Deconfinement and Gluon Plasma Dynamics in Improved Holographic QCD

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The thermodynamics of 5D dilaton gravity duals to confining gauge theories is analyzed. We show that they exhibit a first order Hawking-Page type phase transition. In the explicit background of improved holographic QCD of [U. Gursoy and E. Kiritsis, J. High Energy Phys. 02 (2008) 032] [U. Gursoy, E. Kiritsis, and F. Nitti, J. High Energy Phys. 02 (2008) 019], we find $T_c = 235$ MeV. The temperature dependence of various thermodynamic quantities such as the pressure, entropy, and speed of sound is calculated. The results are in agreement with the corresponding lattice data.

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Large-N_c techniques have provided a promising approach to the strongly coupled physics of QCD, based on an effective string theory description of glue. This route took an interesting twist in 1997 with the advent of the Maldacena conjecture [1], with the unexpected result that the string theory must live in more than four dimensions. In particular there is one extra direction, known as the holographic dimension, that plays the role of (renormalization group) energy scale of the strongly coupled gauge theory.

Since [1] there has been a flurry of attempts to devise such correspondences for gauge theories with less supersymmetry with the obvious final goal: QCD. A phenomenological approach was in the meantime developed and is now known as AdS/QCD. The original idea was formulated in [2] and it was successfully applied to the meson sector in [3]. The bulk gravitational background consists of a slice of AdS_5, and a constant dilaton. There is a UV and an IR cutoff. The confining IR physics is imposed by boundary conditions at the IR boundary. This approach, although crude, has been partly successful in studying meson physics, despite the fact that the dynamics driving chiral symmetry breaking must be imposed by hand via IR boundary conditions. Its shortcomings however include a glueball spectrum that does not fit well the lattice data, the fact that magnetic quarks are confined instead of screened, and asymptotic Regge trajectories for glueballs and mesons are quadratic instead of linear. The thermodynamics of this model have been analyzed in [4] where it was shown that the system exhibits a first order deconfinement phase transition.

Improved holographic QCD.—In [5] an improved model for QCD was proposed. It reunited inputs from both gauge theory and string theory while keeping the simplicity of a two-derivative action. It could describe both the region of asymptotic freedom as well as the strong IR dynamics of QCD.

The basic fields that are nontrivial in the vacuum solution and describe the pure gauge dynamics, are the 5D metric $g_{\mu\nu}$, a scalar Φ (the dilaton) that controls the 't Hooft coupling λ_t of QCD, and an axion a, that is dual to the QCD θ angle. Quarks can be added to the pure gauge theory by adding $D_4 - \bar{D}_4$ brane pairs in the background gauge theory solution. The $D_4 - \bar{D}_4$ tachyon condensation then induces chiral symmetry breaking, [5,6].

The action for the 5D Einstein-dilaton theory reads

$$S_5 = M_P^3 N_c^2 \left(- \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] + 2 \int_{\partial M} d^4x \sqrt{h} K \right), \tag{1}$$

where M_P is the Planck mass. The second term in the action is the Gibbons-Hawking with K being the extrinsic curvature on the boundary.

The only nontrivial input in the two-derivative action (1) is the dilaton potential $V(\lambda)$, where $\lambda = e^{\Phi}$ is identified with the 't Hooft coupling of the gauge theory. The potential is directly related to the gauge theory β-function once a holographic definition of energy is chosen. Although the shape of $V(\lambda)$ is not fixed without knowledge of the exact gauge theory β-function, its UV and IR asymptotics can be determined.

In the UV, the input comes from perturbative QCD. We demand asymptotic freedom with logarithmic running. This implies, in particular, that the asymptotic UV geometry is that of AdS_5 with logarithmic corrections. It requires a (weak-coupling) expansion of $V(\lambda)$ of the form $V(\lambda) = 12/\ell^2(1 + v_1\lambda + v_2\lambda^2 + \cdots)$.

Demanding confinement of the color charges restricts the large-λ asymptotics of $V(\lambda)$. In [5] we focused on potentials such that, as $\lambda \to \infty$, $V(\lambda) \sim \lambda^{4/3}(\log\lambda)^{(\alpha-1)/\alpha}$ where α is a positive parameter. The IR asymptotics of the solution in the Einstein frame are

$$ds_0^2 \to e^{-C(r/\ell)^\alpha}(dr^2 + dx_4^2), \qquad \lambda_0 \to e^{3C/2(r/\ell)^\alpha}\left(\frac{r}{\ell}\right)^{(3/4)(\alpha-1)}, \tag{2}$$

where the constant C is related to Λ_{QCD}. Confinement requires $\alpha \geq 1$. The parameter α characterizes the large excitation asymptotics of the glueball spectrum, $m_n^2 \sim n^{2(\alpha-1)/\alpha}$. For linear confinement, we choose $\alpha = 2$.

