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ICME-14 Topic Study Group 1

Mathematics education at preschool level

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TSG 1. Mathematics education at preschool level

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CONTENTS

MATHEMATICS EDUCATION AT PRESCHOOL LEVEL

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After having been shelved for more than one year, the papers in this booklet can happily finally see the light. One year later than planned they can be presented at the *14th International Congress on Mathematical Education* held in Shanghai from 11 to 18 July 2021. Unfortunately, for most authors, participating in this conference, and presenting and discussing their papers will be only a virtual pleasure.

ICME-14 has 62 Topic Study Groups in total, which are each the arena for exchanging and conversing new developments in dedicated aspects of the teaching and learning of mathematics. This large number of TSGs clearly indicates the richness of mathematics teaching and learning as a field of study. One of these 62 TSGs is devoted to where this teaching and learning all starts. This is *TSG 1 Mathematics education at preschool level*.

The focus of TSG 1 is on the foundations of learning mathematics and the contexts in which the first steps are taken towards achieving mathematical understanding. The group is set up with the aim to offer participants an opportunity to inform each other about their research findings on early childhood mathematics teaching and learning and to have deliberations about their approaches and theoretical and methodological frameworks. The studies discussed in TSG 1 involve mainly research on children's mathematical development in the years before they enter in formal schooling in first grade. The nurturing of this development can take place in various environments: care centers, preschool, kindergarten, and at home.

The 17 submitted papers to TSG 1 can be divided in two categories:

- Investigations of children's learning (papers 1-7)
- Investigations of children's learning environment (papers 8-17).

The papers in the first category are all based on data collected from children. For several mathematical content areas and competences it is investigated what children are capable of. The papers of the second category are based on data collected through observing classrooms and interviewing early childhood teachers and educators, prospective preschool teachers, and other adults. Interestingly, in one study the learning environment was also investigated by interviewing children themselves. In this second category are also two papers which have a more theoretical stance. One is proposing a revision of a framework for mathematical activities and the other is recommending the use of mathematical games in kindergarten.

The one-slide overview that precedes every paper highlights its key findings and serves as a guide into the study.

Investigations of children's learning

The collection of papers in this section addresses mathematical competences in the domain of early number, symmetry, patterns, and representation of data.

With respect to early number, one of the papers (#1) investigated children's competence in making estimations on the number line. This is a topic that is not everyday dealt with in kindergarten classes. By eye-tracking technology the study showed that the ability of kindergarten children (aged five to six) of making estimations on a 0-10 number line can be effectively improved by using a midpoint marker instead of a marker at every quartile.

Another study (#2) aimed to unravel the composition of the quantitative competence of kindergartners. By analysing data from a collection of paper-and-pencil items it was revealed that in addition to counting, subitizing, and additive reasoning, also multiplicative reasoning belongs to this early number ability. Furthermore, an implicative analysis at item level showed that in general, multiplicative reasoning and conceptual subitizing items were found at the top of the implicative chain, counting and perceptual subitizing items at the end, and additional reasoning items in the middle.

Three studies investigated the development of the notion of symmetry in kindergartners. In one study (#3) a sequence of 16 symmetry-related activities was developed in the context of art work. In these activities kindergartners had to work with various axes of symmetry. The authors found that the designed sequence can constitute a hypothetical path by which children in early childhood education can progress in their learning of symmetry.

The second study on kindergartners' competence in symmetry (#4) looked for an alternative for the often used "butterfly" pictures. To make the context more meaningful for the children they had to work with drawings of their portraits which they had to analyse for features of line symmetry and mathematical structure. The authors found that over thirty percent of children represented explicit structural features such as equal spacing, congruence, partitioning and alignment of facial features.

The third symmetry study (#5) explored the assessment of early symmetry knowledge. In the study an intervention with symmetry software took place in which first and second grade children were taught reflection, translation and rotation. After the intervention the children were assessed by a paper-and-pencil test and by interviewing them. The authors found that children who reached higher scores on reflection and translation tasks, in the interviews also provided explanations indicating conceptual understanding of the symmetric transformations. The similar relationship was reported for girls and boys.

Recognizing and being able to work with patterns is considered a vital element of young children's mathematical development. To know more about children's understanding of patterns, a study (#6) investigated in a sample of 134 four-year-olds preschool children how able they are in solving tasks on repeating patterns. The results showed that the children could fill and expand repeating patterns, but also difficulties came to the fore in the abstraction of the pattern, especially in identifying the unit of a repeating pattern.

The last content domain that is reported in this section is the representation of data (#7). The paper describes a study in which it was investigated whether kindergartners (aged five-six) can sort and group objects, identify the quantity of each group of objects, and then can draw pictures or write names and numbers to organize and present the data. One of the results of the study is that half of the 35 children involved failed to represent the quantities using numerals and pictures.

Investigations of children's learning environment

The papers in this section lift in different ways the veil of the conditions and circumstances in which the early learning of mathematics can come about. To gain knowledge about this, in most studies data were collected by interviewing early childhood teachers. In one study (#8) a broader response group was surveyed and adults (not being preschool teachers but including grade school and high school teachers, psychologists, occupational therapists, engineers, municipal workers, and accountants) were asked what types of activities they perceive as the ones that can promote numerical skills. Many participants suggested counting objects. Sub-skills such as counting forward from some number other than one or focusing on one-to-one correspondence, were less mentioned.

When 23 early childhood teachers were asked about their use of finger patterns in their daily interaction with children (#9) it was found that they all use finger patterns in a variety of everyday (such as age/birthday, finger games, board games) and mathematical contexts (verbal counting, object counting, refering to quantities or number signs, and when calculating). The frequency and type of used finger patterns varied among the teachers. Only four teachers used finger patterns doing calculations. Two of them used the fingers in a dynamic way and two in a static way. No more then ten teachers used finger patterns as a visualization to help children develop an understanding of numbers.

Because what early childhood educators think about the mathematical abilities of their children may influence the learning environment they offer to them, an international study (#10) was set to investigate the performance expectations of early childhood educators in five countries. The focus was on shapes and space. The data of 1343 early childhood educators revealed that the expectations for this content area were more accurate in Austria and Zwitserland than in China, Vietnam and the USA. Also, the estimations for 3-6 year old children were more appropriate than those for the 1-3 year olds.

In addition to learning through focused activities, young children's learning of mathematics also takes place to a large extent through free play. To figure out what interactions between preschool/kindergarten teachers and preschoolers (two-six years) can be considered as useful for stimulating young children's language and mathematical development a professional learning community (PLC) was set up consisting of preschool and kindergarten professionals and researchers (#11). Based on discussions held within PLC-meetings and the analysis of the mentioned interaction charateristics three guidelines for interactions were identified that can stimulate children's mathematical development during children's spontaneous play: observing (understand the child's interest and feelings), connecting (confirm what the child is playing) and enriching (cooperatively construct mathematical meaning).

Preschool teachers' positive feeling about mathematics is a determining factor of the quality of the early childhood learning environment. Therefore, in a longitudinal study (#12) with an experimental pre-test post-test control group design, it was investigated whether and how a preservice teacher training can change prospective early childhood teachers' emotions about mathematics. The study was carried out with full-time and part-time teacher students. Only the part-time students showed after the training an increase in mathematics enjoyment and a reduction of mathematics anxiety. For almost all the part-time students the lessons at the university were the most important reason of this change. For only half of the full-time students this was the case, while 35% indicated that it was the five-weeks practical period they spent in an early childhood institution.

In the two following papers, instead of an empirical approach, the learning environment is considered from a theoretical point of view. In the first study (#13) it is discussed with what mathematics young, pre-verbal children might be engaged. The authors used for this Bishop's framework of the six mathematical activities which are fundamentally mathematical: counting, locating, measuring, designing, playing, and explaining. By reframing each of these activities by putting the focus rather on actions than on language, the framework is made appropriate for pre-verbal children and may provide assistance in identifying the mathematical thinking that is evident in pre-verbal children's actions.

The second paper (#14) focuses on games as the basic form of activity for preschool children and describes the mathematics that children can meet in games and through which they can achieve the ability to think mathematically. Questions to be answered are how the gameplay and the core mathematics experience are related and how the fun of games can be combined with the effectiveness. The paper continues by giving examples of teachers playing games with children and children playing alone or cooperatively.

A tool to measure the quality of the early childhood learning environment in a standardized way is the Classroom Assessment Scoring System (CLASS). With this tool, among other things, the given instructural support can be investgated with respect to three dimensions: the development of concepts, the quality of the feedback and the language modelling. When using this tool in a kindergarten class and analyzing the classroom interaction in an observed lesson (#15) it was revealed that there was a low score on Instructional Support: the teacher did not prompt children to explain their strategy, did only focus the feedback on the correctness of the answers, and did mostly asking close-ended questions. By proving this information, the tool can give indications in what way the teacher may develop.

In a large national study (#16), the quality of the learning environments in the child groups of Early Childhood Education and Care centres were investigated by means of data based on observations with the Infant/Toddler Environment Rating Scale—Revised and the Early Childhood Environment Rating Scale—Revised. The focus in these observations was on the learning area "Number, Spaces and Shapes". In addition, questionnaires were used to collect from directors of ECEC centres. A comparison of the results with a study done some seven years ago showed that the centres worked more systematically on this learning area. However, the quality of the learning environments as measured through the observations varies greatly and are to a large extent qualified as inadequate. For example, most of the centres only provided one kind of blocks on a daily basis, giving little opportunity for children to investigate different kinds of properties of space and shape.

The booklet ends with a study (#17) in which an alternative research perspective was chosen. In this study children themselves was given a voice when investigating the quality of the early childhood learning environment. The focus was on the quality of educational software. In particular five early numeracy apps were investigated, which were uploaded onto the classroom iPads. Data from 12 children (4 to 6 year-olds) were collected through multiple sources, including observations, interviews and videotaped child-led 'tours' of their favorite apps. As criteria for good apps were identified the quality of the game experience (frequent positive verbal reinforcement and earning rewards) and the automony in making choices.

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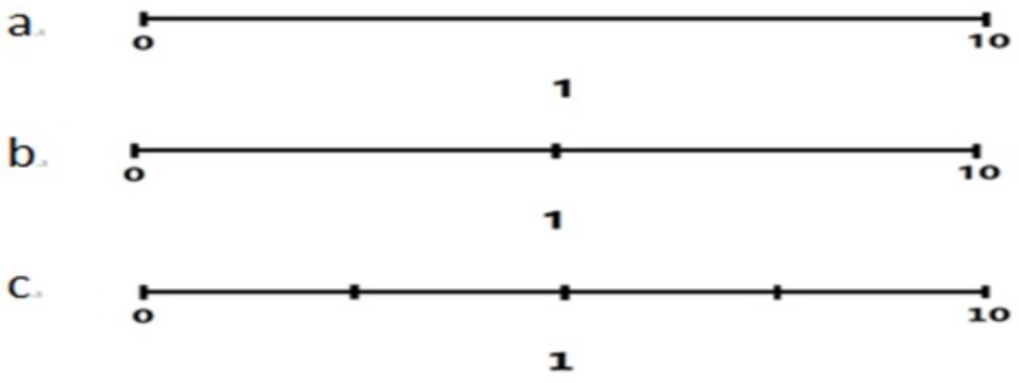
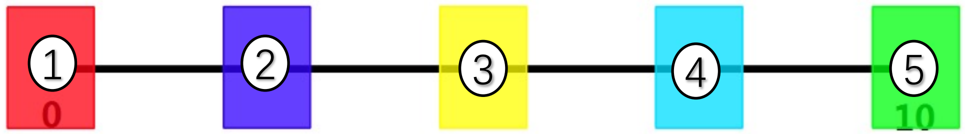
APPLICATION OF NUMBER LINE ESTIMATION STRATEGY FOR 5-6 YEARS OLD CHILDREN : EFFECT OF REFERENCE POINT MARKING

Participants： 90 children, aged 5-6 years (*M*=67.07 months, *SD*=3.27)

Stimuli and procedure: Presented number line in **a** endpiont, **b** midpoint, and **c** quartile condition



Analysis: fixation count, fixation duration and regression count

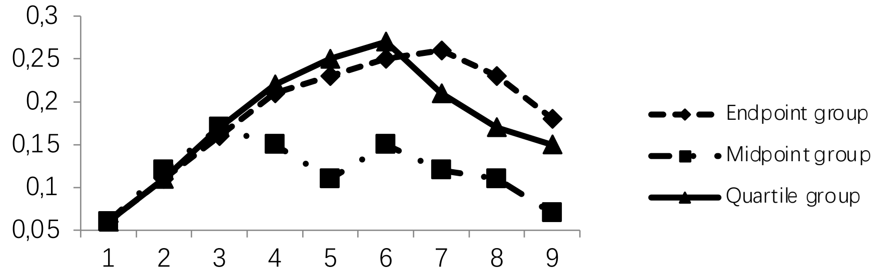


RESULTS:

Overall estimation accuracy

Group	Mean	SD	F	Post-hoc
1 Endpoint	.19	.11	4.79	2<1**,3*
2 Midpoint	.12	.07		
3 Quartile	.18	.09		

The estimation accuracy of each number



Conclusion:

1.The midpoint marker in the midpoint condition can indeed increase the frequency of children using the midpoint strategy.

2. The midpoint condition significantly improves the estimation accuracy of children aged 5-6 years compared with other two conditions.

APPLICATION OF NUMBER LINE ESTIMATION STRATEGY FOR 5-6 YEARS OLD CHILDREN: EFFECT OF REFERENCE POINT MARKING

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To verify whether the reference point marker is effective in improving preschool children' reference point strategies use, this study adopts the eye-tracking technology to investigate usage of reference point and the change of the estimate ability in the number line estimation (NLE) task for 5 to 6 years old children, in order to help preschool children improve reference points strategy use, to further promote the development of children's NLE ability. According to the estimated performance, 90 children aged 5-6 were divided into three groups with the same estimated level. 0-10 number line task was divided into three conditions: endpoint condition, midpoint condition and quartile condition. Each group completed one of the conditions. The results showed that children aged 5-6 years used different estimation strategies with different frequencies, and the estimation accuracy of midpoint group children was significantly higher than other.

INTRODUCTION

In recent years, researchers pay more attention to individual estimation strategies to understand the internal psychological mechanism of children's number mental representation. Peeters' results(2015) revealed that external reference point markers could increase the frequency of use of child reference points, and the mid-point reference points could improve the estimated performance of children. Some researchers have proposed the idea of using external reference points to help estimate numbers, but there is little research on the relevant content. Studies have proved that preschool children can use the reference point strategy, so whether the external reference point marker can increase the frequency of the application of the reference point strategy for preschool children, that is, whether the reference point marker is equally effective for preschool children needs to be verified.

This study intends to use 0-10 NLE tasks, combined with Tobii X3-120 portable eye movement equipment to collect 5 to 6 years old children eye movement and behavior data in number line of three different reference point mark, and analyze their reference points use. The study aims to solve the following key problems. First, is the frequency of three reference points use different among the three reference point conditions for children aged 5-6? Second, are the estimation strategies different? Third, how is the estimate accuracy? Which condition is more accurate?

METHOD

Participants

90 children aged 5-6 years were randomly selected from two kindergartens in Beijing (M=67.07 months,SD=3.27), all of whom had no previous experience of NLE tasks. Children performed one of three NLE tasks(endpoint condition, median condition, quartile condition, and your condition), with 30 children in each condition, ensuring that the level of estimation ability of children in the three groups was basically the same before experiment.

Stimuli and procedure

According to the study of Peeters, the NLE task is divided into three conditions: endpoint condition, midpoint condition and quartile condition.In the endpoint condition, number lines were bounded at both sides by the corresponding marks (0 and 10). In the midpoint condition, endpoint number lines with an additional mark at the midpoint (at 5) were presented. In the quartile condition, children were provided with a endpoint number line with a mark at every quartile (at 2.5, 5, and 7.5). The test time was about 25-40 minutes and the number of stimuli was 1, 2, 3, 4, 5, 6, 7, 8, 9. All tasks are completed on the computer. The length of the number line is 23 cm, and the stimulating number is displayed 3 cm below the number line.

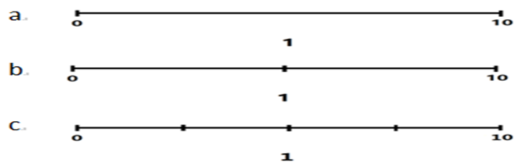

Figure 1: Presented number line in **a** endpiont, **b** midpoint, and **c** quartile condition

The study was conducted on children one on one by the experimenter, and the experimental process of each group of children was consistent. Before the test, all the participants went through the mouse operation test. To ensure that the child understands the NLE task, practice before the formal test begins. In the test, a number line was shown in the center of the screen, and the number stimuli appeared in random order. Participants selected the estimated position by sliding the mouse over the number line, determined the position, and then clicked the mouse to submit the answer, stimulating the update. After all stimuli are presented, the program will prompt the task to complete and exit, and the eye movement recording will stop at the same time. The test was conducted independently by the children, without any verbal cues.

RESULTS

Strategy use

To investigate which strategies children used in the three conditional NLE tasks, the number line divided into five equal area of interest (AOI): AOI 1 (starting point area), AOI 2 (1/4 area), AOI 3 (midpiont area), AOI 4 (3/4 area), AOI 5 (end point area). See figure 3 for details. The eye movement indexes of fixation count and the fixation duration and regression count were calculated in the five AOI.

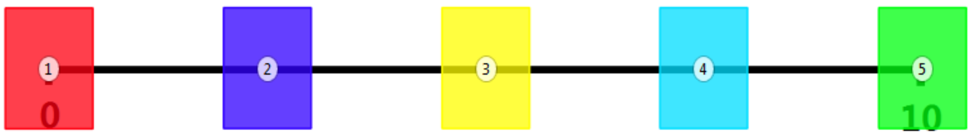

Figure 2: Area of Interest division

When fixation count, fixation duration and regression count were at least two items significantly higher in a AOI than other AOI, it was determined that children used this reference point strategy.

Repeated measures ANOVA was used to investigate the differences in fixation count(FC), fixation duration(FD) and regression count(RC) when children estimated each number.

In the endpoints group, when estimating 1 and 2, FC and FD of the children in AOI 1 was significantly higher than other AOI, indicating that children used the starting point strategy. when estimating 3-9, FC, FD and RC of the children did not focus on the same AOI, manifesting that the children may not have adopted the reference point strategy. In the midpiont group, FC and FD of the children were significantly higher when estimate 1, indicating that children used the starting point strategy. The reference point strategy was not used for 2 and 3, because FC, FD and RC of the children did not focus on the same AOI. When estimating 5-8, FC and RC of the midpoint group children in AOI 3 were significantly higher, indicating that the midpoint strategy was used. When estimating 9, the FC, FD and RC of the midpoint group children in AOI 5 were significantly higher, indicating that the children used the end-point strategy. In the quartile group, when estimating 1, FC, FD and RC of the children in AOI 1 was significantly higher, indicating that children used the starting point strategy. The FC, FD and RC of the children in AOI 2 were significantly higher when estimating 5, this indicates that children may have used the quartile strategy. When estimating 2-4 and 6-9, they may be does not adopt the strategy of reference point.

Estimation accuracy

Overall estimation accuracy. A one-way ANOVA assessing the effect of groups (endpoint, midpoint, quartile) on overall PAE was significant, $F(2,87)=4.79$, $p<.05$. PAE of children in the midpoint group was significantly lower than other group, and there was no significant difference between the endpoint group and the quartile group children.

Group	Mean	SD	F	Post-hoc
1 Endpoint	.19	.11	4.79	2<1**,3*
2 Midpoint	.12	.07		
3 Quartile	.18	.09		

Table 1: Comparison of PAE among different group children

The estimation accuracy of each number. A MANOVA assessing the effect of groups (endpoint, midpoint, quartile) on PAE of each number was significant, $F(18,160)=1.89$, $p<.05$, $\eta^2=.18$. The PAE of 4, 5, 6 and 7 for midpoint group children was significantly lower than the endpoint group and the quartile group, while there was no significant difference between the latter two groups.

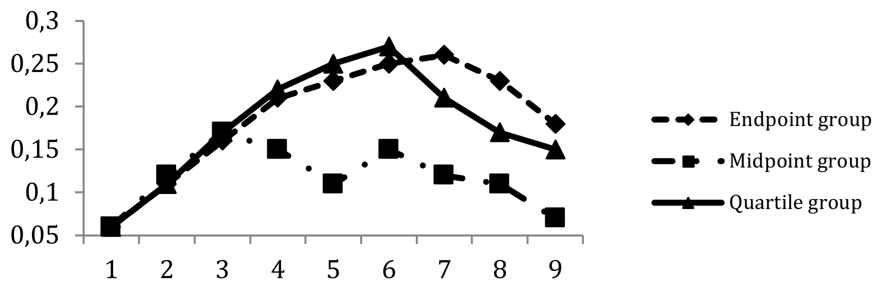

Figure 3: The estimation accuracy of each number among different groups children

DISCUSSION

The study found that the midpoint children used the midpoint strategy more frequently than the endpoint and quartile children when estimating each number, indicating that the midpoint marker in the midpoint condition can indeed increase the frequency of children using the midpoint strategy. The quartile marker also included the midpoint, but the midpoint marker did not increase the frequency of use of the midpoint strategy in children as the midpoint condition did. The reason may be that the quartile condition also contains two quartile markers, which increases the complexity of the marker. Moreover, most children aged 5-6 are still unable to carry out quartile classification. Therefore, two marks with unknown meanings will affect children's judgment of the midpoint.

The result of estimation accuracy,.it shows that the midpoint condition significantly improves the estimation accuracy of children aged 5-6 years compared with other two conditions. This is consistent with Peeters' results(2015) on the 0-200 NLE task for second graders. The reason for this may be that the 0-10 are familiar to 5-6 year olds, they can count every number, Therefore, under the endpoint condition, the child will superimpose the psychological length of ‘1‘ with the help of the starting point marker to get other numbers. When the length of "1" is not accurate, this superposition will result in lower accuracy. In the midpoint condition, the external reference point marker can attract the attention and thinking of children to accurately judge the position of ‘5‘, Children estimate with the help of the midpoint marker, which improves the estimation accuracy. In the quartile condition, the estimation accuracy was not higher than the other two marker conditions. The reason was that the three externalized reference points provided to children caused obstacles. Children can't understand the numbers represented by three reference points. The child used the first quartile as an aid in estimating the midpoint number (5). This may be that the child did not choose the appropriate reference point strategy, or it may be because the child mistakenly believed that a quarter represented 5, Therefore, the reference point under the quartile condition did not play its due role.

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Unraveling the quantitative competence of kindergartners

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Background

What mathematics should we teach beginning learners of mathematics?

⇩

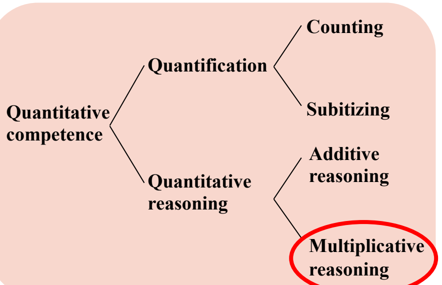
Unraveling the components of early quantitative competence and how they are related.

⇩

Theoretical basis: The two main components, quantification and quantitative reasoning correspond with the division in two as found by Jordan et al. (2006).

⇩

New in our model: Quantitative reasoning is extended with multiplicative reasoning.



Research Questions

R1. Can early quantitative competence be modelled as a four-factor structure?

R2. What are the relations between items assessing the components of early quantitative competence?

