CLASSIFICATION OF MOMENTUM PROPER EXACT HAMILTONIAN GROUP ACTIONS AND THE EQUIVARIANT ELIASHBERG COTANGENT BUNDLE CONJECTURE

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Abstract. Let G be a compact and connected Lie group. The Hamiltonian G-model functor maps the category of symplectic representations of closed subgroups of G to the category of exact Hamiltonian G-actions. Based on previous joint work with Y. Karshon, the restriction of this functor to the momentum proper subcategory on either side induces a bijection between the sets of isomorphism classes. This classifies all momentum proper exact Hamiltonian G-actions (of arbitrary complexity).

As an extreme case, we obtain a version of the Eliashberg cotangent bundle conjecture for transitive smooth actions. As another extreme case, the momentum proper Hamiltonian G-actions on contractible manifolds are exactly the symplectic G-representations, up to isomorphism.

Contents

1. The main result and applications

Let G be a compact and connected Lie group. In order to define the Hamiltonian G-model functor, we need the following. For every $g \in G$ we denote by

$$c_g : G \to G, \quad c_g(a) := gag^{-1},$$

the conjugation by g. We define $\mathrm{SympRep}_{\leq G}$ to be the following category:

- Its objects are the tuples $(H, \rho) = \big(H, V, \sigma, \rho\big)$, where H is a closed subgroup of G, (V, σ) is a (finite dimensional) symplectic vector space and ρ is a symplectic H-representation.
- Its morphisms between two objects (H, ρ) and (H', ρ') are pairs (g, T), where $g \in G$ and $T : V \to V'$ is a linear symplectic map, such that

$$c_g(H) = H', \tag{1.1}$$

$$T\rho_h = \rho'_{c_g(h)}T, \quad \forall h \in H. \tag{1.2}$$

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(The dimension of V' may be bigger than the dimension of V. In this case T is not surjective.) The composition of two morphisms is defined by

$$(g', T') \circ (g, T) := \big(g'g, T'T\big). \tag{1.3}$$

Remark. A morphism (g, T) is an isomorphism in the sense of category theory if and only if T is surjective (and hence bijective). In this case the inverse of (g, T) is given by (g^{-1}, T^{-1}).

Let $\psi = \big(M, \omega, \psi\big)$ be a Hamiltonian G-action. We call ψ exact if there exists a ψ-invariant primitive of ω.[1] We call ψ *momentum proper* iff every momentum map for ψ is proper.[2]

We define $\mathrm{Ham}_G^{\mathrm{ex}}$ to be the following category:

- Its objects are the exact Hamiltonian G-actions $\big(M, \omega, \psi\big)$ with M connected (and without boundary[3]).
- Its morphisms between two objects (M, ω, ψ) and (M', ω', ψ') are proper symplectic embeddings Φ from M to M' that intertwine ψ with ψ'. (The dimension of M' may be bigger than the dimension of M.) Composition is the composition of maps.

Remark. The isomorphisms between two objects are equivariant symplectomorphisms.

We define the *Hamiltonian G-model functor*

$$\mathrm{Model}_G : \mathrm{SympRep}_{\leq G} \to \mathrm{Ham}_G^{\mathrm{ex}}$$

as follows:

- For every object (H, ρ) we define

  $$\mathrm{Model}_G(H, \rho) = \big(Y_\rho, \omega_\rho, \psi_\rho\big)$$

  to be the centred Hamiltonian G-model action induced by (H, ρ). This action is defined as follows. (For details see [KZ18, Section 3].) We define ψ_ρ^D to be the diagonal H-action on $T^*G \times V$ induced by the right translation on G and by ρ. We denote by $\mathfrak{g}, \mathfrak{h}$ the Lie algebras of G, H and by

  $$\nu_\rho : V \to \mathfrak{h}^* \tag{1.4}$$

  the unique momentum map for ρ that vanishes at 0. For $a \in G$ and $\varphi \in \mathfrak{g}^*$ we denote by $a\varphi \in T_a^*G$ the image φ under the derivative of the left translation by a. We define

  $$\mu_{H,\rho}^D := \mu_\rho^D : T^*G \times V \to \mathfrak{h}^*, \quad \mu_\rho^D\big(a, a\varphi, v\big) := -\varphi|\mathfrak{h} + \nu_\rho(v). \tag{1.5}$$

