History & Heritage

rewaq

ISSN 2414-6005

Issue 5, January 2018

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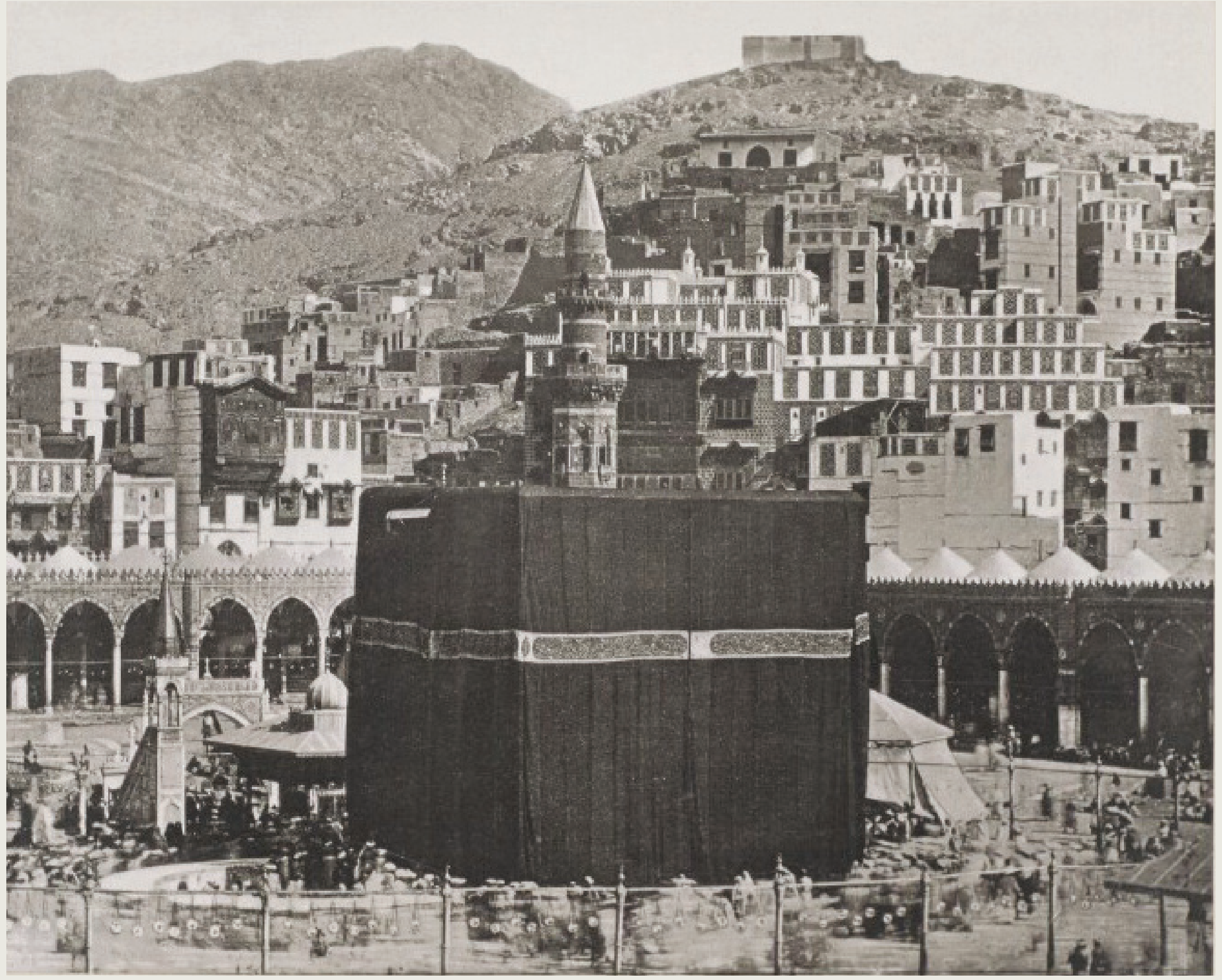

Christiaan Snouck Hurgronje: Bilder-Atlas Mekka, 1888 (من مقتنيات المركز)

Three instruments for finding the direction and distance to Mecca: European cartography or Islamic astronomy?

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All images in this article were chosen by the writer

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Jan P. Hogendijk, researches the history of mathematical sciences in antiquity until the 17th century. He has edited and translated a number of unpublished Arabic manuscripts, such as the Completion of the Conics of Ibn al-Haytham. He also specializes in workshops for a general audience on topics in medieval science, such as the astrolabe.

Many new discoveries in the history of Islamic science have been made in the last 25 years. The year 1989 saw the discovery of a new and hitherto unknown metal instrument for finding the direction and distance to Mecca for all localities in the medieval Islamic world [King 1999, 199]. Three instruments of this type are now known to exist. They were manufactured in Iran, probably in the seventeenth century, and perhaps more such instruments can be found in private collections or museums in Iran or elsewhere.