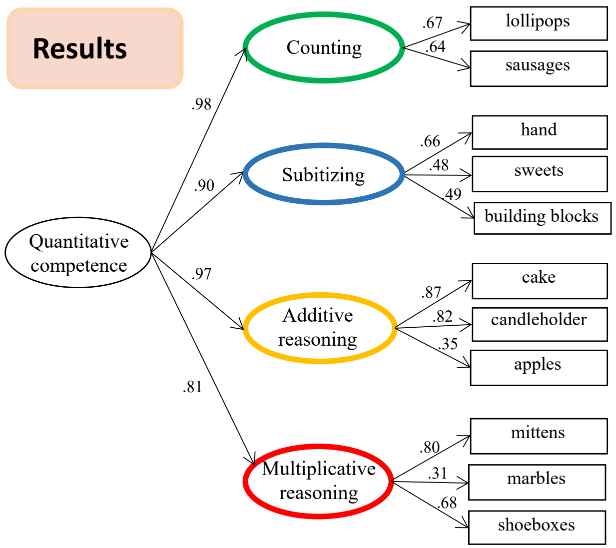
Method

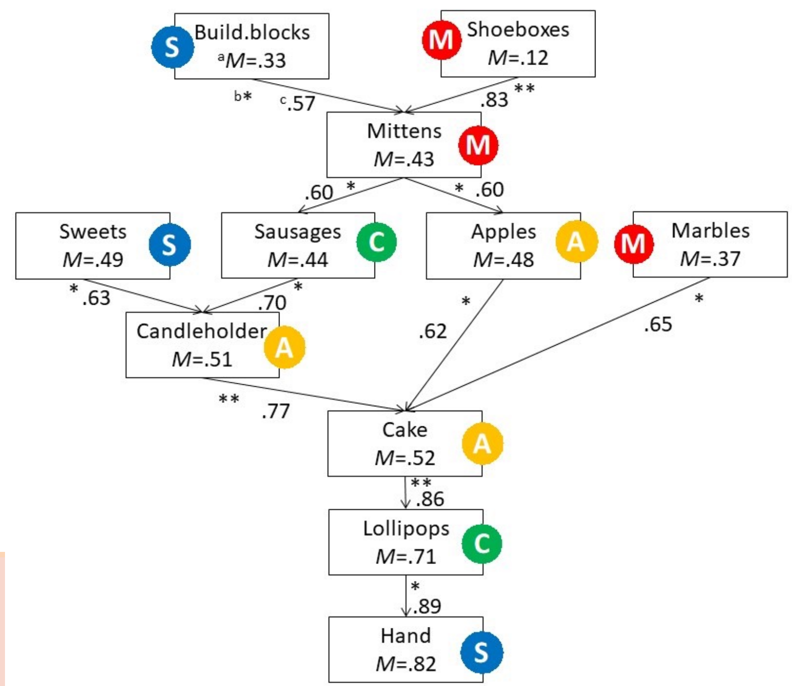
Participants: K1 children (*n*=123, average age = 4.67) and K2 children (*n*=211, average age = 5.69)

Instrument: A set of paper-and-pencil items about counting (C), subitizing (S), additive reasoning (A) and multiplicative reasoning (M)

Analysis: CFA and Statistical Implicative Analysis at item level

Results





Conclusions

- The four-factor model was quite well reflected in the empirical data.

We see this study as a first step to further unravel early quantitative competence of kindergartners in which multiplicative reasoning is included.

- The implicative chains show that in general, multiplicative reasoning and conceptual subitizing items were found at the top and counting and perceptual subitizing items at the end, with additional reasoning items in the middle.

UNRAVELING THE QUANTITATIVE COMPETENCE OF KINDERGARTNERS

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In this study we investigated the structure of quantitative competence of kindergartners by testing a hypothesized four-factor model of quantitative competence consisting of the components counting, subitizing, additive reasoning and multiplicative reasoning. Data were collected from kindergartners in the Netherlands (n = 334). A confirmatory factor analysis showed that the four-factor structure fit the empirical data. A statistical implicative analysis at item level revealed that the found implicative chain reflects by and large the sequential steps mostly followed in teaching kindergartners early number: starting with counting and subitizing, then additive reasoning and finally multiplicative reasoning. These implicative chains also clearly show that the development of early quantitative competence is not linear. There are many parallel processes and cross-connections between components of quantitative competence.

BACKGROUND OF THE STUDY

Recently much awareness has grown that young learners' future understanding of mathematics requires an early foundation based on a high-quality, challenging, and accessible mathematics education (Duncan et al., 2007; NCTM, 2013; Geary, 2011). Necessary for developing this education is a good understanding of what mathematics we want beginning learners of mathematics to get acquainted with and need to teach them. To feed this understanding, in this study we focus on early number and try to unravel its components and how they are related.

For doing this we investigated a model of quantitative competence consisting of two main components, each with two specific components (see Figure 1).

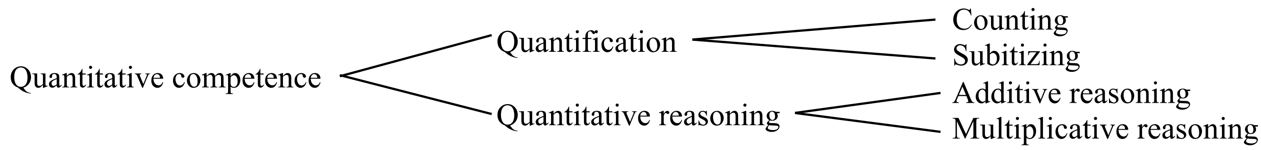

Figure 1: Model of quantitative competence

The two main components, quantification and quantitative reasoning, correspond with the division in two as found by Jordan et al. (2006), but what is new in our model is that we extended the quantitative reasoning with multiplicative reasoning. Our research questions were:

R1. Can early quantitative competence be modelled as a four-factor structure containing the components counting, subitizing, additive reasoning and multiplicative reasoning?
R2. What are the relations between items assessing the components of early quantitative competence?

METHOD

Sample

The study was carried out with children from 18 kindergarten classes from 18 schools in the Netherlands. The classes included both first-year (K1) (*n*=123, average age = 4.67) and second-year kindergartners (K2) (*n*=211, average age = 5.69). The mathematics program mostly consists of playful activities about number and shape and space.

Instrument

The quantitative competence was assessed in a whole-class setting with a set of paper-and-pencil items originally developed for the PICO project (Van den Heuvel-Panhuizen et al., 2016).

Component	Name test item	Format	Description
Counting	Lollipops	Open response	A picture of 10 lollipops. The children are asked to circle 5 lollipops.
	Sausages	Multiple choice	A picture of 10 dogs and 4 boxes with respectively 8, 9, 10 and 11 sausages. The children are asked which box has so many sausages that each dog can get one.
Subitizing	Hand	Multiple choice	Four drawings of a hand with respectively 4, 5, 6 and 3 fingers. The children are asked to select the hand which is like their own hand. (perceptual subitizing)
	Sweets	Multiple choice	Four drawings of 6 sweets in different arrangements (respectively next to each other in a horizontal row, in a diagonal row, placed criss-cross, and placed in two groups of three). The children are asked to select the drawing by which they can identify the fastest where there are 6 sweets. (conceptual subitizing)
	Building Blocks	Multiple choice	Four drawings of 10 blocks in different arrangements (next to each other in a horizontal row, two times as an irregular building, in two rows of five blocks). The children are asked to select the drawing by which they can identify the fastest where there are 10 blocks. (conceptual subitizing)
Additive reasoning	Cake	Open response	Picture of a birthday cake with 6 marked places for the candles and next to it six boxes with candles (two boxes with 2 candles each, two with 4 candles each and another two with 3 candles each). The children have to select the boxes that would make exactly 6 candles together.
	Candleholder	Open response	Picture of a candleholder with 5 empty places for the candles and next to it five boxes with candles (two boxes with 3 candles each and three boxes containing 2 candles each). The children have to select the boxes that would make exactly five candles.
	Apples	Multiple choice	Picture of a rectangular case with room for 9 apples. The case contains only six apples. Next to the case there are four drawings with respectively 2, 3, 4 and 5 apples. The children are asked how many additional apples fit in.
Multiplicative reasoning	Mittens	Multiple choice	Picture of three children and next to them four boxes with respectively 3, 4, 6 and 8 mittens. The children have to indicate which box is needed to give these children mittens. (multiplication)
	Shoeboxes	Multiple choice	Picture of a shoebox with next to it 10 shoes in a horizontal line. Below them four boxes with respectively 5, 4, 10, and 7 shoeboxes. The children have to indicate which box has the needed number of shoeboxes. (quotative division)
	Marbles	Multiple choice	Picture of 3 children with next to them 9 marbles in row. Below them four boxes with respectively 1, 2, 3 and 4 marbles. The children have to indicate how many marbles each child will get. (partitive division)

Figure 2: Overview of test items

The set includes items about counting (2), subitizing (3), additive reasoning (3) and multiplicative reasoning (3) (see Figure 2), each covering one page with a picture illustrating the problem. The instruction is given orally. Answers can be given by underlining pictures. Cronbach's alpha is α = .72.

Analysis

To answer our first research question a confirmatory factor analysis (CFA) was applied, using MPLUS. For the second research question, we investigated the implicative relations between the used items by means of CHIC (Classification Hiérarchique, Implicative et Cohésitive) software (Bodin et al. 2000).

RESULTS

Figure 3 shows the found higher-order CFA model of four first-order factors standing for the four early quantitative components counting, subitizing, additive reasoning and multiplicative reasoning. The used items loaded adequately on each of the four factors. The model involves a second-order factor on which all the first-order factors are regressed. This second-order factor stands for the general quantitative competence underlying the solution of items involving counting, subitizing, additive and multiplicative reasoning. The model reflected the empirical data quite well. The descriptive-fit measures supported the hypothesized model (χ^2=42.40, *df*=40, χ^2/df=1.06, *CFI*= .997; *RMSEA*= .01).

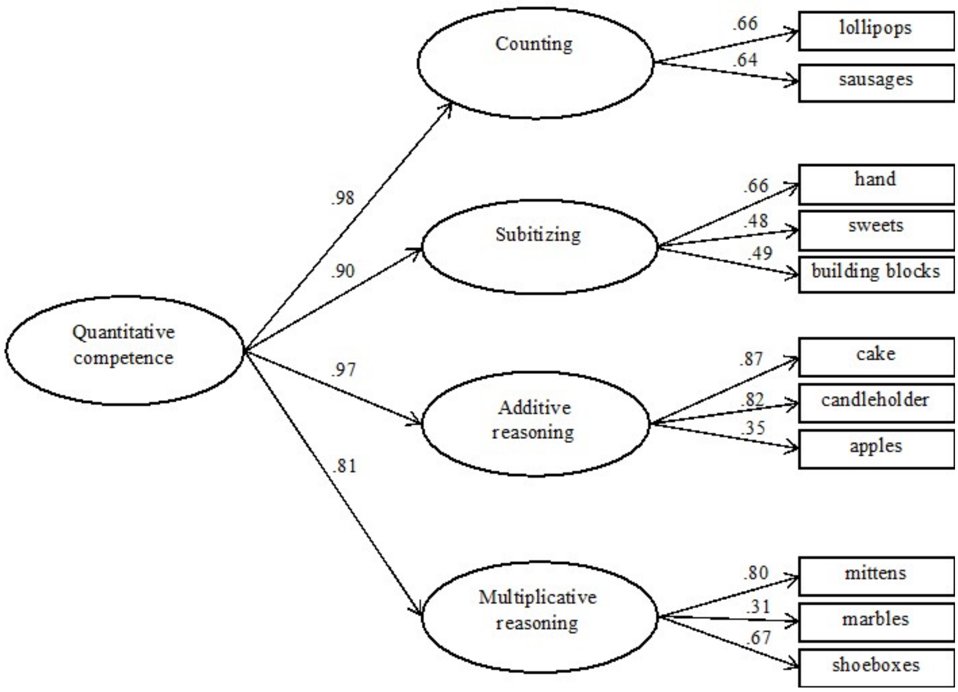

Figure 3: CFA model for early quantitative competence components

The diagram in Figure 4 graphically shows the implicative relations we found between the early quantitative competence items. In general, the multiplicative reasoning and conceptual subitizing items were found at the top of the chain and the counting and perceptual subitizing items at its end, whereas the three additive reasoning items are in the middle of the chain. As an example, a strong direct implicative relation was obtained between the *multiplicative reasoning* items Shoeboxes and Mittens, indicating that children who were successful in the Shoeboxes item, figuring out how many pairs of shoes can be made by ten shoes (quotative division), were successful also in the Mittens item where they had to figure out the total amount of mittens needed for three children (multiplication).

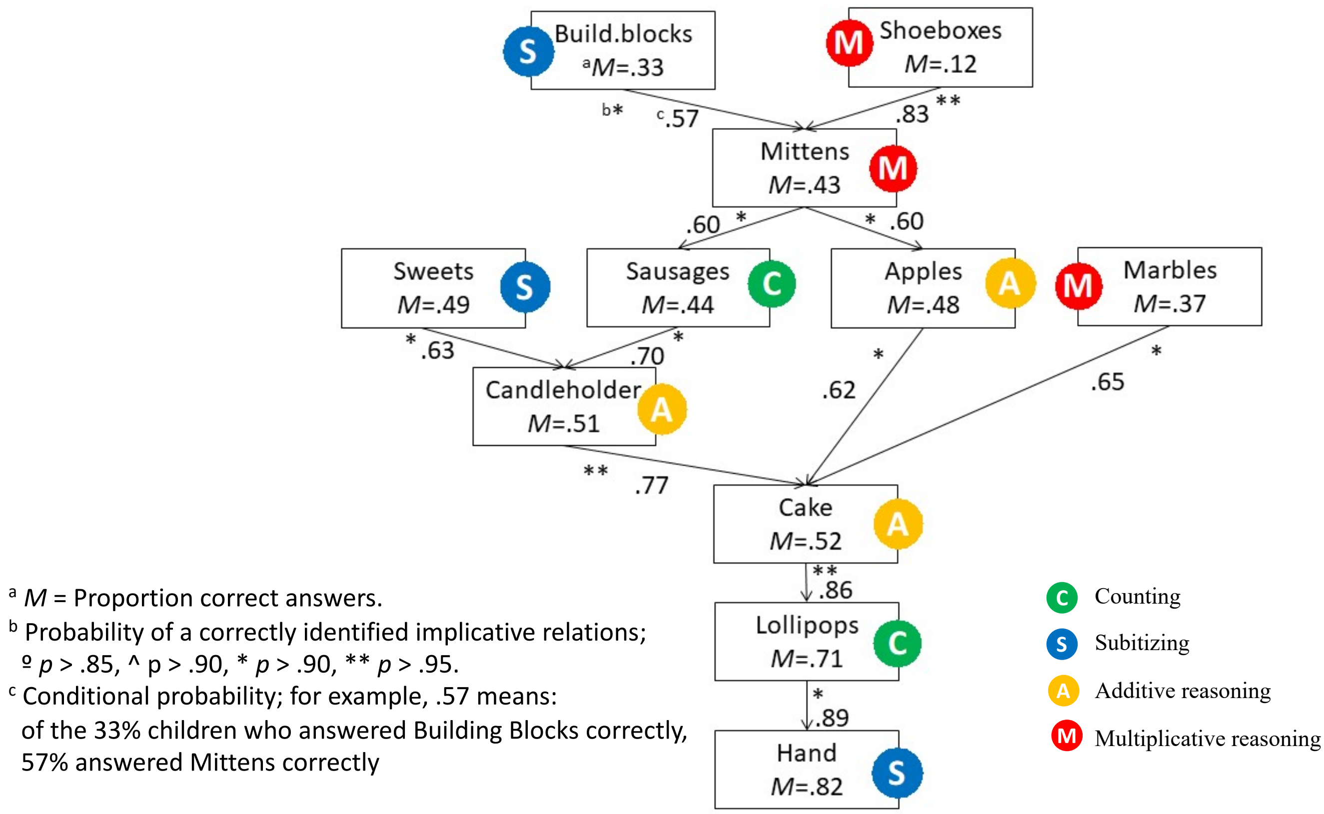

Figure 4: Implicative diagram early quantitative competence items based on the children's responses

TO CONCLUDE

For us this was a first step to unravel the early quantitative competence of kindergartners. Although the study's design was not perfect, we think it is worthwhile to continue with this line of research and further investigate the four-factor model of young children's quantitative competence which besides the components counting, subitizing and additive reasoning, also includes multiplicative reasoning.

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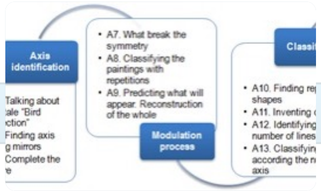
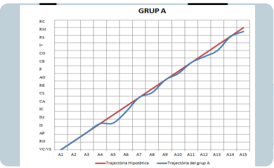
INSIGHTS ABOUT CONSTRUCTING SYMMETRY WITH 5-YEAR-OLD CHILDREN IN AN ARTISTIC CONTEXT

The objective of this communication is to describe an initial approach to the construction of a learning trajectory of symmetry for early childhood education



- **Learning trajectories** (Sarama & Clements, 2009)
- **Symmmetry in early childhood education** (Seo & Ginsburg, 2004; Streefland, 1991 and Sámuel, Vanegas & Giménez, 2016)

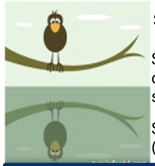
- A sequence of 16 activities considering the components of the LT
- Two groups of 25 students aged 5-6 years old.
- The implementation of the sequence was recorded audio-visually.



- Children explain the change of orientation.
- Children classify according to the number of axis.
- Children do predictions about figures referring equal shapes

- The dialogues are opportunities for children visualizing some elements that "break the symmetry".
- The designed sequence can constitute a hypothetical path of symmetry.

HLT OF SIMMETRY

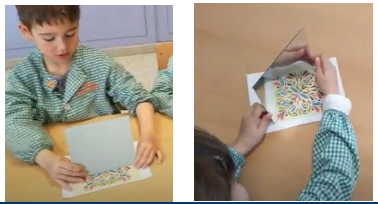


S1: Why it is like this (on an opposite side)

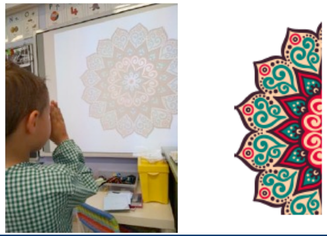
S2: I know, because always you put a mirror or in a lake, you will see on the contrary side.

S3: Yes. Here the tree goes up and here (mark the bird below) goes down.

REPETITIONS AND SIMILARITIES



AIXIS IDENTIFICATION



MODULATION PROCESS

CLASSIFYING

STRUCTURING AND APPLYING

UNIVERSITAT DE BARCELONA
Universitat de Lleida

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ICME-14

INSIGHTS ABOUT CONSTRUCTING SYMMETRY WITH 5-YEAR-OLD CHILDREN IN AN ARTISTIC CONTEXT

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The objective of this communication is to describe an initial approach to the construction of a learning trajectory of symmetry for early childhood education. A sequence of activities was designed based on found in previous research, in which art is introduced as a specific context. It describes the implementation of the proposal, which was carried out with 5-6 year olds and the subsequent analysis. The results obtained from the study have allowed us to recognize that children can classify artistic works according to the number of axes and how they consider the line of symmetry as an axis. At the same time the results reveal that the path followed in the sequence may constitute as a hypothetical learning trajectory of symmetry for early childhood education.

INTRODUCTION

Spatial thinking skills and geometric reasoning play a critical role in the development of problem-solving skills, mathematical learning, and reading comprehension for children (Van den Heuvel-Panhuizen & Buys, 2008). Numerous studies also indicate that spatial skills are far more important to academic achievement than many elementary educators previously thought. It's important to have a variety of tried and tested spatial and geometric learning activities in which children discuss and evidence their observations about the phenomenon of symmetry. In the case of symmetry, several researches show us that cultural issues are important references for symmetry (Giménez & Vanegas, 2019). First one is the idea of identifying a pattern of repetition starts since early experiences based upon congruence observations. More difficulties appear about the location of symmetry axis and identification of properties (Sámuel, Vanegas & Giménez, 2016). In this presentation we describe the path followed by pupils aged five years old in order to construct the notion of symmetry when working on mathematical activities in artistic contexts.

LEARNING TRAJECTORIES AND SYMMETRY IN EARLY CHILDHOOD EDUCATION

Children come to school with a large repertoire of informal spatial understandings that should be developed. Brenneman, Boyd and Frede (2009) highlight the importance of children learn and develop mathematical skills from an early age, as it is considered that this will provide a solid basis for further learning.

It is clear that geometric knowledge is not acquired by receiving information, nor does it limit in observing and recognizing certain forms and knowing their correct name, it implies developing very diverse capacities in each person. It involves a long process that requires: exploring, comparing, decomposing/recomposing, visualizing, symmetrising, and transforming among other aspects.

Symmetry, among other concepts, is related to the creation of patterns that help us organize our conceptual world (Knuchel, 2004). Working with the notion of symmetry at early ages is important for the development of geometrical thinking (Sarama & Clements, 2009). Seo and Ginsburg (2004) found that pattern and shape, including symmetry, were frequently explored in free play related to the mathematics of 4-5 years old. However, despite this spontaneous and recurrent inclusion of symmetry in children's construction and play activities, symmetry is a notion to which little time is devoted in early childhood education and when addressed it is generally done in a limited way. An important aspect to build the notion of symmetry is to relate to the idea of module of repetition, and the fact that the symmetry axis allows the children to reconstruct the whole, as it was observed in halving process (Streefland, 1991).

Sarama & Clements (2009) state that children follow natural processes of development when learning mathematics. These developmental paths form the basis of learning trajectories (LT). These authors consider that learning trajectories involve three essential components: *a mathematical goal*; *a developmental path*; and *a set of instructive activities or tasks typical of the levels of thinking of the path*. The use of LT can help answer key questions concerning teaching and learning processes: the goals to be set; where to start; how to decide the direction of the next step; and how to achieve that next step.

METHODOLOGY

The research is based upon qualitative observation as a case study and designed based research process using realistic situations. A sequence of 16 activities considering the components of the LT was designed to respond to research goal (see Figure 1). This proposal was carried out with two groups of 25 students aged 5-6 years old in a public school in Spain. The activities featured the use a wide variety of spatial and geometry education materials. The implementation of the sequence was recorded audio-visually.

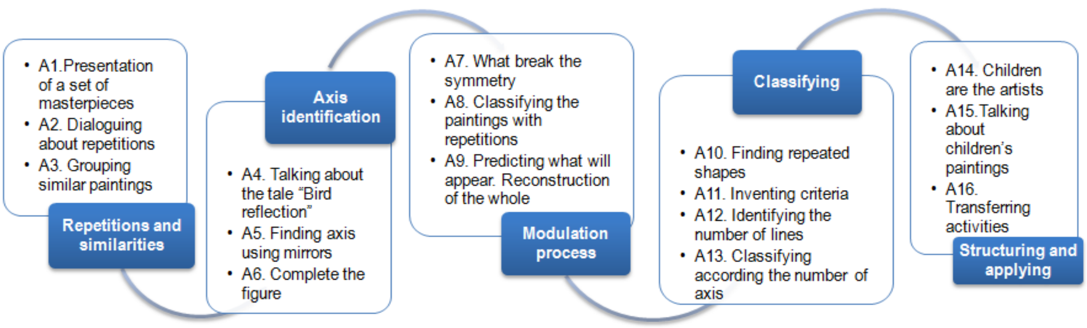


Figure 1: Set of activities for LT for symmetry

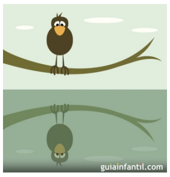
In the design of the activities, we consider that the construction of the notion of symmetry in early childhood education involves the following processes: identification of repetition phenomena; identification of symmetry lines; visualizing elements for a broken symmetry and module making; classifying according number of axis and constructing and structuring ideas using personal representations. To analyze the discourse we focusing in different small groups of four-five children, to look their constructions, mediated by the language. We also see if they ask for the material, or simply answer the questions proposed by the teacher. We use a presentation as card-game with big format to give opportunities for children to see details and manipulate them when using mirrors. By way of an example, Table 1 presents activities 1 and 5, consisting of the goal, the task, the description and a series of questions that guide the dialogue during its implementation.

	Goal	Task	Description	Image	Dialogue
A1	Introducing a set of paintings as a provocation of geometry findings	Initial discussion about feelings	Children talk about the paintings and their meaning		Why did you choose such painting? What is painter's thinking?
A5	Find out whether the child recognizes the line of repetition. Describe the type of recognition	Discussion about where the lines of symmetry are	Children should find several lines of symmetry if it is possible. To ensure it, they should reconstruct the global image		Could you find other lines than vertical? Please, continue. Do you find the global image?

