Roel Nicolai [*]

Analysing MapAnalyst and its application to portolan charts

Keywords: Cartometric analysis, distortion grid, MapAnalyst, map accuracy, map projection, portolan chart

Summary: The objective of this paper is to demonstrate that the distortion grid generated by MapAnalyst, a free software package for the cartometric analysis of historical maps, should be computed and interpreted judiciously and not be seen as revealing the immutable structure of implicit parallels and meridians of the map. Awareness of the limitations as well as the capabilities of this software tool is essential. This paper explains the processing method of MapAnalyst and demonstrates in what way this imposes limitations on the analysis of portolan charts. The paper concludes with recommendations on how MapAnalyst can be successfully applied to the analysis of portolan charts and demonstrates this with an example analysis.

Introduction

It is some 120 years ago now that the German geographer Hermann Wagner introduced the cartometric method in the context of research into portolan charts. Wagner proposed to take measurements on a portolan chart in order to establish indisputable facts about that chart that would allow hypotheses to be tested, thus preventing a sequence of "indefinite suppositions" (Wagner 1896, p 695 (476)). He suggests two broad methods:

1. measurement of distances between points on a historical map, comparing those with modern reference values;
2. generation of a graticule of meridians and parallels from the geographic features displayed on the map.

Computers were not available in Wagner's time and therefore cartometric analysis in principle has nothing to do with computers. However, computers have increased its potential enormously and have also led to increased complexity of cartometric methods. Whereas more and more researchers are using numerical analysis methods nowadays, the degree of understanding of these methods among many traditional researchers of historical maps has not always kept pace with these technological developments and it is still an open question whether the implications and limitations of these methods are understood properly. There appears to be a gap between portolan charts researchers with a background in the social sciences or humanities on the one hand and researchers with a background in the exact sciences, such as myself, on the other. It does not help that some cartometric studies have produced results that are inconsistent with one another: Scott Loomer analysed 27 charts and concluded the Mercator projection provides the best match for the coastline shapes of portolan charts (Loomer 1987, 149), while Duken felt that the oblique stereographic projection yields the best fit (Duken 1988). Mesenburg concluded that a conformal map projection provides the best match, but was unwilling to be more specific than that. Accuracy estimates of the charts in the same publications also vary, to which may be added the accuracy estimates for five charts in (Nicolai 2016, 162). Such vari-

[*] Utrecht University, Department of Mathematics, *r.nicolai@uu.nl*

ations may undermine trust in numerical methods of people who do not understand these methods well enough. Some authors even claim that numerical methods add no value (Gautier Dalché 2011, 11 footnote 5 and Pujades 2007, p. 460 and 506), while others will not accept results generated by cartometric analysis on the grounds that its methods are poorly or not understood (Campbell 2016).
During the First International Workshop on the Origin and Evolution of Portolan Charts in Lisbon, June 2016 it became clear that more work is needed to clarify cartometric methods in terms understandable to all. This paper intends to make a contribution to that process.
A number of cartometric studies have been executed using MapAnalyst (e.g. Gaspar 2008 and 2010; Pacheco 2017). MapAnalyst, a free-of-charge open-source software package written in Java, runs on MS-Windows PCs, Apple Macs (Mac OS X) and Linux platforms. It is easy to use and produces graphical output in the form of a distortion grid, which is highly appreciated by many map historians. A distortion grid is derived from the computed position displacements of geographical point features on the old map and therefore reflects the accuracy of the old map to some extent.

The objective of this paper is to demonstrate that the distortion grid generated by MapAnalyst should not be regarded as a kind of 'X-ray image' that reveals the immutable structure of the old map in the form of its hidden implicit parallels and meridians. A distortion grid is indeed a valuable way of visualising aspects of the map's accuracy, but it must be computed and interpreted judiciously. MapAnalyst does not automatically produce results that are correct and repeatable in an absolute sense, because these results depend on the input data and on the way this data has been processed subsequently. For some problems MapAnalyst is a suboptimum or even unsuitable tool.

Variations in the results of cartometric analysis can be a help or a hindrance in establishing knowledge about an old map. They will be a help if the researcher understands the software tool and uses that understanding to find answers to specific questions he or she has asked prior to the analysis. They will, on the other hand, hinder an understanding of the old map's properties if the software tool is not understood well enough or when a researcher has omitted to ask specific questions. One cannot apply a cartometric tool blindly and hope that it will answer a question one did not ask. Alternatively, the analysis tool may be unsuitable or suboptimal for answering the researcher's questions. However, asking the right question may involve iteration. An example is the existence of regional scale variations on a portolan chart, which may be suspected in initial processing results and proven or disproven subsequently after modification of the processing approach.

Conceptual classification of cartometric analysis methods[1]

Knowledge of the characteristics of the method used by any cartometric tool is a prerequisite for numerical analysis of a historical map and that also holds for MapAnalyst.
Cartometric analysis methods can be classified broadly into *interpolation*, *smoothing* and *adjustment* techniques. MapAnalyst uses a combination of adjustment and interpolation.

[1] This section is a abbreviated rendering of Section 4.2 "Quantitative Analysis methods: a conceptual classification" in Nicolai (2016).

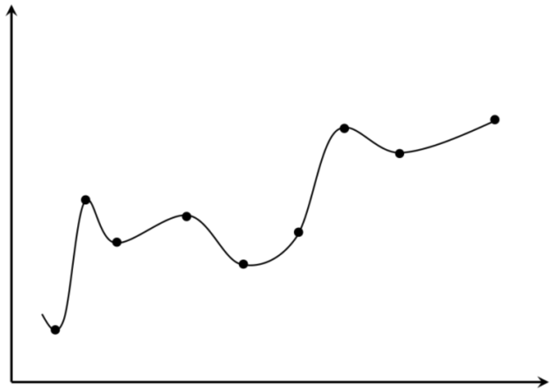

Figure 1: The principle of interpolation
(source: Nicolai (2016), 94)

Interpolation techniques do not correct the sampled, or measured, data, which are represented in Figure 1 by the black dots. They estimate the values in between, assuming that errors change continuously and gradually. Figure 1 shows a fictitious one-dimensional example, that is, one variable along the vertical axis that changes as a function of the horizontal reference value. In Figure 1 not only the interpolated values are continuous, but also the rate of change of the interpolated values. Not all interpolation methods generate a gradually changing rate of change of the interpolated values. It is relatively easy to imagine an extension to the two-dimensional case of a map, in which errors in two directions with respect to two reference values exist.

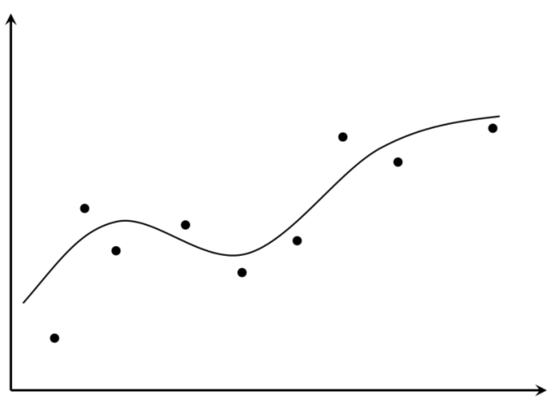

Figure 2: The principle of smoothing
(source: Nicolai 2016, 97)

Smoothing techniques assume that the error in a sampled point has a *systematic* or *trend* component that is transferred partly to nearby points, in addition to a *random* component that is unique to the sampled point only. Smoothing techniques attempt to filter out the random errors whilst retaining or estimating the trend in the errors, often using the least squares criterion to optimise the solution. They would be suitable for the analysis of historical maps but are hardly used or not used at all.