The discovery was unexpected and exciting, because no documents containing descriptions or drawings of such instruments were known in the medieval Islamic tradition. The three instruments were published by David King, now emeritus professor of the history of science of the University of Frankfurt in Germany, in [King 1999] and [King 2004, 831-846].

Photos of two instruments are available (Fig. 1, Fig. 2, [King 1999, 199, 201]), but King was apparently not allowed to publish a photo of the third instrument. The first instrument is now in the Museum of Islamic Art in Kuwayt, which is in the process of reconstruction, and the two other instruments are in private collections. The third instrument was at one time on display in the Sackler Museum of Harvard University [King 2004, 836], but it does not belong to the permanent collection. Thus, none of the three instruments was available at the moment of writing this article.

The instruments consist basically of three parts: ruler, axis and plate. The ruler pivots on the axis on the plate, which is engraved with a rectangular grid and around 150 small dots with names of cities. Figure 3 is a photo of a detail of the first instrument with the dot indicating Istanbul, and the name Qustantiniyya.

I will now explain the working of the instruments with Istanbul as my example, and referring to Figure 4, suppose we want to find the "qibla", that is the direction of Mecca, at Istanbul. We turn the ruler until the side of the instrument through the axis, passes through the point indicating Istanbul. Then the qibla of Istanbul is indicated on the circular scale of the instrument by the tip of the other half of the ruler, as in Figure 4.

This circular scale is divided in degrees and minutes and the four cardinal directions (North, East, South and West) are indicated. The distance between Istanbul to Mecca is indicated on the ruler by the dot on the plate, which corresponds to Istanbul. This is the distance, which a bird would fly, not the distance along the road. The unit of measurement is the "Farsang", which is about 6 kilometers. The first two instruments once contained a compass. Once the qibla has been found by means of the ruler, the instrument can be held horizontally in such a way that "north" on the instrument corresponds to the northern direction in reality. Then the ruler points to the real qibla.

For cities whose name is not indicated, one can use the grid on the instrument, provided that the geographical coordinates of one's locality are known. The grid divides an area containing the whole Islamic world into little pieces which resemble small squares and rectangles. The horizontal and vertical lines of the grid correspond to meridians and parallels on earth, with intervals of two degrees. The longitude and latitude concepts in medieval Islamic science are almost the same as today: the latitude is Northern latitude with respect to the equator, and the longitude was measured with respect to a zero meridian through the Canary Islands, which were considered to be the westernmost part of the inhabited world. The horizontal curved lines on the instrument correspond to the parallels on earth for Northern latitudes between 8 and 52 degrees, and the vertical lines correspond to the meridians for longitudes between 29 and 125 degrees according to the medieval Islamic convention. Figure 1 and 2 show that, the vertical lines are not equidistant, the horizontal lines are curved, and the markings on the ruler, which indicate the distance to Mecca, are not equidistant either, but approach one another near the end. So clearly the instrument is based on sophisticated mathematics.

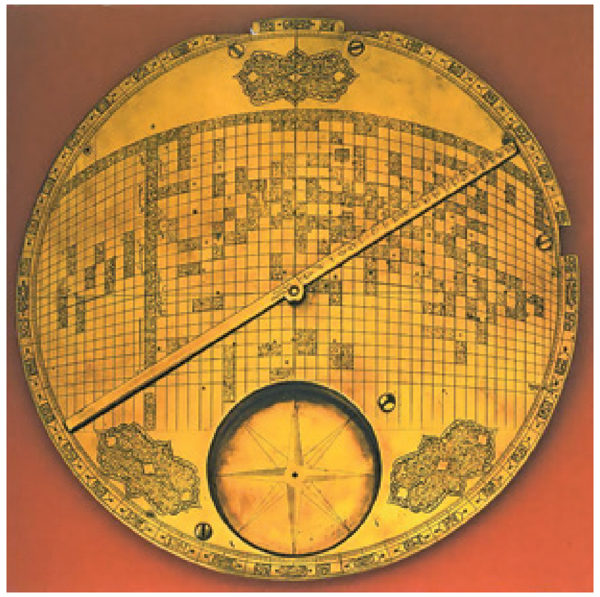

Figure 1: The first instrument, discovered in 1989.