Table 1: Example of part of the learning trajectory for symmetry

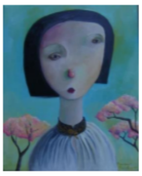
SOME FINDINGS

After analyzing the classroom process, as design-based research, some trends of hypothetical trajectory were found. Learners identify the horizon line in the picture. They recognize the phenomenon when using the tale "The Bird's Reflection". Children explain the change of orientation. Many interventions reveal consecutive deep approaches, assuming the production of hypothesis, explanations and confirmations or refutations, as shown in the following dialogue of a group of children



1 S1: Why it is like this (on an opposite side)
2 S2: I know, because always you put a mirror or in a lake, you will see on the contrary side.
3 S3: Yes. Here the tree goes up and here (mark the bird below) goes down.

The dialogues are opportunities for children visualizing some elements that "break the symmetry". Let's see an example when children are talking about a about paintings that showing a face. They value the consideration of repetition without complete symmetry. In fact they discuss whether all the cards (that reproduce different paintings) having or not some line of symmetry. In such dialogue we assume that children have recognized the property.



1 T: Ameli says that we should consider also these paintings
2 S1: No, because they have an eye up, and another eye down
3 S2: No, do not consider these ones
4 T: Who says yes? Are they equal? Raise your hands. [Only three children say yes]. Now raise your hands if you consider they are not completely symmetric? Almost the majority say it. Therefore, why you did that?
5 S2: Because here you have "an eyebrow" and the other part no
6 S3: But this one yes. They are equal (mark the face with the trees).
7 S4: No because they have an eye up, and another eye down
8 T: Therefore, do you consider these ones?

Some children found other lines as diagonal ones, as it is possible to look in the figure. Other children imitate the actions and explanations of their colleagues. They are surprised of the modular idea, that the figure appears from a triangle to form the square as the complete painting.

When the children did the prediction task, they even explain that some images are equal shapes, but colours are different. The group discussion reveals achievements very close to the symmetry observations. It appears some language difficulties.

SOME FINAL REMARKS

It seems that almost all of the children discover the four possibilities of the classification according the number of axis. In some different tasks, we observe that the arguments evoke the difficulties when representing symmetries. The importance of self-communicating to see that many abstract ideas are intentional, but children need to complete technological experiences in which you see what you desired.

It is noteworthy that in this type of study is relevant to consider the influence of the interaction between children and the tutor, as it could make a difference in the evaluation of actions in the various activities. We could observe that children learn from the interaction in a small group. In some cases, this could be considered a limitation of the study. In our case, the tutor who participated is a teacher who knew the children and was involved in the research.

The results reveal that the designed sequence can constitute a hypothetical path by which children in early childhood education can progress in their learning of symmetry.

Acknowledgement

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Kindergartners use of symmetry and mathematical structure in representing self-portraits

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MACQUARIE University

Background

Pattern and Structure Mathematics Awareness Projects (2009-2016)

Connecting Mathematics Learning through Spatial Reasoning (2017-2021)

Research question

What is the relationship between identification of symmetry and development of mathematical structural features?

Symmetry is fundamental (and possibly causal) to future development; link between symmetry and spatial reasoning

Literature

Elia's (2018) Thom et al. (2015; 2018) Bruce et al., 2015; model-based reasoning (Lehrer & Schauble, 2000, Oslington et al., 2018)

Method

Kindergarten study of symmetry: n=44, 4y5m-5y11m (analysis of one task and consistency of structural level for individuals)

Results

30% of children represented explicit structural features such as equal spacing, congruence, partitioning and alignment of facial features.

Children's Representations

- Analysis of symmetry— matching key features of the face in alignment reflecting symmetrical features (eyes, nose, mouth and ears).
- Drawings categorised for one of five levels of structural development.

Mulligan, J., Oslington, G., & English, L. (2020). Supporting early mathematical development through a 'pattern and structure' intervention program. *ZDM Mathematics Education, 52*(4), 663-676. doi:10.1007/s11858-020-01147-9

KINDERGARTNERS' USE OF SYMMETRY AND MATHEMATICAL STRUCTURE IN REPRESENTING SELF-PORTRAITS

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As part of a large study, Connecting Mathematics Learning through Spatial Reasoning, 44 Kindergarten children's drawings of their portraits were analysed for features of line symmetry and mathematical structure. Children's portraits were initially drawn by observing the face of a partner. The drawing was folded along a central line, placed on blank paper, with only half the portrait visible along a line of symmetry. The child completed the portrait matching the explicit features of the initial drawing. Evidence of symmetry was analysed such as matching key features of the face in alignment and with details reflecting the symmetrical features of the eyes, nose, mouth and ears. Drawings were categorised for one of five levels of structural development utilised in a related study on mathematical pattern and structure with Kindergartners. There were wide qualitative differences found in the drawings, with half of the drawings showing incomplete symmetrical features. However, over thirty percent of children represented explicit structural features such as equal spacing, congruence, partitioning and alignment of facial features.

INTRODUCTION

The analyses of children's drawings of their responses to spatial tasks have featured in many studies of early childhood mathematics education over the past decades (Brizuela, 2004; Elia, 2018; Mulligan & Mitchelmore, 2018; Thom, 2018). In a suite of studies Mulligan and colleagues found that the analysis of patterns and structural features of children's representations across domains provides reliable evidence of their developing spatial structures. Elia's (2018) longitudinal study of a kindergarten class found that gestures, together with oral language and semiotic inscriptions, played a critical role in kindergartner's development of geometric awareness of 2D shapes, their attributes and shape deconstruction. Another study by Thom and colleagues (2015) gave further insight into young children's spatial-geometric reasoning by elucidating the role of embodied actions in children's drawings. Curriculum developers and professional development programs have also promoted broadly the development of representational thinking through children's drawings and justifications often portrayed as 'work samples'.

Recent research perspectives based on 'embodied action' have highlighted the role of drawings as more than products that provide evidence as snapshots of learning. Thom asserts that children's drawings reveal "*re*presentations that reveal their cognitive schema— what they 'know' about geometry, such as their cognitive capabilities, spatial awareness, and conceptual understanding" (Thom, 2018, p.134). Thom and McGarvey (2015) describe children's mathematical drawings as both acts and artifacts where the act of drawing serves as a means of developing conceptual awareness rather than being a product of that awareness. Although the research process in many studies aims to capture the embodied process of drawing, there remain few studies that provide analyses of both the process and the artefact or product, along with the child's explanation and sense making of the process.

Symmetry is a fundamental concept that young children encounter in their early years prior to formal schooling. Children often recognise symmetry in their everyday environment and they can spontaneously construct symmetrical figures during play. Exploring the line of symmetry is often the focus of early mathematics curriculum and embedded in spatial reasoning programs in both static and dynamic ways (Bruce, Sinclair, Moss, Hawes, & Caswell, 2015; Schuler, 2001). In connection with a large study of children's spatial reasoning, Kindergarten children's concept of symmetry was explored in a task requiring them to observe symmetry in the faces of others. Two research questions were raised:

1. How do children notice the spatial and structural features of their face including line symmetry?
2. How accurate and detailed are their drawings of facial features reflecting line symmetry?

METHOD

Context and participants

The study was conducted in one metropolitan school, in Sydney, in the state of New South Wales (NSW) Australia with two groups of Kindergarten children. The school served families from socio-economic backgrounds of middle to high incomes and drew children from a wide range of cultures. The children were aged between 4 years 5 months and 5 years 11 months (mean 5 years 4 months). The teacher administering the task was a specialist teacher at the school and acted as 'teacher as researcher' (second author) and research assistant to the project. She was assisted by two regular teachers.

Symmetry task

As part of a larger project on spatial reasoning one area of activity was focused on the body as symmetrical. Over a series of four activities on line symmetry children were challenged with a task where they observed their facial features in a mirror and discussed what was "the same and different" about their face. Preliminary tasks included drawing or cutting simple two-dimensional shapes and folding a line of symmetry to show halves. The children, working in pairs, then drew their partner's face. The teachers and researcher helped them find the approximate line of symmetry and fold the portrait so that half the features were hidden. This half was attached to a blank piece of A4 paper, and photocopied. The children then swapped portraits with another pair and then drew the missing half of the portrait with the child's face in view.

The children were allowed to make several attempts before completing the task. They discussed their observations with their peers and the teacher drew attention to aspects such as partitioning the face by visualising the position of features along the line of symmetry . For example, children were encouraged to carefully look at the shape of the missing half of the face, and to match individual features on each side.

RESULTS AND DISCUSSION

Table 1 shows the percentage of drawings categorised by structural level and a description of the drawings' features. The majority of drawings fell in the Emergent and Partial Structural categories. Emergent responses showed at least one symmetrical feature, although the majority of features were repeated or obviously not symmetrical. Interestingly some children included different features on each side of the face at times including features that they "thought" should be there, rather than focusing on creating a symmetrical drawing of the features that were observed.

Structural Level	%	Description
1.Pre-structural	9	Icons, marks or repetition of a whole face without awareness or use of symmetry
2. Emergent	9	Some matching icons, marks and facial features drawn on corresponding half of face with some alignment but without symmetry
3.Partial Structural	50	Some icons, marks and facial features drawn on corresponding half of face with alignment and symmetry but partially complete
4. Structural	27	Icons, marks and facial features drawn on corresponding half of face with alignment and use of line symmetry
5. Advanced	5	Icons, marks and facial features drawn accurately and with explicit detail on corresponding half of face with alignment and use of line of symmetry and with explanation of the symmetry

Table 1: Percentage of drawings by structural level (n=44)

Pre-structural | Emergent | Partial | Structural | Advanced

Fig 1: Five different children's symmetrical drawings of their face by structural level

Partial Structural features were easily discerned through a direct attempt to match the features of the other half of the face. However, symmetrical features were not consistently applied with some features such as the shape of the head asymmetrical or alternatively, missing completely. Structural and Advanced Structural drawings both explicitly used symmetrical features in the representations, with Advanced Structural drawings demonstrating equal spacing, partitioning and accurate copying of features such as number of teeth, eyelashes, locks of hair and patterned clothing. Fig. 1 illustrates some typical drawings of the children's faces that range from non-symmetrical to showing accurate symmetry (Pre-structural to Advanced Structural). Pre-structural responses were often a repetition of whole faces without recognising symmetry in the original drawing.

The exemplars analysed in this paper present drawings produced at one point in time. These indicated the presence or absence of important developing structural features such as equal spacing, partitioning, halving and estimating. Furthermore, depictions in the drawings may have reflected a response that their classroom teacher encouraged or expected, even though the children were at the early stage of schooling. Although it was not feasible in the tasks to capture the drawing process for every child as a digital recording, we did capture pertinent examples of the process of drawing to demonstrate spatial structuring. Despite the valuable insights that can be gained from explicit interpretation of drawings, we can only make inferences about the child's external representations as reflections of their internal conceptual structures.

Limitations and Implications

The levels of structural development depicted by the drawings provided descriptors of the children's increasingly sophisticated development of mathematical structures and l symmetry. There were powerful examples of children's awareness of symmetry and spatial processes such as embedding shapes, equal spacing, partitioning, congruence and alignment. However, the analysis from one pertinent task should be viewed as only part of the child's developing spatial reasoning. Although the analysis did show the child's awareness of matching of facial features, it could not be assumed that their portrait attempts provided conclusive evidence of their conceptual understanding of line symmetry. In fact, some children ignored the fold or line and still produced accurate drawings. Analysis of children's explanations in conjunction with the drawing process is necessary to provide deeper insight into their development of symmetry.

When children begin formal schooling in Australia there seems to be a transition from the pre-school's focus on spontaneous self-initiated drawing and using a broad and unrestricted range of media, to teacher-directed more formal drawings of mathematical ideas or situations. These are sometimes limited to traditional activities such as drawings or paintings of "butterfly" pictures by paper folding and can be restricted by the size and shape of the media and tools for expression. This paper raises the issue that there needs to be more opportunities for children to self-initiate their representations of mathematical ideas in contexts that are meaningful and challenging. Raising this awareness with both researchers and professionals is critical for further reliable use of drawings as evidence of mathematical development.

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INVESTIGATING EVIDENCE OF GIRLS' AND BOYS' EARLY SYMMETRY KNOWLEDGE THROUGH MULTIPLE MODES OF ASSESSMENT

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- Intervention using software to teach symmetric transformations— reflection, translation, and rotation—to young children
- Prior study (Fletcher, 2015) found intervention group was better able to identify, explain, and create symmetric transformations than control group
- Purpose of current study:
  - (1) to explore the convergence of qualitative and quantitative evidence collected in the study
  - (2) to explore any potential differences between girls and boys in this convergence
- Results:
  - (1) Moderate positive correlations were found. Children who demonstrated conceptual understanding in explanations of symmetric transformations at post-test also scored higher on paper-and-pencil post-test tasks.
  - (2) Similar correlations were found for girls and for boys.
- Discussion:
  - Importance of multiple means of assessment when assessing knowledge of young children
  - Verbal tasks may reveal understanding not observed in traditional written assessments
  - Early exposure to symmetry concepts can benefit both girls and boys

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INVESTIGATING EVIDENCE OF GIRLS' AND BOYS' EARLY SYMMETRY KNOWLEDGE THROUGH MULTIPLE MODES OF ASSESSMENT

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Symmetry is a fundamental geometric concept that receives minimal attention in early childhood curriculum. This study sought to explore the relationships between children's accurate identification and explanation of symmetric transformations after a symmetry software intervention and children's performance on a post-test that asked students to draw symmetric transformations and identify lines of symmetry. This study also sought to explore whether the relationships between these measures differed for girls and for boys. Positive correlations between accurate symmetry identification and the domains assessed at post-test and between explanations of symmetries and the domains assessed at post-test were found for participants overall, and similar patterns were found for girls and for boys. The findings have implications for software learning opportunities and modes of assessment for all children.

INTRODUCTION

A computer program was developed to expand young children's understanding of three symmetric transformations—reflection, translation, and rotation. Results from a previous study evaluating the program's effectiveness showed that children assigned to the experimental condition, in which they used the software, were better able to accurately identify and explain symmetric transformations and had higher post-test translation scores and overall scores than the control group, which did not use the software, controlling for pre-existing ability (Fletcher, 2015). The purpose of the current study is to explore the convergence of the qualitative and quantitative evidence collected in the study and to explore any potential differences between girls and boys in this convergence. To achieve its purpose, this study sought to answer the following questions: (1) Is there a relationship between children's identification and explanation of symmetric transformations after the symmetry software intervention and post-test performance on key measures of symmetry understanding? (2) Does the relationship between children's accurate identification and explanation of symmetric transformations, on the one hand, and post-test performance, on the other, differ for girls and for boys?

THEORETICAL FRAMEWORK

Symmetry is present in everyday life, and the cognitive processes linked to it develop over the lifespan. Children begin to develop the ability to perceive symmetry in infancy (e.g., Bornstein, Ferdinandsen, & Gross, 1981), and symmetry aids adults in processing visual information (Bornstein and Stiles-Davis, 1984). Preschool-aged children often experiment with ideas of shape, pattern, and spatial relationships in their play. Such activities in early childhood help to build a foundation for concepts important in later mathematics (Clements and Sarama, 2007). Despite symmetry's relevance across mathematics, learning standards addressing symmetry do not appear in the United States' Common Core State Standards until grade four (National Governors Association, 2010).

Because geometry is often taught in a cursory manner in early childhood (Clements, 2004), children's experiences with symmetry often occur in informal contexts. Gender differences favoring boys in symmetry-related spatial tasks such as mental rotation have been documented (e.g., Maeda & Yoon, 2013), but some researchers argue that these gender differences are likely substantially attributable to socio-cultural or experiential factors (Fennema & Sherman, 1977; Terlecki & Newcombe, 2005) rather than biological factors. Certain informal play experiences may help children develop understanding of symmetry, but gender differences in the frequency of these types of play may contribute to gender differences in symmetry understanding.

METHODS

Materials: Symmetry Software

A computer program was designed to teach three symmetric transformations—reflection, translation, and rotation—to young children. Cognitive principles for the design of mathematics software for young children (Ginsburg, Jamalian, & Creighan, 2013) guided the software development. The program's exploratory mode allows children to move, stretch, or shrink shapes on the screen and observe corresponding changes in the symmetric transformation. In prediction mode, the program prompts the child to place a shape on the screen to create a specified symmetric transformation. Visual and audio feedback identify mistakes and provide solution strategies.

Setting and Participants

A study of the computer program's effectiveness was conducted in one urban public elementary school in the Eastern United States. Participants included 86 first and second grade children—43 were randomly assigned to the experimental group (24 females and 19 males) and 43 were assigned to the control group (21 females and 22 males). Participants' ages ranged from 5.8 to 7.8 years.

Research Design and Procedure

The study utilized a pre- and post-test between-subjects randomized experimental design with exposure to the symmetry software as the treatment condition and a control group that experienced non-symmetry math software and was conducted over 9 weeks. A video transformation task was implemented at post-test to assess children's ability to identify and explain symmetries. The treatment condition consisted of 9 symmetry software sessions (three each for reflection, translation, and rotation); the control condition had 9 sessions using a non-symmetry-related mathematics software.

Measures

A paper-and-pencil instrument designed by the primary investigator to measure children's understanding of reflection, translation, and rotation was used for pre- and post-testing. The instrument included items that asked students to draw symmetric transformations and identify lines of symmetry. The instrument included explanations or examples for the symmetric transformations to ensure that it assessed symmetry concept understanding rather than familiarity with relevant vocabulary.

A task designed by the primary investigator to measure participants' ability to identify and explain reflection, translation, and rotation was implemented at post-test. Participants were shown six short videos of symmetric transformations of shapes. Participants were then asked to identify the symmetric transformation and explain their reasoning for selecting the stated symmetry ("How do you know?").

Participants' identification of the symmetric transformations in each video were scored for accuracy (0 for incorrect or 1 for correct). Participants' explanations of each symmetric transformation were coded for the presence of words or phrases indicating conceptual understanding of the symmetric transformation. This emergent coding scheme was developed by reviewing children's explanations and identifying words and phrases appearing in student responses that indicated conceptual understanding of that symmetric transformation. Explanations were scored 1 for indicating conceptual understanding or 0 for not indicating conceptual understanding of the symmetric transformation.

RESULTS

Previous research found that there was a statistically significant treatment effect when controlling for pre-intervention symmetry knowledge (Fletcher, 2015). Previous research also found that there was a statistically significant difference benefitting the treatment group in accurate identification and explanation of symmetric transformations at post-test. In order to explore the relationship between children's identification or explanation of symmetric transformations and post-test performance, the accuracy and explanation scores were correlated with the mean post-test scores on the paper-and-pencil instrument. With the exception of rotation post-test scores, there are moderate positive correlations between accuracy or explanation and the domains assessed by the post-test, indicating that children who accurately identified symmetric transformations or provided explanations indicating conceptual understanding of the symmetric transformations also tended to reach higher scores on post-test reflection and translation tasks as well as post-test total score (see Table 1). A similar relationship between children's accurate identification and explanation of symmetric transformations and post-test performance was found for girls and for boys (see Table 1).

	Accuracy			Explanation		
	All (N = 43)	Girls (N = 24)	Boys (N = 19)	All (N = 43)	Girls (N = 21)	Boys (N = 22)
Post-Test Total Score	0.423*	0.418*	0.430*	0.365*	0.383*	0.342*
Post-Test Reflection	0.360*	0.342*	0.381*	0.317*	0.314*	0.308
Post-Test Translation	0.475*	0.483*	0.469*	0.449*	0.489*	0.421*
Post-Test Rotation	0.243*	0.351*	0.107	0.057	0.208	-0.088

Table 1: Correlations among dependent variables and post-test scores by group (Note. * $p < .05$)

DISCUSSION

In examining the relationships between post-test performance, accuracy of symmetric transformation identification, and explanations of symmetric transformations, moderate positive correlations were found between post-test reflection and translation scores and accuracy or explanations on the video transformation task. Children who achieved higher scores on reflection and translation items at post-test also tended to identify symmetric transformations accurately and demonstrate conceptual understanding in their explanations. Because prior research has found gender differences favoring boys in symmetry-related spatial tasks, we separated the data by gender to see if the pattern we discovered in our results was being driven by boys' performance on the measures. We found the relationships between post-test scores, accurate symmetry identification, and conceptual understanding of symmetric transformations conveyed in explanations to be similar for girls and boys.

The convergence of the post-test scores, accuracy of symmetry identification, and explanations of symmetries points to the importance of multiple means of assessment—and in particular, the use of interview-like questions—when assessing the skills and knowledge of young children. While some children may feel comfortable completing a paper-and-pencil task, others may not yet have the skills or confidence to accurately complete a written task. Including verbal tasks that allow children to explain their thinking when assessing children provides an important opportunity to reveal understanding that may not observed in traditional written assessments.

In this study, qualitative and quantitative evidence of girls' and boys' symmetry knowledge shows similar patterns of convergence, suggesting that early exposure to symmetry concepts can benefit both girls and boys. Teaching symmetry concepts to young children — either as part of the standard curriculum or through supplementary educational tools such as the computer program utilized in this study — allows teachers to build on children's natural interest in symmetry and prepare children for success in higher level mathematics and career opportunities both in and out of mathematics.

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4-Year-Olds Chinese Children's Understanding of Repeating Patterns

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Current Study

The current research focuses on Chinese preschoolers' understanding of repeating patterns and its association with numerical ability.

Materials

- Repeating Pattern Skills.
- Visual-spatial Working Memory.
- Numerical Ability.
- Spatial Skills.

Discussion

- 4-year-old children were difficult to explain the inner structure of a repeating pattern
- Patterning skills were unique predictors of numerical ability

Participants

Study 1: N=134

(71 boys, 63 girls; M_{age} = 4.35 years)

Study 2: N=117

(57 boys, 60 girls; M_{age} = 4.34 years)

Results

- Evidence for reliability and validity of the patterning assessment
- Characterizing Children's patterning knowledge
- Association between patterning and numerical ability

Implications

- Encouraging preschool children to explain example patterns, by providing children with explanations and by using abstract labels to identify the unit of repeat.
- Providing appropriate patterning activities to support children identifying the repeat unit.

4-YEAR-OLDS CHILDREN'S UNDERSTANDING OF REPEATING PATTERNS: A REPORT FROM CHINA

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Young children have an impressive amount of mathematics knowledge, but past psychological research has focused primarily on their number knowledge. Preschoolers also spontaneously engage in a form of early algebraic thinking—patterning. 134 children （male=71，female=63） aged 4 were administered a repeating pattern, visual-spatial working memory and early mathematics assessment. According to the results of our study, children aged 4 could copy, fill and expand the repeating pattern, but there are still some difficulties in abstracting pattern, especially in identifying the repeat unit of the pattern. Children's repeating patterning skills were significantly linked to spatial skills and numerical ability. Preschoolers' patterning skills independently and significantly predicted their numerical ability above age, visual-spatial working memory and spatial ability.

INTRODUCTION

Preschoolers also spontaneously engage in a form of early algebraic thinking—patterning. Indeed, mathematics has been defined as the science of patterns (Steen,1988). Patterns are a pervasive and important, but understudied, the component of early mathematics knowledge. Although evidence is limited, knowledge of repeating patterns has been shown to support mathematics knowledge (Rittle-Johnson, Zippert, & Boice, 2019), particularly early algebra (Papic, Mulligan, & Mitchelmore, 2011). Children's knowledge of repeating patterns (e.g., ABBABB) is a central component of early mathematics, but current preschool education standards give minimal attention to the role of patterning skills in China. The development of patterning skills has been overlooked in Chinese preschool curricula. There is a lack of research support for children's patterning learning and teaching in China. The current research focuses on Chinese preschoolers' understanding of repeating patterns and its association with numerical ability.

