + \sqrt{1 - \frac{32ad\beta(\mu^* - \mu)}{9b^2}} \right) \quad (3)$$

for $\mu > \mu_{IN}$. In order to describe the ‘‘Onsager’’-type I - N phase transition of uniaxial hard needles [2,38] we follow Ref. [34] and set $a = 1.436$, $b = 5.851$, $d = 3.693$, and $\beta\mu^* = 6.855$. As shown in Ref. [35], the uniaxial N phase becomes unstable with respect to an N_{TB} or N_{SB} phase at a critical μ where the renormalized bend elastic constant $K_{33}^{\text{eff}} = 0$. The N_{TB} phase is described by a chiral nematic director and polarization vector

$$\hat{\mathbf{n}}_{\text{TB}}(z) = [\sin\theta \cos(qz), \sin\theta \sin(qz), \cos\theta], \\ \mathbf{P}_{\text{TB}}(z) = P(z)[\sin(qz), -\cos(qz), 0], \quad (4)$$

with q and θ the variational wave number and tilt angle, respectively, and $P(z)$ the variational magnitude of the polarization. The N_{SB} phase is described by an achiral nematic director and polarization vector

$$\hat{\mathbf{n}}_{\text{SB}}(z) = [\sin\phi(z), 0, \cos\phi(z)], \\ \mathbf{P}_{\text{SB}}(z) = P(z) \cos(qz) [-\cos\phi(z), 0, \sin\phi(z)], \quad (5)$$

where $\phi(z) = \theta \sin(qz)$. The system either stabilizes an N_{TB} phase, if l_1 and l_2 are such that $K_{11} > 2K_{22}$, or an N_{SB} phase, if l_1 and l_2 are such that $K_{11} < 2K_{22}$ [35]. The N - N_{TB} as well as N - N_{SB} phase transitions are continuous and proceed in both cases without any jump in $S(\mu)$.

We employ the thermodynamic identity $\partial(\Omega/V)/\partial\mu|_{V,T} = -\rho$ to perform the conversion between chemical potential μ and number density ρ . To this end, we introduce the grand-potential density ω_I of the isotropic I state and define $\omega \equiv \omega_I + \Delta\omega_b$, where $\Delta\omega_b$ is the excess bulk grand-potential density (1). From $\partial(B_2\omega)/\partial\mu = -c$, with $c \equiv B_2\rho$ the dimensionless particle concentration, we find

$$c(\mu) = c_I(\mu) + aS^2(\mu). \quad (6)$$

The particle concentration of the I phase, $c_I(\mu) = -\partial(B_2\omega_I)/\partial\mu$, can be calculated within Onsager theory, by using an isotropic distribution function, such that $\beta\mu(c_I) = \log(c_I/4\pi) + 2c_I$ [38]. By inverting this relation we obtain $c_I(\mu)$ straightforwardly. Together with Eq. (6), we can determine c , for every value of μ for which $S(\mu)$ is known. We observe that Eq. (6) not only allows us to convert a dependence on the chemical potential into a dependence on the particle density, but also expresses a coupling between the scalar order parameter S and the particle concentration c . In this Letter we employ this coupling to obtain insight into the structure of the N_{SB} phase.

In contrast to earlier works [6,28–33,35] we consider here a z -dependent nematic order parameter $S(z)$ in the definition of

the tensorial order parameter $Q_{\alpha\beta}(z) = S(z)[3n_\alpha(z)n_\beta(z)/2 - \delta_{\alpha\beta}/2]$, as well as a z -dependent magnitude $P(z)$ of the polarization vectors \mathbf{P}_{TB} and \mathbf{P}_{SB} in Eqs. (4) and (5). In order to study the relative stability of the N_{TB} and N_{SB} phases, we insert $\hat{\mathbf{n}}_{\text{TB}}$ and $\hat{\mathbf{n}}_{\text{SB}}$, respectively, into \mathbf{Q} . After insertion of \mathbf{Q} and \mathbf{P} into $\Delta\Omega(\mathbf{Q}, \mathbf{P})$, we perform, at fixed chemical potential μ , a full grand-potential minimization. To this end we numerically solve the system of Euler-Lagrange equations

$$\frac{\delta\Delta\Omega}{\delta S(z)} = \partial_z \frac{\delta\Delta\Omega}{\delta(\partial_z S(z))}, \quad \frac{\delta\Delta\Omega}{\delta P(z)} = \partial_z \frac{\delta\Delta\Omega}{\delta(\partial_z P(z))}, \quad (7)$$

for many combinations of fixed wave number q and tilt angle θ , from which we identify the grand-potential minimum of the N_{TB} phase characterized by $q_{\text{TB}}(\mu)$, $\theta_{\text{TB}}(\mu)$, $S_{\text{TB}}(\mu; z)$, and $P_{\text{TB}}(\mu; z)$, and that of the N_{SB} phase at $q_{\text{SB}}(\mu)$, $\theta_{\text{SB}}(\mu)$, $S_{\text{SB}}(\mu; z)$, and $P_{\text{SB}}(\mu; z)$.

