Higher-Order QCD Corrections in Prompt Photon Production

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We exhibit a method for simultaneously treating recoil and threshold corrections in single-photon inclusive cross sections, working within the formalism of collinear factorization. This approach conserves both the energy and transverse momentum of resummed radiation. At moderate p_T, we find the potential for substantial enhancements from higher-order perturbative and power-law nonperturbative corrections.

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Collinear factorization for short distance, inclusive cross sections is an important tool of particle physics. Next-to-leading order (NLO) corrections in the strong coupling α_s are known for a wide, and still growing, set of reactions. NLO cross sections, however, are not uniformly successful in describing hard-scattering data. This has suggested the necessity of examining yet higher orders in α_s. Among the widely discussed examples is prompt, or direct, photon production [1,2].

In this Letter, we describe a new approach for studying higher-order corrections in the context of collinear factorization for the prompt photon cross section,

$$p_T^3\,\frac{d\sigma_{AB\to\gamma X}(x_T^2)}{dp_T} = \sum_{ab}\int dx_a\,\phi_{a/A}(x_a,\mu)\int dx_b\,\phi_{b/B}(x_b,\mu)\,p_T^3\,\frac{d\hat\sigma_{ab\to\gamma X}(\hat x_T^2,\mu)}{dp_T}\,, \tag{1}$$

where $d\hat\sigma_{ab\to\gamma X}/dp_T$ is the hard-scattering function at fixed p_T. Hadronic and partonic scaling variables are $x_T^2 \equiv 4p_T^2/S$ and $\hat x_T^2 \equiv 4p_T^2/\hat s$, respectively, with $\hat s = x_a x_b S$ the partonic center-of-mass (c.m.) energy squared. As $\hat s$ approaches its minimum value at $\hat x_T^2 = 1$, the phase space available for gluon bremsstrahlung vanishes, which results in large corrections to $d\hat\sigma/dp_T$ at all orders. Threshold resummation [3,4] organizes this singular, but integrable, behavior of $d\hat\sigma/dp_T$.

Another source of higher-order corrections is the recoil of the observed particle against unobserved radiation. Thus, for Eq. (1), beginning with NLO in $d\hat\sigma/dp_T$, a soft gluon may be radiated before the hard scattering, say a QCD-Compton process $gq\to\gamma q$. The outgoing γq pair recoils against the soft gluon, and, as a result, the Compton process may be softer than would be the case without initial-state radiation.

Valuable insights have emerged in studies of direct photon production in which the partons initiating the hard scattering are described by generalized parton distributions [5,6]. Such distributions include transverse momentum that is partly nonperturbative and partly perturbative [7]. Nevertheless, it is difficult to state confidently whether intrinsic transverse momentum is required by the data, which themselves allow varied interpretations [8]. Li [9] first showed how to develop a joint resummation in both leading threshold and transverse momentum logarithms in parton distributions.

Our approach remains within the formalism of collinear factorization. Contributions to the hard-scattering function associated with threshold resummation are redistributed over soft gluon transverse momenta, simultaneously conserving energy and transverse momentum. Accounting for recoil leads to an additional enhancement.

To compute higher-order recoil effects in Eq. (1), we consider the partonic cross section [3] for $ab\to\gamma c$, and fix the c.m. rapidity η of the photon. Near threshold, the overall process consists of a hard $2\to 2$ subprocess, along with soft radiation, which can be factorized. To leading power, $1/Q_T^2$, this soft radiation does not change the flavor of the initial-state partons, and the hard-scattering subprocess recoils from soft radiation with a transverse momentum $\mathbf{Q}_T$. For fixed $\mathbf{p}_T$, the transverse momentum of the photon relative to the c.m. of the hard scattering is $\mathbf{p}_T - \mathbf{Q}_T/2$. The c.m. rapidity is thus related to $\mathbf{Q}_T$ by

$$\frac{1}{\cosh^2\eta} = \frac{4|\mathbf{p}_T - \mathbf{Q}_T/2|^2}{\tilde s} \equiv \frac{4|\mathbf{p}_T'|^2}{\tilde s} \equiv \tilde x_T^2\,, \tag{2}$$

where $\tilde s \le S$ is the c.m. energy squared of the $2\to 2$ hard subprocess. The relative transverse momentum $\mathbf{p}_T'$ thus sets the minimum value of $\tilde s$, and the scaling variable for the subprocess is $\tilde x_T^2$. This kinematic linkage between transverse momentum and partonic energy drives the quantitative effects of recoil. We must limit the size of Q_T in this analysis to avoid going outside the region where the singularities in Q_T dominate. For small Q_T, however, we can jointly resum large corrections to $d\sigma_{ab\to\gamma c}/d^2\mathbf{Q}_T\,dp_T$, in logarithms of $1-\hat x_T^2$ and Q_T. The latter cancel in the hard-scattering functions. Like threshold corrections, however, they may leave finite remainders.