METHOD

Participants

Time 1 repeating pattern assessment. The participants were 134 Chinese preschool children (71 boys, 63 girls; M_{age} = 4.35 years) recruited from 4 public preschools in Shanghai (a developed city in China) and Xi'an (a developing city in west China). All preschools had a publicly funded. 80 of 134 preschoolers were measured 2 weeks later for Time 2 repeating pattern assessment. To further explore the relationship among variables, 117 Chinese children (57 boys, 60 girls; M_{age} = 4.34 years) through stratified random sampling in Shanghai. They were from 4 preschools in three municipalities with different levels of rank.

Materials

Repeating Pattern Skills. Adapted from Research-based patterning, which is consisted of nine items varying in difficulty, validated in previous studies (Rittle-Johnson et al., 2013; 2015), adding 2 pattern completion items and 2 borders pattern extension (Papic, 2007). Cronbach's α = 0.84.

Visual-spatial Working Memory. The visual-spatial working memory subtest (18 items) from the Test

of Visual-Perceptual Skills Revised (Gardner, 1996; TVPS-4th edition, 2017). Cronbach's α = 0.77.
Numerical Ability. A revised Chinese version of the Test of Early Mathematical Ability (Ginsburg & Baroody, 2003; Zhou, 2016). Cronbach's α = 0.92.
Spatial Skills. *Spatial perception*. A subtest (18 items) from the Test of Visual-Perceptual Skills Revised (Gardner, 1996; 4th Edition, 2017). The test has been used successfully in previous studies with Chinese preschoolers (Zhang, 2016). Cronbach's α = 0.79.
Spatial visualization. Block Design, a subtest of the Wechsler Preschool and Primary Scale of Intelligence–4th Edition (Wechsler, 2012) was used. Cronbach's α = 0.78.

Procedure

For the Children participated individually in a quiet room at their preschools on two occasions. Session 1 involved the assessment of children's ability with repeating patterns and Visual-spatial Working Memory. Numerical ability and Spatial Skills assessments were measured in session 2. During the pattern measuring, children were then prompted to explain, "*What is my pattern?*" and "*How is your pattern the same as the model pattern?*" after solving each abstract and identify item. The feedback session was videotaped for later coding. We coded children's errors and self-explanations on the abstract and identify items. The code framework was based on a system used in previous research (Rittle-Johnson et al., 2013). Two coders independently classified all errors, with a Cohen's Kappa of .89, and all explanations, with a Cohen's Kappa of .84 (Cohen, 1960).

RESULTS

Evidence for reliability and validity of the patterning assessment

Evidence for Validity Evidence based on test content. Table 1 shows the item-total correlation was relatively high for all items. Experts' ratings of items provided evidence in support of the face validity of the test content. Eight experts (4 are professor, 4 are kindergarten teachers, 2 are instructional coaches) rated all of the test items to be important (rating of 3) to essential (rating of 5) for tapping the knowledge of repeating patterns, with a mean rating of 4.75.

Item Type	M	SD	Item total correlation	Expert rating
copy1	0.99	0.12	0.30**	4.8
copy2	0.95	0.22	0.22*	4.8
fill1	0.84	0.37	0.48**	4.8
fill2	0.90	0.30	0.43**	4.8
extend1	0.82	0.39	0.68**	4.7
extend2	0.83	0.38	0.72**	4.8
abstract1	0.61	0.49	0.66**	4.8
abstract2	0.69	0.46	0.77**	4.5
abstract3	0.55	0.50	0.70**	4.4
memory	0.62	0.49	0.83**	4.7
identify	0.60	0.49	0.82**	4.7
extend3	0.49	0.50	0.43**	4.8
extend4	0.46	0.50	0.40**	4.8

Note：Copy indicates copy a pattern like the model. $^{**}p<0.01$.

Table 1: Description of and summary statistics for repeating pattern assessment items

The results of the Exploratory Factor Analysis suggested that five factors explained 75.38% of the variance, χ^2 =920.53, df =78(p<0.05), KMO=0.76, MSA>0.50, all factor loadings were relatively high, ranging from 0.68 to 0.89. The assessment was internally consistent as assessed by Cronbach's alpha=0.84. Test-retest correlation *r* (80) =0.78(p<0.001), indicating good internal consistency. The inter-item correlation was relatively significant for all items of the assessment, ranging between 0.22 and 0.86. Evidence based on relations to other variables. Success on our measure was related to Visual-spatial working memory (*r=0.41, p<0.001*).

To further explore the validity of the measure, we investigated the structure underlying the patterning measure was present in the data by means of confirmatory factor analysis (CFA)in Mplus version 8.3 (Muhtén & Muhtén, 2017). Results showed that the five-factor model had a good fit, χ^2/df =2.29 < 3 (Kline, 2005), TLI>0.09, CFI>0.90(Bentler,1990), RMSEA≤0.08 (Hair et al., 2006).

Characterizing children's patterning knowledge

Error Analysis. In addition to considering accuracy, we examined children's errors to gain further insight into their repeating-pattern knowledge. Errors on all items could be classified into one of six categories (see Table 2).

Error type	Description	abstract *N* (%)	identify *N* (%)
No response	Correct. May have partial unit at beginning or end.	72 (43.37%)	42 (29.79%)
Sort	Sorts by color or shape.	29 (17.47%)	14 (9.93%)
Non-pattern	Makes linear sequences of blocks in random order.	32 (19.28%)	21 (14.89%)
Partial Correct	At least one full unit of model pattern; includes errors as well	7 (4.22%)	4 (2.84%)
Wrong Pattern ABAB	Produces an AB pattern. May contain errors at beginning or end	22 (13.25%)	37 (26.24%)
Wrong Pattern Other	Produces a wrong three or four element patterns. May contain errors at the beginning or end.	4 (2.41%)	23 (16.31%)

Table 2: Errors: Descriptions of trials on which the error was produced on extend and abstract items

Self-Explanation. Among pattern abstraction items, Table 3 presents three types of children's feedback. 74.5% could make them correctly. 19.2% of children completed the pattern abstraction task by using the strategy of object-matching or replacement. Only 6.3% of the children were able to identify the internal structure of the pattern model.

Association between patterning and numerical ability

Significant positive correlations were found among patterning skills, visual-spatial working memory, spatial ability and numerical ability. Hierarchical linear regression analyses were performed with the numerical ability which including informal and formal numerical ability serving as the dependent variables. The first regression block included age and numerical ability. Adding the spatial measures in Block 2 resulted in significant improvement in model fit for predicting numerical ability ΔR^2= 0.34, p = .01. Spatial perception was a predictor for numerical ability (β = 0.34, p < 0.01) and informal numerical ability (β = 0.29, p < 0.01). Spatial visualization was a reliable and unique predictor for numerical ability (β = 0.40, p < 0.01), including both informal (β = 0.33, p < 0.01) and formal numerical ability (β = 0.35, p < 0.01). Overall, spatial skills predicted numerical ability, but visual-spatial working memory tended not to be a unique predictor. Repeating patterning measures in Block 3 resulted in significant improvement in model fit for predicting numerical ability ($\Delta R^2 = 0.397$, $p < 0.01$) including both informal ($\Delta R^2 = 0.35$, $p < 0.01$) and formal ($\Delta R^2 = 0.35$, $p < 0.01$) numerical ability.

Feedback type	Example	*N*（%）
Correct, could not explain	*"I don't know!"* *" I think it's a different pattern."*	175（74.50%）
Object-matching/replace	*"Yellow block is like the triangle."* *"I replaced the triangle with a blue block"* *"I just match the model."*	45（19.20%）
Identify the unit of repeat	*"These blocks are placed two by two (pointing to model)"* *"Two yellows, two blues, then two yellows... My pattern is also two by two."*	15（6.30%）

Table 3: Self-explanation types and frequency of abstract items

DISCUSSION

Children' early repeating pattern skills

According to the results of our study, children aged 4 could fill and expand repeating patterns, but there are still some difficulties in the abstraction of the pattern, especially in identifying the unit of repeat of the pattern. It also suggested that 4-year-old children were difficult to explain the inner structure of a repeating pattern, although they could solve the abstract items. It is suggested that adults should provide explanations and more opportunities to talk about patterns to support preschoolers to understand and describe the inner structure of the repeating pattern.

Patterning skills was related in part to visual-spatial working memory. Working memory has been shown to impact success on mathematics achievement and repeating patterns (Bull, Espy, & Wiebe, 2008; Rittle-Johnson et al., 2016) which is consistent with our findings.

Patterning skills were unique predictors of numerical ability

The result indicated that children's patterning skills were significantly linked to their performance on spatial skills and numerical abilities. Preschoolers' patterning skills independently and significantly predicted their numerical abilities in line with previous findings (e.g., Rittle-Johnson et al., 2019; Wijns et al., 2019). The current study further highlights the predictive role of repeating patterning skills for numeracy abilities in particular.

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Investigating How Kindergartners Represent Data with Early Numeracy & Literacy Skills through a Performance Task

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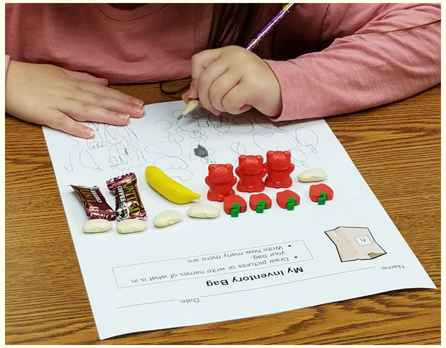
Purpose of the Study

1) Sort each small concrete item according to the property of the object.
2) Identify the quantity of each group of the objects.
3) Record the results with drawings, writing object names, and writing numbers.

Performance Task



Results: 35 kindergartners (20 girls and 15 boys)

Sort Items	Identify the Quantity	Record the Results	
Sorting before counting: **14** sts.	Correct: **18** sts. (51%)	A picture & numbers: **8** sts.	Numbers & a picture: **1** st.
Pointing to the same item: **7** sts.		Pictures & numbers: **4** sts.	A list, numbers, a picture, & letters: **1** st.
Not sorting at all: **14** sts.	Incorrect: **17** sts. (49%)	Letters & numbers: **2** sts.	Numbers & words: **2** sts.

Numerals written Backwards	Letters or Words
#3 (14 sts.), #2 (13 sts.), #4 (2 sts.), & #5 (2 sts.)	App, bmN, cz, APPLES, BA, BS, APO, KADES, BANR, BhR, Se, I, CADE, BEG, BAPS, B, BDH, BH, Hb, BC, & D.

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INVESTIGATING HOW KINDERGARTNERS REPRESENT DATA WITH EARLY NUMERACY AND LITERACY SKILLS THROUGH A PERFORMANCE TASK

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This study investigated kindergartners' early numeracy and literacy skills to complete a performance task utilizing data collection skills. The purpose of the study was to examine how kindergartners were able to sort each set of the concrete items according to the property of the item, identify the quantity of each set of the objects, and record the results with drawings, writing object names and numbers to organize and represent the data in a worksheet. The responses of the children were analyzed qualitatively and quantitatively to accomplish the objectives of the study. The data was collected from thirty-five kindergartners enrolled in a public district. Analysis of children's responses resulted in eighteen children (51%) providing accurate quantity of each set and successfully recording the quantity of each while Seventeen children (49%) did not produce accurate answers.

INTRODUCTION

Early literacy skills are alphabet knowledge, phonological awareness, letter writing, print knowledge, and oral language. Children begin to develop these skills in the preschool years (National Institute for Literacy, 2009; Whitehurst & Lonigan, 1998). Early numeracy skills are defined as counting, knowing the number symbols, recognizing quantities, discerning number patterns, comparing numerical magnitudes, and estimating quantities (in Toll & Van Luit, 2014). According to Jean Piaget (1963), during the preoperational stage (ages 2-7), children begin to develop classification, one-to-one correspondence, representation skills, and skills for manipulating concrete objects and symbols and numerically representing their world.

Various research studies reported that early numeracy and literacy skills are related to and predictors of children's academic success in later school years and career success (e.g., National Institute for Literacy, 2009). Children with inadequate early numeracy and literacy skills experience academic difficulties later (Purpura et al., 2015).

A previous study done by Chung and Higgs-Coulthard (2019) with kindergartners to examine early numeracy and literacy skills revealed that eighty-four percent of children accurately sorted, sixty-eight percent identified the quantity of the objects, and forty-nine percent successfully recorded the data using letters or words and drawings of the objects. The results suggested a need for further research with a more in-depth analysis of kindergartners' representation skills.

PURPOSE OF THE STUDY

The specific objectives of this study were to investigate how kindergartners were able to 1) sort each small concrete item according to the property of the object, 2) identify the quantity of each group of the objects, and 3) record the results with drawings, writing object names, and writing numbers to organize the data in a worksheet to represent the data.

METHODOLOGY

Population and sample

Kindergartners enrolled in public schools in a northern area of the USA participated in the study. Classroom teachers selected thirty-five children (n=35) from two kindergarten classes. There were twenty girls and fifteen boys whose average age was five years and nine months in the fall's mid-semester. The children's academic ability levels identified by the classroom teacher were nine children as above average, sixteen as average, and ten children were categorized as below average.

Procedures

The researcher individually interviewed all children with a researcher-made worksheet that contained an open-ended performance task used in the previous study (Chung & Higgs-Coulthard, 2019). Children's responses and progress of the task were videotaped during the interview. The task was to have children record the quantity of five sets in a bag (e.g., one banana counter, two candies, three teddy bear counters, four apple counters, and five lima beans). Children were to sort items into five groups of the quantity from one to five and record each set's quantity by writing a number, drawing a picture/pictures of the item, or writing the item's name. The verbal directions were given during the interview. It took about 10 to 15 minutes for each child to complete the task. This study was replicated in the same public school (Chung & Higgs-Coulthard, 2019), altering the data collection method by using individual interviews instead of small group activity employed in the previous study.

The children's responses on the worksheet were collected and videoclips were analyzed according to the categories of accuracy as quantitative data, classifying and representation skills using numerals and letters to provide each set's quantity and name of the items as qualitative descriptive data. The qualitative data were classified according to the three categories: 1) Sorting and grouping items; 2) Recording the numbers and representing the quantity of each set; and 3) Drawing pictures or writing words to organize the data. A trained examiner cross-examined the data after participating in a training session about the data analysis technique and purpose of the study.

RESULTS

Eighteen children (51%) accurately sorted the 15 items in the bag to make five groups of objects and identified one to five items in each set. They recorded the exact quantity (numbers one to five) of the items in each bag. The first group of children provided a correct amount; eight of them drew one picture to represent the item and wrote a number next to it; four children illustrated the quantity of each set and labelled each image with a number; two children wrote letters and a number for each set; one child wrote numbers 1, 2, 3, 4, & 5 to illustrate the quantity of each set with a picture of the item, and one child created a list by writing numbers 1 to 5 (off sequence) vertically and drew a picture to illustrate the item and wrote apo for apple and bmN for banana. Two children wrote numbers 1 to five out of sequence and wrote letters next to each number such as cz, APPLES, BA, BS, APO, KADES, BANR, BAR, & BhR. Of 16 children, twelve of them sorted items first to group the same items close to each other to count each set, record the quantity, and draw a picture/pictures to illustrate each set item. Four children picked one item and collected the same item to record the amount. The second group of two children wrote accurate numbers 1 to 5 randomly on the worksheet. One child counted all items first and recorded the result without sorting and wrote letters Ade next to number 4. Another child drew a large apple in the middle of the worksheet and wrote numbers 1,4, 2, 3,3,5,3, & 1.

Seventeen children (49%) did not record the correct quantity of all five sets. Two children wrote the accurate quantity for several sets, for example, 1,4, & 5, or I & 4. Both of them draw some pictures to illustrate the items, such as one picture of a banana and four picture apples but failed to write the quantity next to the pictures. One child sorted first to record the quantity while the other child counted each item by pointing to the same one. The second child wrote on the worksheet; I see the Bnan, the BER, BES, PBu, nEY around a picture of an apple with two eyes. Six children wrote several numbers, drew some pictures, and letters seem to be random ones. None of them sorted the items before counting each set. The letters shown on the worksheet were the Se, I, CADE, BEG, BAPS, B, BDH, BH, Hb, BC, & D. Three children wrote several numbers correctly. The first child wrote numbers 1 to 10 in sequence. The second child wrote numbers 1, 3, and 5. The third child wrote number 1, 3, 4, & 5, but they were not easily recognizable. Six children draw pictures of some items such as apples, heart shapes, irregular shapes, and lines on the worksheet.

Fourteen children (40%) wrote the number 3, thirteen children (37%) the number 2, two children the number 4, and two children number 5 backward.

DISCUSSION AND CONCLUSION

Half of the children (51%) successfully sorted and recorded the quantity of five sets, while the rest could not complete the task with the accurate quantity of the items in each set. When the interviewer explained the directions and gave the bag with the items (15 objects) to the individual child, most of them seemed to be confused about the directions until hearing the second verbal instruction that said, Mrs. (classroom teacher's name) wants to know what kind of items are in the bag. Twelve of eighteen children who completed the task successfully separated items into each group of the same objects before recording the quantity and illustrating them on the worksheet. This indicates sorting is a critical early numeracy skill that children should acquire to complete a data collection task. No child constructed a table to report the quantity of each set. One child created a list vertically, but not in the order of the amounts.

Kindergartners are to learn counting and writing numbers, connecting counting to cardinality, comparing two numbers between 1 to 10 as written numerals, and classifying objects and the number of objects in each category according to the Kindergarten Math Standards (The Common Core Standards Initiative, 2010). The children who participated in the study related counting to cardinality by identifying the quantity of the objects up to 5. However, half of the children failed to represent the quantities using numerals and pictures.

Ten children (29%) attempted to write words by writing some alphabetical letters. One child used a random assortment of letters, five children wrote the first or the last sounds heard, three children wrote the first, middle, and final sounds to label the set of the items, and one child used combinations of consonants and vowels to write a sentence such as I see BNana.

In the previous research (Chung & Higgs-Coulthard, 2019), the majority of children (84%) completed the sorting and recording the quantities task successfully. The study used the same classroom teachers and the grade, but the data was collected through a small group activity in their classroom. In contrast, the current research was done through individual interviews with the same task but in a separate room. The setting and data collection method might affect the children's performance. The classroom teacher was not present in the room. The interviewer was not familiar with the children even though the classroom teacher explained the project and introduced the interviewer and the researcher before the interviews.

The children's responses indicated that they had difficulty representing the quantity at Bruner's symbolic learning mode. Bruner recommended using concrete objects (Enactive mode) first to introduce new concepts and have children represent the concepts with diagrams and drawing (Iconic mode), and finally with numbers and letters (Symbolic mode) (in Smith, p. 2006). The children struggling in connecting and representing the concepts and skills between or among these modes should learn new concepts using concrete objects to explore them.

Comparing the results to the previous study in 2019 (Chung & Higgs-Coulthard), small cooperative group work for children might booster children's learning better than individual work. A further study on how individual and small group work make a difference can be conducted in the future. A follow-up study with the same group of kindergartners at the end of the school year or when they become first graders can be conducted since the current study was done during the first semester of the kindergarten year.

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Counting activities for young children: Adults' perspectives

Dina Tirosh, Pessia Tsamir, Ruthi Barkai, Esther S. Levenson

Rationale and aims for this study:

The home environment is important when considering young children's mathematics (Anders et al., 2012).

Counting and enumerating are fundamental skills learned during early childhood (Baroody, 1987).

This study investigates the types of activities that adults perceive as ones that can promote these numerical skills.

Results :

Counting activities (verbal counting):

(1) counting physical objects *("counting toys, beads");* (2) counting and movement *("jump till the number 30");*
(3) counting without any physical accompaniment (*"counting forward from the number one"; "from the end till the beginning")*

Enumerating activities (object counting):

(1) general activities ("*count chairs*"); (2) detailed activities ("*count crayons placed in a row*"; "*bring 8 toys from the box*")

Summary:

Many participants suggested counting objects when asked to promote verbal counting.
Less mentioned were the sub-skills necessary for counting and enumerating, such as counting forward from some number other than one, or focusing on one-to-one correspondence.

The findings of this study (as part of our larger project) can help professionals offer enrichment workshops for adults, increasing adults' awareness of early mathematical skills that can be promoted at home.

COUNTING ACTIVITIES FOR YOUNG CHILDREN: ADULTS' PERSPECTIVES[1]

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Taking into consideration that young children engage with mathematics outside of the preschool classroom, this study focuses on adults who are not preschool teachers and their perspectives regarding activities that may promote children's counting and enumerating skills. Finding indicated that adults may not differentiate between verbal counting skills and object counting skills. Many activities did not take into account sub-skills necessary for counting objects. Results of this study may be used to plan enrichment workshops for adults.

INTRODUCTION AND BACKGROUND

Researchers agree that promoting numerical skills, such as counting, comparing sets, number composition and decomposition, and recognition of number symbols, is important during early childhood (Nguyen et al., 2016; Sarama & Clements, 2009). Several studies have investigated the types of numerical activities that are implemented in preschools[2] by trained teachers (Clements et al., 2011). However, young children often spend much of their day in the care of other responsible adults who are not necessarily trained preschool teachers. Studies suggest that for children to take advantage of the academic opportunities provided at preschool, some level of support from the home environment, such as toys that stimulate learning number and shapes, is necessary (Anders et al., 2012). This paper investigates the types of activities these adult perceive as ones that can promote numerical skills.

Counting and enumerating are fundamental skills learned during early childhood (Baroody, 1987). In this paper, we refer to verbal counting as counting, and to object counting as enumerating. Both counting and enumerating involve several sub-skills. Counting includes being able to say the number words in the proper order and knowing the principles and patterns in the number system as coded in one's natural language (Baroody, 1987). Fuson (1988) differentiated between two learning phases. The first phase is the acquisition phase, when children learn the conventional number words, in order, and can repeat them consistently. The second stage is the elaboration phase, when children become aware that the chain of numbers can be broken up, and that parts of the chain may be produced starting from a number other than one. Counting backwards is learned more slowly and is based on first mastering the forward sequence.

Enumerating refers to counting objects for the purpose of saying how many. Gelman and Gallistel (1978) listed five enumeration principles: the stable-order principle, the one-to-one principle, the

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[2] In this paper, preschool children refer to children between the ages of 3 to 6 years, before first grade.

cardinality principle, the abstraction principle, and the order-irrelevance principle. Skillfully and competently enumerating may depend on both the size and setup of the objects to be counted. In one study of preschool children (Tsamir et al. 2018), only half of the children who were able to count eight items placed in a row, were able to keep track of where they began and where they ended when seven items were placed in a circle. Two children, when faced with counting items in a circle, claimed that they did not know what to do. Young children may also err by pointing to an object without saying a word, or by pointing to an object and saying more than one counting word (Fuson, 1988).

METHODOLOGY

This paper focuses on one part of a larger study, intended to investigate the knowledge and beliefs of adults, who are not preschool teachers, regarding early childhood mathematics education. The participants in this part of the study were 44 adults; all but 10 said that they had some connection with children 6 years old and under. Participants were between the ages of 20 and 60, and included teachers (grade school and high school), psychologists, occupational therapists, engineers, municipal workers, and accountants. Questionnaires were handed out individually to the adults in a comfortable environment. Two questions are analyzed here: (1) In the preschool mathematics curriculum it states that by the end of kindergarten, children should be able to count till 30. What counting activities would you implement with children to promote this skill? (2) What enumerating activities would you implement with children to promote their ability to enumerate eight objects?

Data analysis began with directed content analysis according to the counting and enumerating sub-skills mentioned in the background, such as saying the counting words in the accepted sequence and knowing the cardinality of a set. We then used inductive content analysis to devise additional categories. Decisions about interpretation were made collaboratively by discussion.

FINDINGS

Counting activities

Categorization of counting activities was conducted in several stages. First, we differentiated between an activity that encouraged children to say counting words in a sequence, and those that did not. For example, one participant wrote the following activity: coloring numerals. While identifying numeral symbols is important, coloring numerals does not require saying the number sequence. Among activities that did require reciting a sequence of numbers, we differentiated between (1) activities that included counting physical objects, (2) activities that involved counting and movement, and (3) activities where there was counting without any physical accompaniment. Activities that included the mention of objects related to counting toys, beads, rocks, candies, and the like. Activities that mentioned movements included counting steps, strides, and swings (the back-and-forth motion of the swing). Some participants offered a more detailed description of their suggested movement activities. For example, A16 suggested to “jump till the number 30,” and “collect the toys laying around while counting till 30.” A5 suggested, “when playing in the playground, walk to another child and count the steps to that child,” Note that in many of the movement activities, it is not clear if the aim is to know, for example, how many times I actually jumped, or if the jumping is merely a rhythmic prop to keep the counting going.