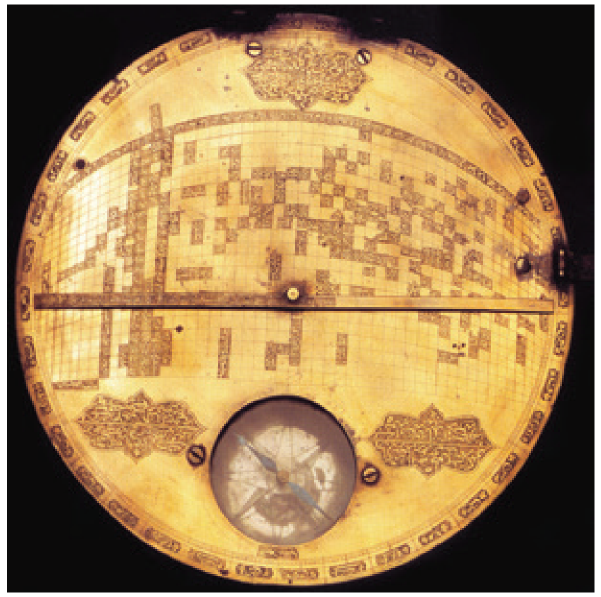

Figure 2: The second instrument, discovered in 1995.

David King and Francois Charette, showed in [King 1999, 240, 254] that the instrument produces essentially correct qibla values. This is a really exciting result because finding the qibla is not an easy mathematical problem, as we shall see below. King called the instrument a world map, because approximately 150 cities are engraved at the instrument, even though coast lines and mountains are not indicated.

The theory behind the instrument was further investigated by the Dutch historian of science Elly Dekker. She showed that, the curves on the instruments are in theory arcs of ellipses, which can be approximated as circles [Dekker, 2000]. For practical reasons, the instrument makers must have used circles in their construction of the instrument [King 2004, 838]. Dekker showed that, if the instrument is interpreted as a world map, the projection is a so-called 'retro-azimuthal projection' which was invented in western cartography in [Jackson, 1968]. My student Eelco Nederkoorn reprogrammed the instrument in 2003, using the same retro-azimuthal projection that was used in the original instrument, on the basis of modern satellite images. This project was carried out at King Fahd University of Petroleum and Minerals in Dhahran (Saudi Arabia) and it was jointly supervised by Dr. Hassan Azad (KFUPM) and me.

Nederkoorn's modern version of the instrument [Figure 5] consists of a plastic ruler which can rotate around an axis (a button) over a plate. The plate contains a projection of half of the Earth, on which some important cities have been indicated by dots, just as in the three instruments which were published by King. If you turn the ruler until the side through the central axis passes through a city, the tip of the ruler will indicate the qibla of the city, and the distance between the city and Mecca is indicated on the scale on the ruler (Farsang have been changed to kilometers).

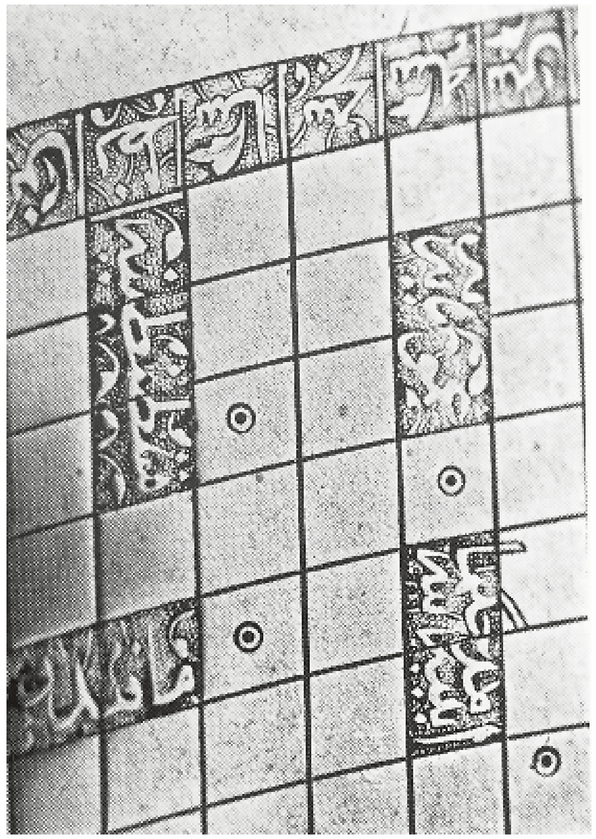

Figure 3: Detail of the first instrument, showing Istanbul in the upper left

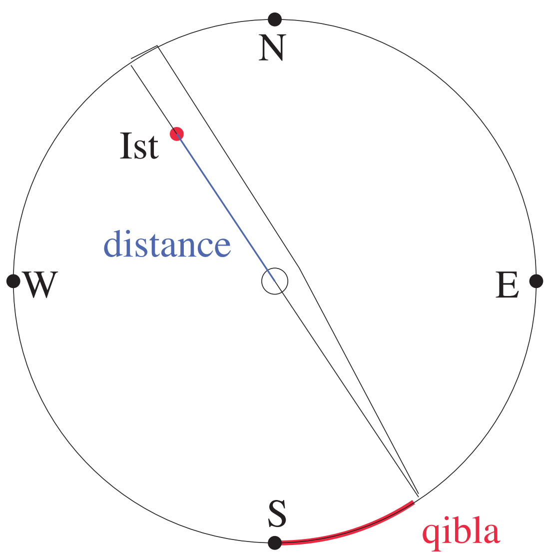

Figure 4: Finding the qibla at Istanbul.

