

Dynamics of lattice-pinned charge stripes

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We study the transversal dynamics of a charged stripe (quantum string), and show that zero-temperature quantum fluctuations are able to depin it from the lattice. If the hopping amplitude t is much smaller than the string tension J , the string is pinned by the underlying lattice. At $t \gg J$, the string is depinned and allowed to move freely, if we neglect the effect of impurities. By mapping the system onto a one-dimensional array of Josephson junctions, we show that the quantum depinning occurs at $(t/J)_c = 2/\pi^2$. In addition, we exploit the relation of the stripe Hamiltonian to the sine-Gordon theory, and calculate the infrared excitation spectrum of the quantum string for arbitrary t/J values. [S0163-1829(99)00325-2]

The existence of a striped phase in doped two-dimensional (2D) antiferromagnets (AF's) has recently been a subject of intense experimental and theoretical investigations. Experimentally, elastic^{1,2} and inelastic³ neutron-diffraction measurements in nickelates¹ and cuprates^{2,3} have revealed the presence of charge and spin order. In addition, muon spin resonance and nuclear quadrupole resonance results⁴ have also been successfully interpreted within the picture of charged domain walls separating antiferromagnetic domains. Striped phases have repeatedly been found in numerical investigations of t - J and Hubbard models.⁵ It is possible that the striped phase is responsible for many of the unusual properties of the cuprate superconductors.⁶

In the present paper, we study the transversal dynamics of a single stripe (quantum string) within a phenomenological model.⁷⁻⁹ By performing a canonical transformation in the quantum string Hamiltonian, we map the system onto a 1D array of Josephson junctions, which is known to exhibit an insulator/superconductor transition at $(t/J)_c = 2/\pi^2$. This transition is also known to represent the unbinding of vortex/antivortex pairs¹⁰ in the equivalent XY model. Further, by exploiting the relation of these models to the sine-Gordon (SG) theory,¹¹ we study the spectrum of the quantum string in a sector of zero topological charge of its Hilbert space, and reveal the meaning of the transition in the "string" language. At $(t/J)_c$ the (insulating) pinned phase, corresponding to an energy spectrum with a finite gap, turns into a (metallic) depinned phase where the spectrum becomes gapless. In doing this, we have connected two important and different classes of problems, i.e., the transversal dynamics of stripes in doped AF's and a system with the well-known properties of the SG theory.

Let us consider a single vertical string on a $N \times L$ square lattice [see Fig. 1(a)]. The linear concentration of holes in the string is assumed to be one hole per site. The string is composed of N charged particles elastically interacting with the neighbor ones and constrained to move along N horizontal lines. The lattice constant is taken as the unit of length.

The classical state of the system is described by the N -dimensional vector $\vec{x} = \{x_1, x_2, \dots, x_N\}$. Here x_n is the x coordinate of the n th particle, $x_n = 1, 2, \dots, L$. The corresponding quantum state $|\vec{x}\rangle$ is defined as an eigenstate of all the coordinate operators \hat{x}_n , $n = 1, 2, \dots, N$: $\hat{x}_n |\vec{x}\rangle = x_n |\vec{x}\rangle$. The phenomenological Hamiltonian describing this system is

$$\hat{H} = -t \sum_n (\hat{\tau}_n^+ + \hat{\tau}_n^-) + \frac{J}{2} \sum_n (\hat{x}_{n+1} - \hat{x}_n)^2. \quad (1)$$

The translation operators $\hat{\tau}_n^\pm$ are defined by their action on the coordinate states, $\hat{\tau}_n^\pm |\vec{x}\rangle = |\vec{x} \pm \vec{e}_n\rangle$, where $(\vec{e}_n)_m = \delta_{nm}$.

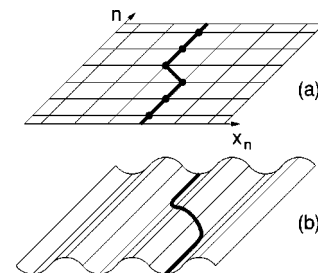


FIG. 1. Two models of stripe: (a) discrete string model; (b) continuous sine-Gordon model.