Several participants often suggested more than one type of activity, and two participants did not reply to this question. As can be seen from Table 1, approximately half of the participants suggested counting objects as a verbal counting activity. Of the 23 who suggested object counting activities, two also suggested verbal counting activities, and six others suggested movement activities. Verbal counting only activities were further categorized into sub-skills of counting. Seven (out of 11) stated counting forward from the number one. Two mentioned counting backwards "from the end till the beginning." Two mentioned counting forward from a number other than one, and two included the act of saying what number comes before or after some number. Finally, one participant (A44) related to skip counting, writing, "to count groups of 10, and another 10, and another 10."

Activity	Not relevant	Object counting	Movement	Verbal counting only
Frequency	7 (16)	23 (52)	10 (23)	11 (25)

Table 1: Frequencies (in %) of types of counting activities (*N*=44)

Enumerating activities

To begin with, all suggested activities included some form of counting objects, and not merely recitation of the counting sequence. The first stage of categorization differentiated between general enumerating activities, and those that offered details. From Table 2 we see that approximately half of the participants offered some details of how an activity was to be implemented, with approximately half of those taking into consideration the specific placement of items. In the first category were statements such as "count chairs," "count fingers," "count candies", and the like. Among the more detailed activities, some participants described how the objects to be counted should be placed. For example, A33 wrote, "put different colored crayons in a row." A37 emphasized that it is important to vary sets of objects to be counted. She wrote, "count identical items and non-identical items." This relates to the abstraction principle (Gelman & Gallistel, 1978). Others emphasized a specific question that should be asked. For example, instead of merely saying "count cucumbers," A17 wrote, "Ask children, how many cucumbers did we buy, or how many plates are on the table." Asking the question of "how many" may infer a focus on cardinality. Some participants incorporated additional activities along with enumeration, such as 'counting out' activities. For example, A4 suggested to request children to "bring eight toys from the box," and A44 suggested to "build a tower with eight blocks." Another activity incorporated with enumeration was sorting. A5 suggested, "Show a picture with several items, such as a playground with children. The task is to count how many items can be found with the same characteristic. How many children are there? How many red shirts are there?" A few participants explicitly said that objects should be counted as the child places them into a box. A9 presented an activity focused on one-to-one correspondence without mentioning the terms count or enumerate, "set a table with eight chairs, eight plates, and on each plate place one slice of bread."

	General activities	More-detailed enumerating activities			
		Placement of items	Question	Object types	Additional activity
Frequency	19 (43)	9 (20)	3 (7)	2 (5)	9 (20)

Table 2: Frequencies (in %) of types of enumerating activities (*N*=44)

DISCUSSION

In a previous study (Anderson & Anderson, 2018), parents were requested to video their children engaging in mathematical activities. In that study, it was found that all families engaged with some form of counting activity. However, only half engaged in rote counting activities, such as singing a counting song. In that study, parents were not asked specifically to describe the mathematics involved. In this study, adults were specifically asked to describe activities that could promote counting and enumerating. In Hebrew (the language of the participants) there are two separate words for counting and enumerating. That being said, while the term for enumerating (meniya) is only used for object counting, the term for verbal counting (sefira) is often used for both verbal and object counting. Thus, it is not surprising that so many participants suggested counting objects when asked to promote verbal counting. Furthermore, counting objects is often one way to motivate children to recite the counting numbers; activities that involve movement may also be said to promote verbal counting.

Less mentioned were the sub-skills necessary for counting and enumerating, such as counting forward from some number other than one, or focusing on one-to-one correspondence. Yet, these skills can also be promoted at home – if an adult is aware of them. During early childhood, children also learn composition and decomposition of numbers, and begin to recognize number symbols (Sarama & Clements, 2009). In our project, we are also investigating adults' perspectives regarding these skills. Results of this study can help professionals offer enrichment workshops for adults, increasing adults' awareness of early mathematical skills that can be promoted at home.

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ASKING EARLY CHILDHOOD TEACHERS ABOUT THEIR USE OF FINGER PATTERNS

Everyday contexts

age/birthday (11), finger games (9), board games (8), songs, checking attendance, reading books, setting the table, free play, countdown, lightning advent candles

→ What about non-mathematically framed communications with individual children?

Mathematical contexts

- ordinal and cardinal contexts
- arithmetical contexts
- fingers as number signs
- visualization for developing understanding of numbers

→ Dynamic finger patterns are most often used in counting; static finger patterns are most often used for showing quantities and as number signs.

n = 23
early childhood teachers

Adaptive use of finger patterns
dependent on age, verbal competencies, mathematical development

ASKING EARLY CHILDHOOD TEACHERS ABOUT THEIR USE OF FINGER PATTERNS

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Young children often use finger patterns to show numbers during communication with peers and adults. In play-based early childhood settings, informal mathematics learning takes place when teachers identify teachable moments and react accordingly to children's ideas and questions. The aim of this paper is to examine early childhood teachers use of finger patterns in their daily interaction with young children. Interviews with 23 early childhood teachers show that all teachers use finger patterns in a variety of everyday and mathematical contexts. The frequency and formation of finger patterns (static versus dynamic), however, varies among the teachers.

Recent mathematics education and neuroscientific studies highlight a connection between children's use of fingers and their mathematical development (see e.g., Björklund, Kullberg, & Kempe, 2019; Moeller et al., 2011). Mathematic education research especially emphasizes the use of finger patterns as important for a sound understanding of numbers. As mathematical contents, and in particular finger patterns, play no dominant role in German pre- and in-service early childhood teachers' education (Gasteiger, 2012), we were interested in teachers' implicit knowledge about the use of fingers. We specifically aimed to learn from early childhood experts, how they integrate finger patterns in informal everyday situations.

THEORETICAL BACKGROUND

Like number words and written numerals, fingers can be used to represent numbers, i.e. holding up a certain number of fingers to represent a set size. Finger patterns can be constructed in two different ways, either by raising the fingers successively (by counting) or simultaneously (Gaidoschik, 2007). The first would be called a dynamic, the second a static finger pattern (Lorenz, 1992). With both procedures, the finger patterns which are formed represent numbers with cardinal value.

A large body of research suggests a close relation between the use of fingers and the development of numerical and arithmetical competencies (see e.g., Berteletti & Booth, 2015; Moeller et al., 2011). Didactical considerations see the potential of finger patterns in the children's development of the cardinal principle, subitizing, part-whole concept (Resnick, 1983), and the concept of base (Fayol & Seron, 2005). Björklund, Kullberg, and Kempe (2019) advocate finger patterns for helping young children to recognize number structures. In their study on different ways of using fingers in solving a subtraction task, they found that using finger patterns to model the specific numbers was related to a high rate of correct answers.

German pre-school education mostly follows – like in other European countries – a play-based approach, where (mathematical) learning evolves from children's play and everyday situations. In this approach, early childhood teachers do not formally teach (and are called *educators*) but rather observe and informally pick up on mathematics in the children's activities (Gasteiger, 2012). As the use of finger patterns is important for numerical and arithmetical learning, and young children use finger patterns in the communication with peers and adults, we were interested in how far early childhood teachers incorporate finger patterns in their daily work. The aim of this paper is to highlight early childhood teachers' routines in enhancing young children's understanding of numbers. The results might also have an influence on pre- and in-service training with regard to the mathematical content of finger patterns. The specific research questions are: How and in which contexts do early childhood teachers use finger patterns? On what kind of factors does finger pattern use depend?

METHODS

Participants and procedure

A total of n=23 early childhood teachers from four different day care centers in Germany were interviewed by the second author. The sample is a good mixture of teachers at all points of their professional careers. All teachers have experiences in working with children from 1-6 years of age.

The interviews were conducted in the form of guided expert interviews, where the early childhood teachers are considered experts in the field of early childhood development and education. The interview guideline comprised open questions on how and in which situations young children use finger patterns as well as if and how the early childhood teachers themselves use finger patterns in their work with children. The interviews took place during the mornings in a quiet room at the day care centers. They lasted between 4½ and 11½ minutes and were videotaped.

Data analysis

All interviews were transcribed and analyzed by means of "qualitative content analysis" (Mayring, 2008), using the qualitative data analysis software MAXQDA 12 (VERBI). In a first deductive step, the data was assigned to the two superordinate categories *child* and *teacher*. Subordinate categories for each of the two were then developed inductively from the empirical data. Subject of this paper are the three categories which comprise the teachers' use of finger patterns: *Everyday context in which teachers use finger patterns, Mathematical context of finger pattern use* and *Addressee-related dependence of finger pattern use*. Additional to a qualitative description of the categories, the frequency of each subcategory was quantified.

RESULTS

All 23 early childhood teachers told during the interviews that they use finger patterns in their work with young children. Two teachers (7 & 12), however, explicitly limited their use (7: "I use them rarely."). Three teachers (9, 18 & 20) stressed their regular and frequent use of finger patterns (20: "Well, I use the fingers very much.").

Everyday context in which teachers use finger patterns Twenty teachers named at least one everyday context for their finger pattern use (43 explicit references in total; see table 1). In their explanations, finger patterns are always used as gestures which accompany number words. The most frequent denomination with 26% was the context of children's age which is shown by finger patterns, often in connection with their birthday. Teachers either model the (birthday) child's age (3: "You are one year old, you are two years old.") or ask for children's age (7: "When the children come and tell me 'It's my birthday' and I ask. Then I may perhaps show 'Are you one? No! Two? No!' and so on."). 21% of the responses named the context of playing finger games. Finger games are a commonly used activity to engage children in chanting and reciting rhymes. In some of the answers it became evident, however, that fingers are not always used in a mathematical relevant way during playing finger games (13: "When I play finger games, somehow, if leaves fall [*moves hands up and down*], if we sing autumn songs or if rain falls, you have the hands."). Teacher 1 differentiated finger games in which fingers are used as mathematical objects or as decorative accessory: "There are finger games […] where you really title the names or […] where simply the fingers show to advantage." It remains unclear, though, if teacher 1 refers to the names of finger patterns in a cardinal sense or the number-related name of each finger in an ordinal sense, like teacher 6: "When we play finger games. This is the first, this is the second, this is the third, this is the fourth, this is the fifth [*raises fingers one by one*]". A third context, which was named quite often (19%), was board games. Teachers use finger patterns when dice are involved (9: "Or you roll the dice and say: 'Here, this is a three [*shows finger pattern of three*], this is a four' [*shows finger pattern of four*].") or quantities of game objects have to be determined (16: "If it's a game where you have to take something. This means, you can take three apples, then I would show them [finger patterns]."). The other seven everyday contexts are mentioned three times or less.

Context	Absolute frequency	Context	Absolute frequency
age / birthday	11	reading books	2
finger games	9	setting the table / lunch	2
board games	8	free play	2
songs	3	start signal / countdown	2
checking attendance	3	lightning advent candles	1

Table 1: Everyday contexts in which early childhood teachers (n=23) use finger patterns

Mathematical context of finger pattern use At the beginning of each interview, finger patterns were introduced as the topic we were interested in, and the interviewer accompanied her explanations by modeling a few static finger patterns, raising the respective fingers simultaneously. However, the analyses of the interviews showed that most early childhood teachers talked about their finger use in general, some even referred to pointing gestures at number symbols as "finger use". Focusing on mathematical contexts (see table 2), finger patterns are most commonly used in a dynamic way, accompanying teachers' verbal counting (1: "Yes, let me think of an example: Rocket, when you 10, 9, 8 [*shows ten fingers and folds them one by one*]") or object counting (2: "Well, also to show the children, if they count something, right. One, two, three. I do it automatically that I show my fingers with it."). Static finger patterns are used by eleven teachers when referring to quantities (see 16 above) and three teachers gave examples for using static finger patterns as number signs (15: "I ask the child 'Show me the three!' [*shows finger pattern for three simultaneously*]"). Only four teachers reported on doing calculations with children. Two teachers use their fingers dynamically when solving arithmetic tasks, two use static finger patterns when calculating with children. As a general perspective on the mathematical function of finger patterns, ten teachers mentioned during the interview that they use finger patterns as a visualization to help children develop an understanding of numbers (23: "[I use finger patterns] to visually represent children that. So that they really can visualize these numbers. […] And if you hold up your fingers like that, it is probably a bit more understandable.”).

Mathematical context	Formation of finger pattern	Absolute frequency
verbal counting	dynamic	12
object counting	dynamic	11
referring to quantities	static	11
referring to number signs	static	3
calculating	dynamic & static	4

Table 2: Mathematical context and formation of finger patterns

Addressee-related dependence of finger pattern use 19 early childhood teachers use finger patterns dependent on the age of the children, with 17 teachers using finger patterns more frequently with younger children (age 1-4), two teachers more frequently with older children (age 4-6). Furthermore, 14 teachers reported on varying the frequency of finger pattern use dependent on the child’s verbal competencies; nine hinge their finger pattern use on the child’s mathematical development.

DISCUSSION

In most of the stated everyday contexts in which teachers use finger patterns, numbers are paramount to the situation. Still, there are a few additional examples how finger patterns can be used in non-mathematically framed situations. It is remarkable that nearly half of the teachers related to the importance of finger patterns as a visualization for (the structure of) numbers. Further research is needed on the reasons for an addressee-related use of finger patterns.

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PERFORMANCE EXPECTATIONS IN THE AREA OF *SHAPES AND SPACE* OF EARLY CHILDHOOD EDUCATORS IN AN INTERNATIONAL COMPARISON

Theoretical Background

- Early math abilities and understanding influence on children´s later school success (Lehrl, Kluczniok, & Rossbach, 2016; Watts, Duncan, Siegler, & Davis-Kean, 2014)
- Age-specific performance expectations – part of early childhood educators **M**athematical **P**edagogical **C**ontent **K**nowledge (MPCK) (Rettenbacher et al., in press) - Little is known, so far on performance expectations (Brown & Richard Wong, 2017; Kilday et al., 2012)
- Early childhood educators' estimation of individual children in their groups are less accurate than estimation of children's developmental level in comparison to the whole group (Kilday et al., 2012)

Method

International five-country-project BELMI 3-6 (https://vorstellung-erziehung-mathematik.uni-graz.at/de/projekt/)

Sample: pre- and in-service early childhood educators (N = 1343)

Austria	Switzerland	China	Vietnam	USA
n = 177	*n* = 522	*n* = 312	*n* = 232	*n* = 100

Questionnaire: Age-cohort estimation of 27 different mathematical items in 4 different mathematical content areas from KiDiT® (https://www.kidit.ch/index.php?id=72)

Analysis: Descriptive analysis and differences in central tendency between countries, using dependence analyses, were calculated.

Research Question

How adequate are early childhood educators age-specific performance expectations in the content area *shapes and space*?

Results

1. Majority of early childhood educators expectations are “inaccurate”
2. Rather underestimate children
3. Same results for pre- and in-service sample
4. Country differences in expectation for math abilities that occur at an younger age (European countries more accurate)
5. More appropriate estimations for age-cohort of 3-6 year old children than for younger age-cohort (1-3 year old’s)

Discussion and Conclusion

1. & 2. Performance expectations important for planning, to meet children´s zone of proximal development -> inaccuracy could lead to inadequate educational offers (e.g. learning environment and activities)

3. Early mathematics still needs more attention in professional training of early childhood educators
4. Curriculums likely to have a great influence
5. More research needed on what shapes performance expectations (experience, training etc.) and children´s math development

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PERFORMANCE EXPECTATIONS IN THE AREA OF "SHAPES AND SPACES" OF EARLY CHILDHOOD EDUCATORS IN AN INTERNATIONAL COMPARISON

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In an international comparative study, the performance expectations in early mathematics in the field of "shapes and space" of qualified early childhood educators and students in Austria, German-speaking Switzerland, China (Shanghai), Vietnam (Hanoi) and the USA (Denver, CO) are examined using the example of construction games. The performance expectations of approximately a third of the qualified early childhood educators and students correspond to the current state of science. In other words, more than two thirds of qualified early childhood educators have performance expectations that are in some cases too low and in others too high. The performance expectations are not more appropriate on average due to the training. However, the assessments tend to be more appropriate for kindergarten children than for toddlers.

PERFORMANCE EXPECTATION

From a European point of view, early mathematics education is an area which found its way into the curricula rather late and is still partly treated with neglect or deliberately not mentioned at all (Wustmann Seiler & Simoni, 2012; Charlotte Bühler Institut, 2009). In other countries, early mathematics education has been anchored for some time and is partly underpinned by competences (Ministry of Education, 2001; 2012; Vietnam Ministry of Education, 2017; Scott-Little, Kagan, Reid, & Castillo, 2011). This could lead to fundamental differences in what capabilities early childhood educators expect from children in early mathematics education.

In regard of the mathematical content areas “patterns and structures”, “numbers and quantities”, “shapes and spaces”, “measurements” and “data, frequencies and probabilities” literature sources can be mainly found for the content area “number and quantities”. For the content area “shapes and spaces” and here in particular “constructions” empirical evidence about the age range can only be partly found: At the age of 1;2-2;0 years children can stack blocks on top of each other and at the age of 1;7-3;0 years fences and walls can be constructed. In the study of Largo (1987) children learn to build bridge-like things with about three years. These studies were carried out experimentally.

Therefore, an additional dataset from the observational tool KiDiT (Walter-Laager, Pfiffner & Schwarz, 2012) was used for validation purposes. This dataset includes more than 100’000 cases. Concerning the KiDiT-data the sample is incidental and the early childhood educators use the tool based on their own professional knowledge. The data from the tool corresponds with the literature.

RESEARCH

In the context of a larger study, the question as to whether early childhood educators have realistic performance expectations of children in the mathematical content area "shapes and spaces" arose?

The sample consists of students of early childhood education and early childhood educators from five countries on three continents. It was drawn by random principle. The distribution in the countries is as follows. Austria: 91 pre-service and 86 qualified; Switzerland: 427 pre-service and 95 qualified; China: 126 pre-service and 186 qualified; Vietnam: 94 pre-service and 138 qualified and USA: 41 pre-service and 59 qualified. The average work experience of the early childhood educators in the tested countries varies from 13,5 years to 19,5 years. In all cases, they are therefore experienced pedagogues.

	Pre-service (undergraduate)	Pre-service (graduate)	Qualified	Total
Austria	091		086	0177
Switzerland	427		095	0522
China	066	060	186	0312
Vietnam	094		138	0232
USA	023	018	059	0100
Total	701	078	564	1343

Table 1: Sample of all countries with education level

In order to answer this question of investigation the participants filled out a questionnaire about the performance expectations in their national language. The probands had to choose in what age range they would expect the child to achieve each construction activity-item .

The data analysis has been carried out using IBM SPSS version 24. Descriptive and inferential statistics were used.

RESULTS

A small extract of the results refers to the performance expectations for the topic "construction". Four statements were made about skills that occur at different ages. The probands assigned these to age segments that they considered appropriate (from 1;0-1;6 / 1;7-2;0 /2;1-2;6/ 2;7-3;0 / 3;1-4;0 / 4;1-5;0 / 5;1-6;0).

The following table shows the percentage of correct performance expectations for the "construction" statements per country.

Countries	The child can put four building blocks on top of each other.	The child builds walls and fences with construction material.	The child can build bridge-like formations (for example: using a beam and two columns).	The child can build a construction with max. 20 pieces by using a plan.
Austria	25,1%	21,8%	41,3%	54,7%
Switzerland	28,3%	19,4%	29,1%	38,9%
China	02,6%	08,7%	38,5%	35,3%
Vietnam	09,9%	18,9%	24,0%	20,6%
USA	13,6%	14,4%	32,2%	28,0%
Average of the correct estimations (all countries)	17,9%	16,9%	32,2%	36,2%

Table 2: Correct estimations in the countries and overall average (trained early childhood educators and early childhood educators in training); $p = .000$

The first statement is about building a simple tower construction using a few blocks. This ability already occurs at the beginning of the first year of life (Largo, 1987). About a quarter of European early childhood educators are aware of this, but professionals from other countries are not. The same applies when it comes to building horizontal structures. This ability also occurs very early in the years of childhood.

At the beginning of the age of four, children usually start to build bridge-like buildings. Here, the proportion of adequate performance expectations is around one third of the early childhood educators. The number of adequate performance expectations even increases when it comes to the item building according to a simple plan.

Overall, it is noticeable that the majority of early childhood educators in all countries does not expect children's abilities in the correct age segment. The results do not improve if a group comparison is carried out between trained early childhood educators and students of early childhood education**.** However, the assessments tend to be more appropriate for kindergarten children than for toddlers.

DISUSSION

Regarding the performance expectations the results show an interesting but inconsistent picture: Practically all performance expectations were rated too high or too low from both the trained and in training early childhood educators. Only a few exceptions are in line with the findings in the scientific research literature.

This raises certain issues: Is it because of the experiences early childhood educators get in their daily practice? Is it because of the professional training, that – even though we have data about it – isn´t relying on theoretical models and key figures? This is unlikely, even if we extend the age segments to at least two years, more than 50% of the early childhood educators in the researched subarea are still wrong. This shows that a considerable part of them do not have a realistic image of developmental processes. The present results definitely build a starting position for the control level, for the professional training as well as for the early childhood educators in charge, to check how play and learning opportunities in the field of early mathematics for children is secured and can be made accessible.

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Mathematics in play

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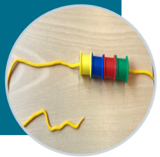
Context

- Mathematical development in spontaneous play vs. isolated activities

Background



- Learning and playing
- Interaction: Observing, connecting, enriching
- Mathematizing and mathematics domains
- Professional learning community

Research Question

- Which characteristics of the interaction between ECE teachers and preschoolers (2-6 years) are useful for stimulating young children’s language and mathematical development in the context of play?

Method

- Design based research, cooperatively performed in a PLC-setting by professionals and researchers

Results

- Professionals elaborate ideas from stimulating mathematical development in spontaneous play

Conclusion

- Observing, connecting and enriching are crucial in stimulating young children’s development in mathematics.
- Stimulating mathematical development in spontaneous play presumes professionals’ deep knowledge of young children’s mathematics. This knowledge helps professionals decide on using their interaction repertoire.

MATHEMATICS IN PLAY

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This paper reports on a practice-oriented research and developmental trajectory exploring children's spontaneous play in the perspective of their mathematical development. The research question was: Which characteristics of the interaction between preschool/kindergarten teachers and preschoolers (2-6 years) are useful for stimulating young children's language and mathematical development in the context of play? We found that three main consecutive characteristics of interaction as part of spontaneous play i.e., (a) observing, (b) connecting, and (c) enriching, can be mathematically deepened and interpreted in order to stimulate young children's development in mathematics. Moreover, the study revealed that stimulating mathematical development in spontaneous play presumes professionals' deep knowledge of young children's mathematics, and that this knowledge helps professionals deciding on the form of child-professional interaction.

INTRODUCTION

Professionals in preschool (for 2-4 year olds) and kindergarten (for 4-6 year olds) are aimed at supporting young children in their groups in developing mathematics as described in national objectives for young children in the Netherlands (SLO, 2018a; SLO, 2018b). These professionals want to stimulate young children's mathematical development though their spontaneous play. Professionals in preschool and kindergarten find it difficult to combine these two (Leseman & Veen, 2016). This situation often results in isolated goal focused mathematics activities. Both teachers and researchers involved in the presented study questioned how they can do more justice to what children's interest is focused on.

This paper reports on a practice oriented research and developmental trajectory, wherein researchers and professionals from preschool and kindergarten cooperatively worked on exploring children's spontaneous play in the perspective of children's mathematical development. In this research project we combined theory on interaction in preschool and kindergarten, with theoretical notions on learning mathematics as process of mathematizing.