In Figure 6, which was also programmed by Mr. Nederkoorn, the retro-azimuthal projection is extended to the other half of the earth. The United States and other areas approximately 90 degrees away from Mecca have a rather insignificant projection, and something strange happens near the North and South poles. We will not discuss these phenomena here because they are irrelevant, when the projection is limited to the medieval Islamic world.

This paper will focus on the origin of the instrument. King and Charette thought that, the instrument was invented in the Islamic world. Dekker, who analyzed the instrument, argued that "[i]t seems therefore legitimate that the maps, [i.e., the grid] reflect a European influence.

Could it be that this.... projection had already been discovered, [meaning: by European cartographers] centuries before Jackson described it in 1968? And if so, might this have happened as part of the, as yet little studied cultural exchanges which took place between Europe and Persia in the seventeenth and later centuries?" [Dekker 2000, 116]. Thus, Dekker thinks that the grid on the instrument could possibly have been invented in Europe. She also states that "the projection underlying the maps on the instruments cannot be linked to Islamic mappings, known from the ninth century".

In support of Dekker's suggestion it can be remarked that, a sundial, which is certainly influenced by European workmanship, was attached to the second instrument (see the photo in [King 1999, 212]). However, this sundial may not have belonged to the instrument, as it was originally designed. So our main question is whether the instrument, meaning the [plate with the ruler and the axis, represents an European or Islamic invention.

To prove that the Instrument was invented in the Islamic world, one would need a similar instrument which can be dated back to the fifteenth century or before, when cartography was almost non-existent in Western Europe. Alternatively, a description of the instrument in an Islamic text before 1500 would also suffice. No such text has yet been found. In the rest of this paper, I will give a new interpretation in which the instrument is related to Islamic astronomy, rather than cartography. My new interpretation suggests that the instrument is of Islamic origin. As an introduction, I will briefly review the history of medieval Islamic methods for finding the qibla.

The early Muslims were not familiar with sophisticated mathematics and in the first century of Islam, finding the qibla was not regarded as a mathematical problem. At that time, the qibla was a direction, but not a number of degrees and minutes. For example, in Al-Madina and Damascus, the qibla was due south. At the time of Caliph al-Ma'mun or before, Muslim astronomers began to determine the qibla by mathematical means.

They knew that the earth is a sphere and defined the qibla of a locality as the direction of the shortest distance (as a bird would fly) between the locality and Mecca. In order to determine the qibla at Baghdad, Caliph al-Ma'mun had various measurements made, such as the geographical latitude of Baghdad and Mecca, and the difference in geographical longitude between Baghdad and Mecca. From these data, his astronomers determined the qibla by an approximate method. The result of their computation has not been preserved, but they must have found a "deviation" (inhiraf) of the qibla in Baghdad from the southern directions of approximately 13 degrees towards the West. [King 2000].

Probably around the same time, Muslim astronomers rephrased the problem of finding the qibla in an astronomical way. The following may serve as an introduction. Mecca lies south of the tropic of Cancer. Thus, on two days of every solar year (in the Western calendar on May 27 and July 15), the sun will pass through the zenith of Mecca, that is to say that, it will be right above the head of the inhabitants of Mecca at noon[(1)]. If we know the difference in geographical longitude between our city and Mecca, we can determine that moment, in our local time. According to modern values, Mecca is located 10 degrees and 48 minutes East of Istanbul.

Since the (apparent) daily rotation of the sky is one degree per four minutes of time, the sun is in the zenith of Mecca 43 minutes and 12 seconds before it reaches its highest point in Istanbul. So if you are in Istanbul and watch the sun on May 27 or July 15, exactly 43 minutes and 12 seconds, before local (solar) noon time, the direction of the sun will be the same as the direction of Mecca. The Islamic astronomers defined the zenith of Mecca, using the following methodology, which they had inherited from their Greek predecessors.