In summary, the resummed inclusive cross section for the leading contributions to prompt photon production is

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of the form:

$$p_T^3 \frac{d\sigma_{ab\to\gamma c}^{(\mathrm{resum})}(\bar{\mu})}{dp_T} \sim \int d^2\mathbf{Q}_T\, p_T^3 \frac{d\sigma_{ab\to\gamma c}^{(\mathrm{resum})}}{d^2\mathbf{Q}_T\, dp_T}\, \Theta(\bar{\mu} - Q_T), \tag{3}$$

where $\bar{\mu}$ is a cutoff. On the right-hand side, the direction of $\mathbf{p}_T$ may be chosen arbitrarily, because of the azimuthal symmetry of the overall process. The inclusive partonic cross section $p_T^3 d\sigma_{ab\to\gamma c}^{(\mathrm{resum})}/dp_T$ in Eq. (3) determines the resummed hard-scattering function $p_T^3 d\hat{\sigma}_{ab\to\gamma c}^{(\mathrm{resum})}/dp_T$ in Eq. (1), after collinear factorization.

The arguments leading to a jointly resummed formula for $d\sigma_{ab\to\gamma c}^{(\mathrm{resum})}/d^2\mathbf{Q}_T dp_T$ are similar to those for transverse momentum resummation in Drell-Yan production. The possibility of joint resummation for singular behavior in Q_T and $1 - \hat{x}_T^2$ is ensured by the factorization properties of the partonic cross section near threshold, which we assume here. The resummed cross section that results from these considerations is

$$\frac{p_T^3 d\sigma_{AB\to\gamma X}^{(\mathrm{resum})}}{dp_T} = \sum_{ij} \frac{p_T^4}{8\pi S^2} \int_C \frac{dN}{2\pi i}\, \tilde{\phi}_{i/A}(N,\mu)\tilde{\phi}_{j/B}(N,\mu) \int_0^1 d\tilde{x}_T^2\, (\tilde{x}_T^2)^N \frac{|M_{ij}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} \times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta(\bar{\mu} - Q_T) \left(\frac{S}{4\mathbf{p}_T'^2}\right)^{N+1} P_{ij}\left(N, \mathbf{Q}_T, \frac{2p_T}{\tilde{x}_T}, \mu\right), \tag{4}$$

in terms of moments of the physical, hadronic parton distributions and of the squared $2 \to 2$ amplitude $|M_{ij}|^2$, with $\mathbf{p}_T'$ defined in (2). The function $P_{ij}(N, \mathbf{Q}_T)$ is a "profile" of $\mathbf{Q}_T$ dependence for fixed N. Recoil is incorporated in Eq. (4) through the interplay of the profile function and the factor $(S/4\mathbf{p}_T'^2)^{N+1}$. Singular $\mathbf{Q}_T$ behavior is most easily organized in impact parameter space, where logarithms of both the transform variables b and N exponentiate. We may thus write the profile functions in Eq. (4) as

$$P_{ij}(N, \mathbf{Q}_T, Q, \mu) = \int d^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\mathbf{Q}_T} \times \exp[E_{ij\to\gamma k}(N, b, Q, \mu)], \tag{5}$$

where azimuthal symmetry insures that E is a function of $b = |\mathbf{b}|$ only. Here and below, $Q \equiv 2p_T/\tilde{x}_T$ denotes the hard scale in the exponent. The parameter μ represents both the renormalization scale and the factorization scale, whose explicit dependence we denote by μ_f below. Equation (4) reverts to a threshold-resummed prompt photon cross section when recoil is neglected, by setting b to zero in the exponents E of (5).

We now construct the exponents $E_{ij\to\gamma k}$, to NLL in both b and N. As already noted by Lai and Li [6], logarithmic recoil corrections (logarithms of b) are generated entirely through initial-state radiation. Final-state interactions, however, do produce logarithms of N. We may thus conveniently split each exponent into initial- and final-state parts:

$$E_{ij\to\gamma k}(N, b, Q, \mu) = E_{ij}^{\mathrm{IS}}(N, b, Q, \mu) + E_{ijk}^{\mathrm{FS}}(N, Q, \mu). \tag{6}$$

In general, resummation of perturbative logarithms leads to nonperturbative power corrections in both b and N/Q. Because our primary interest is in transverse momentum distributions, however, we incorporate only a term proportional to b^2 in E^{IS} [7], associated with the region of strong coupling [10]. The initial-state exponents E_{ij}^{IS} will therefore be a sum of perturbative and nonperturbative contributions. The final-state exponent, E^{FS} in Eq. (6) is essentially the same as for pure threshold resummation [3].