  This is a momentum map for ψ_ρ^D. The pair (Y_ρ, ω_ρ) is defined to be the symplectic quotient of ψ_ρ^D at 0 w.r.t. μ_ρ^D. This means that

  $$Y_{H,\rho} := Y_\rho = (\mu_\rho^D)^{-1}(0)/\psi_\rho^D. \tag{1.6}$$

  (The subgroup H is compact, since it is closed and G is compact. Therefore, the restriction of ψ_ρ^D to $(\mu_\rho^D)^{-1}(0)$ is proper. Since it is also free, the symplectic quotient is well-defined.) The left translation by G on G induces a G-action on T^*G and hence on $T^*G \times V$. Since left and right translation commute, this action preserves $(\mu_\rho^D)^{-1}(0)$ and descends to a G-action ψ_ρ on Y_ρ. This defines $\mathrm{Model}_G(H, \rho) = \big(Y_\rho, \omega_\rho, \psi_\rho\big)$.

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[1] This condition is satisfied if ω is exact, because we assume that G is compact. (We obtain a ψ-invariant primitive from an arbitrary primitive by averaging w.r.t. the Haar measure on G.)

[2] By definition every momentum map is equivariant w.r.t. ψ and the coadjoint action.

[3] In this article every manifold is assumed to have empty boundary.

- For every $g \in G$ we denote by $R^g : G \to G$, $R^g(a) := ag$, the right translation by g, and by $R^g_* : T^*G \to T^*G$ the induced map. The map Model_G assigns to every morphism $(g,T) : (H,\rho) \to (H',\rho')$ the morphism $\mathrm{Model}_G(g,T)$ given by

$$\mathrm{Model}_G(g,T)(y) := \big[R^{g^{-1}}_*(a, a\varphi), Tv\big], \tag{1.7}$$

where $(a, a\varphi, v)$ is an arbitrary representative of y. (Here on the right hand side we denote by $\big[a', a'\varphi', v'\big]$ the equivalence class of $\big(a', a'\varphi', v'\big)$.)

The main result is the following. (As always, we assume that G is compact and connected.)

1.8. **Theorem** (Hamiltonian G-model functor). *(i) (well-definedness on objects) The map* Model_G *is well-defined on objects, i.e.,* ψ_ρ *is indeed an exact Hamiltonian* G*-action.*

(ii) (well-definedness on morphisms) The map Model_G *is well-defined on morphisms, i.e.,*

$$\Big(R^{g^{-1}}_* \times T\Big)\big((\mu^D_\rho)^{-1}(0)\big) \subseteq (\mu^D_{\rho'})^{-1}(0), \tag{1.9}$$

the right hand side of (1.7) does not depend on the choice of a representative $(a, a\varphi, v)$*, and* $\mathrm{Model}_G(g,T)$ *is a morphism of* $\mathrm{Ham}^{\mathrm{ex}}_G$*.*

(iii) (functoriality) The map Model_G *is a covariant functor.*

(iv) (essential injectivity) The map between the sets of isomorphism classes induced by Model_G *is injective.*

(v) (morphisms) Let (H,ρ) *and* (H',ρ') *be objects of* $\mathrm{SympRep}_{\leq G}$*, and* $(g,T), (\widehat{g}, \widehat{T})$ *be morphisms between these objects.* Model_G *maps these morphisms to the same morphism if and only if*

$$h' := \widehat{g}g^{-1} \in H', \quad \widehat{T} = \rho'_{h'}T. \tag{1.10}$$

(vi) (momentum properness and morphisms) Let A *and* A' *be objects of* $\mathrm{SympRep}_{\leq G}$ *or* $\mathrm{Ham}^{\mathrm{ex}}_G$*, such that* A' *is momentum proper and there exists a morphism from* A *to* A'*. Then* A *is momentum proper.*

(vii) (momentum properness and model functor) An object of $\mathrm{SympRep}_{\leq G}$ *is momentum proper if and only if its image under* Model_G *is momentum proper.*

(viii) (essential surjectivity) Every momentum proper object of $\mathrm{Ham}^{\mathrm{ex}}_G$ *is isomorphic to an object in the image of* Model_G*.*

Remarks.
- Theorem 1.8(v) characterizes the extent to which the functor Model_G is faithful.
- In (vi,vii) an object (H,ρ) of $\mathrm{SympRep}_{\leq G}$ is called momentum proper iff ρ is momentum proper, i.e., if every momentum map for ρ is proper.