As discussed in [5], once the dilaton potential is fixed, the solutions of the model are parametrized by a single scale Λ that plays the role of $\Lambda_{\rm QCD}$. The AdS length ℓ is not a physical parameter but only a choice of scale: only $\Lambda\ell$ enters into physical observables. A specific choice for $V(\lambda)$ was made in [5] with the appropriate asymptotic properties. The scale Λ was fixed by matching to the lattice data for the first 0^{++} glueball mass. Once Λ is fixed, all other interesting scales, like the fundamental string scale ℓ_s and the effective QCD string tension σ are also fixed.

The deconfinement transition.—At finite temperature there exist two distinct types of solutions to the action (1) with AdS asymptotics: (i) The thermal graviton gas, obtained by compactifying the Euclidean time in the zero temperature solution with $\tau \sim \tau + 1/T$: $ds^2 = b_0^2(r) \times (dr^2 + d\tau^2 + dx_3^2)$, $\lambda = \lambda_0(r)$. This solution exists for all $T \geq 0$ and it corresponds to the confined phase, if the gauge theory at zero T confines. (ii) The black-hole (BH) solutions (in Euclidean time) of the form

$$ds^2 = b^2(r)\left(\frac{dr^2}{f(r)} + f(r)d\tau^2 + dx_3^2\right), \qquad \lambda = \lambda(r), \quad (3)$$

with $f(0) = 1$. There exists a singularity in the interior at $r = \infty$ that is now hidden by a regular horizon at $r = r_h$ where f vanishes. Such solutions correspond to a deconfined phase.

As we discuss below, in confining theories the BHs exist only above a certain minimum temperature, $T > T_{\rm min}$.

The thermal gas as well as BH solution has two parameters: T and Λ. Near the horizon, $f \to f_h(r_h - r)$ with $4\pi T = f_h$. From Einstein's equations, [7]:

$$4\pi T = b^{-3}(r_h)\left(\int_0^{r_h} \frac{du}{b(u)^3}\right)^{-1}. \quad (4)$$

In the large-N_c limit, the physics is dominated by the saddle point with minimum free energy. For a given temperature we must therefore compare the free energies of solutions (i) and (ii)

We introduce a cutoff boundary at $r/\ell = \epsilon$ in order to regulate the infinite volume. The difference of the two scale factors is given near the boundary as

$$b(\epsilon) - b_0(\epsilon) = \mathcal{C}(T)\epsilon^3 + \cdots. \quad (5)$$

By the standard rules of AdS/CFT we can relate $\mathcal{C}(T)$ to the difference of VEVs of the gluon condensate: $\mathcal{C}(T) \propto \langle {\rm Tr}F^2\rangle_T - \langle {\rm Tr}F^2\rangle_0$.

The free energy difference is given by

$$\frac{\mathcal{F}}{M_P^3 N_c^2 V_3} = 15\frac{\mathcal{C}(T)}{\ell} - \pi T b^3(r_h) = 15\frac{\mathcal{C}(T)}{\ell} - \frac{TS}{4M_P^3 N_c^2 V_3}, \quad (6)$$

where, in the last equality, we used the fact that the entropy is given by the area of the horizon. It is clear that the existence of a nontrivial deconfinement phase transition is driven by a nonzero value for the thermal gluon condensate $\mathcal{C}(T)$.

For a general potential we can prove the following statements, that only require the validity of the laws of black-hole thermodynamics: (i. There exists a phase transition at finite T, if and only if the zero-T theory confines. (ii) This transition is of the first order for all of the confining geometries, with a single exception described in (iii): (iii) In the limit confining geometry $b_0(r) \to \exp(-Cr)$ (as $r \to \infty$), the phase transition is of the second order and happens at $T = 3C/4\pi$. (iv). All of the nonconfining geometries at zero T are always in the black-hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.

We now sketch a heuristic argument, limited to asymptotics of the type (2). A general, coordinate independent proof valid for all confining geometries will appear in [7].