Rather than giving a formal derivation of the NLL perturbative exponent, we motivate our expression by requiring it to reproduce both threshold and k_T resummations in the appropriate limits. The following expression for the initial-state exponent, including a nonperturbative term, gives all leading and next-to-leading logarithms in b and N:

$$E_{ij}^{\mathrm{IS}}(N, b, Q, \mu) = \int_{Q\chi^{-1}(N,b)}^{\mu_f} \frac{d\mu'}{\mu'} \{A_i[\alpha_s(\mu'^2)] + A_j[\alpha_s(\mu'^2)]\} 2\ln\frac{\bar{N}\mu'}{Q} - b^2 F_{ij}(N, Q), \tag{7}$$

where $\bar{N} \equiv Ne^{\gamma_E}$ with γ_E the Euler constant. At this point, the coefficient $F_{ij}(N, Q)$ is arbitrary. The function $A_a(\alpha_s) \equiv \sum_n (\alpha_s/\pi)^n A_a^{(n)}$ is, as usual, $A_a^{(1)} = C_a$ and $A_a^{(2)} = (C_a/2)[C_A(67/18 - \pi^2/6) - 10T_R N_f/9]$, with $C_q = C_F$, and $C_g = C_A$.

The perturbative exponent in Eq. (7) is similar to initial-state contributions in threshold resummation [3,4], but with a new lower limit for the integral over the coupling scale μ', which we denote $Q\chi^{-1}(N, b)$. The following simple choice of χ is accurate to NLL:

$$\chi(N, b) = \bar{N} + bQ/c_1, \tag{8}$$

where $c_1 = 2e^{-\gamma_E}$. This doubly resummed NLL exponent has the important property of respecting momentum conservation, not only for transverse components as in k_T resummation, but for energy as well.

The exponent in (7) is a function of the logarithmic variables,

$$\lambda = b_0 \alpha_s(\mu^2) \ln\bar{N}, \quad \beta = b_0 \alpha_s(\mu^2) \ln[\chi(N, b)], \tag{9}$$

where $b_0 = (12\pi)^{-1}(11C_A - 4T_R N_f)$. The initial-state contributions are given in these terms by

$$E_{ij}^{\rm IS}(N,b,Q,\mu) = \sum_{a=i,j}\left[\frac{1}{\alpha_s(\mu^2)}\,h_a^{(0)}(\lambda,\beta) + h_a^{(1)}(\lambda,\beta,Q,\mu,\mu_f)\right], \tag{10}$$

where, adopting the notation of Ref. [4],

$$h_a^{(0)}(\lambda,\beta) = \frac{A_a^{(1)}}{2\pi b_0^2}\,[2\beta + (1-2\lambda)\ln(1-2\beta)], \tag{11}$$

$$h_a^{(1)}(\lambda,\beta,Q,\mu,\mu_f) = \frac{A_a^{(1)}b_1}{2\pi b_0^3}\left[\frac{1}{2}\ln^2(1-2\beta) + \frac{1-2\lambda}{1-2\beta}[2\beta + \ln(1-2\beta)]\right] + \frac{1}{2\pi b_0}\left[-\frac{A_a^{(2)}}{\pi b_0} + A_a^{(1)}\ln\left(\frac{Q^2}{\mu^2}\right)\right]$$
$$\times\left[2\beta\,\frac{1-2\lambda}{1-2\beta} + \ln(1-2\beta)\right] - \frac{A_a^{(1)}}{\pi b_0}\,\lambda\ln\left(\frac{Q^2}{\mu_f^2}\right), \tag{12}$$

with $b_1 = (17C_A^2 - 10C_A T_R N_F - 6C_F T_R N_F)/24\pi^2$.

Given the above, we are in a position to illustrate the role of recoil, using Eq. (4), through the evaluation of the N and Q_T integrals, with profile functions at fixed N defined by Eqs. (5)–(12). As in threshold resummation, it is necessary to define the integrals in a manner that avoids the ("Landau") singularities in the running coupling of Eq. (7) at large N and/or b. For pure threshold resummation, a number of methods have been employed to define the N integral in (4). We will use a "minimal" approach [4,11], defining the contour C in (4), to extend to infinity through the negative real half plane. For the b integral in the profile function, Eq. (5), we have developed a new integration technique.