We start by considering the coefficients $l_1 = 0.165L^2$, $l_2 = 0.427L^2$, $e_2 = 1$, $S_0 = 0.99$, $e_4 = 0.5$, $\kappa = 0.3L^2$, and $\lambda = 0.18L$. It follows that $K_{11}/K_{22} = 2.294$ and that, as shown in Fig. S2(a) in the Supplemental Material (SM) [39], a second-order N - N_{TB} phase transition takes place at $\beta\mu = 5.663 \equiv \beta\mu_{NN_{\text{TB}}}$. We plot S_{TB} and P_{TB} as a function of $z \in [0, 2\pi/q_{\text{TB}}]$ for several values of the chemical potential $\mu \equiv \mu_{NN_{\text{TB}}} + \Delta\mu$ in Figs. S4(b) and S4(b), respectively. We clearly find that S_{TB} and P_{TB} are constant along z for every μ . As a consequence, the corresponding density profile c_{TB} is also a spatial constant. Interestingly, we show in the SM and in Fig. S5 [39] that the extremal solutions $q_{\text{TB}}(\mu)$, $\theta_{\text{TB}}(\mu)$, $S_{\text{TB}}(\mu)$, $P_{\text{TB}}(\mu)$, and $c_{\text{TB}}(\mu)$ exactly coincide with those obtained when S and P are assumed z independent in \mathbf{Q} and \mathbf{P}_{TB} , respectively. In particular we observe from Fig. S5(a) that $S_{\text{TB}}(\mu)$ coincides with $S_N(\mu)$ of Eq. (3). We conclude that spatial twist and bend modulations do not affect the scalar nematic order.

Subsequently, we reset the elasticity parameters to $l_1 = 0.1L^2$ and $l_2 = 0.0427L^2$ such that $K_{11}/K_{22} = 1.214$ and, as shown in Fig. S2(b), a second-order N - N_{SB} phase transition takes place at $\beta\mu = 5.354 \equiv \beta\mu_{NN_{\text{SB}}}$. In Fig. 1(a) we plot for several chemical potentials $\mu \equiv \mu_{NN_{\text{SB}}} + \Delta\mu$ the (shifted) scalar nematic order parameter $S_{\text{SB}} - S_N$ as a function of $z \in [0, 2\pi/q_{\text{SB}}]$. The periodic modulation of S_{SB} is manifest, with an amplitude that vanishes at $\Delta\mu = 0$ and increases to 0.0075 at $\Delta\mu = 0.5k_B T$ (see also SM [39]). The inset shows the minimum and maximum of $S_{\text{SB}}(z)$ as a function of μ , the difference being small but clearly discernible. Note that the wavelength of the modulation of the scalar order parameter equals half of the pitch $2\pi/q_{\text{SB}}$ of the nematic director. This can be explained by the fact that one period of a nematic director modulation corresponds to two alternating bend and splay domains with opposite polar order. Figure 1(b) shows the corresponding (shifted) density profiles $c_{\text{SB}}(z) - c_N$, obtained from Eq. (6) with c_N the density of the metastable N phase. Figure 1(b) clearly reveals spatial density modulations with the same wavelength as $S_{\text{SB}}(z)$ and an amplitude that grows from zero at $\Delta\mu = 0$ to a few percent of c_N at $\Delta\mu = 0.5k_B T$; the inset shows the minimum and the maximum of the density profile as a function of μ , which confirms the density variations on the order of a few percent. From the periodic modulation of c_{SB} along z , it follows that the N_{SB} phase has all