BACKGROUND

In the sociocultural view on learning it is emphasized that young children's development is a largely cultural-historical process based upon the appropriation of cultural tools like language in the interaction with adults and more knowledgeable peers (for example Vygotsky 1978). In mathematics pedagogy, this social constructivist view implies that learning mathematics is seen as a process of enculturation with interaction as a core element (Sfard, Nesher, Streefland, Cobb, & Mason, 1998). With regard to young children's learning, Vygotsky conceived the zone of proximal development particularly in children's play. By play, the child can imitate adult activities and meaningfully enter a zone of proximal development that creates opportunities for development-promoting learning (Vygotsky, 1978).

The current research builds upon the following theoretical and empirical notions.

First, it is based on research findings pointing out that learning in a play-based setting can be effectively useful for the attainment of positive outcomes on mathematics learning (Van Oers, 2014). In this view, learning and playing are inseparably connected and play is considered as a potentially rich educational context for young children's learning (e.g., Poland, 2009).

Second, this research follows a chronological trinity of guidelines for interaction in young children's play. This trinity is derived from practice oriented research on stimulating young children's language and thinking skills in a so-called spontaneous play-based setting. This trinity of guidelines for professionals consists of (a) carrying out *observations* of children's spontaneous play, with the aim to understand children's interests and feelings, (b) making a *connection* by getting involved in children's play activities and by playing along, for example by mirroring the child's activities, with the aim to confirm for oneself what the play activities are about, and (c) *enriching* the play situation by remaining in the play and provide an impulse, with the aim to cooperatively construct new meaningful experiences (Van der Zalm, Boland, & Damhuis, 2018).

Thirdly, the current research embraces the notion of young children's mathematizing (Freudenthal, 1991) which is at the core of Realistic Mathematics Education (in short RME) (Streefland, 1991). Mathematizing implies the activity of structuring, modeling and interpreting one's world mathematically. In RME, rich 'realistic' situations serve as a source for initiating the development of mathematical concepts and strategies, but also as a context in which students can apply their mathematical knowledge, which over time becomes more formal and general and less context specific (Van den Heuvel-Panhuizen & Drijvers, 2014). The TAL teaching-learning trajectories for primary school mathematics (Van den Heuvel-Panhuizen, 2008; Van den Heuvel-Panhuizen & Buys, 2008) and curriculum documents from the Dutch national curriculum organization (SLO, 2018a; SLO, 2018b) are used in this research to indicate and interpret young children's spontaneous mathematics in a play-based setting.

Finally, in order to support professionals in developing new practices and knowledge, the model of a professional learning community (PLC) (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006) is used, as teachers' collaborative routines are important in securing improved student learning outcomes (Resnick, 2010). At both the Universities of Applied Sciences iPabo and Marnix Academie, the PLC-model is frequently used in research and innovation trajectories as it offers a system approach to school improvement (e.g. Keijzer, Smit, Bakker, & Munk, 2016; Van Popta-Erkelens, Van der Wal-Maris, & Peltenburg, 2018).

RESEARCH QUESTION

This research focuses on supporting professionals in preschool and kindergarten in stimulating young children's development in mathematics in the context of spontaneous play. This results in the following research question for this study:

Which characteristics of the interaction between preschool/kindergarten teachers and preschoolers (2-6 years) are useful for stimulating young children's language and mathematical development in the context of play?

METHOD

This study is a design based research, cooperatively performed in a PLC-setting by professionals in preschool and kindergarten and researchers (Van den Akker, Gravemeijer, Mc Kenney, & Nieveen, 2006; Engeström, 2011). It uses a mixed methods approach in conducting data collection and analysis and has a practice based oriented character (e.g. Commissie Pijlman, 2017). Researchers involved in this study followed the Dutch Code of Conduct for Research Integrity (VSNU/Vereniging Hogescholen, 2018).

In this study, professionals in preschool and kindergarten from four locations in three urban areas in the Netherlands are participants. At these locations preschool and kindergarten are housed in one building. In the Netherlands, professionals in preschool teach and care for children aged two to four years old, whereas professionals in kindergarten teach four to six year old children. In both locations involved in this study four professionals from preschool, four from kindergarten and three researchers participated in five to seven two and a half hour PLC-meetings. These PLC-meetings focused on retrieving and discussing the participating professionals' experiences in stimulating children's play and mathematical development. The researchers fueled these discussions with theoretical input.

All participants' activity in the PLC is video-recorded. The videos are transcribed. Purposive sampling is used in order to select fragments from the setting where mathematics is stimulated in children's spontaneous play (Lavrakas, 2008). These fragments are coded using the three guidelines for interaction in young children's play, that is (a) observing, (b) connecting and (c) enriching (Van der Zalm, Boland, & Damhuis, 2018), and mathematical domains, namely (i) number, (ii) measurement, (iii) geometry, (iv) graphs and representations and (v) ratio and proportion (SLO, 2018a; SLO, 2018b; Van den Heuvel-Panhuizen, 2008; Van den Heuvel-Panhuizen & Buys, 2008).

From all the transcribed videos from the five to seven PLC-meetings in the four locations 97 video clips were selected, from which 29 video clips described a situation in a preschool (2-4 year olds) and 68 clips captured a situation in kindergarten (4-6 year olds). Criteria for selecting and coding video clips were (1) in the clip a professional describes a concrete situation in her own teaching, (2) the professional or researcher speaks explicitly about a child's or children's mathematical activity, when summarizing group discussions that took place in small groups they participated in. Video clips differ in length from a two to 75 minute discussion.

A classification scheme was developed in which all professionals' statements in the video clips could be coded. This classification scheme was developed in five coding rounds. As discussed in the Results section, a number of categories were distinguished in how professionals combine mathematics in interaction with the children in their groups. More specifically, the classification scheme indicates combinations of mathematical domains (number, measurement, geometry, graphs and representations, and ratio and proportion) and types of interaction between the professional and one or more children (i.c., observing, connecting and/or enriching) and interaction between the child and its environment (i.c., without the professional being involved). The final coding was based on the classification scheme and was carried out by four authors of this paper. There were only a few cases of disagreement (<5%). After discussing these cases, full agreement was reached.

RESULTS

Frequencies of interaction in stimulating mathematics in children's spontaneous play by professionals or their learning environment

Table 1 provides an overview for how frequent professionals in the 97 video clips combine mathematical domains with interaction characteristics or characteristics of the learning environment. The numbers indicate frequencies of situations wherein particular mathematical domains – made explicit in the PLC-meetings – and interaction form coincide. Numbers in the table do not add up to 97, as professionals often describe different forms of interaction between themselves and children in one situation. Moreover, in several video clips more than one mathematical domain is involved.

	Type of interaction			
Mathematical domain	learning environment	observing	connecting	enriching
(i) number	14 (3)	12 (3)	10 (5)	11 (4)
(ii) measurement	26 (10)	37 (14)	31 (9)	22 (5)
(iii) geometry	18 (6)	31 (11)	26 (9)	22 (5)
(iv) graphs and representations	3 (0)	0 (0)	0 (0)	0 (0)
(v) ratio and proportion	1 (1)	1 (0)	1 (0)	1 (0)

Table 1: Interaction characteristics and domains in all 97 video clips
(29 video clips preschool (2-4 year olds) in parenthesis)

In the 97 video clips that were analyzed we observed professionals arguing on aspects of the learning environment, considering how children's play might be stimulated *(n=40)*. When considering children's actual spontaneous play, we distinguished observing, connecting and enriching. When the professional shows real interest in the child's play, the professional observes the play *(n=52)*, plays along, connect with the child *(n=41)* or enriched the child's play mathematically *(n=33)*. When enriching the children's play professionals specifically prepared an activity focused on a goal or when they aimed to develop play further from connecting, aiming at a goal that is not easily reached by connecting alone.

Measurement was coded most. This was done when children referred in language or gestures to things being small or big, or more or less, made constructions using wooden blocks or clay and referring to length, width, surface, or volume, compared objects and/or children's length, referred to time in playing. Geometry in spontaneous play came forward in the video clips when children refer in language or gestures to relative positions in space, take a specific point of view or take a geometrical form into account. About 25 percent of the clips focus on 'number'. This came forward when a professional introduces a unit when measuring, children or professionals indicate a number of things or people, or count a number of things or people (e.g. counting shells). Number was also coded when children explore using money or numerals. The formal nature of number probably makes that mathematical enriching is here more prominent then for measurement and geometry. Researchers in several occasions mentioned graphs or ration, when professionals described situations from their classrooms. However, only in a few cases professionals recognized these domains in the children's spontaneous play.

Characteristics of interaction in stimulating mathematics in children's spontaneous play by professionals or their learning environment

In coding how the professionals stimulated mathematics in children's spontaneous play, the three guidelines for interaction in young children's play ((a) observing, (b) connecting and (c) enriching) offered a helpful framework. Based on the professionals statements, responses and examples discussed during the PLC-meetings, it was found that within these three categories of interaction particular acts by the professionals were carried out in order to stimulate children's mathematical thinking, reasoning and understanding. Figure 1 shows the classification scheme that was developed based on the coding of the video clips of the PLC-meetings.

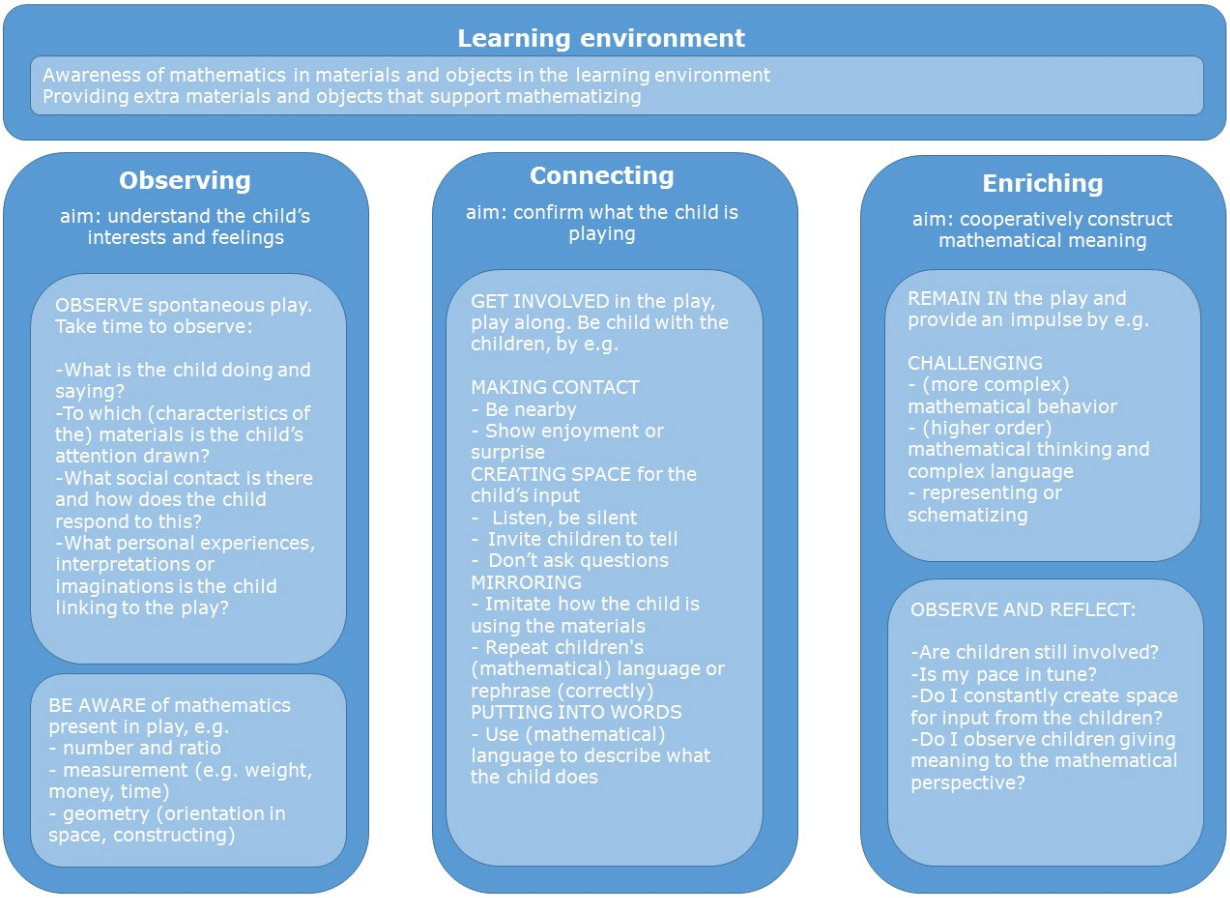

Figure 1: Classification scheme of interaction characteristics in stimulating mathematics in the spontaneous play of children.

The following narrative illustrates how professionals stimulated mathematics in children's spontaneous play. It shows how a professional elaborated on the three guidelines for interaction in combination with an awareness of the mathematics involved in the children's play. In one of the PLC-meetings one of the professionals discusses a recent experience in her working practice in kindergarten. In this situation children decide to build a mosque using wooden blocks. First, the professional decides to observe the children's spontaneous play and discovers what the children are doing. With respect to mathematics, the professional recognizes aspects of geometry (i.c., construction) in the children's play. Second, the professional connects to the children's play with the aim to get involved. She plays along by building together with the children and by mirroring their play. Third, the professional enriches the play by asking the children where the particular mosque is located, which challenged the children's to think and make modifications with the aim to build a mosque similar to the one they know their parents visit. This encourages the children to discuss and elaborate on the position and amount of minarets as part of the mosque, making a mental model of the mosque, constructing with and rotating blocks.

As stated before, we found that – apart from discussions on the mathematical domains number, measurement, geometry during the PLC-meetings – two other domains were discussed as well, namely graphs and representations, and ratio and proportion. However, when describing situations from their classrooms, the professionals in only a limited number of cases indicated these domains or objectives within these domains, whereas there were experiences in their classrooms in which graphs or representations or ratio played a central role. In fact, the professionals gave illustrations of children who reasoned proportionally when playing with toy buildings, people and animals in their classes. This playing is recognized by the professionals but they interpreted this as a geometrical activity and rarely than proportional reasoning. In coding the video clips this ignorance for ratio was followed as it was the aim of the study to support professionals in stimulating mathematics in their children's play. This finding shows that in general the professionals involved in this study are able to recognize and indicate mathematics to a certain degree.

DISCUSSION

The results of the study presented in this paper show that professionals use particular interaction guidelines in order to stimulate mathematics in children's spontaneous play. These professionals were stimulated to recognize mathematics and experiment with mathematics in children's spontaneous play in their groups. The PLC-trajectory offered an excellent opportunity for professionals and researchers to learn from each other in a heterogeneous composed setting. However we also observed that professionals experience difficulty recognizing ratio and proportion, and graphs and representation in children's spontaneous play. We assume that ratio might be too obvious to recognize as mathematics. We, further, think graphs are not recognized in spontaneous play, as it generally is not typical for this play.

Results in this study are in a sense indirect, as they do not from direct practice. Strict European regulations for video recording in kindergarten and preschool made systematically obtaining video recordings of children's spontaneous play impossible. Therefore, we instead discussed illustrative video from some of the professionals in the PLC-meetings. Doing so, we not only observed what professionals did in their group, but were also able connecting this with professionals' arguments and considerations.

CONCLUSION

This research searched for characteristics of the interaction between professionals and children in play aimed at stimulating mathematics and language development in spontaneous play. We found that observing, connecting and enriching are crucial in stimulating young children's development in mathematics. We found that stimulating mathematical development in spontaneous play presumes professionals' deep knowledge of young children's mathematics, and that this knowledge helps professionals deciding using their interaction repertoire.

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DOES PRESERVICE TEACHER TRAINING CHANGE PROSPECTIVE PRESCHOOL TEACHERS' EMOTIONS ABOUT MATHEMATICS?

Dr. Oliver Thiel

		Start Autumn 2017		End Spring 2018			End Spring 2015	
Variable	Subsample	N	Mean (SD)	N	Mean (SD)		N	Mean (SD)
MJOY	Full-time	116	3.67 (1.20)	29	3.92 (.91)		117	3.98 (.90)
	Part-time	36	4.05 (1.09)	21	4.96** (.80)		46	4.94 (.70)
	Control	19	3.69 (1.19)	13	3.83 (1.57)			
MAS-R	Full-time	116	43.0 (12.2)	29	43.3 (9.9)		116	41.5 (10.8)
	Part-time	36	42.3 (10.5)	21	36.1* (9.0)		46	34.6 (9.8)
	Control	19	44.4 (11.8)	13	44.1 (11.3)			

$F(2,54) = 6.016$, $p < .01$; $V = 0.182$;

** $\Delta M_{MJOY} = -.91$, $F_{MJOY}(1,55) = 11.223$, $p < .01$, $\eta_p^2 = 0.169$;

$F_{MJOY,2018}(2,60) = 7.259$, $p < .01$; $\eta_p^2 = 0.195$;

* $\Delta M_{MAS\text{-}R} = 6.2$, $F_{MAS\text{-}R}(1,55) = 5.155$, $p < .05$, $\eta_p^2 = 0.086$;

$F_{MAS\text{-}R,2018}(2,60) = 3.910$, $p < .05$; $\eta_p^2 = 0.115$

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OF EARLY CHILDHOOD EDUCATION

DOES PRESERVICE TEACHER TRAINING CHANGE PROSPECTIVE PRESCHOOL TEACHERS' EMOTIONS ABOUT MATHEMATICS?

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A major aim in many curricula for Early Childhood Education and Care (ECEC) is that children shall develop positive attitudes towards mathematics. In order to facilitate this, teachers need to have positive feelings about mathematics. Therefore, ECEC teacher training has to deal with prospective teachers' emotions about mathematics. The aim of this small-scale study is to investigate if and how ECEC teacher training changes prospective ECEC teachers' emotions about mathematics, especially mathematics-related enjoyment and mathematics anxiety. 171 students from a Norwegian University College participated in a longitudinal quantitative study with an experimental pre-test post-test control group design. The findings suggest that a positive change of emotions only succeeds when practical and theoretical training are intertwined.

INTRODUCTION

Although mathematics is important in science, business, engineering and everyday life, many people dislike or even hate this subject (Larkin & Jorgensen, 2016). Emotions and attitudes towards a subject develop from experiences. In a dynamic interaction with the environment, attitudes guide approach to and avoidance of the subject (Metje et al., 2007). People often persist in negative attitudes and emotions because of a lack of experiences that contradict their prior experiences (Eiser et al., 2003). Early experiences of failure can lead to fear that is difficult to dispel later (Bandura, 1977). Therefore, it is an aim in many curricula for Early Childhood Education and Care (ECEC) that children shall have positive experiences with mathematics. To facilitate such experiences, teachers need to have positive feelings by themselves, because teachers' and students' enjoyment in the classroom are positively related to each other (Frenzel et al., 2009), and teachers' attitudes affect children's mathematical learning (Beilock et al., 2010). Thus, ECEC teacher training focuses not only on mathematical and pedagogical content knowledge but has to deal with prospective teachers' attitudes and emotions as well. The aim of this small-scale study is to investigate if and how ECEC teacher training at a Norwegian University College changes prospective ECEC teachers' emotions about mathematics.

THEORETICAL FRAMEWORK

The term 'emotion' is difficult to define because it has rather disparate and unspecified meanings (Izard, 2010). However, it is possible to provide operational definitions for discrete emotion terms like joy and fear. This study focusses on mathematics-related enjoyment and mathematics anxiety.

Mathematics-related enjoyment (MJOY)

Mathematics-related enjoyment (MJOY) is the motive why someone engages in a mathematical activity for his or her own sake and not as a means to gain other rewards (Deci & Ryan, 2000). In

Norwegian ECEC institutions, MJOY is important because the Norwegian Framework Plan for kindergartens' content and tasks demands that staff shall encourage children to enjoy mathematics (Ministry of Education and Research, 2017). To measure prospective ECEC teachers' MJOY, the present study used the same instrument as Blömeke et al. (2019), six items that the participants had to rate on a six-point Likert scale from 'strongly disagree' to 'strongly agree'.

Mathematics anxiety

People with mathematics anxiety have 'feelings of tension and anxiety that interfere with the manipulation of mathematical problems in a wide variety of ordinary life and academic situations' (Richardson & Suinn, 1972). Anxiety has cognitive (e.g. thoughts about own failure), affective (e.g. fear), physiological (e.g. increased muscle tension), and behavioural (e.g. avoidance) aspects (Bessant, 1995). In an education research study, we can focus only on the cognitive and affective level. We used the revised Mathematics Anxiety Scale MAS-R (Bai et al., 2009). It has 14 items, six positive items that represent the cognitive aspect and eight negative items that represent the affective aspect. Participants had to rate the items on a five-point Likert scale. The positive statements have been scored in reverse so that a high sum score (possibly range from 14 to 70) indicates high anxiety. The same instrument was used by Thiel and Jenßen (2018).

THE STUDY

This is a small-scale quantitative longitudinal study with an experimental pre-test post-test control group design and a convenience sample. An online questionnaire to measure MJOY and MAS-R was sent to 392 prospective ECEC teachers from one Norwegian University College at the beginning and at the end of the academic year 2017/18. At the end, we asked in addition, 'If your attitude has changed, what was the main reason for this?' Participation in the study was voluntary and anonymous. In August/September 2017, the response rate was 43 %, but in May/June 2018, it was only 16 %. We compare three subsamples:

1) **full-time** ECEC teacher students, 2nd year of study, who took a course of 9 ECTS credits in ECEC mathematics education during the project period;
2) **part-time** ECEC teacher students, 3rd year of study, with the same course in ECEC mathematics education;
3) **control** group: part-time ECEC teacher students, 2nd year of study, without any lessons in mathematics during the project period.

FINDINGS

Table 1 shows the measured mathematics-related enjoyment and mathematics anxiety of the three subsamples. There are no significant differences between the subsamples at the beginning of the academic year. At the end of the academic year, only the part-time ECEC teacher students who had lessons in mathematics education show higher MJOY and lower MAS-R. The differences are significant both compared to the beginning of the academic year [$F(2,54) = 6.016$, $p < .01$; $V = 0.182$; $\Delta M_{\text{MJOY}} = -.91$, $F_{\text{MJOY}}(1,55) = 11.223$, $p < .01$, $\eta_p^2 = 0.169$; $\Delta M_{\text{MAS-R}} = 6.2$, $F_{\text{MAS-R}}(1,55) = 5.155$, $p < .05$, $\eta_p^2 = 0.086$] and compared to the other subsamples [$F_{\text{MJOY,2018}}(2,60) = 7.259$, $p < .01$; $\eta_p^2 = 0.195$; $F_{\text{MAS-R,2018}}(2,60) = 3.910$, $p < .05$; $\eta_p^2 = 0.115$].

Asked about the main reason for changing their attitude towards mathematics, almost all part-time students in the treatment group answered that it was the mathematics lessons at the university college. Only one student chose the practical period in an ECEC institution as the main reason. On the contrary, only 54% of full-time students stated the mathematics lessons as the main reason, while 35% thought it was the five weeks practical period in an ECEC institution.

		Start Autumn 2017		End Spring 2018		End Spring 2015	
Variable	Subsample	N	Mean (SD)	N	Mean (SD)	N	Mean (SD)
MJOY	Full-time	116	3.67 (1.20)	29	3.92 (.91)	117	3.98 (.90)
	Part-time	36	4.05 (1.09)	21	4.96** (.80)	46	4.94 (.70)
	Control	19	3.69 (1.19)	13	3.83 (1.57)		
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	Part-time	36	42.3 (10.5)	21	36.1* (9.0)	46	34.6 (9.8)
	Control	19	44.4 (11.8)	13	44.1 (11.3)	* – p < .05; ** – p < .01	

Table 1: Sample size, mean and standard deviation (SD) of mathematics-related enjoyment (MJOY) and mathematics anxiety (MAS-R) by subsample and time (data from 2015 for comparison)

LIMITATIONS

A major limitation of this study is the small sample size and that we used a convenience sample. Our findings are not representative but show an interesting trend that should be studied in more detail. Especially the very low response rate at the end of the academic year is a problem. There might be a bias if only the most interested students responded. Therefore, we compared our findings with data from an earlier study that used the same instruments at the end of the academic year 2014/15 (Blömeke et al., 2019; Thiel & Jenßen, 2018). In our sample, MJOY was almost the same and MAS-R was only insignificantly ($p > .36$ for full-time and $p > .45$ for part-time students) higher than in 2015 (see Table 1). Therefore, we assume that our sample is not biased.