[Figure 7] Imagine a very large (Imaginary) sphere around us, so large that the earth is only a tiny ball in the centre. This

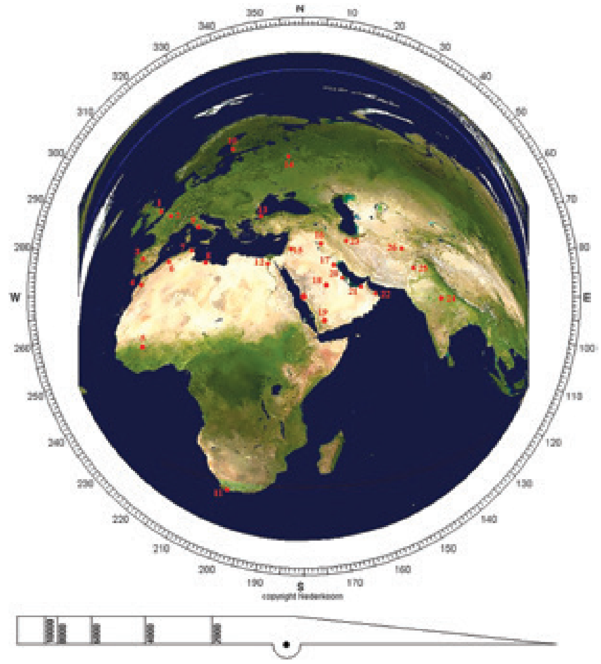

Figure 5: Nederkoorn's modern version of the instrument.

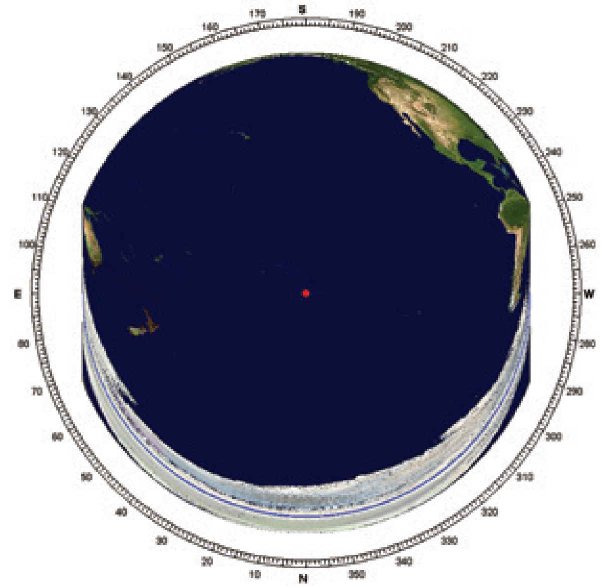

Figure 6: Nederkoorn's map of the other half of the earth

1 Details: This happens when the declination δ of the sun is equal to the geographical latitude of Mecca ($\O M = 21° 25'$) IF we take the obliquity of the ecliptic as $\varepsilon = 23° 30'$, we can show that, this is the case if the sun is in 6 ½ degrees Gemini or in 23 ½ Cancer, using the formula $\sin \delta = \sin \lambda \sin \varepsilon$ with λ the ecliptical longitude of the sun. Of course the center of the sun will never be exactly above the head of the people of Mecca, but the difference will only be a few minutes of arc. Between May 27 and July 15, the center of the sun will be slightly North of the zenith in Mecca at noon.

sphere is called the celestial sphere. We consider ourselves to be in Istanbul. On the sphere we consider the zenith of Istanbul, which is the point exactly above our head. It is the point where the straight line through the center of the earth and us, intersects the celestial sphere.

We can also draw our horizon, that is, the horizon of Istanbul. The straight line from the center of the earth to the zenith of Istanbul is exactly perpendicular to the horizon of Istanbul. We indicate the four cardinal directions (North, West, South, East) on the horizon of Istanbul. Figure 7 also displays a great circle arc[1], through the north point on the horizon and the zenith of Istanbul. This arc passes through the celestial North Pole (the zenith of the North Pole on Earth). As a result of the rotation of the Earth, all celestial bodies seem to rotate around the celestial North Pole. In modern times, the pole star is near the celestial North Pole, and the two are sometimes identified in popular accounts.

In the same way, the straight line through the center of the earth and an observer in Mecca will intersect the celestial sphere in the zenith of Mecca. This is the point at which the sun can be found on May 27, when the time at Istanbul is 43 minutes and 45 seconds before local noon time. We now draw a great circle arc through the zenith of Istanbul and the zenith of Mecca. This arc will intersect the horizon of Istanbul in the qibla.

In modern terms, we have not gained anything, because the big celestial sphere is an enormously blown-up image of the Earth. But for the medieval Islamic astronomers, this transition from the earth to the celestial sphere, was very important. All the mathematical tools which had been developed in ancient Greece and India for astronomical applications, now became available for finding the qibla. A variety of trigonometrical solutions was found soon after the time of Caliph al-Ma'mun, and perhaps even in or before his time.