The existence of a minimum black-hole temperature $T_{\rm min}$ in confining theories follows from the small and large r_h behavior of the geometries. On one hand, the black-hole approaches an AdS-Schwarzschild geometry near the boundary, which obeys $T = 1/\pi r_h$. On the other hand, as the horizon approaches the deep interior, i.e. $r_h \to \infty$, the mass of the black hole vanishes and the black-hole solution approaches the zero-T geometry in this limit. This implies that $\mathcal{F}$ vanishes in this limit. Using the large r_h limit in (4), we find the following asymptotics for T:

$$T \to \frac{3C\alpha}{4\pi} r_h^{\alpha-1}, \quad r_h \to \infty; \qquad T \to \frac{1}{\pi r_h}, \quad r_h \to 0. \quad (7)$$

The large r_h behavior in Eq. (7) is valid under the assumption that the zero-T solution, with IR asymptotics (2), can be continuously deformed into a black hole with arbitrarily small mass and arbitrarily large value of r_h. This assumption indeed holds, as we will show elsewhere [7] for a more general class of confining backgrounds.

Equation (7) shows that for $\alpha \geq 1$, that there exists a minimum temperature $T_{\rm min} > 0$ above which the black-hole solutions exist. Here, for simplicity, we assume a single extremum of the function $T(r_h)$. We illustrate the function $T(r_h)$ schematically in Fig. 1. The simple convex shapes in (a) are due to our assumption of a single minimum. In general the function $T(r_h)$ may exhibit multiple extrema. Our demonstration here can be generalized to these cases [7]. In the confining geometries $\alpha > 1$, for a given $T > T_{\rm min}$, there exist a big and a small black-hole solution. The big BH has positive specific heat hence it is thermodynamically stable, whereas the small BH is unstable. In the borderline confining geometry $\alpha = 1$, there is a single BH solution.

Existence of a critical temperature $T_c \geq T_{\rm min}$ for $\alpha \geq 1$ follows from the physical requirement of positive entropy. From the first law of thermodynamics, it follows that $d\mathcal{F}/dr_h = -SdT/dr_h$. Since $S > 0$ for any physical sys-

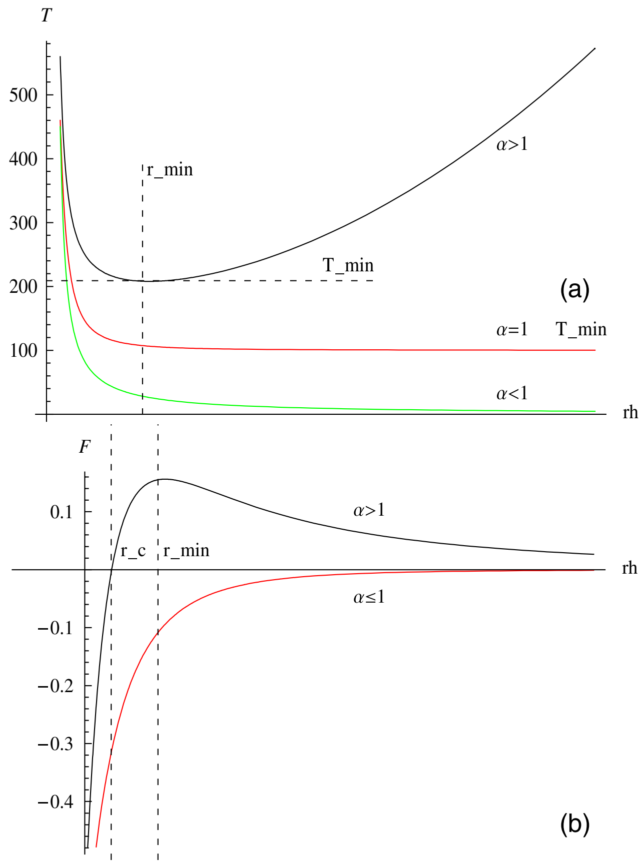

FIG. 1 (color online). Schematic behavior of temperature (a) and free energy density (b) as a function of r_h, for the infinite-r geometries of the type (2), for different values of α.

tem, extrema of $\mathcal{F}(r_h)$ coincide with the extrema of $T(r_h)$. Using also the fact that $\mathcal{F}(r_h) \to -\infty$ for $r_h \to 0$ and $\mathcal{F}(r_h) \to 0$ near $r_h \to \infty$, we arrive at conclusion (ii) described above: There is a first order transition for all of the confining geometries (this becomes second order for the borderline case $\alpha = 1$).