To perform the b integral, we rewrite Eq. (5) as

$$P_{ij}(N,\mathbf{Q}_T,Q,\mu) = \pi\int_0^\infty db\,b[h_1(bQ_T,v) + h_2(bQ_T,v)]e^{E_{ij\to\gamma k}(N,b,Q,\mu)}. \tag{13}$$

Here we introduce two auxiliary functions $h_{1,2}(z,v)$ related to Hankel functions and defined in terms of an arbitrary real, positive parameter v by integrals in the complex θ plane:

$$h_1(z,v) \equiv -\frac{1}{\pi}\int_{-iv\pi}^{-\pi+iv\pi} d\theta\, e^{-iz\sin\theta}, \qquad h_2(z,v) \equiv -\frac{1}{\pi}\int_{\pi+iv\pi}^{-iv\pi} d\theta\, e^{-iz\sin\theta}. \tag{14}$$

The $h_{1,2}$ become the usual Hankel functions $H_{1,2}(z)$ in the limit $v \to \infty$. They are finite for any finite values of z and v. Their sum is always $h_1(z,v) + h_2(z,v) = 2J_0(z)$, independent of v. The utility of the h functions is that they distinguish positive and negative phases in Eq. (13), making it possible to treat the b integral of the profile function as the sum of two contours, one for each h_i. These contours avoid the Landau pole by a deformation into either the upper half plane (h_1), or the lower half plane (h_2). Such a definition of the integral is completely equivalent to the original form, Eq. (5), when the exponent is evaluated to finite order in perturbation theory. It defines the resummed integral "minimally" [11], without an explicit cutoff.

Proton-nucleon cross sections computed in this fashion are illustrated in Fig. 1. Here we show, for several values of photon p_T, $d\sigma^{\rm (resum)}_{pN\to\gamma X}/dQ_T dp_T$, the distribution of the cross section in the recoil momentum Q_T. The kinematics are those of the E706 experiment [2]. Since this is a "demonstration" calculation, we pick a nominal value of $F_{ij}(N,Q) = 0.5$ GeV^2 for the Gaussian coefficient in Eq. (7), independent of parton type. We leave for future work a more realistic determination of this coefficient, including its Q dependence. The parton distributions are those of Ref. [12], and we treat NLO N-independent ("hard virtual") terms as in [4]. For simplicity, we approximate Q in Eq. (5) by $2p_T$. Finally, we set $v = 1$ in Eq. (14).

The dashed lines are $d\sigma^{\rm (resum)}_{pN\to\gamma X}/dQ_T dp_T$ from Eq. (4), but with recoil neglected by fixing $S/4\mathbf{p}_T'^2$ at $S/4\mathbf{p}_T^2$. The dashed lines thus show how each Q_T contributes to threshold enhancement. The profiles are subtracted in their b-dependent one-loop corrections, which produce the peaks and dips at low Q_T for the smaller values of p_T. Every curve shows a peak (near 2 GeV) associated with resummation, and falls off as a power with increasing Q_T. Integrating the dashed lines reproduces the threshold-resummed cross sections of Ref. [4], after a subtraction at order α_s to recover exact NLO.

The solid lines of Fig. 1 show the N-integrated distributions in Q_T, $d\sigma^{\rm (resum)}_{pN\to\gamma X}/dQ_T dp_T$, now found by including the true recoil factor $(S/4\mathbf{p}_T'^2)^{N+1}$ in Eq. (4). These curves therefore describe the fully resummed cross section. The resulting enhancement is clearly substantial. For small p_T, the enhancement simply grows with Q_T (it must diverge at $Q_T = 2p_T$), while for p_T above 5 GeV it has a dip at about $Q_T = 5$ GeV, which becomes more pronounced as p_T increases, but which never really gets to zero. This behavior makes it problematic to identify a unique cutoff $\bar{\mu}$ in (4) that gives a stable prediction for the cross section. A full analysis requires a procedure for "matching" Eq. (4) at $Q_T = \bar{\mu}$ to a cross section appropriate to $Q_T = \mathcal{O}(p_T)$ [7].