The coefficients t and J denote the hopping amplitude and the string tension, respectively. The operators $\hat{\tau}_n^\pm$ can be expressed through the momentum operators \hat{p}_n , which obey the canonical relation $[\hat{x}_n, \hat{p}_m] = i\delta_{nm}$. We then find $\hat{\tau}_n^\pm = \exp(\pm i\hat{p}_n)$, and Hamiltonian (1) becomes

$$\hat{H} = -2t \sum_n \cos \hat{p}_n + \frac{J}{2} \sum_n (\hat{x}_{n+1} - \hat{x}_n)^2. \quad (2)$$

Hereafter, we classify the state of the quantum string by the value of the topological charge $\hat{Q} = \sum_n (\hat{x}_{n+1} - \hat{x}_n)$. In the case of open boundary conditions (BC's), the topological charge is an arbitrary integer, $Q = 0, \pm 1, \pm 2, \dots$. The states with positive and negative charges are called kinks (K's) and antikinks (AK's), respectively. Here we consider periodic BC's, $\hat{x}_{N+1} = \hat{x}_1$. Hence, the total topological charge of the string is zero.

Since we are interested in the conducting properties of the system, we have to determine the current operator $\hat{j}_n = e\dot{\hat{x}}_n$, where e is the charge of the particle and the dot denotes the time derivative. Using the equation of motion $\dot{\hat{x}}_n = i[\hat{H}, \hat{x}_n]$, we obtain $\hat{j}_n = 2et \sin \hat{p}_n$.

At this point, it is convenient to perform a dual transformation to new variables referring to the segments of the string, i.e., to a pair of neighbor holes,

$$\hat{x}_n - \hat{x}_{n-1} = \hat{\pi}_n, \quad \hat{p}_n = \hat{\varphi}_{n+1} - \hat{\varphi}_n. \quad (3)$$

The new local variables also obey the canonical relation $[\hat{\varphi}_n, \hat{\pi}_m] = i\delta_{nm}$. Furthermore, we take the limit $L \rightarrow \infty$ in order to deal with all operators in the φ representation, $\hat{\varphi}_n \Rightarrow \varphi_n$, $\hat{\pi}_n \Rightarrow -i\partial/\partial\varphi_n$. The continuous variable φ_n is restricted to the interval $0 \leq \varphi_n < 2\pi$. Finally, the Hamiltonian and the transverse current operator acquire the forms

$$\hat{H} = -2t \sum_n \cos(\varphi_{n+1} - \varphi_n) - \frac{J}{2} \sum_n (\partial/\partial\varphi_n)^2, \quad (4)$$

$$\hat{j}_n = 2et \sin(\varphi_{n+1} - \varphi_n),$$

which are known from the theory of superconducting chains. Equations (4) describe a Josephson-junction chain, with the Coulomb interaction taken into account. The solution of this problem at $T=0$ was found by Bradley and Doniach.¹² Depending on the ratio t/J , the chain is either insulating (small t/J) or superconducting (large t/J). The results arise from the standard mapping of the 1D quantum problem onto the 2D classical one. One obtains the XY model with Euclidean action¹¹

$$S_E = \sqrt{\frac{2t}{J}} \sum_{\langle r, r' \rangle} \cos(\varphi_{\vec{r}} - \varphi_{\vec{r}'}), \quad (5)$$

where the vectors $\vec{r} = (n, \tau)$ form a rectangular lattice in space and imaginary time.

At $t/J = 2/\pi^2$ the Josephson chain undergoes a Kosterlitz-Thouless (KT) transition.¹⁰ For small t/J values, the two-point correlator $\langle \exp i(\varphi_{\vec{r}} - \varphi_{\vec{r}'}) \rangle$ decays exponentially. Then the frequency-dependent conductivity exhibits a resonance

$\text{Re } \sigma(\omega) \propto \delta(\omega - J)$. Since there is no conductivity at $\omega = 0$, this is an insulating state with a gap $\Delta = J$. In the opposite case, when t/J is large, the same correlator decays algebraically. Then the conductivity is singular at $\omega = 0$, $\text{Re } \sigma(\omega) = 2\pi e^2 t \delta(\omega)$, and the array is superconducting.