DISCUSSION

This study shows how difficult it is to change prospective ECEC teachers' emotions about mathematics, but it reveals as well a way that possibly works. Part-time students' MJOY and MAS-R changed positively after one year of study and they stated the mathematics lessons as the main reason for the change. However, full-time students who attended the same kind of lessons did not change their emotions. In Norway, only about half of the educational staff in ECEC institutions are trained ECEC teachers. The other half have other or no formal qualification. The predominant majority of part-time students in ECEC teacher education are already working in ECEC institutions and want to become ECEC teachers. Those students already have a lot of practical experience from ECEC but not with teaching mathematics, and they can directly apply to their workplace what they have learnt during the lessons. Therefore, it seems reasonable to assume that a combination of theory and practice is the most promising approach. Whether practical experience nor theory lessons alone have an effect but reflecting on experiences from practice based on sound mathematical knowledge for teaching leads not only to professional competence but will also increase enjoyment and reduce fear. This has a distinct practical implication. If ECEC teacher training shall positively affect prospective ECEC teachers' emotions about mathematics, practical and theoretical training should be intertwined. However, more research is needed to confirm this hypothesis. We suggest a quasi-experimental study comparing three groups: (1) theory lessons followed by a practical period; (2) a practical period followed by theory lessons; (3) intertwined practical and theoretical training.

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Bishop's (1988, 1991) Mathematical Activities Reframed for Preverbal Young Children's Actions

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- Young children's engagement with mathematics
- The very young preverbal child
- Bishop's (1988, 1991) mathematical activities and their reframing
- Specific examples from video of very young preverbal children
- Questions?

BISHOP'S (1988, 1991) MATHEMATICAL ACTIVITIES REFRAMED FOR PRE-VERBAL YOUNG CHILDREN'S ACTIONS

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Educators who work with young, verbal children can use Bishop's (1988, 1991) Six Mathematical Activities to identify mathematical thinking. But what mathematics might young, pre-verbal children engage with? To help unlock the mathematical thinking of young, pre-verbal children, the authors analysed hours of video to identify children's actions that could indicate which of Bishop's activities was evident. The authors used these visible actions to reframe Bishop's activities for young, pre-verbal children, focusing on what the educator observes rather than what the child says.

YOUNG CHILDREN'S ENGAGEMENT WITH MATHEMATICS

Björklund (2018) states that the child making sense of their environment and the situations within that environment will involve mathematical thinking. In engaging with their world, very young children (birth – 3 years of age) who are often pre-verbal, use mathematical thinking. When Bishop's (1988, 1991) Six Mathematical Activities were used to identify the mathematical thinking of children in an education and care setting, it became apparent that although they provided a suitable lens for identifying mathematical understandings of older children, it was less effective for young, preverbal children. As a result, Bishop's activities were reframed to focus on actions rather than language.

REFRAMING BISHOP'S (1988, 1991) MATHEMATICAL ACTIVITIES

The authors reframed Bishop's (1988, 1991) activities to focus on very young children's actions. Each of Bishop's activities are provided below, followed by the reframed activity and examples observed from the videos. These examples typically demonstrate more than one activity, reflecting the overlap that Bishop (1991, p. 108) described was inherent across the concepts of the activities.

Counting

In his 1988 paper, Bishop describes counting as "the use of a systematic way to compare and order discrete phenomena." (p. 182). Bishop notes counting "is stimulated by, and in turn effects, the cognitive processes of classifying and pattern-seeking" (Bishop, 1991, p. 28). It shows an indication of whether there is more or less, many or some, or each of a discrete quantity (Bishop, 1991, p. 100). Counting is a consideration of 'how many', it can relate to family members, objects, the environment, and the universe itself – whenever a quantity is considered (Bishop, 1991, p. 104).

Reframed

Although not aware of the numeric order that is associated with counting, very young children repeatedly and frequently manipulate sets of objects in a deliberate way. They explore and consider individual and multiple objects in terms of how many may be moved or held. The consideration of individual items may involve a child using *one* hand to pick up *one* object or deliberately selecting an individual object from a collection of objects. Counting reframed includes young children recognising a pattern in items, movement, or music. It also includes children recognising change in the quantity of a collection of items or objects. Video evidence shows a child selecting one object from a collection of objects, matching one item to one item when reaching out with one hand to pick up one block; or using one hand to add items to their collection one at a time.

Locating

Bishop (1988) describes locating as "exploring one's spatial environment and conceptualising and symbolising that environment" (p. 182). He states that locating "seemed necessary to demonstrate the significance of the spatial environment" (Bishop, 1991, p. 28). Actions on the environment are included, such as opening or closing an object, positioning objects, orienting oneself, and moving in a direction, as well as recognising borders or boundaries, separating, moving forwards or backwards, and maneuvering around the environment (Bishop, 1991, pp. 29-33). Environmental locations, descriptions of movements, specific locations, navigation of the environment, and descriptions of objects and places and their relationships to each other are also involved (Bishop, 1991, p, 105).

Reframed

'Locating' activities are evident in the pre-verbal child's positioning of their body within the environment. When a child attempts to move in a specific way or in a specific direction, their actions show their realisation that something cannot be accessed from their current position. Young children purposefully move objects in an effort to examine them in detail, make them useful, or to non-verbally communicate (for example, handing an adult something they want opened). Although they are not able to give verbal directions, pre-verbal children follow them to place, position, retrieve, or move items in their environment. Examples from the videos are a child reaching for a toy but, when not able to access it, crawling to move themselves closer to pick up the toy; and moving the flaps of a box out of the way to make sufficient room to crawl into the box.

Measuring

The concept of measuring is described by Bishop as "quantifying qualities for the purposes of comparison and ordering" (Bishop, 1988, p. 182). It is "concerned with comparing, ordering, and quantifying qualities which are of value and importance in the immediate environment which furnishes the qualities to be measured" (Bishop, 1991, p. 34). Included are "ideas like 'more than' and 'less than' … comparing more than two or three objects … clearly estimating 'by eye'" (Bishop, 1991, p. 35). Measuring involves "comparing things according to a shared quality, and developed through paired comparisons to many comparisons, through convenient units to standardised units" and includes uni-dimensional, two-dimensional, and three-dimensional (Bishop, 1991, p. 101). The child's environment impacts their measuring activities, in both physical and temporal features (Bishop, 1991, p. 105).

Reframed

Activities for pre-verbal young children that incorporate 'measuring' include the investigation of their environment and the objects that inhabit their environment. They involve consideration of attributes, such as texture and shape, as well as recognising familiar objects and identifying and exploring unfamiliar objects. These behaviors provide the opportunity for the children to 'catalogue' the environment and the objects within it. This 'cataloguing' behaviour enables young children to understand their world and creates a 'representation' against which they can compare new environments or objects or identify preferred or favoured objects. Examples in the videos involved a child cataloguing an object by shaking it and carefully turning it while looking intently at it; reaching out for an object and, when not successful, moving closer to get the object; and a child who, after trying to move a toy by pulling on the attached cord, moved their body further from the toy and pulled the string harder to get the object to move.

Designing

Bishop (1988) describes designing as "creating a shape or design for an object or for any part of one's spatial environment" (p. 183) and "the conceptualisations of objects and artefacts, which lead to the fundamental idea of 'shape'" (p. 23). Bishop (1991) explains "the essence of designing is transforming a part of nature … imposing a particular structure on nature" (p. 39), where "the designing of objects offers the possibility of imagined form, shape and pattern in the environment; when the shapes are drawn, made, and designed that the form itself becomes the focus of attention" (p. 40). It is "the perceived spatial relationship between object and purpose … the plan, the structure" (p. 39). Seeing shapes in the environment and determining similarities and differences enables children to identify properties in shape (Bishop, 1991, p. 102), comparing "by properties of form, aesthetics, large, small, proportions, similarity, and congruence", whether in nature or man-made (Bishop, 1991, p. 102).

Reframed

When exploring known objects and situations within their environment, very young children can recognise the patterns and features that these objects and situations have. Very young pre-verbal children identify the attributes of an object, a situation, or elements in their environment that function in a preferred way. Their knowledge is used in their known environment and with new objects, environments, or situations. Examples in the videos involved a child who, when holding one green block, picked up a different block and looked at it, then after cataloguing its attributes, she dropped it and picked up a block that did match the green block, keeping one block in each hand as she crawled away; a child picked up a larger block, shook it to make a smaller block inside fall out, and then put another block inside the larger block; and placing toys with wheels on the ground wheel-shape first before 'driving' the toy across the floor.

Playing

Playing is "devising, and engaging in, games and pastimes, with more or less formalised rules that all players must abide by" (Bishop, 1988, p. 183). In 1991, he stated "playing is concerned with social procedures and rules of performance, and also stimulates the 'as if' feature of imagined and hypothetical behavior" (Bishop, 1991, p. 23), speculating whether the characteristics of play might "be at the root of hypothetical thinking" (1991, p. 43). `Bishop (1991) notes "mathematics is a game" and "chance and prediction were key ideas in developing the notion of serious play, and predicting outcomes in relatively more or less structured situations … predicting generalisations can be significant" (pp. 106-107).

Reframed

Very young pre-verbal children demonstrate joy when they play; an element of 'fun' is present and they will sometimes replicate everyday procedures and processes, often of a social nature. Examples from the videos show a child unsuccessfully trying to switch on the light box by pressing against the box but, after watching another child successfully switch on the light box, moved around to reach where the switch was located, and turned the light box on and off several times; a child picking up a book, turning it so that the pages are towards them, then opening the pages to look inside the book.

Explaining

Bishop describes 'explaining' as "finding ways to account for the existence of phenomena" and as "the various cognitive aspects of enquiring into, and of conceptualising, the environment and of sharing those conceptualisations" (Bishop, 1991, p. 23). It focuses attention on the actual abstractions and formalisations themselves exposing relationships between phenomena" (Bishop, 1991 p. 48). Similarity, considered "the security of things familiar which probably makes us seek 'sameness'", is highlighted as significant in explaining relationships and creates the opportunity to use labels (Bishop, 1991, p. 48). Bishop considers classifying and classification systems as behaviours within explaining as are conventions, generalisability of logical relationships, explanations, and arguments that often build on and represent outcomes of the previous activities (p. 107).

Reframed

Pre-verbal children can be observed 'explaining' their world in behaviours by using what they know about the environment, the objects, or situations – they are not sharing what they know verbally with others but can be seen acting on what they know through their engagement with the world. They form conceptions and propose hypotheses as they investigate the world. Examples from the videos show a child who crawled into a large box, turned their body around, and then, using one hand on each of the two side flaps, closed the flaps so that they are enclosed in the box; a child crawling to a bookcase who then pulls themselves up to standing by holding on to the bookcase; and a child who picks up two large blocks joined together and pulls them apart, then repeats this with another pair of joined blocks.

CONCLUSION

Pre-verbal children engage continually with mathematical thinking. Reframing Bishop's (1988, 1991) Six Mathematical Activities to focus on actions rather than language may provide assistance in identifying the mathematical thinking that is evident in pre-verbal children's actions during their engagement with the environment.

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Presenter:
Jianqing Wen
Shanghai Jing’an Anqing Kindergarten

When Math Meets Games

——The active construction of children’s core mathematics experience in games

Math games for kindergarten

Consider

- How to integrate the logic and play of mathematics?
- What is the relationship between gameplay and the core mathematics experience?
- How do different children learn in games?

Teachers playing games with children

Math game: Playing Cards

1. Which card was drawn
2. Guessing game of cards within the same suit
3. Guessing game of cards within two suits

Children playing alone or cooperatively

Math game:Red and Yellow Paper Tube Chess

Observe the process of children’s math games

What we can see

- What children can do
- How they approach the problem; How they try to solve the problem
- Children's contentment

What we can understand through observation

- Try to understand how each child thinks
- Why they learn in this way
- See children's contentment from their perspective

Children’s nature of games

Rules of mathematics learning

WHEN MATH MEETS GAMES—THE ACTIVE CONSTRUCTION OF CHILDREN'S CORE MATHEMATICS EXPERIENCE IN GAMES

Jianqing Wen
Shanghai Jing'an Anqing Kindergarten

Games are how children understand the world. Through games, Children obtain perceptual mathematics experience and achieve mathematical thinking ability. The curriculum of Shanghai kindergarten no longer takes field experience but children's life experience as a logical starting point since many years ago. The math activities have been focusing more on mathematics problems and phenomena in life, such as ranking in number, sub-groups, sorting toys by category. We are studying the characteristics of children's mathematics learning and are paying more attention to children's development of thinking in mathematics learning.

Games are a basic form of activity for preschool children. We are using games to advance children's mathematics learning, stimulate children's interest, meet children's operation and exploration needs, and promote the construction of children's mathematical experience in an exciting and natural state.

MATH GAMES FOR KINDERGARTEN

Games are a way for children to understand the world. When games become more valued and wider applied, we continue to think about several questions: How to effectively integrate the logic and play of mathematics? What is the relationship between gameplay and the core mathematics experience? How do different children learn in games?

To combine the fun of games with the effectiveness of learning, it is not a simple addition of games and learning content, but an organic fusion of both reflecting in each other so that children can continue to experience games and improve mathematics learning.

TEACHERS PLAYING GAMES WITH CHILDREN

In kindergartens, there are many math games that teachers and children play together, which teachers have designed based on the needs of most children and around the core experience of mathematics. Teachers will design every aspect of the game and promote core learning experience from the perspectives of the needs in children's development. Teachers will consider: a) The game structure: how to play the game and whether the children are interested. b) Problem designing in the games: how to correspond to the core experience construction of children.

Math game: Playing Cards

The teacher used poker cards as game materials and designed a 25-minute game activity. The game has three designed stages based on 6-year-old children's numerical experience.

1) Which card was drawn

The teacher took out five cards and asked the children to draw a card. The teacher then guessed which card the children just drew. It arouses children's interest in the game and leads to initial attempts to solve the problems through observation and exclusion, thus letting children understand that the card not in the teacher's hands is the drawn card.

Figure 1: Which card was drawn

2) A guessing game of cards within the same suit

The teacher selected a card from the cards of Hearts 1-10 and recorded the number. Children guessed which card it was by asking questions about the number. The teacher answered either "too large" or "too small" to provide clues to the children, and children could make logical reasoning with the clues to flip those cards, which they assumed were not the right card.

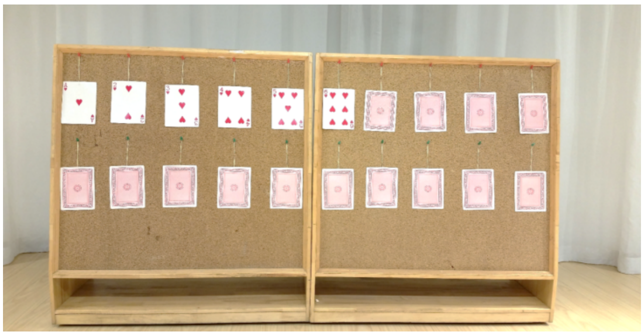

Figure 2: A guessing game of cards within the same suit

3) A guessing game of cards within two suits

Ten more cards of another suit were added. The teacher provided "yes" or "no" answers to children's questions, offering clues. After increasing the suit and the number of cards, children must combine their existing numerical sequence experience with observation to try to reason, conclude, and generalize the clues in order to exclude and flip more cards that were not the answer, and express the thinking process in mathematical language at the same time. Therefore, children would gradually ask questions that reflect their observation and mathematical understanding: Is it a red Spade? Is it singular? Is it larger than 5?

Within 25 minutes, the teacher and the 6-year-old children have played three card-related games, step by step. The children used the existing numerical sequence experience to carry out thinking activities such as excluding, logical reasoning, and concluding while challenging their reverse thinking and mathematical language expression ability.

CHILDREN PLAYING ALONE OR COOPERATIVELY

China's *3-6 Years Old Children's Learning and Development Guide* regards problem-solving as the core of mathematics cognition, which requires children's mathematics learning to focus on discovering, analyzing, and solving problems. The essence of using games to teach children's mathematics is to regard children's learning process as a process of discovery, exploration, and problem-solving.

In kindergartens, besides the games designed by teachers to play with children, there are also numerous game materials and environments where children can play alone or with each other. Teachers provide the materials, and in the process of games, children learn and explore independently, cooperate with each other to solve problems and develop thinking abilities.

Math game: "Red and yellow paper tube chess"

The teacher provided a 3*3 chessboard and two sets of different color paper tubes with five sizes in the classroom. Two children each chose one color, putting the tubes in the squares in turn. The goal is to connect three tubes of the same color in a line. Children can capture the smaller tubes the opponent put on board using their own bigger tubes to change the color.

Figure 3: Red and yellow paper tube chess

The game material has a simple structure, producing variables throughout the game. During the game, 5-year-old children usually only considered the tube's placement based on space, while 6-year-old children not only considered the position but also found strategies from the various sizes of the tubes. At first, children would pay attention to whether they could connect their tubes into three. However, during the game development, they gradually discovered that it was useless to focus only on their own materials but to pay attention to the sizes of the opponent's tubes as well. It would be the key to success or failure in the game, focusing on what sizes of tubes the opponent has already placed and what sizes are left.

This game is combined with the concept of comparison in mathematical experience so that children not only need to understand sizes in the game but also use the method of "smaller sizes fitting the larger ones" to play strategically and learn and perceive through the repeated experience.

OBSERVING THE PROCESS OF CHILDREN'S MATH GAMES

Observing and interpreting children’s behaviors in math games and learning will enable teachers to better discover the characteristics of each child in learning mathematics, thus provide corresponding support. For example, in the card-guessing game, the teacher discovered and focused on children’s thinking methods and logical judgments in the comprehensive use of numerical experience, and saw different cognitive levels and problem-solving methods. Some children only flipped one card, instead of excluding the cards of larger numbers simultaneously according to the sequence of numbers; in the guessing game with two suits, some children often asked questions about one particular card, resulting in only excluding one answer. The teacher would then ask if there was any question that could help them flip more cards. Some children gradually changed their strategies in the development of comprehension, asking questions based on the cards’ characteristics and the experience of the number sequence to exclude more cards.

What we can see

When observing children’s math games, teachers can discover: what children can do; how they approach the problem; how they try to solve the problem; and the state of children’s contentment.

What we can understand through observation

Observing the children’s learning process, teachers try to understand how each child thinks and why they learn in this way, and see children’s contentment from their perspective.

Learning mathematics through games will encourage children’s active learning. The current mathematics games are no longer just for solving a single learning problem in the construction children’s mathematical experience but also for allowing teachers to consistently discover the development of thinking process brought about by children’s mathematical experience construction. For example, children showing initiative when exploring the world of mathematics; planning to formulate strategies according to their own intentions and implementing them; continuously focusing on the activities of interest; initiatively solving problems in games through various ways; reflecting, questioning, criticizing their experience.

In the study of children’s mathematics learning, it is necessary to respect the nature of children’s growth in games while studying the rules of children’s mathematics experience learning. Play math games, and motivate thinking.

ANALYSING A DANISH KINDERGARTEN CLASS TEACHER'S INSTRUCTIONAL SUPPORT IN MATHEMATICS WITH THE TOOL CLASS

- The first Danish research study in teaching and learning mathematics below first grade.
- The Danish kindergarten class has recently become mandatory and a national curriculum for mathematic has been designed. Teachers are generalists.

- A general, systematic and quantitative observation system - Classroom Assessment Scoring System (CLASS).
- Analysing quality of Instructional Support using CLASS: Concept Development; Quality of Feedback; Language Modeling.
- CLASS measured the quality of Instructional Support in this case as low (a score of 2.17).

- Possibilities and challenges using CLASS in the present study.

ANALYSING A DANISH KINDERGARTEN CLASS TEACHER'S INSTRUCTIONAL SUPPORT IN MATHEMATICS WITH THE TOOL CLASS

Birgitte Henriksen
Aarhus University

The Danish kindergarten class has recently become mandatory and a national curriculum for mathematic has been designed for the age group 5-7 years. The aim of this study is to employ the Classroom Assessment Scoring System (CLASS) developed by Hamre and Pianta (2007) in order to analyse the quality of instructional support a Danish kindergarten class teacher offers when teaching mathematics. Employing CLASS Instructional Support is associated with pupils' cognitive development concerning higher-order thinking, process-oriented feedback and language development. The analysis according to CLASS shows that the quality of Instructional Support is low.

BACKGROUND INFORMATION

The Danish kindergarten builds the bridge between informal and formal learning for 5-7 year old pupils. Like the policy efforts in several other countries to strengthen the quality of the preschool (Blömeke, Dunekacke and Jenßen, 2017) the Danish kindergarten class changed from voluntary to mandatory in 2009, and earlier emphasis on so-called play-based activities has changed to emphasize so-called learning through play and children's cognitive development (Vejleskov, 2017). In 2014, the Danish Ministry of Education presented standards divided into six areas of competence and one area covering mathematics. The teacher in a kindergarten class is a generalist without any training in teaching mathematics. This study, which is a part of a PhD Thesis, gives an insight into instructional support in practice in order to generate ideas on how to design a teacher questionnaire and an intervention study focusing on teacher-pupil mathematical dialogues.

THEORETICAL AND METHODOLOGICAL FRAMEWORK

Building on systems theory and empirical research in social development in classroom environments Pianta (1999) investigates the role of child-adult relationships in the development of social and academic competencies. Hamre & Pianta (2007) suggest that interactions between pupils and adults are the primary mechanism of children's development and learning. The development of CLASS is based on extensive literature review as well as on scales used in large-scale classroom observation studies (Hamre, Pianta, Mashburn & Dower, 2007). The design of CLASS offers a general and systematic observation and supports a common metric and vocabulary in order to describe quality as a number of aspects of the interaction. The aspects are organized as three domains with a number of dimensions, indicators and behavioral markers. The three domains are Emotional Support, Classroom Organization and Instructional Support. In the present paper, focus is on data from a micro level in the domain Instructional Support and the three dimensions within this domain: Concept Development, Quality of Feedback and Language Modeling. Metrically CLASS is based on a scoring system using a 7-point range: Low range 1, 2; Middle range 3, 4, 5; High range 6, 7. A set of scores is the result of the observed quality of each dimension during each observation cycle (20 min). These scores are averaged across cycles and consolidated to create domain scores. A certification as observer is required as well as a recertification every year.

CONTEXT AND ANALYSIS

The participant is an experienced kindergarten class teacher trained for 3½ years (210 ECTS) for a Bachelor in Social Education at a University. An introductory interview with the kindergarten class teacher showed a highly-trained teacher dedicated to improving teaching and learning in mathematics. The teacher expressed concern that she lacks a theoretical background to guide her instruction in mathematics. The school is in a middle-class area. About 20 pupils (5-7 years old) attended the kindergarten class. The pupils sit at two-person tables. In the observed lessons, the topic was numbers. As a certified observer, I carried out the observations in April and May 2019. I videotaped 20 min. four times, according to the recommendations in CLASS manual K-3 (Kindergarten through third-grade) (Pianta, La Paro & Hamre, 2008), which cover the age group of the observed pupils. The analysis of the domain Instructional Support is coded below and the score in each dimension is stated. The phrases in the extracts from the observations were originally spoken in Danish and later translated into English.

Concept development (Score 2)

Analysis and reasoning: Twice the teacher uses discussions and activities that encourage analysis and reasoning. She asks: “When you are in a store, how do you know whether you have enough money?” and “how do you work out how much a coin is worth?” The majority of the teacher’s questions are rote, such as: how many do you have all together? The questions do not prompt pupils to explain e.g. their addition or subtraction strategy or to engage in extended problem solving or higher-order thinking. The teacher moves through the activities focusing on facts, recall and repetition.

Creating: It is not observed that the teacher provides opportunities for the pupils to be creative or generate their own ideas and products. The focus is on having pupils complete activities and provide correct answers rather than on helping to stimulate their creativity and ability to plan.

Integration: The teacher makes one attempt to integrate ideas across the curriculum. The teacher links the activity of building a ten bar of centicubes to the activity the previous day.