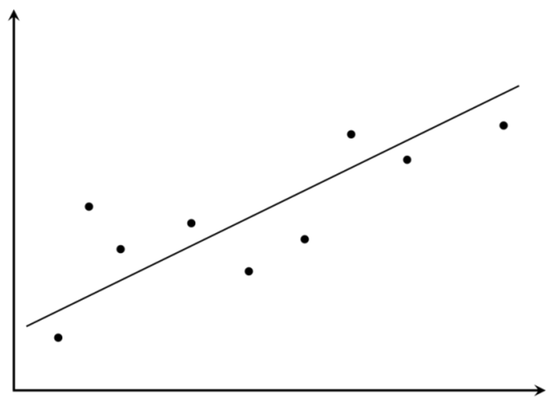

Figure 3: The principle of adjustment
(source: Nicolai 2016, 99)

Adjustment techniques assume that a priori knowledge exists about the function that is being measured. In Figure 3 that is a straight line, which has been best-fitted through the measured points. The slope of the straight line and the point at which it intersects the vertical axis are unknown and are computed in the adjustment process. In the case of a map the straight line may be thought of as a conceptual way of representing the systematic errors of the map, such as the distortions introduced by a map projection and/or the stretch or shrink of the carrier material. The remaining deviations from the straight line are called 'residuals' or 'residual errors', which are the basis of accuracy estimates and statistical testing in most adjustment techniques. The best-fitting process is often defined according to the least squares criterion. Least Squares Adjustment plays an important role in cartometric analysis methods and is also used in a range of geodetic problems, such as the computation of the position of a geodetic control network, points in the terrain between which distances and directions have been measured.

How does MapAnalyst work?

Nearly all types of cartometric analysis are based on identifying common points on the old map and a modern reference map, which is considered to be absolutely correct and against which the old map is evaluated. These common points are often called 'control points' or 'identical points'; in MapAnalyst they are simply referred to as 'points'. After loading the old map the user may begin straight away by defining these two sets of linked corresponding points.

The coordinates of corresponding points on the old map and the reference map are subtracted pairwise from one another and the resulting coordinate differences are called 'displacements'. These displacements form the basis of all analysis results that MapAnalyst produces.

However, coordinates can only be subtracted meaningfully when they are referenced to the same coordinate system. A number of computational steps are required before the displacements can be derived. The old map and the reference map must be based on the same map projection to begin with and even then the coordinate systems of the old map and the reference map may be incompatible.

MapAnalyst executes a series of processing steps, some without any user interaction. These steps are as follows:

Allocation of a map projection to the 'old' map.

MapAnalyst was designed to be used for the analysis of historical maps that were not constructed on a map projection in the modern geodetic-cartographic sense and for maps of which the projection is not known. In order to satisfy the requirement of equality of map projections of the old and reference maps, MapAnalyst *assumes* the old map to correspond with the Transverse Cylindrical Equal-Area projection. The user cannot influence this choice, unless he or she imports a different reference map than the default, which is OpenStreetMap. In that case MapAnalyst assumes reference map and old map use the same, but unknown, projection. Once the process of creating two sets of corresponding linked points has been completed and assuming that OpenStreetMap is used MapAnalyst converts the coordinates of the points on the reference map to the Transverse Cylindrical Equal-Area projection centred on the average longitude of the set of points. OpenStreetMap is an Open Source global geographic dataset and it is published in the Mercator projection[2]. MapAnalyst therefore converts the Mercator coordinates of the points on the reference map to Transverse Cylindrical Equal-Area coordinates. Both sets of points are now either defined or assumed to be in the Transverse Cylindrical Equal-Area projection. The assumption of the map projection for the old map is evidently incorrect in nearly all cases, but some assumption needs to be made and the developers of MapAnalyst justify their choice in the user manual and in (Jenny 2010, 179).

Calculation of a coordinate transformation

After the step described in the previous paragraph the coordinates of the points on the old map and the reference map are still incompatible. The old map may be rotated relative to the reference map; it may have a different scale, may be shifted and its (implicit) coordinate axes may not even be orthogonal. A transformation is therefore required between the coordinate system of the 'old' map and that of the modern reference map. The parameters of this coordinate transformation are computed from the coordinates of the two sets of linked points. Only after the coordinate transformation has been computed and the coordinates of points on the reference map have been transformed to the coordinate system of the old map (or vice versa) may the displacements of corresponding points be computed.

MapAnalyst allows the user to select one of the following three types of transformations:

- a Helmert transformation; this transformation, also known as a *similarity transformation*, describes the relationship of the coordinate systems of the old and the new map by a horizontal and a vertical shift, a scale difference and a rotation, that is, *four* parameters in total;
- an affine transformation with *five* parameters; a horizontal and a vertical shift, a horizontal and a vertical scale difference and a single rotation of both axes;
- an affine transformation with *six* parameters; a horizontal and a vertical shift, a horizontal and a vertical scale difference, a rotation about the horizontal axis and a separate rotation about the vertical axis.

[2] The full coordinate reference system is WGS 84 / Pseudo-Mercator (EPSG:3857), where Pseudo-Mercator, also known as Web Mercator is a simplified approximation of the Mercator projection. This approximation has no noticeable impact on the analysis of old maps. See http://openstreetmapdata.com/info/projections.

By default the parameters of the selected transformation type are computed by Least Squares Adjustment (LSA), which is a standard method for dealing with situations where more data is available than calculation of the desired parameters requires as a minimum. LSA has the drawback that it is sensitive to outliers in the data, but that drawback can be countered effectively by supplementary statistical testing, aimed at rejecting such outliers. MapAnalyst allows outliers to be marked by a different colour of the displacement vector (see Figure 4) and it marks outliers in the numerical results of the calculation (under menu option '*Analyse –Show results of last computation*'). The rejection of outliers is a manual process in MapAnalyst. Only one point at a time should be rejected, viz. the point with the largest displacement (residual) that exceeds the threshold value. The computation should then be redone without the rejected point. This cycle should be repeated until no more points are flagged as exceeding the threshold value, which in MapAnalyst has been set at three times the standard deviation of the residuals. It is important to remove only one point at a time and then recompute, because a single identical point that is a genuine outlier will 'drag' nearby points with it, even when those points are valid.

MapAnalyst offers an alternative approach to this statistical testing problem by opting for a *robust adjustment method* to compute the transformation: Huber, Hampel or V-estimator. This option is presented as if it were a fourth type of transformation, but it is not. It is an alternative way of calculating a Helmert or similarity transformation. All three of these robust methods are based on LSA, but they use an automated technique of reducing the damaging effects of outliers that differs from the manual statistical testing process described above. Therefore, if one does not want to get involved in manual statistical testing, which can indeed be cumbersome, a robust adjustment method may be the better option. The drawback of robust methods is that the statistics of the computation are somewhat compromised, while LSA allows statistically meaningful statistics to be computed.

Analysis of the displacement; the distortion grid

The displacements are the basis of the accuracy analysis of the old map. Map Analyst offers two ways to display them:

- as vectors of which the length corresponds with the magnitude of the displacement;
- as circles, of which the area corresponds to the magnitude of the displacement, centred on each identical point.

The lengths of the displacement vectors are easier to compare than the areas of the circles and the vectors contain more information, as they also show the direction of the displacement. However, the most well-known and most-used output from MapAnalyst is a *distortion grid*, generated from the point displacements, which can be shown with or without the displacement vectors or circles. Figure 4 shows an example of a portolan chart with a distortion grid and displacement vectors.