Part (viii) of Theorem (1.8) was proved in joint work [KZ18, 1.5. Theorem] with Y. Karshon. The other parts will be proved in the next section. The proof of (iv) (essential injectivity) is based on Lemma 2.19, which provides criteria under which the symplectic quotient representation of the model action $\mathrm{Model}_G(H,\rho)$ at a given point is isomorphic to (H,ρ). We also use the fact that if two compact subgroups of a Lie group are conjugate to subgroups of each other then they are conjugate to each other. (See Lemma 2.8 below.)

Remark. Naively, in the definition of a morphism of $\mathrm{SympRep}_{\leq G}$, one could try to weaken the condition (1.1) to either the condition $c_g(H) \subseteq H'$ or $c_g(H) \supseteq H'$. With this modification the model functor would no longer be well-defined on morphisms. ("$\supseteq$" is needed in order for (1.9) to hold and "$\subseteq$" is needed for the right hand side of (1.7) not to depend on the choice of a representative. See the proof of Theorem 1.8(ii) below.)

We denote by

$$\mathrm{SympRep}^{\mathrm{prop}}_{\leq G}, \quad \mathrm{Ham}^{\mathrm{ex,prop}}_G$$

the full subcategories of $\mathrm{SympRep}_{\leq G}$ and $\mathrm{Ham}^{\mathrm{ex}}_G$ consisting of momentum proper objects. Theorem 1.8 has the following application.

1.11. **Corollary** (classification of momentum proper exact Hamiltonian actions)**.** *The functor* Model_G *induces a bijection*

$$\left\{\textit{isomorphism class of } \mathrm{SympRep}^{\mathrm{prop}}_{\leq G}\right\} \to \left\{\textit{isomorphism class of } \mathrm{Ham}^{\mathrm{ex,prop}}_G\right\}. \tag{1.12}$$

Remarks.

- It follows from Theorem 1.8(vi) that the isomorphism class of any object of $\mathrm{SympRep}^{\mathrm{prop}}_{\leq G}$ is its isomorphism class in the bigger category $\mathrm{SympRep}_{\leq G}$. A similar remark applies to $\mathrm{Ham}^{\mathrm{ex,prop}}_G$.
- Corollary 1.11 classifies all momentum proper exact Hamiltonian G-actions up to isomorphism.
- The inverse of the classifying map (1.12) is induced by assigning to a Hamiltonian action its symplectic quotient representation at any suitable point, see Proposition 4.1 below.
- In contrast with Corollary 1.11 the map induced by Model_G between the sets of isomorphism classes of $\mathrm{SympRep}_{\leq G}$ and $\mathrm{Ham}^{\mathrm{ex}}_G$ is not surjective. To see this, let Q be a connected compact manifold of positive dimension, without boundary. We define ω to be the canonical symplectic form on T^*Q and ψ to be the trivial G-action on T^*Q.

  We claim that the isomorphism class of (T^*Q, ω, ψ) does not lie in the image of Model_G. To see this, assume that $\big(H, V, \sigma, \rho\big)$ is an object of $\mathrm{SympRep}_{\leq G}$ for which ψ_ρ is trivial. Then $H = G$ and therefore, Y_ρ is canonically diffeomorphic to V. If $\big(Y_\rho, \omega_\rho, \psi_\rho\big)$ is isomorphic to $\big(T^*Q, \omega, \psi\big)$ then it follows that Q is a singleton. This proves the claim.
- Many classification results are known for Hamiltonian group actions whose complexity is low. (By definition, the complexity is half the dimension of a generic non-empty reduced space. For references see [KZ18].) What makes Corollary 1.11 special is that it classifies Hamiltonian actions of *arbitrary* complexity.

Proof of Corollary 1.11. By Theorem 1.8(i,ii,vii"$\Rightarrow$",iii) the map (1.12) is well-defined. By Theorem 1.8(iv,viii,vii"$\Leftarrow$") the map (1.12) is bijective. This proves Corollary 1.11. $\square$

By considering the extreme case of the full subgroup $H = G$, this corollary implies that the momentum proper Hamiltonian G-actions on contractible manifolds are exactly the momentum proper symplectic G-representations, up to isomorphism. See Corollary 1.17 below. On the other hand, by considering the extreme case in which the vector space V is trivial, using Corollary 1.11, we can classify the *critical* momentum proper exact Hamiltonian G-actions in terms of transitive G-actions on manifolds.