These complicated computations usually consisted of several steps, which involved the sine and cosine functions and which were based on the sine and cosine tables available in the Islamic astronomical works.[2] Even though, the methods were theoretically correct, they did not always work in practice. Thus, the Iranian astronomer Al-Nayrizi found a theoretically correct computation method, but when he wanted to apply his method for Baghdad, he had bad luck with one of his formulas, so a small computational error which he made in the beginning exploded in

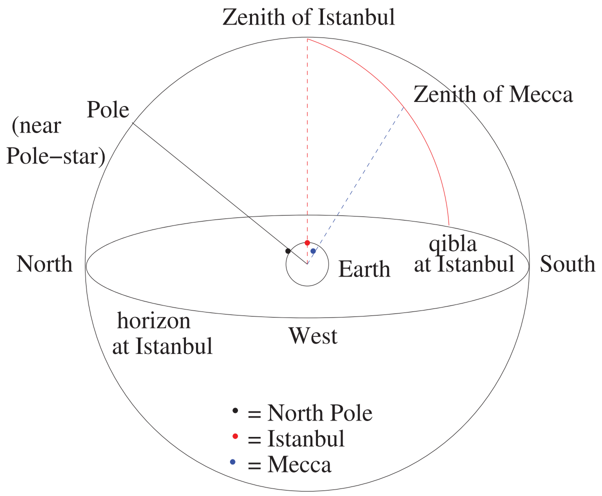

Figure 7: The celestial sphere with the zenith and horizon of Istanbul and the zenith of Mecca.

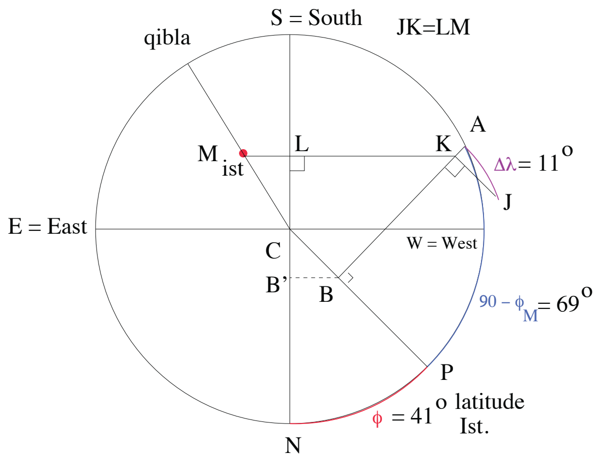

Figure 9: The geometrical construction by al-Biruni.

1 A great circle is a circle whose center is the same as the center of the sphere. Because the celestial sphere is very large, compared to the earth, the radius of the earth can be ignored so the Islamic mathematicians considered the horizon of Istanbul as a great circle.

2 In modern terms, if Ø is the geographical latitude of the locality *ØM*, the geographical latitude of Mecca, Δλ the longitude difference, and *q* the deviation of the qibla from the south, *q* can be found by means of the formula 1/tan *q* = sin Ø / tan Δλ - cos Ø tan *ØM* / sin Δλ.