The distortion grids created in MapAnalyst are generated using the so-called *multiquadric interpolation* method, developed by Rolland Hardy (Hardy, 1971), following Dieter Beineke's suggestion to implement Hardy's method. The formulae in MapAnalyst are taken from Beineke (2001). Hardy developed his method to enable automated generation of height contours by interpolating measured values of terrain heights. When applied to the analysis of old maps the terrain height in Hardy's method is replaced by the point displacements, which are split into an X-component (east-west on a north-oriented map) and a Y-component (north-south on a north-oriented map. The interpolation algorithm is therefore applied twice; first to the X-component of the displacements and then separately to the Y-component. The results of the two interpolations are combined to create the distortion grid.

Figure 4: Portolan chart of Petrus Roselli (1466) with distortion grid and displacement vectors in 393 identical points shown as small blue squares. Points with residuals exceeding the threshold value are shown in red. (Portolan chart image by courtesy of the John Ford Bell Library, University of Minnesota).

The term *multiquadric* refers to the properties of the surfaces created by the interpolated values. When one imagines for a moment that the horizontal map error is a kind of terrain height the term 'surface' becomes understandable. Multiquadric interpolation generates interpolated values that are located on sections of hyperboloids between data points (Hardy 1971, 1906-1907; Beineke 2007, 21).

Limitations of the MapAnalyst approach: the map projection

Any rendering of geometries on the curved surface of the earth onto a flat map surface involves some kind of projection. When no attempt is made to manage the projective distortions in a systematic manner as is done with geodetic-cartographic map projections, one might call such a projection arbitrary. Many large-scale historical maps have not been drawn to any geodetic-cartographic map projection; MapAnalyst was developed for the analysis of such maps. MapAnalyst *must* assume a geodetic-cartographic map projection for such a map, because only then can the transformation between old map and new map be computed and only after that transformation is known and applied can the displacements and the distortion grid be computed. The developers of MapAnalyst consider that the Transverse Cylindrical Equal-Area projection centred on the mean longitude of the analysed map is an acceptable approximation for most historical maps. However, as stated above, the default reference map in the display pane of MapAnalyst 1.3 is OpenStreetMap (Jenny 2010, 179; see also www.openstreetmap.org).

The user may load up his or her own (custom) reference map, in which case a different scenario emerges: MapAnalyst will assume in this case that the map projection of the old map and the custom reference map are identical, but unknown. MapAnalyst therefore cannot convert the custom reference map to the Transverse Cylindrical Equal-Area projection. It will also compute a transformation between the old map and the reference map, but that will not result in georeferencing the old map, because the reference map's projection is unknown. The resulting distortion grid merely visualises the differences between the two maps; a graticule cannot be computed. Two portolan charts may thus be compared and differences in their geometry visualised.

If the map projections of the reference map and the old map differ, the transformation between the old map and the new map will be incorrect, because it will include these systematic differences between the two map projections. This does not imply *a priori* that the analysis is unusable; if the effects of the map projection mismatch are small enough to be negligible compared with the magnitude of the displacements of the old map, the map projection mismatch will not invalidate the cartometric analysis. This condition can only be satisfied if the old map covers a relatively small part of the earth's surface. For old maps that are not based on a geodetic-cartographic map projection, the question of which map projection is used for the reference map is largely irrelevant, but the same condition applies: the map projection of the reference map should not introduce additional systematic deviations that exceed the magnitude of the errors on the old map.

The computed values of the transformation parameters, the displacement vectors and consequently the estimated accuracy of the old map will contain errors, the magnitude of which depends on how severe the mismatch of the map projections is. The computed distortion grid tends to be more resilient than the map errors to mismatches of the map projection, as will be shown below.

Referring to the effect described in the previous paragraph, Bernhard Jenny, one of the authors of MapAnalyst, warns that:

> "The Transverse Cylindrical Equal Area projection is much better suited than the Mercator projection for most map analyses, as it introduces much smaller distortions. *Yet, it does not perform well when analysing old maps at small scales showing large countries, entire continents or even the complete globe.* For such small-scale analyses, a reference map using a projection that approximates the projection of the old map must be used, as considerable distortions would be added by the Transverse Cylindrical Equal Area projection" (Jenny 2010, 179 – italics by the author).

The bad news is that portolan charts firmly belong to the problematic category described by Jenny, as do the portolan-style nautical charts from the Age of Discovery that depict one or several continents.

Loomer analysed 27 portolan charts for nine different map projections and found that portolan charts are best approximated by the Mercator projection, that is, evaluation against the Mercator projection resulted in the smallest sum of the squared residuals (Loomer 1987, 149). Loomer included the part of the African Atlantic coastlines, but only the coastlines of the Iberian peninsula up to Cabo Ortegal.

Figure 5 shows the agreement between a portolan chart and the Mercator projection. The outlines of the portolan chart of Angelino Dulcert (1339) and the outline of the Mercator reference map in Figure 5 were taken from the cartometric analysis of this portolan chart in (Nicolai 2016, 147-148). A total of 836 points were identified in this cartometric analysis. In Figures 5 and 6 only the points on the perimeters of the Black Sea and Mediterranean are shown and are joined by line pieces, so that an outline of the coasts of these two seas is created.

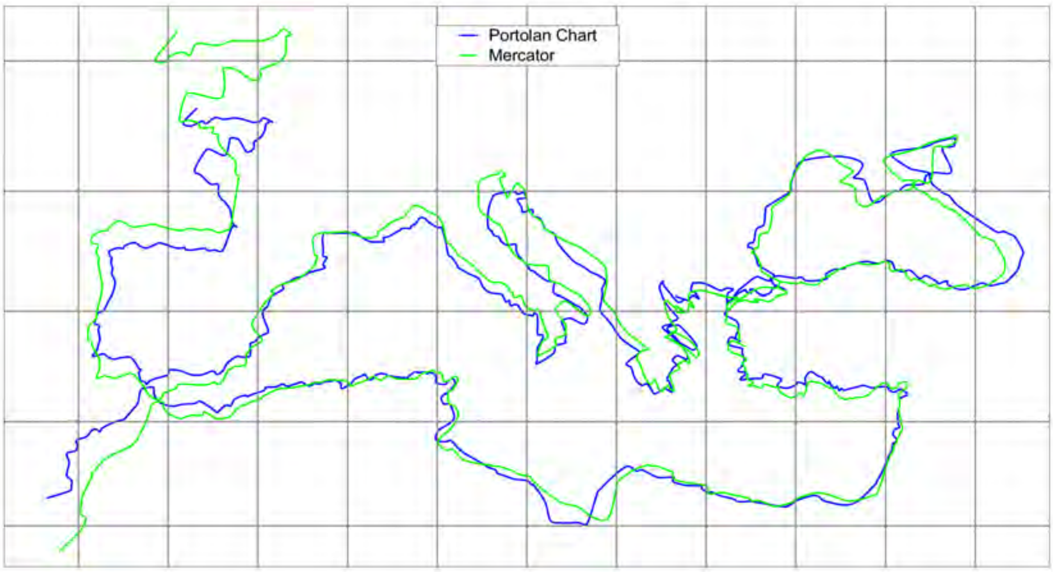

Figure 5: Outline of the 1339 portolan chart of Angelino Dulcert (blue) and the corresponding outline on a modern Mercator map.