To explain the latter application, we define $\mathrm{Act}^{\mathrm{trans}}_G$ to be the category whose objects are the transitive smooth G-actions on connected closed[4] manifolds and whose morphisms are

[4]This means compact and without boundary.

the G-equivariant diffeomorphisms. We call an object (M, ω, ψ) of $\mathrm{Ham}_G^{\mathrm{ex,prop}}$ *critical* iff M is homotopy equivalent to some closed manifold of dimension equal to $\dim(M)/2$.

1.13. *Remark* (criticality). By Corollary 1.11 there exists an object $(H, \rho) = \big(H, V, \sigma, \rho\big)$ of $\mathrm{SympRep}_{\leq G}^{\mathrm{prop}}$, such that $\mathrm{Model}_G(H, \rho)$ is isomorphic to (M, ω, ψ). The manifold part of $\mathrm{Model}_G(H, \rho)$ is homotopy equivalent to the closed manifold G/H and has dimension $2(\dim G - \dim H) + \dim V$. It follows that M is not homotopy equivalent to any closed manifold of dimension bigger than $\dim(M)/2$. This justifies the terminology *critical*.

We denote

$$\mathrm{Ham}_G^{\mathrm{crit}} := \text{full subcategory of } \mathrm{Ham}_G^{\mathrm{ex,prop}} \text{ consisting of critical objects.}$$

For every manifold Q we denote by ω_Q the canonical symplectic form on T^*Q. We define the *G-cotangent functor T_G^** to be the canonical functor from the category of G-actions on manifolds and G-equivariant diffeomorphisms to the category of Hamiltonian G-actions and G-equivariant symplectomorphisms. It takes an object (Q, θ) to (T^*Q, ω_Q) together with lifted G-action θ_*, and a morphism $f : Q \to Q'$ to the lifted map $f_* : T^*Q \to T^*Q'$.

1.14. **Corollary** (classification of critical momentum proper exact Hamiltonian actions). *The functor T_G^* induces a bijection*

$$\big\{\textit{isomorphism class of } \mathrm{Act}_G^{\mathrm{trans}}\big\} \to \big\{\textit{isomorphism class of } \mathrm{Ham}_G^{\mathrm{crit}}\big\}. \tag{1.15}$$

Remarks (classification of critical actions, Eliashberg cotangent bundle conjecture).

- Part of the statement is that T_G^* maps $\mathrm{Act}_G^{\mathrm{trans}}$ to $\mathrm{Ham}_G^{\mathrm{crit}}$.
- The isomorphism class of any object of $\mathrm{Ham}_G^{\mathrm{crit}}$ in $\mathrm{Ham}_G^{\mathrm{crit}}$ is its isomorphism class in the bigger category $\mathrm{Ham}_G^{\mathrm{ex,prop}}$ (or in $\mathrm{Ham}_G^{\mathrm{ex}}$).
- Corollary 1.14 classifies the critical momentum proper exact Hamiltonian G-actions in terms of transitive G-actions on manifolds.
- The *cotangent functor T^** is the canonical functor from the category of connected closed smooth manifolds and diffeomorphisms to the category of symplectic manifolds and symplectomorphisms. It agrees with $T_{\{e\}}^*$. The Eliashberg cotangent bundle conjecture states that T^* is essentially injective, i.e., it induces an injective map between the sets of isomorphism classes. See [MS17, Problem 31, p. 561]. Very little is known about this conjecture. See [Abo12, EKS16, ES16] for some results.
- By Corollary 1.14 the restriction of the functor T_G^* to the category $\mathrm{Act}_G^{\mathrm{trans}}$ of transitive G-actions is essentially injective. This proves an equivariant version of the Eliashberg cotangent bundle conjecture. In fact, Corollary 1.14 provides more information, namely it also specifies the image of the class of objects of $\mathrm{Act}_G^{\mathrm{trans}}$ under T_G^*, up to isomorphism.
- The philosophy behind this application is that symmetry makes problems more accessible. In the present situation it allows for a classification of the structures at hand (transitive G-actions and critical Hamiltonian G-actions). The same philosophy was for example used recently in [FPP18], where the authors used Delzant's classification of symplectic toric manifolds to prove that certain equivariant symplectic capacities are (dis-)continuous. (Without symmetry the question whether a given symplectic capacity is continuous is hard in general.)

We will prove Corollary 1.14 in Section 3.