These results are also valid for the quantum string on the lattice. Now, it remains to reveal their physical significance for the striped phase. In order to achieve this aim, we first analyze the problem in two limiting cases: $t \ll J$ and $t \gg J$.

In the limit of weak fluctuations, $t \ll J$, the energy spectrum is discrete with spacing $\approx J$. The first excitation is separated from the ground level by a gap $\Delta \approx J$. This is the minimal energy required to create the doublet K-AK excitation, i.e., to change the initially flat configuration of the string. Hence, the ground state is insulating and the elementary excitations are bound K/AK pairs. The dimension of the pair can be estimated as the correlation length $\xi = 2/\ln(J/2t) \ll 1$.

In the limit of strong fluctuations, $t \gg J$, we can expand the cos term in the Hamiltonian (4) up to second order $\cos(\varphi_{n+1} - \varphi_n) \approx 1 - (\varphi_{n+1} - \varphi_n)^2/2$, and diagonalize the quadratic Hamiltonian. Then we obtain the phononlike spectrum $E_k = -2tN + \sqrt{8tJ} |\sin(k/2)|$ with a finite bandwidth $\sqrt{8tJ}$ and no gap. Therefore, the ground state is conducting and the stripe is depinned. The calculations of the conductivity are straightforward, since in this case the time dependence of the current $\hat{j}_n \approx 2et(\hat{\varphi}_{n+1} - \hat{\varphi}_n)$ follows from the standard relation

$$\hat{\varphi}_n(\tau) = \sum_k \sqrt{\frac{J}{2N\omega_k}} [e^{i(kn - \omega_k\tau)} \hat{a}_k + e^{i(\omega_k\tau - kn)} \hat{a}_k^\dagger]. \quad (6)$$

Here \hat{a}_k and \hat{a}_k^\dagger are Bose operators. Using these expressions, we calculate the current-current correlator

$$\Pi(k, \omega) = -i \int_0^\infty d\tau e^{i\omega\tau} \langle [\hat{j}_k^\dagger(\tau), \hat{j}_k(0)] \rangle \quad (7)$$

and the uniform conductivity

$$\sigma(\omega) = -\frac{1}{\omega_{k \rightarrow 0}} \lim \text{Im } \Pi(k, \omega) = 2\pi e^2 t \delta(\omega). \quad (8)$$

The phase correlator exhibits quasi-long-range order,

$$\langle \exp i(\hat{\varphi}_n - \hat{\varphi}_m) \rangle \propto |n - m|^{-\alpha}, \quad (9)$$

with $\alpha = \sqrt{J/8\pi^2 t}$. Hence, in the limit $t \gg J$, the average dimension of the K/AK pair diverges, $\xi \rightarrow \infty$, providing the conducting ground state. Now the elementary excitations are phononlike excitations of the phase. This transformation is similar to what occurs in the Josephson-junction (JJ) array: gapped charge excitations in the insulating state transform into gapless phase excitations in the superconducting state.

Next, we consider the quantum dynamics of the stripe at arbitrary t/J . The calculation of the complete energy spectrum corresponding to the string Hamiltonian (2) is a difficult task. However, in the long-wavelength limit $k \rightarrow 0$ the physics of the stripe can be described by a continuous SG model [see Fig. 1(b)] with Hamiltonian

$$\hat{H}_x = \int dn \left[t \hat{p}_n^2 + \frac{J}{2} \left(\frac{\partial \hat{x}_n}{\partial n} \right)^2 - \eta \cos(2\pi \hat{x}_n) \right]. \quad (10)$$

Here the restriction to the integer values of the coordinate \hat{x}_n is provided by the potential term $-\eta \cos(2\pi \hat{x}_n)$.

It can be shown¹¹ that the partition function of a 2D XY model is the imaginary time version of the action corresponding to the real-time Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{m^4}{\lambda} \left[1 - \cos \left(\frac{\sqrt{\lambda}}{m} \varphi \right) \right], \quad (11)$$

where we have rescaled fields and coordinates as $x_n \rightarrow \beta \varphi / 2\pi$ and $(n, \tau) \rightarrow (n, \tau/c)$. Here, $\beta^2 = \lambda/m^2 = (2\pi)^2 \sqrt{2t/J}$. In Eq. (11), m is the mass of the elementary boson of the theory and λ its coupling constant.