The small r_h asymptotics also allows us to fix the value of the Planck mass in (1). This geometry corresponds to an ideal gas of gluons with a free energy density (We use lowercase letters for the densities of the corresponding functions) $f \to (\pi^2/45)N_c^2 T^4$. As the geometry becomes AdS, Eq. (6) implies that: $f \to \pi^4 (M_P \ell)^3 N_c^2 T^4$. We conclude that $M_P \ell = (45\pi^2)^{-(1/3)}$. Using the value of ℓ in [5], we obtain $M_P \approx 2.3$ GeV.

Numerical results.—Here we present a numerical study of the relevant thermodynamic quantities in the theory advocated in [5] with the choice $\alpha = 2$ in (2). Our general analysis shows that this theory has black-hole solutions above a temperature $T_{\rm min}$ and exhibits a first order phase transition at some $T_c > T_{\rm min}$.

To analyze the behavior of the theory at finite temperature, we have solved numerically Einstein's equations for the metric and dilaton. The integration constants were fixed as explained earlier. We find a minimum temperature for the existence of black-hole solutions, $T_{\rm min} = 210$ MeV.

The resulting free energy as a function of the temperature is shown in Fig. 2, which clearly shows the existence of a minimum temperature, and a first order phase transition at $T = T_c$, where $\mathcal{F}(T_c) = 0$. For $T < T_c$, the thermal gas dominates, and the system is in the confined phase. For $T > T_c$, the (large) black-hole dominates, corresponding to a deconfined phase. The entire small black-hole branch is always thermodynamically disfavored.

The value we obtain for the critical temperature, $T_c = 235 \pm 15$ MeV, is close to the value obtained for large-N Yang-Mills [8], which with our normalization of the lightest glueball would be 260 ± 11 MeV (combining the results in [8,9]).

From the free energy we can determine all other quantities by thermodynamic identities:

$$p = -\mathcal{F}/V_3, \quad s = 4\pi M_P^3 N_c^2 b_T^3(r_h), \quad \epsilon = p + Ts. \qquad (8)$$

Next, we present some of the thermodynamic quantities that are compared with the lattice results.

Latent heat.—The latent heat per unit volume is defined as the jump in the energy at the phase transition, $L_h = T_c \Delta s(T_c)$, and it is expected to scale as N_c^2 in the large N_c limit [8]. From Eq. (8) we note that this expectation is reproduced in our theory. Quantitatively, we find $L_h^{1/4}/T_c \simeq 0.65\sqrt{N_c}$. This is to be compared with the value 0.77 reported in [8].

Equation of state and the interaction measure.—A useful indication about the thermodynamics of a system is given by the relations between the quantities ϵ/T^4, $3(p/T^4)$, $3/4(s/T^3)$. In Fig. 3(a) we compare our results for these quantities with the corresponding lattice results, reported in [10] (for $N_c = 3$). In the low temperature phase, the thermodynamic functions vanish to the leading order in N_c^2 and the jump in ϵ and s at T_c reflects the first order phase transition.

The *interaction measure*, $(\epsilon - 3p)/T^4$, is plotted in Fig. 3(b) b, together with the lattice result from [10]. From Eq. (6), $\epsilon - 3p \propto \mathcal{C}(T)$, consistent with our interpretation of $\mathcal{C}(T)$ as the gluon condensate.

Speed of sound.—This quantity is defined as $c_s^2 = (\partial p/\partial \epsilon)_S = s/c_v$. It is expected to be small at the phase transition, and to reach the conformal value $c_s^2 = 1/3$ at

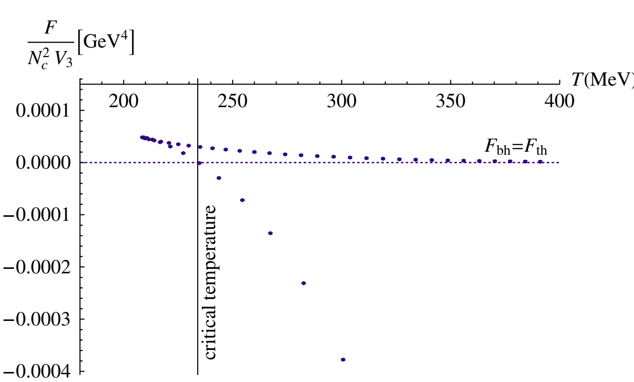

FIG. 2 (color online). Black-hole free energy.