As another application of Corollary 1.11, we now classify the momentum proper Hamiltonian G-actions on contractible manifolds. Here we consider another extreme case, in which the subgroup H equals G. We denote by $\mathrm{SympRep}_G$ the category whose objects are symplectic G-representations and whose morphisms are G-equivariant linear symplectic maps (possibly not surjective), and by

$$\mathrm{SympRep}_G^{\mathrm{prop}}$$

the full subcategory consisting of momentum proper objects. We denote by $\mathrm{Ham}_G^{\mathrm{contr}}$ the full subcategory of $\mathrm{Ham}_G^{\mathrm{ex}}$ consisting of those objects (M,ω,ψ) for which M is contractible, and by

$$\mathrm{Ham}_G^{\mathrm{contr,prop}}$$

the full subcategory consisting of momentum proper objects. We denote by

$$\iota^G : \mathrm{SympRep}_G \to \mathrm{Ham}_G^{\mathrm{contr}}, \quad \iota^{G,\mathrm{prop}} : \mathrm{SympRep}_G^{\mathrm{prop}} \to \mathrm{Ham}_G^{\mathrm{contr,prop}}$$

the inclusion functor and its restriction to the momentum proper subcategories. We denote by $\iota^G_*, \iota^{G,\mathrm{prop}}_*$ the maps between the sets of isomorphism classes induced by $\iota^G, \iota^{G,\mathrm{prop}}$.

1.16. *Remarks.* (i) The isomorphism class of any object in $\mathrm{SympRep}_G^{\mathrm{prop}}$ is its isomorphism class in the bigger category $\mathrm{SympRep}_G$. This follows from Remark 2.18(ii) below. Similar remarks apply to the subcategory $\mathrm{Ham}_G^{\mathrm{contr,prop}}$ of $\mathrm{Ham}_G^{\mathrm{contr}}$ and the subcategory $\mathrm{Ham}_G^{\mathrm{contr}}$ of $\mathrm{Ham}_G^{\mathrm{ex}}$.

(ii) The map ι^G_* extends the map $\iota^{G,\mathrm{prop}}_*$. By (i) this statement makes sense.

1.17. **Corollary** (classification of momentum proper Hamiltonian actions on contractible manifolds). *The map ι^G_* is injective.*

(ii) The map $\iota^{G,\mathrm{prop}}_*$ is surjective.

Remarks.
- It follows from (i) and Remark 1.16(ii) that $\iota^{G,\mathrm{prop}}_*$ is injective. Using (ii), this map is bijective.
- Part (ii) means that every momentum proper Hamiltonian G-action on a contractible symplectic manifold is symplectically linearizable.
- The statement of Corollary 1.17 means that the momentum proper Hamiltonian G-actions on contractible symplectic manifolds agree with the momentum proper symplectic G-representations, up to isomorphism. This classifies these actions.
- Assume that G is non-abelian. In contrast with part (ii) the map ι^G_* is not surjective. This follows from [KZ18, Corollary 8.4].

For the proof of Corollary 1.17(ii) we need the following.

1.18. *Remark.* For every symplectic G-representation (V,σ,ρ) the map

$$\mathrm{I}^\rho_G : V \to Y_\rho, \quad \mathrm{I}^\rho_G(v) := [e,0,v], \tag{1.19}$$

is a G-equivariant symplectomorphism, i.e., an isomorphism from $\iota^G(\rho)=\rho$ to $\mathrm{Model}_G(G,\rho)$ in $\mathrm{Ham}_G^{\mathrm{ex}}$. This follows from a straight-forward argument.

Proof of Corollary 1.17. **(i):** Let R and R' be isomorphism classes of $\mathrm{SympRep}_G$ that are mapped to the same class under ι^G_*. We choose representatives $(V,\sigma,\rho),(V',\sigma',\rho')$ of R,R' and an isomorphism Φ in $\mathrm{Ham}_G^{\mathrm{contr}}$ from $\iota^G(\rho)$ to $\iota^G(\rho')$. The differential $d\Phi(0) : T_0V \to$

$T_{\Phi(0)}V'$ is an isomorphism from $d\rho(0)$ to $d\rho'(\Phi(0))$ in $\mathrm{SympRep}_G$. Since ρ is linear, the canonical identification between V and T_0V is an isomorphism from ρ to $d\rho(0)$ in $\mathrm{SympRep}_G$. Similarly, ρ' is isomorphic to $d\rho'(\Phi(0))$. Combining these three isomorphisms, it follows that ρ and ρ' are isomorphic in $\mathrm{SympRep}_G$, i.e., $R = R'$. Hence the map ι^G_* is injective. This proves (i).