Although the equivalence of the SG model to our starting Hamiltonian (1) is strictly correct only near criticality, both models are dominated by K/AK excitations, so that also away from criticality, the two models should have very similar properties. The SG model is further clearly a natural choice to describe an elastic string in a periodic potential and our derivation of the SG model from the lattice model (1) provides us with a relation of the phenomenological parameters of the SG model to the more microscopic parameters of the lattice Hamiltonian.

The excitations of the SG theory are known exactly, and consist of fermionic solitonlike excitations and bosonic bound states. The quantization about the so-called ‘‘breather’’ or ‘‘doublet’’ solution¹³ leads to a set of discrete states whose energies are the doublet masses

$$M_N = 2M_s \sin \left(\frac{N\gamma}{16} \right), \quad (12)$$

$$\gamma = \frac{\beta^2}{1 - \beta^2/8\pi}$$

is the renormalized coupling constant of the SG model, M_s is the soliton mass and $N=1, 2, \dots, 8\pi/\gamma$. In the weak-coupling regime, $\beta^2 < 4\pi$, $M_s = 8m/\gamma$.

These results suggest that we could regard the doublet as a bound state of the quantum soliton-antisoliton pair. This is valid once $\gamma < 8\pi$ ($\beta^2 < 4\pi$); otherwise no bound state would survive in Eq. (12). The present interpretation can be further exploited if we use the equivalence of Eq. (11) with the massive Thirring model (MTM),¹⁴

$$\mathcal{L}_{\text{MT}} = i\bar{\Psi} \gamma_\mu \partial^\mu \Psi - m_F \bar{\Psi} \Psi - \frac{g}{2} (\bar{\Psi} \gamma^\mu \Psi)(\bar{\Psi} \gamma_\mu \Psi), \quad (13)$$

where $\gamma_0 = \sigma_x$, $\gamma_1 = -i\sigma_y$, and Ψ is a two-component (right and left movers) fermionic field. The constants m_F and g are, respectively, the mass of the fermions and the coupling constant for their self-interaction. This equivalence allows us to identify the soliton of the SG model with the fermion of the MTM, and thereby Eq. (12) can also be regarded as a set of bound states of fermions and antifermions of the latter.

The relationship between the coupling constants of the two models is¹³ $4\pi/\beta^2 = 1 + g/\pi$, which clearly shows us

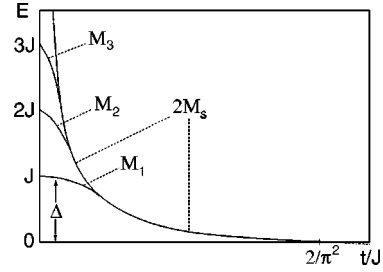


FIG. 2. Infrared energy spectrum of the quantum string and the effective masses of its doublet excitations. Every line shows the lower infrared boundary of continuum. The insulating gap Δ turns to zero at $t/J = 2/\pi^2$.

that when $\beta^2 < 4\pi$, $g > 0$. This implies that particles and antiparticles should attract one another, in agreement with our previous interpretation. For $\beta^2 \rightarrow 4\pi$ one has $g \rightarrow 0$ and, therefore, fermions and antifermions are about to decouple. The last remaining bound state has mass $M_1 = 2m/\pi$, which turns out to be twice the solitonic mass $M_s = 8m/\gamma$ for $\gamma = 8\pi$. At $\beta^2 = 4\pi$ fermions and antifermions are no longer bound and can freely move along the line.