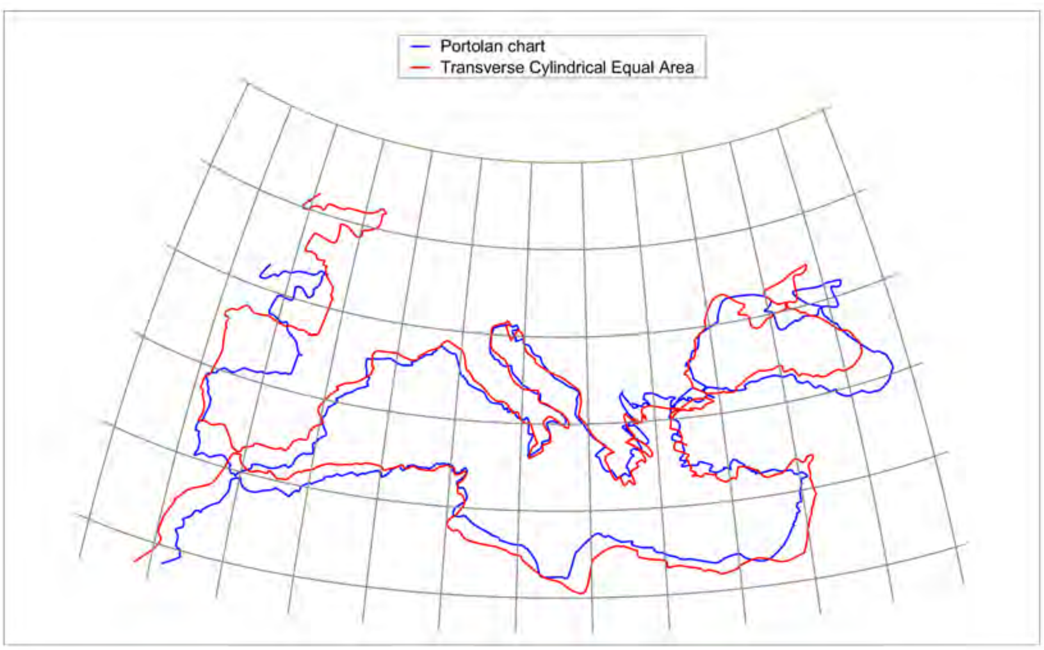

Figure 6: Outline of the 1339 portolan chart of Angelino Dulcert (blue) and the corresponding outline on a modern Transverse Cylindrical Equal-Area projection centred on the mid-longitude (18° E) of the map area (red).

In Figure 6 the outline of the Dulcert chart is the same, but the Mercator coordinates of the reference map have now been converted to the Transverse Cylindrical Equal-Area projection. A Least Squares fit based on a similarity or Helmert transformation was then applied to the coordinates of the identical points both in Figure 5 and Figure 6. Only the Mediterranean and Black Sea points were used to compute the Least Squares fit, as the Atlantic coasts of most portolan charts have differ considerably in scale relative to the Mediterranean coastal outlines (Nicolai 2016).

Figure 5 is intended to facilitate a visual confirmation of the degree to which the outline of a portolan chart agrees with a Mercator chart, whereas Figure 6 is intended to demonstrate that the transformation of the reference chart to the Transverse Cylindrical Equal-Area projection will lead to a much poorer fit with the coastal outline of a portolan chart and therefore will result in unrealistically large displacement vectors, as shown in Figure 4. It is easy to see in Figure 6 that the effect of the map projection on the displacement vectors is largest at the edges of the map, which is why MapAnalyst should only be used for the analysis of large-scale old maps covering a relatively small area. MapAnalyst only displays the reference map in the Mercator and not in the Transverse Cylindrical Equal-Area projection; the results of this re-projection are not shown graphically to the user in MapAnalyst and it is therefore illuminating to see what the effects of this conversion are.

The different distortion characteristics of projection of the portolan chart and the reference map in the Transverse Cylindrical Equal-Area projection) are interpreted by MapAnalyst as displacements of points on the portolan chart. These are added to the portolan chart's mapping errors. Therefore MapAnalyst will compute unrealistically large accuracy estimates for the portolan chart. The accuracy estimates for the portolan chart produced by MapAnalyst will be too large and the transformation parameters (scale and rotation) will also be affected adversely. But the key question is: how is the distortion grid affected by MapAnalyst's usage of the Transverse Cylindrical Equal-Area projection?

Only the map errors in the identical points contain true information; the distortion grid is derived entirely from this relatively small body of real data on the basis of the assumption that the map errors change smoothly across the map, without sudden jumps. When one has defined only a handful of identical points, the distortion grid will show regular intervals between the interpolated parallels or meridians between the sparse data points. When additional identical points are inserted in between that regularity will be disrupted by the map errors of the newly inserted points. The distortion grid therefore depends on the number and distribution of identical points from which it was generated. The fewer the number of identical points, the smoother and more regular the distortion grid will be. The interpolation algorithm does not really 'care' how many identical points there are.

Limitations of the MapAnalyst approach: the distortion grid

How multiquadric interpolation works

The multiquadric interpolation method may be illustrated by two small fictitious 'maps', created in MS-Excel, one of which takes the role of the 'old' map, while the other plays the role of the reference map. For convenience both 'maps' were created in the same coordinate system and seventeen identical points were defined. The blue dots in Figure 7 show the locations of the points on the reference map and the red dots the locations of the corresponding points on the 'old' map. Figure 7 can therefore be seen as an overlay of one imaginary map on top of the other. The values of the coordinates are arbitrary. The Excel spreadsheet calculates interpolated values of the map error according to Hardy's multiquadric interpolation method.