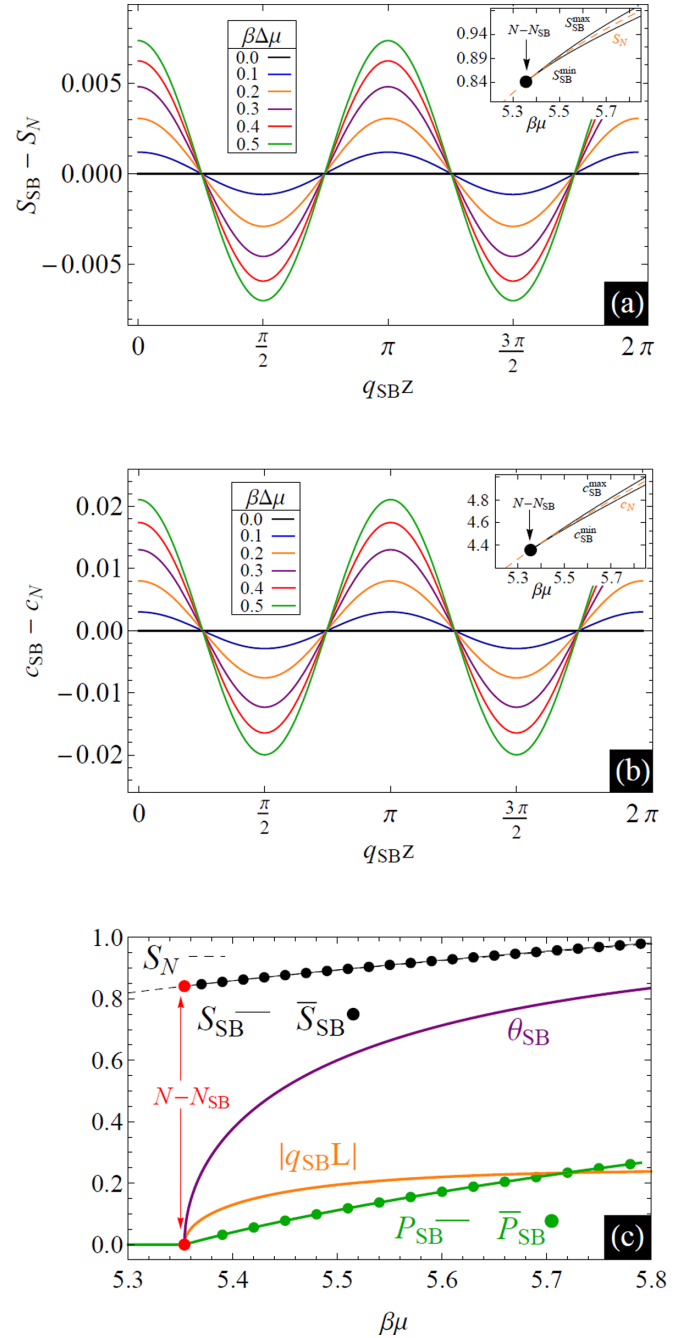


FIG. 1. The excess scalar nematic order parameter $S_{\text{SB}} - S_N$ (a) and the excess particle concentration $c_{\text{SB}} - c_N$ (b) in the splay-bend phase N_{SB} as a function of position $z \in [0, 2\pi/q_{\text{SB}}]$, for several values of the chemical-potential excess $\Delta\mu \equiv \mu - \mu_{NN_{\text{SB}}}$ above the N - N_{SB} transition, at wave numbers $q = q_{\text{SB}}$ of the director modulations [see Eq. (5)] given by the orange curve in (c). The period average \bar{S}_{SB} and \bar{P}_{SB} of the nematic order parameter $S_{\text{SB}}(z)$ and the polarization $P_{\text{SB}}(z)$ are presented by the dots in (c), which are indistinguishable from the corresponding solid curves that follow from the assumption of a spatial constant S_{SB} and P_{SB} . The violet curve in (c) represents the opening angle θ_{SB} . The expansion coefficients are $l_1 = 0.1L^2$, $l_2 = 0.0427L^2$, $e_2 = 1$, $S_0 = 0.99$, $e_4 = 0.5$, $\kappa = 0.3L^2$, and $\lambda = 0.18L$. The insets of (a) and (b) show the μ dependence of the minima and maxima of the modulating profiles $S_{\text{SB}}(z)$ and $c_{\text{SB}}(z)$.