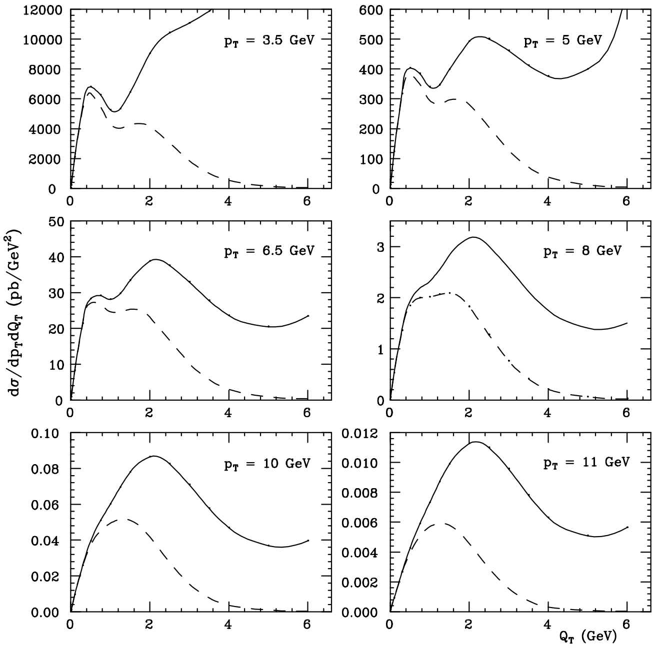

FIG. 1. The prompt photon cross section $d\sigma_{pN\to\gamma X}/dQ_T dp_T$ at $\sqrt{s} = 31.5$ GeV, as a function of Q_T for various values of photon p_T. Dashed lines are computed without recoil [$\mathbf{p}_T' = \mathbf{p}_T$ in (4)]; solid lines are with recoil.

This reservation notwithstanding, and reemphasizing that our calculation is primarily an illustration, rather than a quantitative prediction, we plot in Fig. 2 the resummed cross section for $p_T \geq 3.5$ GeV, with the choice $\bar{\mu} =$ 5 GeV for the calculation of Fig. 1, using the approximate procedure of Ref. [4] to convert $d\sigma_{pN\to\gamma X}/dp_T$ to a cross section integrated over a finite rapidity interval. We include an NLO photon fragmentation component in the cross section, again calculated as in [4]. To show the size of the enhancement that recoil can produce, as well as its potential phenomenological impact, we also exhibit the pure threshold resummed cross section and the E706 direct photon data in this range [2]. For the theoretical curves, we choose $\mu = p_T$; both resummed curves have sharply reduced factorization scale dependence compared to NLO. Evidently, recoil can be phenomenologically relevant.

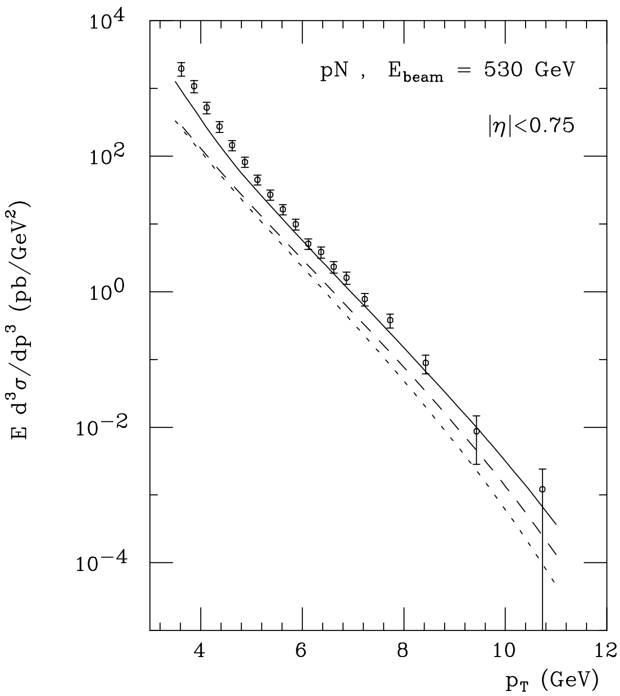

FIG. 2. Prompt photon cross section $Ed^3\sigma_{pN\to\gamma X}/dp^3$ for pN collisions at $\sqrt{s} = 31.5$ GeV. The dotted line represents the full NLO calculation, while the dashed and solid lines, respectively, incorporate pure threshold resummation [4] and the joint resummation described in this paper. Data have been taken from [2].

It will take some time to explore this resummation formalism, including the implementation of practical nonperturbative estimates and of matching procedures. Nevertheless, we hope that this formulation of recoil effects at higher orders, in the language of collinear factorization, is a step toward clarifying what has been a thorny issue in the application of perturbative QCD

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