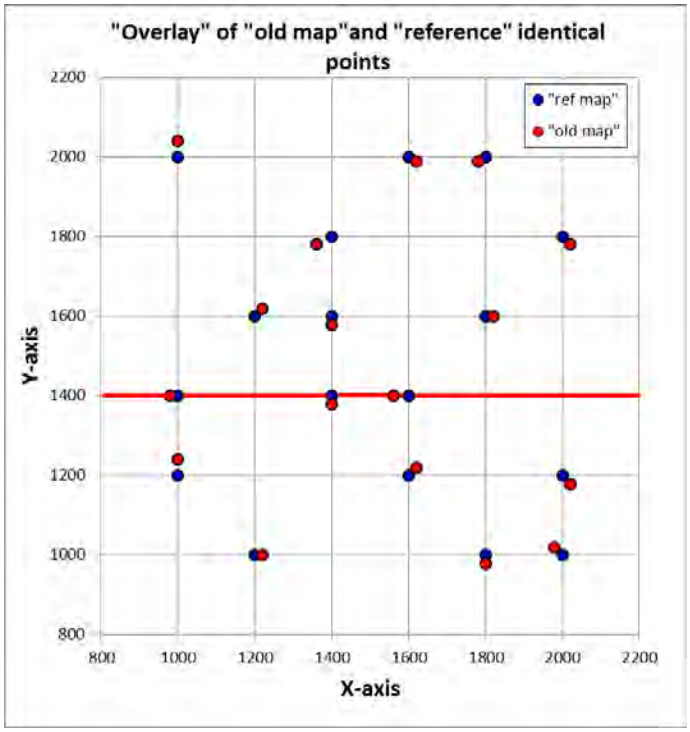

Figure 7: Overlay of the imaginary 'old' map and the 'reference'map

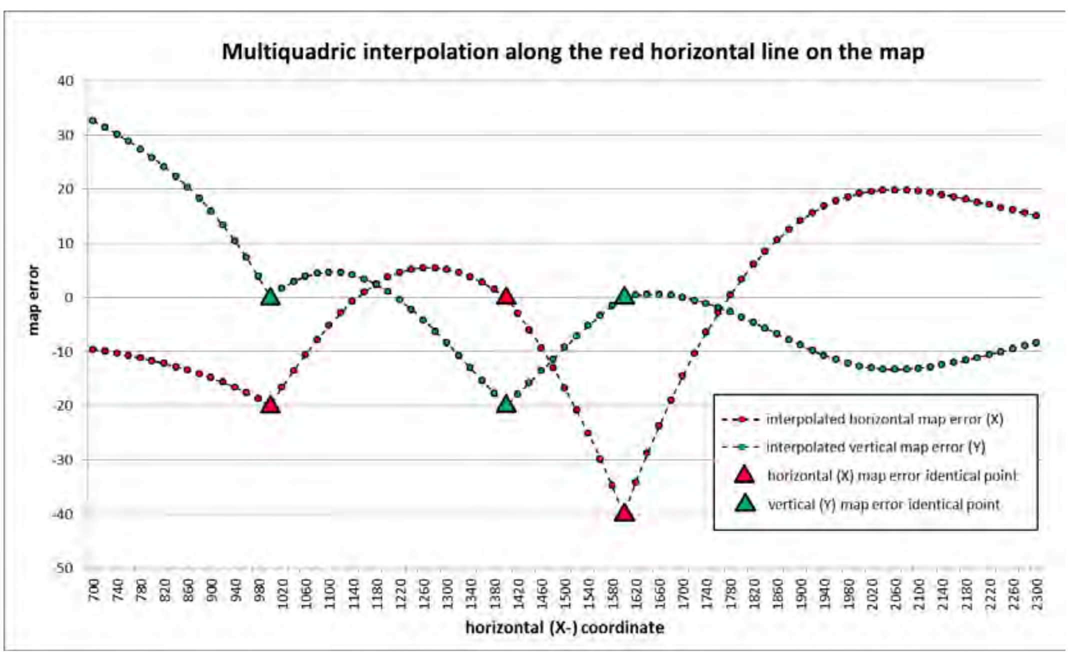

Figure 8: Multiquadric interpolation along the cross section (red line) in Figure 5

Figure 8 shows the interpolated displacements along a cross section of the map: the red line in Figure 7. There are three identical points on that cross section, of which the displacements are indicated in Figure 8 by triangles; red for the X-component of the displacement and green for the Y-component.

The hyperbola-shaped interpolation patterns are clearly visible in Figure 8, showing a smooth interpolation between any two identical points. The property that the *rate of change* of the interpolated values is not smooth is also clearly visible; although the interpolated values between any two data points are continuous there are clear breaks in the rate of change in those points. That is particularly visible in the X and Y values of the first identical point from the left, in the X-value of the last identical point and the Y-value of the middle one.

The sharp peaks in the interpolated points can be avoided by applying a smoothing factor. It is outside the scope of this article to describe this mathematically. MapAnalyst. Sharp peaks and troughs may be realistic for interpolation of terrain heights, which was the application for which Hardy originally developed the multiquadric method, but they are less realistic for the interpolation of map errors of historical maps. Although it is quite conceivable that on a historical map only a single point is misplaced, usually the map errors in adjacent points are strongly correlated. It is reasonable to assume that the cartographer attempted to capture the continuity of the terrain features mapped on the historical map. It is therefore recommended to apply strong smoothing to the generation of the distortion grids in MapAnalyst.

Extrapolation

Another property, however, is also very visible in the example calculation of Figure 8. The multiquadric interpolation method not only interpolates; it also *extrapolates* as far as the user cares to, potentially well beyond the range covered by the identical points. It cannot be stressed enough that the extrapolated values are an *artefact* of the multiquadric method. The extrapolated Y-values in Figure 8 increase to values well above those found in the entire rest of the ‘map’ and there is no reason why this should be so. Extrapolation is discouraged in all mathematical textbooks, unless it is absolutely certain how the estimated parameter will behave beyond the data points. In the case of historical maps and charts we really have no idea how map errors might propagate to areas beyond the coverage of the identical points, since the map or charts often contains no data in those areas. This does assume that the user has defined a well-distributed, dense set of identical points on the map. Extrapolation beyond the area of any meaningful map content can only be qualified as pure fantasy.

The importance of choosing a sufficiently large number of points

Figure 9 shows an example of the same chart used in Figure 4, but now based on thirteen points only, which is far too few for a realistic distortion grid. The resulting grid looks to be a smoothed version of the one in Figure 4; the local irregularities are gone. Showing a grid without the identical points may therefore create the impression the old map is more accurate than it really is.

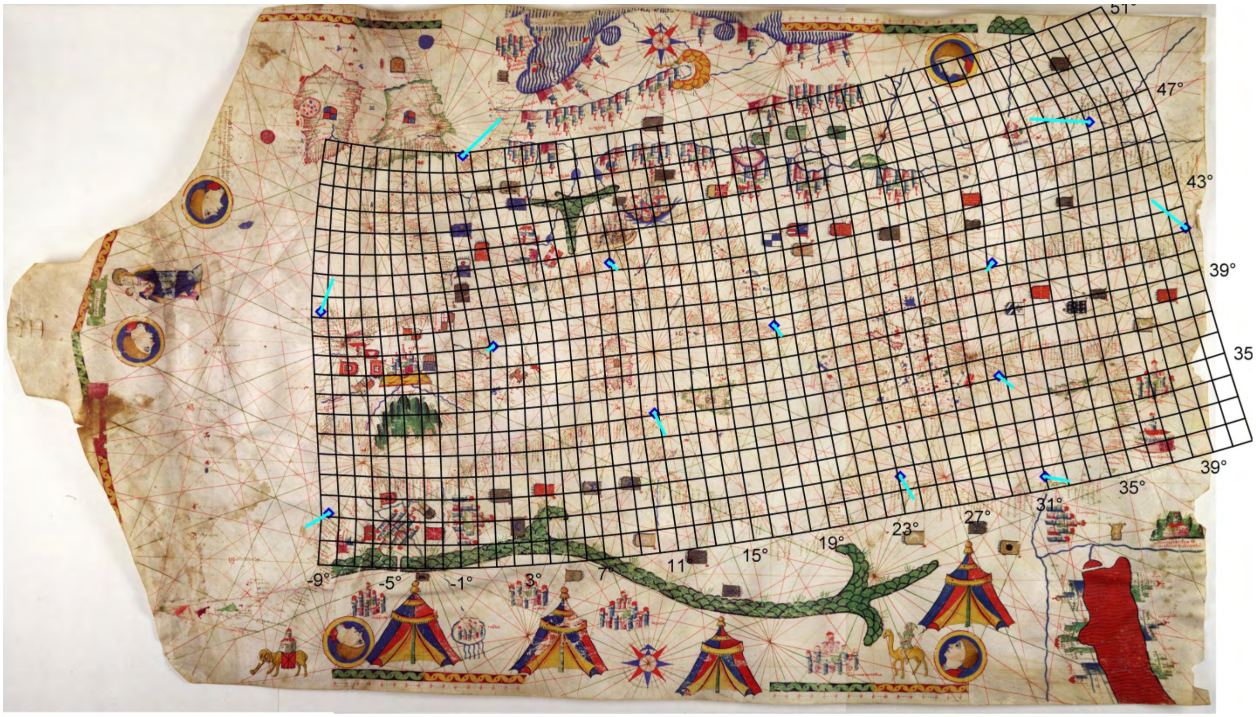

Figure 9: Portolan chart of Petrus Roselli (1466) with distortion grid and displacement vectors in 13 identical points

A sufficiently large number of points, well distributed over the area (if the old map allows that) should always be chosen.