the symmetries of a smectic (or lamellar) phase. However, in contrast to ordinary smectic phases with periods on the order of the particle length L , the wavelength π/q_{SB} of the density modulation in the splay-bend phase is much larger than L . This becomes manifest from the orange curve in Fig. 1(c), which presents the μ dependence of the wave number q_{SB} , which is infinitesimally small at the N - N_{SB} transition and grows to typically $|q_{\text{SB}}L| \simeq 0.2$ in the splay-bend phase; this corresponds to density modulations with wavelengths of the order $\pi/q_{\text{SB}} \simeq 15L$ well in the splay-bend phase and a diverging wavelength upon approach of the transition to the N phase. Figure 1(c) also shows the tilt angle θ_{SB} and the period average of the profiles of the nematic order parameter $S_{\text{SB}}(z)$ and the polarization $P_{\text{SB}}(z)$; Fig. S7 of the SM [39] shows $P_{\text{SB}}(z)$ in the N_{SB} phase, which also exhibits spatial modulations. The infinitesimally small q_{SB} , θ_{SB} , and $P_{\text{SB}}(z)$ at the N - N_{SB} transition are in line with the continuous character of the N - N_{SB} transition. Additionally, Fig. 1(c) shows that the period average of the profiles $S_{\text{SB}}(\mu; z)$, $P_{\text{SB}}(\mu; z)$, and $c_{\text{SB}}(\mu; z)$ as well as q_{SB} and θ_{SB} (see SM [39]) actually coincide with the solutions $S_{\text{SB}}(\mu)$, $P_{\text{SB}}(\mu)$, $c_{\text{SB}}(\mu)$, and $q_{\text{SB}}(\mu)$ and $\theta_{\text{SB}}(\mu)$, that are obtained when S and P are assumed z independent in \mathbf{Q} and \mathbf{P}_{SB} . In particular the period average of $S_{\text{SB}}(\mu; z)$ is very close to $S_N(\mu)$ of Eq. (3), as can be seen in Fig. 1(c), where their tiny difference is only discernible at the largest μ 's and stems from the small- θ approximation of the grand-potential density [35].

Since we only find variations in the local density of the N_{SB} phase and not in that of the N_{TB} phase, we investigate the possibility of a direct connection between density modulations and splay deformations. Referring to the SM [39] for more details, we first note that the $S_{\text{SB}}(z)$ profiles shown in Fig. 1(a) accurately fit the functional form $S_{\text{SB}}(z) = S_{\text{SB}}^{\text{max}} \cos[\theta_s \sin(q_{\text{SB}}z)]$ with θ_s a fit parameter. Next, for several values of μ , we plot $S_{\text{SB}}(\nabla \cdot \hat{\mathbf{n}}_{\text{SB}})$ (solid lines) and $H(\nabla S_{\text{SB}}) \cdot \hat{\mathbf{n}}_{\text{SB}}$ (dashed lines) as a function of z in Fig. S10, with H a suitably chosen spatial constant that varies with μ as shown in the inset of Fig. S10. We find that for each μ the condition $S_{\text{SB}}(\nabla \cdot \hat{\mathbf{n}}_{\text{SB}}) = H(\nabla S_{\text{SB}}) \cdot \hat{\mathbf{n}}_{\text{SB}}$ holds, from which we can derive

$$\nabla \cdot \hat{\mathbf{n}}_{\text{SB}}(z) = \frac{H}{2} \frac{\nabla c_{\text{SB}}(z)}{[c_{\text{SB}}(z) - c_I]} \cdot \hat{\mathbf{n}}_{\text{SB}}(z), \quad (8)$$

through Eq. (6). We observe that an expression similar to the one of Eq. (8) was found by de Gennes [40] and Taratuta and Meyer [41] for polymer nematics. It expresses the coupling between concentration gradients ∇c_{SB} and splay deformations, given by $\nabla \cdot \hat{\mathbf{n}}_{\text{SB}}$, such that splay deformations cannot exist without density gradients.

To conclude, in this Letter we have employed the LdG theory introduced in Ref. [35] for lyotropic colloidal suspensions of bent rods to investigate the repercussions of spatial distortions in the nematic director field on the density of the twist-bend and splay-bend nematic phases. In contrast with the existing theories [6,28–33], our theory allows to analyze the spatial dependence of the nematic order parameter S and provides a natural coupling between S and the particle density ρ . We show that the N_{SB} phase is characterized by a one-dimensional density modulation such that the originally predicted [5,6,28] N - N_{SB} transition is strictly speaking a N -Sm transition. In the case of an N_{TB} phase, instead, we find that S and hence ρ are spatial constants such that this phase is a true nematic phase. Our findings are in agreement with simulations [24] and a Maier-Saupe theory [27]. Furthermore, the existence of a coupling between ∇c and $\nabla \cdot \hat{\mathbf{n}}$, as shown in this Letter, lends strong support that the splay deformations in the nematic director field $\hat{\mathbf{n}}$ are inherently coupled to density modulations.

Finally the theory presented in this Letter could be employed to verify the existence and the nematic nature of a recently experimentally observed splay nematic phase [32,42–47] and a recently postulated twist-splay-bend phase [27]. In addition, biaxiality and external constraints could be taken into account. While we do not expect biaxial order to change the smectic nature of N_{SB} phase, it could influence the “shape” of the density modulations. Understanding the repercussions of external constraints on the local density of the bulk N_{SB} phase, instead, could be very important to connect the findings presented in this Letter to those of Refs. [15–19].

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