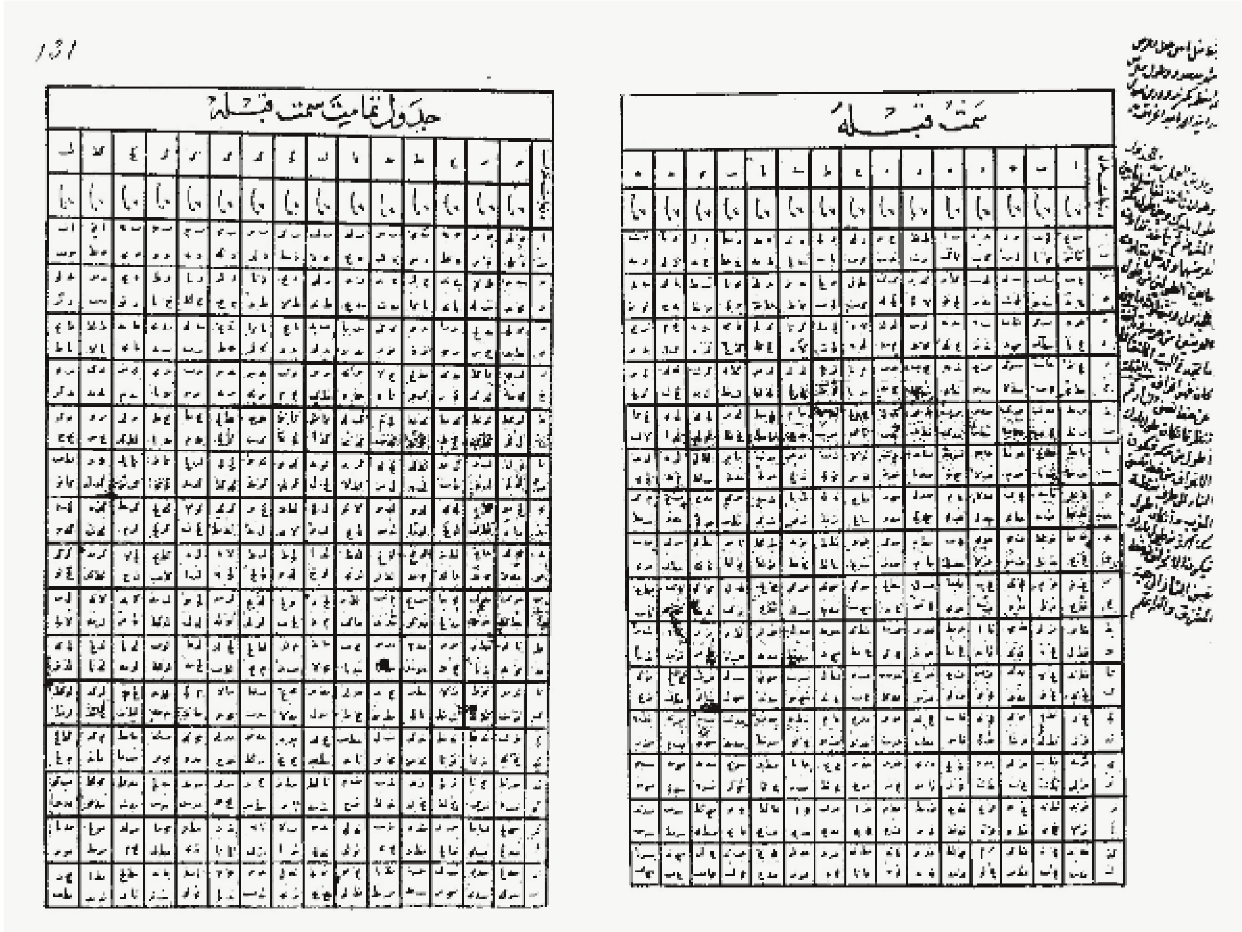

Figure 8: The qibla table in the Zij-I Ashrafi.

the course of his computation. Thus his so-called "correct" value for the qibla at Baghdad was 29° West of South, [Hogendijk, 2000] more than twice as much as the value 13° West of South, which had been found by others. This must have been very confusing for his fellow-scientists, and as far as we know, the cause of the error remained unexplained in the Islamic middle ages. Thus, determining the qibla was a challenging topic in Islamic astronomy and provoked interesting discussions between the Islamic scientists.

From the ninth century onwards, Islamic astronomers intended to solve the qibla problem for the entire Islamic world by putting together the results of their computations in tables. One such table is found in the astronomical handbook Zij-i Ashrafi [Fig 8], which was finished in 1305, although the table is probably much older. The table was computed according to a mathematically exact method.

The table contains for every degree of difference of latitude between one's city and Mecca and for every degree of the longitude difference, the deviation of the qibla from the South, in degrees and minutes. In this table, there are many errors, which is of course not surprising because the computation was difficult [Hogendijk 1994, 94].

The Islamic astronomers also used a geometrical construction of the qibla, which is, as we will see, important in connection with the three instruments. This construction was probably invented by Habash al-Hasib in the ninth century [Kennedy 1974]; it was repeated in many variations in later works.

Fig 9 displays the simple version by al-Biruni (ca. 1025) [Al-Biruni 249252], and the figure is drawn for the example of Istanbul. We begin by drawing a horizontal circle, with the four cardinal directions (North, East, South, West) and the observer at point C in the centre. Using as input, the geographical coordinates of the location (for example Istanbul) and Mecca, a certain point M is constructed in the interior of the circle. Then the straight line CM will be the qibla. The details of the construction can be relegated to a footnote; the only thing that matters here is the fact that, point M depends on the geographical coordinates of Istanbul and Mecca.[1]

The basic idea of this construction is related to astronomy. Al-Biruni and the other astronomers were thinking about the circle NESW as the horizon of Istanbul in the celestial sphere which has been mentioned in Figure 7 before. From the zenith of Mecca, they drop a vertical straight line towards the horizon, and point M is the point where this vertical line hits the horizon.

In Figure 10, we first consider the same point M as in Figure 9, but we write M1st to indicate that this point belongs to Istanbul. Now if we repeat the construction for Konya, with geographical coordinates different from those at Istanbul, so we arrive at another point M, which we will write as MKonya, and another qibla. We can then repeat the construction for Isfahan and Meshed and write MIsfahan and MMeshed and so on.

Thus we obtain a geometrical qibla table, produced by the geometric constructions, as opposed to the numerical qibla table in the Zij-i Ashrafi, which was produced by repeating the computations.