Connections to the real world: There are three examples of the teacher making connection to the real world, however, the examples are limited e.g. the pupils are counting money and the teacher asks questions such as: What do we use money for? When you are in a shop, how do you know if you have enough money?

Quality of feedback (Score 2)

Scaffolding: The teacher often scaffolds for pupils who are having a hard time answering a question or completing an activity. She provides the incremental hints to allow the pupil to complete a task.

Feedback loops: There are some (3-5 times per lesson) feedback loops – back-and-forth exchanges – between the teacher and pupils such as:

1 T: Alma, what do you want to say?
2 S: I have most in the purple group (a group of purple centicubes).
3 T: How many do you have?
4 S: I have 5.
5 T: Do you have other groups of centicubes with just as many?
6 S: No.
7 T: What is the next number you have?
8 S: 3.
9 T: How many should be used so that there are as many as in the purple group?
10 S: 2.
11 T: Very good.

Other times feedback is more perfunctory and entirely focused on whether an answer is correct or not.

Prompting thought processes: It was not observed that the teacher queries the pupils or prompts pupils to explain their thinking and rationale for their answers and actions.

Providing information: Twice the teacher provides additional information e.g. expansions or clarifications to expand on pupils' understanding or actions.

Encouragement and affirmation: The teacher does not offer encouragement of pupils' efforts that increases their involvement and persistence. She appears to measure pupils' progress by how well they conform to her expectations by providing general praise to individual pupils e.g., "That is correct," "Very good" rather than providing them with feedback about their work process.

Language modeling (Score 2.5)

Frequent conversation: There are limited conversations in the classroom. The teacher talks regularly with and to the pupils, but the conversation is initiated by the teacher and typically limited to one or two back-and forth exchanges. The pupils do not initiate conversation with their peers and do not engage in extended conversation with one another. The pupils direct their answers to the teacher.

Open-ended questions: The majority of the teacher's questions are closed-ended. The teacher asks questions that require no more than a one-word answer or a short sentence e.g., "How many centicubes do you have?" "Which coins do you have?" Pupils rarely (twice) have the opportunity to respond to open-ended questions with language that is more complex.

Repetition and extension: A few times the teacher repeats or extends to pupils' responses e.g.

1 T: What is written on the coin?
2 S: It says five and zero.
3 T: Yes, that is 50 øre (Danish currency) or half a crown (Danish currency).

The teaching situation rarely offers pupils the possibility of making comments or asking questions, but offer them mostly the possibility of answering (closed-ended) questions.

Self- and parallel talk: The teacher does not map her own actions and the pupils' actions through language and description. She does not use self-talk i.e. links her actions to language or parallel talk i.e. describes what pupils are doing in conversation or instruction.

Advanced language: The teacher sometimes uses advanced language with pupils. She uses words as: more than, equals, the same number, together, same amount. She also uses the words “take away”, “remove”, “take from” and “subtract” and link them together. At another time she uses the words “take away” and “subtract” but without explicitly linking them to each other.

RESULT AND FINAL REMARKS

The analysis shows that CLASS measures the quality in Instructional Support with a score of 2.17 as low. This implies that important mathematics content such as mathematical reasoning and development of a mathematical language is of a low quality when interacting with the teacher. The observation instrument CLASS provides the opportunity to analyze the Instructional Support in detail and provides knowledge of dimensions the teacher may develop. When it comes to assessing classroom interactions in mathematics with a general instrument, it gives rise to considering its validity. Taut & Rakoczy (2016) suggest that cognitive activation within the domain of instructional support “has to be defined for each academic subject based on specific findings from didactic research and cognitive psychology in that field” (p. 47). Adapting CLASS to mathematics learning requires more observation topics e.g. select, apply, and translate among mathematical representations and a specification of mathematical language modeling. Despite the observation instrument CLASS needs to be further developed, I think the study gives me valuable insight into practice in order to design a teacher questionnaire and an intervention study focusing on how the kindergarten class teacher can facilitate mathematical dialogues with pupils in order to support their development and participation in mathematical communication.

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MATHEMATICAL LEARNING ENVIRONMENTS IN NORWEGIAN ECEC DEPARTMENTS

Øyvind Jacobsen Bjørkås, Dag Oskar Madsen, Anne Grethe Baustad, Elisabeth Bjørnestad

ICME-14, TSG 1. Shanghai, July 11-18, 2021

Background

- In Norway 90% of children age 1–5 attend ECEC centres daily
- Mathematics has since 2006 had an important place in the curriculum

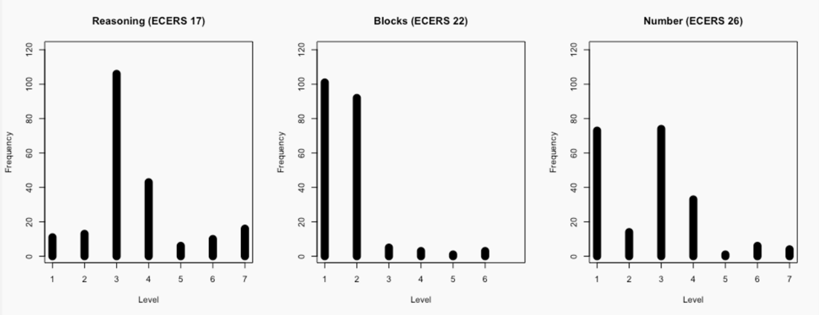
Research question

- What characterizes the mathematical learning environment in Norwegian ECEC child groups?

Methods

- Observational study using ITERS-R (n=206) and ECERS-R (n=205)
- Survey (ECEC directors)

Results



MATHEMATICAL LEARNING ENVIRONMENTS IN NORWEGIAN ECEC DEPARTMENTS

Øyvind Jacobsen Bjørkås, Dag Oskar Madsen, Anne Grethe Baustad, Elisabeth Bjørnestad
Nord University

The mathematical learning environment in Norwegian Early Childhood Education and Care (ECEC) departments is evaluated in a large scale national observational study by using ITERS-R and ECERS-R. The results suggest that especially in the area of block play, the learning environment is inadequate. We find little difference in results even if the ECEC centers report that they work systematically on mathematics.

MATHEMATICS CURRICULUM IN NORWEGIAN ECEC

Mathematics has since 2006 had an important place in the national curriculum for Norwegian ECEC. In the current curriculum (Utdanningsdirektoratet, 2017) the knowledge area "Number, Space and Shape" forms one of seven knowledge areas that teachers in ECEC are committed to educating children about. In the Norwegian context, it is problematic to use the term "teaching" about the education of children in ECEC (Sæbbe & Pramling Samuelson, 2017), and many teachers describe their work with children as a holistic approach to learning and play. Over 90% of children in Norway age 1–5 attend ECEC centers daily ("Barnehager," 2019), and so the quality of the learning environment for mathematics learning is important for the children's mathematical learning.

All departments of an ECEC center must have a qualified teacher. Most qualified ECEC teachers have a 3-year tertiary degree in ECEC that from 1995 to 2015 included mathematics as a separate subject, and from 2016 had "Language, Text, and Mathematics" as an integrated subject area. Before 1995, mathematics was not included in the ECEC teacher education. So it is expected that a large proportion of the departments have personnel with some mathematics education background. Since the proportion of smaller children attending ECEC has increased sharply in the last ten-to-fifteen years, and the demands of providing mathematics education also to these children also has increased in the last curriculum, it is interesting to investigate whether the quality of the learning environment for the smallest children is adequate.

Mathematics has had a place in the national curriculum for ECEC since 2006. It is expected from this curriculum that ECEC centers have policies to develop the mathematics teaching on all levels, for children age 1 to 5. We investigate the implementation of these policies by asking the director of the center: "Has the center worked systematically with number, space and shape during the last year?" The directors answered "Worked quite a lot with it", "Worked some with it", or "Worked little with it". This variable is called *SystWork.* The same question was asked in a national survey of the implementation of the 2006 curriculum (Østrem et al., 2009).

An adequate learning environment for mathematics in ECEC can be of different kinds. However, providing opportunities for children to learn about space and shape through block play, to be physically active, to learn about reasoning and about number must be part of any adequate learning environment for mathematics. Three items from ITERS-R (Harms, Cryer, & Clifford, 2006) and three items from ECERS-R (Harms, Clifford, & Cryer, 2005) provide indicators for the mathematics learning environment in departments providing education for children aged 1–3 and 3–5, respectively (table 1).

Item	Title	Item	
ITERS 16	Active physical play	ECERS 17	Reasoning
ITERS 19	Blocks	ECERS 22	Blocks
ITERS 21	Sand and water play	ECERS 26	Number

Table 1: Items from ITERS-R and ECERS-R

The departments are scored on level 1 (inadequate) to level 7 (excellent) on each of these items. 206 departments were investigated by ITERS-R and 205 by ECERS-R.

The empirical data was collected as parts of the BePro project, a national project which for the first time investigate features of ECEC quality and the relationships between the quality and children's well-being and development (Bjørnestad, Gulbrandsen, Johansson, & Os, 2013).

RESULTS

The results from interviewing the governors of the ECEC centers are given in table 2, and the results from observing the departments are given in figures 1 and 2.

	Fraction of departments (2016, N=184)	Fraction of departments (2009, N=469)
Worked little with it	6%	8%
Worked some with it	43%	50%
Worked quite a lot with it	51%	42%

Table 2: Has the center worked systematically with number, space and shape during the last year? Compared to results from (Østrem et al., 2009)

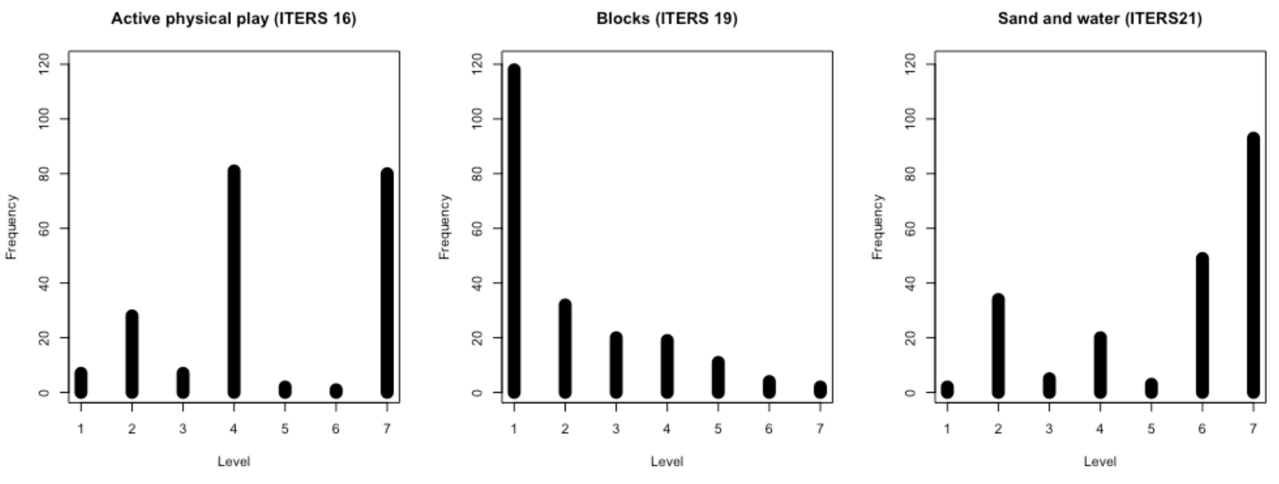


Figure 1: Results from rating departments on the ITERS scales for mathematics (*N*=206)

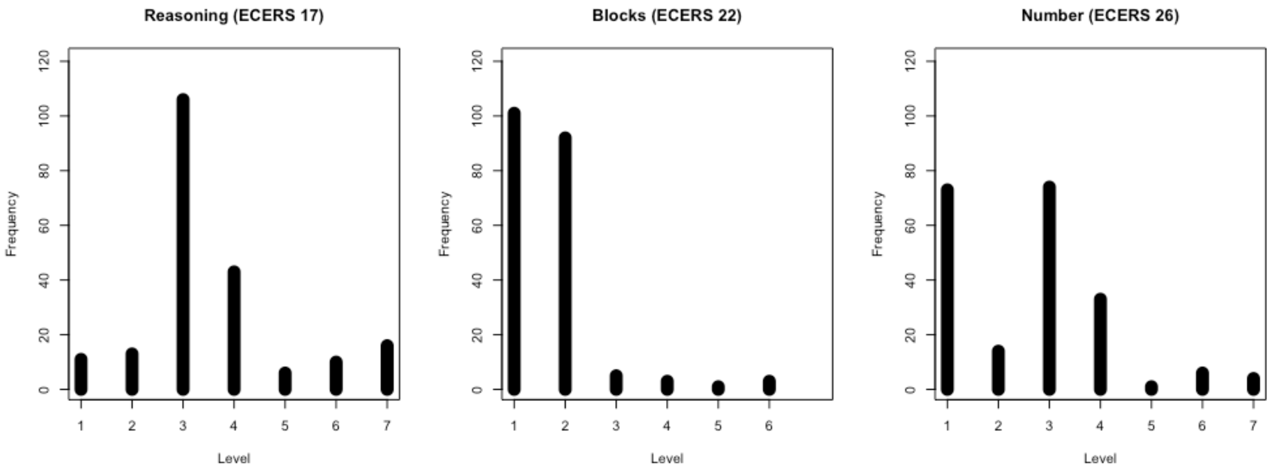


Figure 2: Results from rating departments on the ECERS scales for mathematics (*N*=205)

Most of the departments in both the ITERS and the ECERS investigations score inadequately on the items regarding blocks (figures 1 and 2). In table 3 more detailed results on these items is provided.

Indicator	ITERS (N=206)	ECERS (N=205)
Blocks accessible daily (ITERS 19:3.1)	60%	
Accessories accessible daily (ITERS 19:3.2)	42%	
Blocks and accessories accessible daily (ECERS 22:3.3)		44%
Blocks and accessories sorted by type (ITERS 19:5.2, ECERS 22:5.2)	36%	33%
Special block area set aside and steady surface (ITERS 19:5.3, ECERS 22:5.3)	36%	28%
At least 2 types of blocks accessible (ITERS 19:5.1, ECERS 22:7.1)	16%	12%

Table 3: Relative frequencies of departments satisfying some indicators from the ITERS19 and ECERS 22 items

Even if the variance is quite low in the results in table 2 and in figures 1 and 2, we report the correlations between these variables in table 4 and table 5.

	SystWork	ITERS 16	ITERS 19	ITERS 21
SystWork	1.00			
ITERS 16 Active Physical play	–.02	1.00		
ITERS 19 Blocks	–.10	.19	1.00	
ITERS 21 Sand and water play	–.05	.19	.12	1.00

Table 4: Correlations between systematic work with mathematics and ECERS-R items (N=205)

	SystWork	ECERS 17	ECERS 22	ECERS 26
SystWork	1.00			
ECERS 17 Reasoning	.04	1.00		
ECERS 22 Blocks	.07	.11	1.00	
ECERS 26 Number	.03	.28	.19	1.00

Table 5: Correlations between systematic work with mathematics and ECERS-R items (N=205)

DISCUSSION

The directors report somewhat more systematic work with mathematics compared to a similar survey published in 2009. This could suggest that the learning environments for mathematics had improved in the period 2009–2016. However, the quality of the learning environments varies greatly in 2016 and is to a large extent inadequate, as measured by ITERS-R and ECERS-R.

Block play is the scale with lowest results on both ITERS-R and ECERS-R. This is an important field of play for the development of space and shape skills for children in ECEC (Sarama & Clements, 2009), so this is an important field of improvement for the departments. Most of the departments only provide one kind of blocks on a daily basis, giving little opportunity for children to investigate different kinds of properties of space and shape.

Generally, there is room for improvement on all scales that ITERS-R and ECERS-R investigates, and more generally there is probably room for improvement of the mathematics learning environments in the departments. There is no evidence for the policy on the center level as stated by the directors having an impact of the mathematics learning environment.

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More Gooder:
Children evaluate early numeracy apps

Participants
- JK / SK
- 4-6 yrs (n=36)
- Low SES
- 2 years

Mosaic Approach
- Reggio Emilia philosophy
- 100 languages of children
- Clark & Moss (2001)
- Children are reliable, knowledgeable & trustworthy participants

15 Apps
- DragonBox
- Duck, Duck, Moose
- KinderTEK
- Gracie & Friends
- Curriculum Associates (formally Motion Math)

Essential Design Features
- Appropriate cognitive challenge
- Adaptability
- Touch responsiveness
- Ease of use

Quality of the Gaming Experience
- Play-based learning
- Feedback & Reinforcement
- Meaningful rewards

Learner Autonomy
- Flexibility
- Control

OntarioTech UNIVERSITY

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"MORE GOODER": CHILDREN EVALUATE EARLY NUMERACY APPS

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A growing body of research analyzing the quality of math education apps have criticized their design quality and superficial exploration of numeracy content. However, these studies present an adult's (i.e., researcher, teacher, or parent) perspective on apps designed for young children. This paper adds the voices of young children (4 – 6 year-olds) to this growing body of research. The paper highlights criteria a group of kindergarten children deem important for 'more gooder' math apps. The children in this study suggest the following criteria adults should consider when selecting early numeracy apps: (1) flexibility / choice of games, tasks, and content explored, (2) appropriate cognitive challenge, (3) adaptability, (4) touch responsiveness, (5) ease of navigation, (6) positive verbal reinforcement, and (7) meaningful rewards.

INTRODUCTION

An extensive body of research has highlighted the predictive power of early mathematics on subsequent school achievement (Claessens & Elgan, 2013; Claessens, Duncan, & Engel, 2009; Watts, Duncan, Siegler & Davis-Kean, 2014). In their 2007 landmark study, Duncan and colleagues concluded that "early mathematics skills [are] more powerful predictors of later academic achievement in both mathematics and reading than attentional, socioemotional or reading skills" (Duncan, et al., 2007, p.1428). Since Duncan et al., (2007), other studies have confirmed the predictive value of early math skills on academic achievement to Grade 5 (Claessens, Duncan, & Engel, 2009; Levine, et al., 2010), Grade 8 (Claessen & Elgan, 2013), and secondary school (Watts, Duncan, Siegler & Davis-Kean, 2014). Consequently, finding effective ways to engage young children in mathematical explorations is an important educational consideration. Software applications (apps) that use touch-screen technology have the potential to foster children's early mathematical development.

Although research on touch-screen technologies in early math education is still relatively new, some studies offer optimistic results on the influence of early educational apps on children's numeracy understanding (Baccaglini-Frank & Maracci, 2015; Goodwin & Gould, 2014; Orr, Flannery, Presser, Vahey, & Latimore, 2015; Spencer, 2013) as well as learning performance and efficacy (Moyer-Packenham, Shumway, Bullock, & Tucker, 2015). The gains in student achievement cited in the research were controlled, in part by the quality of the apps used in each study.

Moyer-Packenham, et al. (2015) assert, "an important goal for mathematics education is the design and selection of mathematics apps" (p. 42). The challenge facing educators and parents is determining how to select and evaluate apps used by young children. This task is seemingly insurmountable for adults who may have limited pedagogical knowledge and understanding of technology or instructional design. However, the young users of this technology can provide insight to help adults address the question of educational app quality.

This paper presents one component of a larger study that attempts to capture the story of incorporating early numeracy apps into a kindergarten classroom. For two years, the first author worked with five kindergarten teachers in two low SES schools (Ontario, Canada). We were field-testing early numeracy apps to explore the interface and instructional design features that best engage students while providing appropriate early learning opportunities. As a component of the project, we were interested in the children's perspectives on the characteristics of a '*good quality early numeracy app*'. This paper presents our preliminary analysis of the app selection criteria highlighted by kindergarten students (N=36) from one low SES school.

METHODOLOGY

To understand the views of children, we used an adapted Mosaic approach in this study. The Mosaic approach, inspired by the Italian preschools of Reggio Emilia, centers on "the notion of the competent child and of the pedagogy of listening and the pedagogy of relationships" (Clark, 2005a, p.29). This multimethod approach provides children with various opportunities to offer their voices and perspectives to the research (Clark, 2005a, 2005b). The Mosaic approach acknowledges young children as competent individuals and experts in their own world while recognizing their limitations in ability to articulate their ideas verbally. The Mosaic approach expands traditional data collection methods, such as interviews, while focusing on the researcher's ability to observe, interact and engage with children (Clark, 2005a, 2005b; Merewether & Fleet, 2014).

Procedure

The study participants included junior kindergarten (JK) and senior kindergarten (SK) children (N=36) in one low SES school. I visited the same teacher over two school years, as such approximately one-third of the children (n=12) participated in the study for 2 years (as JK & SK students).

We vetted the early numeracy apps used in this study from approximately 75 apps tested by one of the authors, her graduate students and participating teachers over a 3-year period, including some used during the 2-year study period. Of the apps tested, five research-based early numeracy apps were loaded onto the classroom iPads: *Early Math with Gracie & Friends*™ (eight apps), *Motion Math, Moose Math, KinderTEK,* and *DragonBox*: *Number & Big Numbers.*

To present the children's perspectives as authentically as possible, the first author spent a significant amount of time in their classroom. The author visited the kindergarten classroom at least twice monthly over 2 school years, spending on average 1.5 hours each visit. During classroom visits, I observed, listened to and interacted with the children while they played with the apps. Guided by the Mosaic approach, I collected data from multiple sources, including:

- In-class observations of app use (photos & videos)
- Videotaped researcher – child interactions while using the app (during class)
- Videotaped child-led 'tours' of their favourite apps
- Semi-structured interviews with the children focused on critiquing various apps including providing feedback on features / criteria app developers should consider when designing early math apps for children (conducted multiple times throughout the study)

DISCUSSION / FINDINGS

Qualitative content analysis was used to analyse the collected data. Videos were uploaded into the qualitative data analysis program, NVivo™ that was used to process data, create codes, and analyze and interpret codes by searching for common words, phrases, themes and patterns. NVivo™ provided an exploratory approach to build complex queries and begin to develop a comprehensive understanding of the data. Table 1 presents themes to emerge from the data as criteria adults should consider when selecting early numeracy apps.

App Criteria	Description
Curriculum Content	Freedom to select the curriculum content explored
Flexibility & Control	Freedom to select games and tasks explored
Appropriate Cognitive Challenge	Content explored must be appropriately challenging
Adaptability	Quickly differentiates tasks based on ability
Touch Responsiveness	Responds quickly to children's (irregular) touch
Ease of Navigation / Use	Easy to navigate and intuitive design
Positive Reinforcement	Regular ongoing positive verbal reinforcement
Meaningful Rewards	Variation in rewards for task completion

Table 1: Children's recommended app criteria

The children in this study highlighted two key themes in their critique of the early numeracy apps: quality of the gaming experience and learner autonomy.

The children in this study measured the **quality of their gaming experience**, in part by the frequency of positive reinforcement provided during game play. For these young learners, the combination of frequent positive verbal reinforcement (e.g., "you are awesome", "keep going") and earning rewards motivated them to continue playing the apps. That said different rewards motivated different children. For example, some children preferred the virtual stickers in *KinderTEK*, while others preferred winning supplies to build or open virtual worlds in *MooseMath* or *DragonBox Big Numbers,* while others valued earning coins to unlock challenging puzzles in *DragonBox Numbers*.

Beyond positive reinforcement, interface design features, including ease of navigation and adaptability, influenced the quality of the gaming experiences. In particular, poor response time or lack of touch sensitivity frustrated the children and frequently led to off task behaviours. Similarly, the children preferred apps that differentiated tasks based on their interactions with the software. Differentiation included modifying the task for struggling students and accelerating levels of challenge to maintain task engagement.

Related to the quality of the gaming experience, **autonomy** was important for these young learners. The children in this study wanted control over the curriculum content they explored, the apps they chose to use and the activities they played within the apps.

CONCLUSION

The preliminary findings presented in this paper contribute to the existing body of research on assessing the quality of children's educational software. This paper provides an alternative research perspective by focusing explicitly on the voices of young children. Who is better qualified to speak about their experiences with early numeracy apps than the children who use them? Listening to the views of the young users of technology can provide an alternative perspective to the growing body of research in the field and provide educators with insight into criteria to consider when selecting early numeracy apps for their kindergarten or early years classrooms.

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