The region where $4\pi < \beta^2 < 8\pi$ [see Eq. (12)] means the very strong-coupling regime of the SG theory. The coupling constant g is negative, which means that particles and antiparticles should now repel each other. The spectrum of excitations still presents a gap¹⁵ that vanishes as we approach $\beta^2 = 8\pi$ ($\gamma \rightarrow \infty$), and beyond this point, the quantum-mechanical SG potential becomes unbounded below.¹⁴

Actually, it has been shown by many authors^{10,16} that the system undergoes a Kosterlitz-Thouless phase transition at $\beta^2 = 8\pi$ or $g = -\pi/2$. Close to this transition, the SG model can be obtained from a model consisting of two different kinds of relativistic massless fermions which is nothing but a spinful Luttinger model (LM).¹⁶ Actually, the fermions to which we refer above can be thought of as spin excitations of the LM, once backscattering processes are considered. When the latter becomes relevant, the corresponding excitation has the SG model as a fixed point.

In the string language (see Fig. 2), we find that for $t \ll J$, the excitation spectrum is basically given by $M_N \approx Nm$, which means N elementary bosons of the SG theory. It also allows us to identify $m = J$ (the lowest elementary excitation for $t = 0$) and consequently $\lambda = (2\pi)^2 \sqrt{2tJ^3}$. As t is increased these turn into N particle bound states which are just the excited states of the K/AK pairs. We can imagine the string being pinned by the lattice, and at least an energy M_1 would be necessary to create a bound K/AK pair (the elementary boson of the SG theory). At $t/J = 1/2\pi^2$ ($\beta^2 = 4\pi$) the pair K/AK becomes free. When $t/J > 1/2\pi^2$ there still exists a gap for the formation of the pair but this bosonic gap vanishes as one approaches the critical value $(t/J)_c = 2/\pi^2$. Beyond this point the SG potential is irrelevant; the string is no longer pinned and can freely move over the antiferromagnetic plane. Hence it exhibits a Gaussian dynamics, with associated logarithmic wandering. The spontaneous symmetry breaking of the discrete system is removed, and the string becomes invariant with respect to arbitrary transversal translations. In principle, a transversal sliding mode

exists. The invariance of the state to transversal translation gives rise to acoustic excitations of the form $\omega = c^*k$, with a renormalized velocity $c^* < c$.

The behavior of correlator (9) is in complete agreement with the interpretation that above $(t/J)_c$ the spectrum of the quantum string should be the same as for the LM we mentioned above. As is well known, all the correlation functions of a LM should present an algebraic decay.¹⁵ Whereas for the equivalent model of a JJ chain this really means an insulator-superconductor transition, here it only reflects the depinning of the string or, in other words, an insulator/metal transition. It would only require a vanishingly small electric field perpendicular to the string to depin it. This fact is also reflected in our expression (8) for the perfect conductivity of the system.

Based on the Josephson chain, as well as on the sine-Gordon results, it follows that at $(t/J)_c = 2/\pi^2$ the quantum string undergoes a KT transition. This transition has been previously predicted,^{7,9} and treated as roughening of the string. In addition, Viertiö and Rice¹⁷ calculated the energy for creating a K/AK pair, and showed that for large t/J values this energy becomes negative, leading to a proliferation of K/AK pairs. Here we have shown that at the transition point the gap Δ vanishes, and the bosonic excitation disappears. Notice that our results are based on the single stripe picture, and we do not necessarily expect them to remain valid at higher doping concentrations.

We want to emphasize that at finite temperatures ($T \neq 0$), thermal fluctuations will “spoil” the quasi-long-range or-

dered phase. In this case, the Euclidean action (5) describes an XY model on a 2D lattice, which is finite in the τ direction, with length $L = 2\pi/T$. Then the KT transition disappears, and the long-range phase correlations are suppressed.

Finally, we can summarize our results: at $t=0$, the ground state (GS) of the string is the kink vacuum. At $0 < t/J < 2/\pi^2$, the energy spectrum is gapped and the system is insulating. At $t/J > 2/\pi^2$ there is no gap anymore, the phase is quasi-long-range ordered, the GS is that of the LM, and the system is a perfect conductor. Thus our results at the critical region agree with the ones obtained from the mapping onto the Josephson chain, with the advantage that they clarify the physical meaning of the insulating and superconducting states for the quantum string. In addition, we propose a phenomenological model which provides us with the spectrum of a quantum string for *any value of t/J* .

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