The accuracy of the distortion grid

The arguments presented above invite questions such as: which, in Figure 4, are the real characteristics of the portolan chart and which are the artefacts of the processing method and possibly the extrapolation properties of the multiquadric method? How is the distortion grid affected? One might be tempted to see some meridian convergence in the distortion grid of Figure 4, as well as curvature in the parallels.

The effects of the map projection mismatch and any interpolation/extrapolation artefacts are mashed up with the real distortions of the old map. However, these effects may be separated to some extent by analysing a modern Mercator map as if it were an 'old' map (in MapAnalyst terms) in the same coverage area of a Mediterranean portolan chart. In MapAnalyst I defined 128 pairs of identical points on a modern Mercator chart, which plays the role of 'old' map, and on the reference map, OpenStreetMap, which is also provided on the Mercator projection. Because both sets of coordinates refer pairwise to the same points on a modern map, the map errors should all be zero, or very nearly so (the manual process of picking these points on the two maps will introduce a small error in each pair of points). One would therefore expect the distortion grid of the 'old' map to show the familiar orthogonal pattern of straight parallels and meridians of a Mercator map. However, that is not the case. Figure 10 shows the distortion grid of the 'old' Mercator map superimposed on the undistorted black grid of the reference map. Figure 10 also displays the displacement vectors.

The displacements or map errors ought to be zero, but evidently, they are not. A few points even have such large displacements, that they are marked in red as outliers. But, although the large displacements or 'map errors' in Figure 12 may confuse the uninitiated, the reader is now well prepared for understanding this. The displacement vectors shown are simply the differences in position of the identical points on the Mercator and the Transverse Cylindrical Equal-Area projections. This then is the effect of the mismatch of the map projections. MapAnalyst *displays* the reference map (OpenStreetMap) on the Mercator projection, but it *calculates* in the Transverse Cylindrical Equal Area projection.

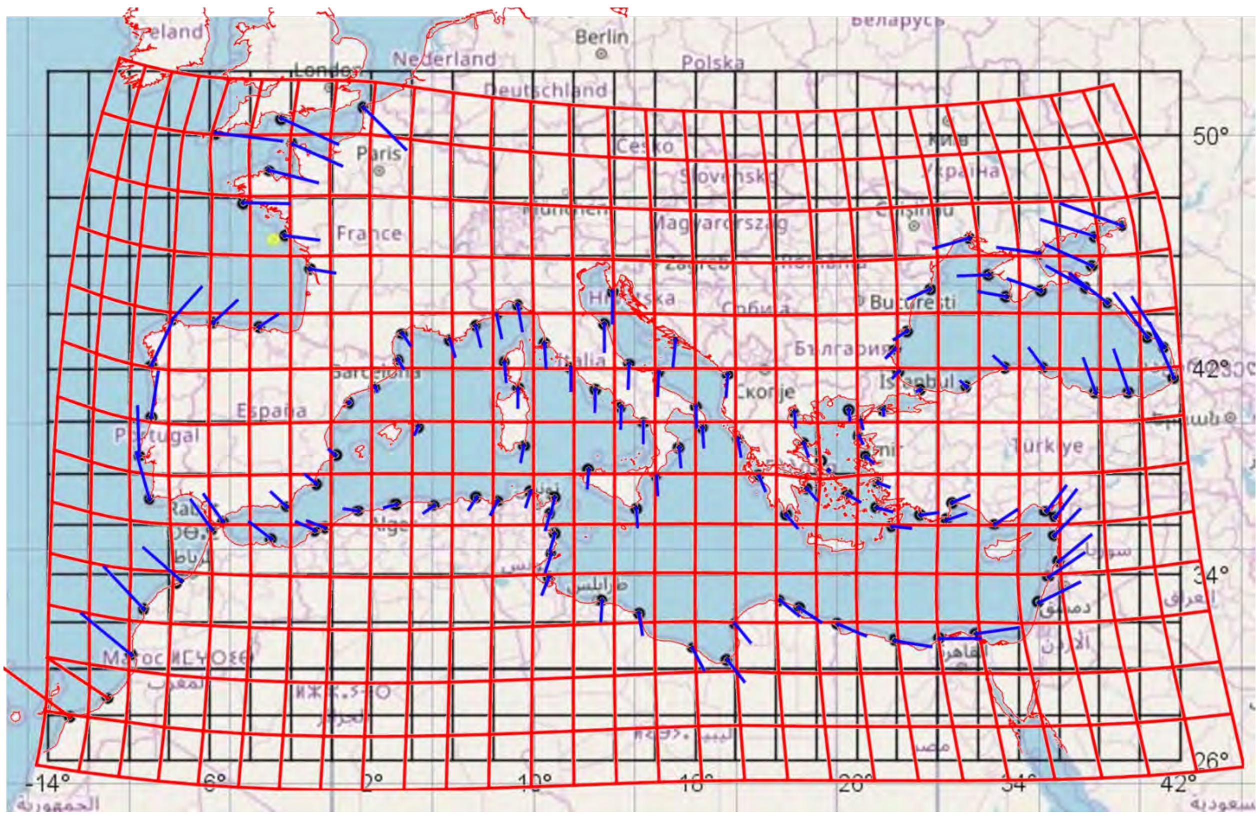

Figure 10: Distortion grid of a modern Mercator map, analysed as if it were an old map (red grid), superimposed on the undistorted grid of the reference map (black grid). The reference map is OpenStreetMap, which is also on the Mercator projection. The grid interval for both grids is two degrees.

At first sight the red distortion grid appears to be wildly off, but that is only partly true. The *interpolated* part of the distortion grid within a tight envelope around the identical points is quite good. The interpolation algorithm does not produce straight parallels and meridians, but, at the scale shown, that is hardly noticeable. This demonstrates that the distortion grid is indeed quite resilient against even dramatic mismatches of the map projections. The interpolated distortion grid is also quite resilient against variations in the transformation method; there is no noticeable effect on the interpolated distortion grid, depending on whether one selects the Helmert transformation or the six-parameter affine transformation. However, the distortion grid *is* wildly incorrect in those places where it is based on *extrapolation*, that is, outside the envelope of the identical points. This demonstrates quite clearly that extrapolation should be avoided. This can be achieved to a large extent in MapAnalyst by selecting the option "around points" for the Extension property of the distortion grid.

The large displacements that are calculated will wreak havoc on statistical testing, because points, marked as outliers may not be outliers at all in the sense that the map errors in those points are too large. It may simply mean that the points are located in an area where the mismatch between the two map projections is large. The accuracy estimates of the 'old' map are based on these displacements and will therefore be in error too. Lastly, the parameters of the transformation between the new map and the old map will be incorrect: for example, MapAnalyst computes a rotation of 1-2° between the two Mercator maps while there is no rotation at all. It also computes a Root Mean Square Position Error ($RMSE_{position}$) of the 'old' map of about 140 km. Both of these values are artefacts of the map projection mismatch and have nothing to do with the analysed modern Mercator map.

Analysis of a portolan charts as a mosaic of subcharts

Do the results shown in this paper and the warning voiced by Bernhard Jenny imply that MapAnalyst cannot be used to analyse portolan charts of the Mediterranean and the Black Sea? It certainly does if one insists on analysing such a chart as a single coherent entity. However, portolan charts have been shown to be composites of sub-charts of which each has its own scale and orientation (Nicolai 2015 and 2016). As early as the end of the nineteenth century Hermann Wagner demonstrated that scale and orientation differences exist on a single chart (Wagner 1896). Easiest to spot is the too-small scale of the Atlantic coastal areas, but that is not the only sub-area. Therefore, analysing a portolan chart by sub-region appears to be a valid way of circumventing the limitations described in the previous sections.