As a final step [Fig 11], we delete the letters M and we shift the cardinal directions. If we work out all mathematical details, it turns out that we obtain precisely the positions of the cities on the instruments which were published by David King. We can also obtain the grid on the instrument and the scale on the ruler from the geometric construction by al-Biruni. This can be done by means of mathematical methods which were available in the Islamic tradition. All one needs to know is that if a circle on the celestial sphere (namely the day circle of Mecca) is projected perpendicularly on the horizon, the projection figure is an

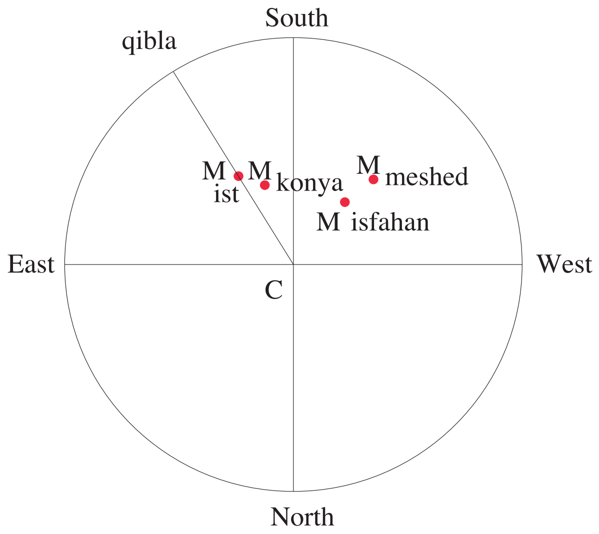

Figure 10: Repeating the geometrical construction by al-Biruni for different cities.

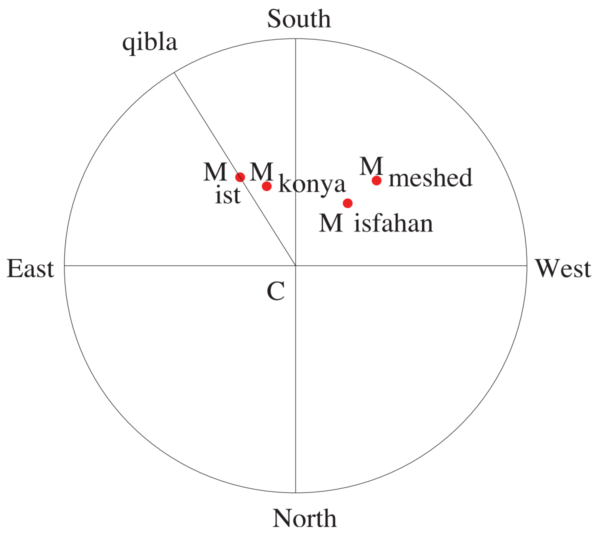

Figure 11: The principle of the instrument.

1 Details: We use rounded modern values for the geographical coordinates of Istanbul (41°, 29°) and Mecca (21°, 40°). Note that, the modern longitudes are measured from Greenwich, but the difference between two longitudes according to modern conventions is the same between the difference between two longitudes according to medieval Islamic conventions. Let N be the north point on the horizon and find P on the circle, such that $\angle NCP = \phi = 41$ [the geographical latitude of Istanbiul]. Then find A such that arc PA is 90° minus the geographical latitude of Mecca, that is 90° – 21° = 69°. Draw the perpendicular AB onto CP, and draw the circle with center B and radius BA on this (small) circle, find point J such that AJ is the longitude difference $\Delta\lambda$ = 40° - 29° = 11° between Mecca and Istanbul. Draw the perpendiculars JK onto AB, and KL onto CS, and let point L be the intersection with CS. Then the required point M is on the rectilinear extension of KL in such a way that $JK=LM$. Line CM points to the qibla in Istanbul.

ellipse. This fact was well known in ancient Greek and medieval Islamic geometry.

In conclusion, the three instruments fit naturally into the Islamic astronomical tradition of qibla determinations. Thus it is not necessary to assume an European origin of the instrument, and it makes sense to continue searching for Arabic or Persian manuscripts, in which, the instrument is described.

I have already found a reference to a method for finding the qibla by means of an ellipse in an Arabic text from the early tenth century – unfortunately the method itself is not explained [King 2004, 842-846]. The idea of the instrument may well date back to Habash al-Hashib in the ninth century, as was suggested by David King for different reasons [King 1999, 345-364]. In any case, I hope that museums of Islamic science and perhaps even metal workers in Isfahan may start producing these qibla finding instrument again, in ancient and modern versions. They can be used to show the splendor of Medieval Islamic science to a wide audience. Thank you very much for your attention.

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