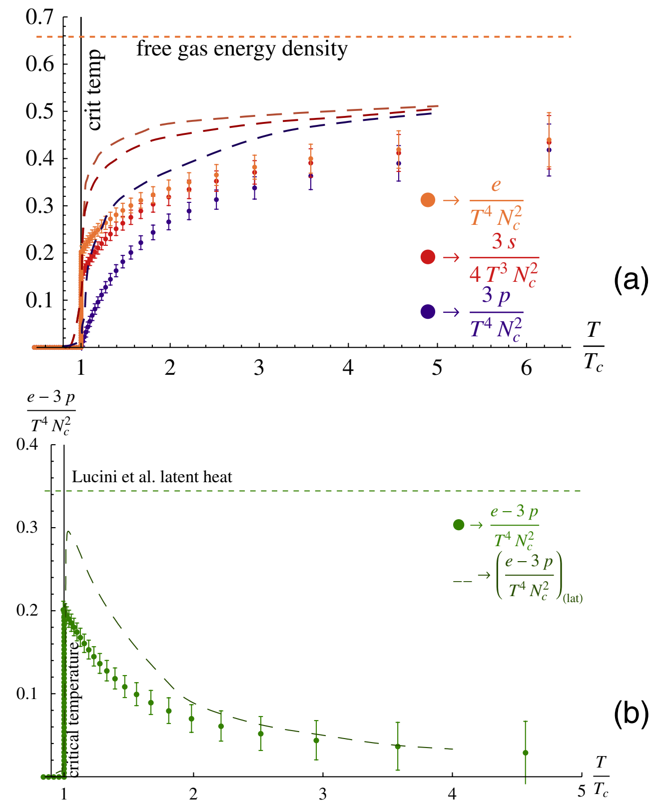

FIG. 3 (color online). (a) Dimensionless thermodynamic functions and (b) interaction measure. The dashed curves correspond to the lattice data of [10].

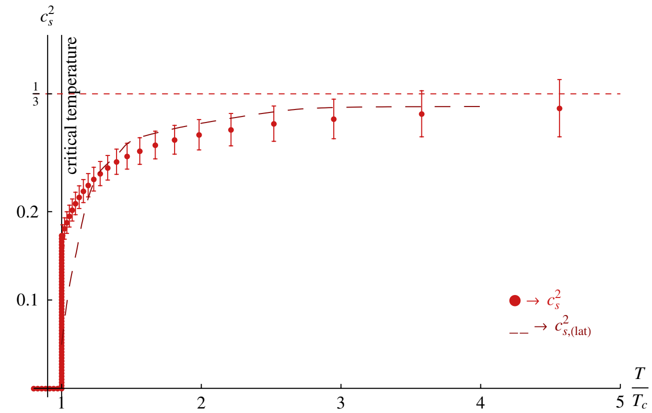

FIG. 4 (color online). Comparison between the speed of sound in our model and the lattice result of [10] (dashed curves).

high temperatures. In Fig. 4 we compare our results with the lattice data, finding good agreement.

Shear viscosity.—In agreement with the general results of [11], the ratio between shear viscosity and entropy density is $\eta/s = (4\pi)^{-1}$.

Discussion.—The model presented here describes well the basic features of large-N_c Yang-Mills at finite temperature: It exhibits a first order deconfining phase transition, and the temperature dependence of the pressure, entropy, energy density, interaction measure and speed of sound in the high temperature phase behave similarly to the corresponding lattice results. Without adding any extra parameter, one obtains a value for the critical temperature 10% off the lattice value.

On the other hand, the model can be improved in many ways. The latent heat L_h/T_c^4 is 40% off the lattice value. Also, our comparison shows that [see, e.g., Fig. 3(a)] approach to the free field limit at high T is slower than the lattice data. This may be traced back to the relative smallness of the latent heat in our potential. Although the UV and the IR asymptotics of the dilaton potential are fixed by general requirements from the field theory, the intermediate region is free to modify. The reason is that the low-level glueball spectrum and the thermodynamics near the phase transition are not controlled by the same regions of the potential. With a suitable deformation one hopes to obtain better agreement with the lattice data. In particular, it is possible to obtain a fit to quantities in Figs. 3 and 4, well within the errors of the lattice data in a temperature range $T_c < T < 5T_c$ [7]. Retrofitting the potential is an interesting challenge that we plan to address in [7].

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Note added.—Recently, we became aware of Ref. [12], which discusses related issues in a similar setup.

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