Although the part of the distortion grid inside the tight envelope of the identical points is hardly affected by the mismatch in map projections, as it covers the undistorted grid almost exactly (see Figure 10), usefulness of a distortion grid is limited, as it conveys no numerical information. The summary of the transformation parameters and its associated estimate of the accuracy of the old map is an underutilized source of information provided by MapAnalyst, but this information is only useful when the errors due to the map projection mismatch are negligible, otherwise the displacements on which the accuracy estimates are based will be too large. Furthermore the scale and orientation parameters will be adversely affected.

It is helpful to know the approximate outlines of sub-regions that have a coherent scale and orientation, because the boundaries between coherent sub-regions do not coincide with the natural boundaries of sub-basins in the Mediterranean area (Nicolai 2015, 2016). The identification of scale/orientation-consistent regions may also be done in MapAnalyst, starting from an initial core area and then adding one point at a time and checking whether the $RMSE_{position}$ remains stable, improves or deteriorates. As the researcher thus approaches the outer edge of that region, the addition of more points will cause the $RMSE_{position}$ to start increasing. When the last added point is flagged as outlier, the user will know that the boundary of the region with a coherent scale and orientation has been reached.

The regions as determined in the analysis in (Nicolai 2016) were used. That also enables direct comparison with the results obtained in that analysis. The six regions distinguished are: the north and south Atlantic coast, the western, central and eastern Mediterranean and the Black Sea. Generation of a distortion grid per region enables the mosaic to be made shown in Figure 11. As mentioned above the extrapolation artefacts of the multiquadric method are largely avoided by choosing the distortion grids to fit around the identical points. Figure 11 is not a standard output of MapAnalyst: the grids with the identical points and the displacement vectors (without the portolan chart) were generated first from MapAnalyst and then manually superimposed on an image of the portolan chart by forcing the identical points to coincide for the separately computed grids.
It is clear that the magnitude of the map errors has dropped dramatically and to a more realistic level because the analysis by region avoids the serious mismatch issues with the map projection discussed earlier in this paper. Should one accidentally divide a sub-area with a coherent scale and orientation into two smaller areas, no error will be made, as the same scale and orientation will be determined (give or take a bit) for both smaller areas.

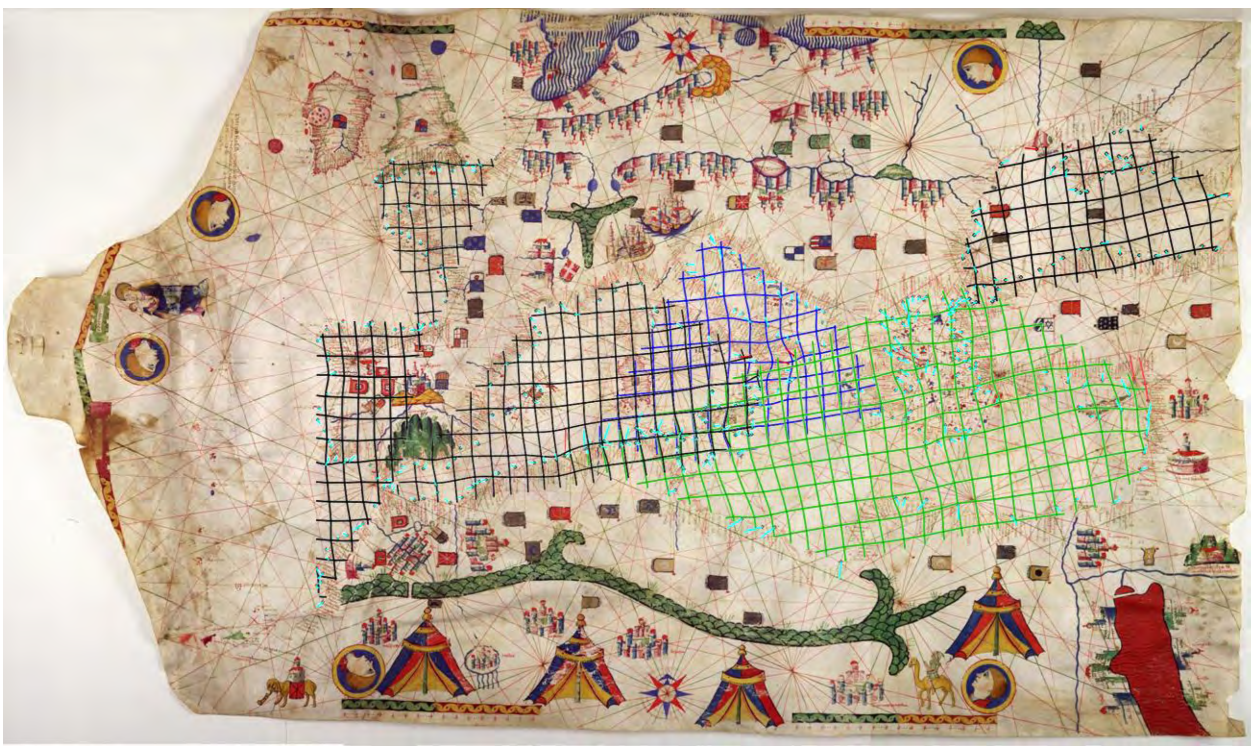

Figure 11: Mosaic of distortion grids by region with the displacement vectors for each of the regions, computed for 393 points. In the overlap areas one identical point will have multiple displacement vectors, one for each region it belongs to. Different colours are used for the distortion grids of the three Mediterranean regions, so the differences are visible more clearly. Portolan chart: Petrus Roselli (1466), by courtesy of the James Ford Bell Library, University of Minnesota.

A six-parameter affine transformation was computed for each of the regions. This is similar to the approach in (Nicolai, 2016 pp. 133-200). However, an important difference with MapAnalyst is that Nicolai computes this transformation using a reference map on the Mercator projection, while MapAnalyst computes the best fit to a reference map on the Transverse Cylindrical Equal-Area projection. Therefore the results from using Nicolai's method and MapAnalyst never will be identical.

The comparison results are nevertheless captured below in three tables, comparing the mean point error, the scale differences of the regions on the portolan chart and the axis rotations.

The estimated mean point error (RMSE) in km is shown in Table 1. The compatibility of the corresponding figures from the MapAnalyst calculation and (Nicolai, 2016) show that the regions selected on the portolan chart are small enough to avoid corruption of the analysis because of the map projection mismatch issue discussed above, with the exception of the eastern Mediterranean, which, on account of its much higher mean point error, appears to be too large and therefore suffers from this effect.

Region	**MapAnalyst**	**Nicolai (2016)**
Atlantic coast north	8.5	9.0
Atlantic coast south	10.6	10.6
Western Mediterranean	13.4	11.9
Central Mediterranean	10.9	9.6
Eastern Mediterranean	20.2	12.4
Black Sea	10.7	12.1

Table 1: Mean point error (RMSE) in km determined by MapAnalyst and in Nicolai (2016)

The scale difference of each sub-chart relative to the western Mediterranean is shown in Table 2 below. Because the scale differences relative to the Western Mediterranean are shown, its percentage shows as 100%. The mean scale of the 'Atlantic coast north' region is 93% of that of the Western Mediterranean region according to Nicolai's method, 97% according to MapAnalyst's calculation, etc.

Region	**MapAnalyst**	**Nicolai (2016)**
Atlantic coast north	97%	93%
Atlantic coast south	92%	89%
Western Mediterranean	100%	100%
Central Mediterranean	104%	104%
Eastern Mediterranean	109%	107%
Black Sea	114%	114%

Table 2: Relative scales of sub-areas on the Roselli 1466 chart, determined with MapAnalyst and in Nicolai (2016)

For the rotation angles the same comparison was made. A positive rotation angle in Table 3 is clockwise, a negative angle anticlockwise.

Region	**MapAnalyst**		**Nicolai (2016)**	
	X-axis rotation	**Y-axis rotation**	**X-axis rotation**	**Y-axis rotation**
Atlantic coast north	+1.4°	-3.7°	+0.5°	-5.4°
Atlantic coast south	+2.4°	-2.9°	+0.4°	-3.6°
Western Mediterranean	-6.4°	-4.9°	-7.5°	-6.0°
Central Mediterranean	-6.6°	-2.9°	-9.4°	-3.3°
Eastern Mediterranean	-11.3°	-8.4°	-10.2°	-10.5°
Black Sea	-9.5°	-8.8°	-13.0°	-10.3°

Table 3: Rotation angles of the horizontal (X) axis and the vertical (Y) axis, determined by MapAnalyst and in Nicolai (2016)

The same pattern emerges in MapAnalyst as shown in (Nicolai, 2016), but the actual rotation values show considerable differences. The Transverse Cylindrical Equal-Area projection contains significant variations in rotation relative to a portolan chart, even when applied to smaller sub-areas of a that chart, which the Mercator projection does not have, which is why the actual values differ more significantly. See also Figure 11.
The difference in the rotations of the horizontal and vertical axis, which is called the shear angle in (Nicolai 2016), is also very much visible in the MapAnalyst results. Although this effect is hardly visible in the distortion grids of Figure 11, these angles are nevertheless there, because the computed six-parameter affine transformation describes the best fit of each of the distortion grids shown in Figure 11 to the corresponding undistorted grid of the reference map. A discussion of this phenomenon is outside the scope of this paper.

Concluding remarks and recommendations

It has been shown that a distortion grid should not be interpreted as if it were an 'X-ray image', reflecting the fixed properties of a portolan chart that lay hidden beneath the visible appearance of the chart. Firstly, the shape of the grid depends on the number and distribution of identical points. Secondly, the part of the grid outside the envelope of identical points is very unreliable, based as it is on extrapolation. No conclusions should be drawn from the extrapolated grid. It is even recommended not to display the extrapolated parts by selecting the option: 'Extension: around points' in the visualisation options of the bottom-left quarter of the MapAnalyst screen. It is also recommended to select strong smoothing for the distortion grid. It is recommended to display the identical points and their displacement vectors together with the distortion grid, as these points and their map errors are the only real data on which the distortion grid is based.

An underutilized source of information in MapAnalyst is the report with the transformation statistics, which will reveal details that are not visible in the distortion grid, but these statistics are only reliable when the coverage area of the analysed map, or part of that map, is small enough. When the area analysed is too large the statistics will be corrupted by the systematic deviations of the Transverse Cylindrical Equal-Area projection of the reference map and the Mercator-like projection of the portolan chart.

MapAnalyst is unsuitable for the analysis of large areas, but what counts as 'large' also depends on the magnitude of the displacements. For a portolan chart this maximum appears to be approximately an area the size of the western Mediterranean. Although the distortion grid is relatively unaffected, the transformation statistics are corrupted when the coverage area is too large, including any indication of what points are outliers. Analysis of a portolan chart should be undertaken by dividing the chart into smaller sub-charts. MapAnalyst can be used to identify the boundaries of sub-charts that have a coherent scale and orientation.

Acknowledgements

I would like to thank Bernhard Jenny and Dieter Beineke for the valuable information and insights they supplied to me on the subject matter and for the feedback they kindly provided on the draft version of this paper.

References

Beineke, Dieter (2007): "Zur Bestimmung lokaler Abbildungsverzerrungen in Altkarten mit Hilfe der multiquadratischen Interpolationsmethode". In *Allgemeine Vermessungsnachrichten* 1/2007, pp. 19-27.

Beineke, Dieter (2001): *Verfahren zur Genauigkeitsanalyse für Altkarte.* PhD Thesis. Heft 71, Studiengang Geodäsie und Geoinformation. Neubiberg: Universität der Bundeswehr Universität der Bundeswehr München.
https://www.unibw.de/ipk/karto/forschung/promotionen/phdbeineke/

Campbell, Tony (2016): personal communication during the First Workshop on the Origin and Evolution of Portolan Charts, Lisbon, 2016.

Gaspar, Joaquim A. (2008): "Dead reckoning and magnetic declination: unveiling the mystery of portolan charts." *e- Perimetron*, Vol. 3, No. 4: 191-203.
http://www.e-perimetron.org/Vol_3_4/Gaspar.pdf.

Gaspar, Joaquim A. (2010): *From the Portolan Chart of the Mediterranean to the Latitude Chart of the Atlantic. Cartometric Analysis and Modelling*. PhD thesis. Lisbon: Universidade Nova de Lisboa, 2010. http://www.ciuhct.com/online/docs/thesis_joaquim_gaspar_2010-v2.pdf.

Gautier Dalché, Patrick (2001) : "Cartes marines, representations du littoral et perception de l'espace au Moyen Âge. Un état de la question". In *Castrum 7*, *Zones côtières et plaines littorales dans le monde méditerranéen au Moyen Age,* 9-32. Rome: École française de Rome.

Hardy, Rolland L. (1971): "Multiquadric Equations of Topography and Other Irregular Surfaces", *Journal of Geophysical Research*, Vol. 76, No. 8, 1905-1915.

Jenny, B. (2006): "MapAnalyst - A digital tool for the analysis of the planimetric accuracy of historical maps", *e-Perimetron*, 1-3, p. 239-245.

Jenny, B., Weber, A. and Hurni, L. (2007): "Visualizing the planimetric accuracy of historical maps with MapAnalyst", *Cartographica*, 42-1, p. 89-94.

Jenny, Bernhard (2010): "New Features in MapAnalyst", *e-Perimetron*, Vol. 5, No. 3, 176-180. http://www.e-perimetron.org/Vol_5_3/Jenny.pdf

Jenny, B. and Hurni, L. (2011): "Studying cartographic heritage: analysis and visualization of geometric distortions", *Computers & Graphics*, 35-2, p. 402–411.

Loomer, Scott A. A (1987): *Cartometric Analysis of Portolan Charts: a Search for Methodology*, PhD thesis, Madison: University of Wisconsin, 1987.

Mesenburg, Peter (1990): "Untersuchungen zur kartometrischen Auswertung mittelalterlicher Portolane", *Kartografische Nachrichten,* 1/1990.

Nicolai, Roel (2015): "The Premedieval Origin of Portolan Charts: New Geodetic Evidence". *Isis; Journal of the History of Science Society*, Vol. 106, No. 3 (September 2015), 517-543, http://dspace.library.uu.nl/bitstream/1874/327279/1/Nicolai.pdf

Nicolai, Roel (2016): *The Enigma of the Origin of Portolan Charts. A Geodetic Analysis of the Origin of Portolan Charts.* Leiden; Boston: Brill.

Pacheco, Luís M.C.P.B. (2017): *A carta-portulano*. PhD thesis. Universidade de Lisboa.

Pujades I Bataller, Ramon J. (2007): *Les cartes portolanes: la representació medieval d'una mar solcada.* Translated by Richard Rees. Barcelona: Lunwerg Editores.

Wagner, Hermann (1896): "The Origin of the Medieval Italian Nautical Charts." In *Report on the Sixth International Geographical Congress*, 1895, 695-702, London (1896). Reprinted in *Acta Cartographica* 5 (1969), 476-483.