(ii): Let Ψ be an isomorphism class of objects of $\mathrm{Ham}_G^{\mathrm{contr,prop}}$. We choose a representative (M, ω, ψ) of Ψ. By Theorem 1.8(viii) there exists an object (H, ρ) of $\mathrm{SympRep}_{\leq G}$, such that $\psi_\rho := \mathrm{Model}_G(H, \rho)$ is isomorphic to ψ in $\mathrm{Ham}_G^{\mathrm{ex}}$. By Theorem 1.8(vi) ψ_ρ is momentum proper. Hence by Theorem 1.8(vii)"$\Leftarrow$" (H, ρ) is momentum proper. Since M is contractible, the same holds for Y_ρ. Therefore, by the proof of [KZ18, 7.6 Lemma] we have $H = G$. Hence by Remark 1.18 $\iota^G(\rho)$ and ψ_ρ are isomorphic in $\mathrm{Ham}_G^{\mathrm{ex}}$ and hence in $\mathrm{Ham}_G^{\mathrm{contr,prop}}$. It follows that $\iota^G(\rho)$ and ψ are isomorphic in $\mathrm{Ham}_G^{\mathrm{contr,prop}}$. Hence $\iota^G_*([\rho]) = \Psi$. Thus ι^G_* is surjective. This proves (ii) and completes the proof of Corollary 1.17. □

Remarks.

- (This remark will be used in the next one.) We define $\widetilde{\mathrm{SympRep}}_G$ to be the category with objects the symplectic G-representations and morphisms between ρ, ρ' given by pairs (g, T), where $g \in G$ and $T : V \to V'$ is a linear symplectic map, such that (1.2) holds. The composition is defined by (1.3). We define the functor

$$i_G : \widetilde{\mathrm{SympRep}}_G \to \mathrm{SympRep}_{\leq G}, \quad i_G(\rho) := (G, \rho), \quad i_G = \text{identity on morphisms.}$$

  We may view $\widetilde{\mathrm{SympRep}}_G$ as a full subcategory of $\mathrm{SympRep}_{\leq G}$ via this functor. We define the map

$$\mathcal{F}_G : \widetilde{\mathrm{SympRep}}_G \to \mathrm{SympRep}_G, \quad \mathcal{F}_G = \text{identity on objects}, \quad \mathcal{F}_G(g, T) := \rho'_{g^{-1}}T.$$

  A straight-forward argument shows that this map is a covariant functor.
- Part (i) of Corollary 1.17 can alternatively be deduced from Theorem 1.8(iv) as follows. Let R, R' be isomorphism classes of $\mathrm{SympRep}_G$ that are mapped to the same class under ι^G_*. We choose representatives ρ, ρ' of R, R'. Then $\iota^G(\rho)$ and $\iota^G(\rho')$ are isomorphic. Using Remark 1.18, it follows that $\mathrm{Model}_G \circ i_G(\rho)$ and $\mathrm{Model}_G \circ i_G(\rho')$ are isomorphic. Hence by Theorem 1.8(iv) there exists an isomorphism (g, T) in $\mathrm{SympRep}_{\leq G}$ from $i_G(\rho) = (G, \rho)$ to $i_G(\rho') = (G, \rho')$. It follows that (g, T) is an isomorphism in $\widetilde{\mathrm{SympRep}}_G$ from ρ to ρ'. Therefore, $\mathcal{F}_G(g, T) = \rho'_{g^{-1}}T$ is an isomorphism in $\mathrm{SympRep}_G$ from $\mathcal{F}_G(\rho) = \rho$ to $\mathcal{F}_G(\rho') = \rho'$. It follows that $R = [\rho] = [\rho'] = R'$. This shows that ι^G_* is injective, i.e., part (i) of Corollary 1.17.
- A straight-forward argument shows that the map $\mathrm{I}_G : \rho \mapsto \mathrm{I}_G^\rho$ is a natural isomorphism between the functors $\iota^G \circ \mathcal{F}_G$ and $\mathrm{Model}_G \circ i_G$,

$$\iota^G \circ \mathcal{F}_G \xrightarrow[\simeq]{\mathrm{I}_G} \mathrm{Model}_G \circ i_G.$$

This means that for every morphism $(g,T):\rho\to\rho'$ of $\widetilde{\mathrm{SympRep}}_G$ the diagram

$$\begin{array}{ccc} \iota^G\circ\mathcal{F}_G(\rho) & \xrightarrow{\iota^G\circ\mathcal{F}_G(g,T)} & \iota^G\circ\mathcal{F}_G(\rho') \\ \mathrm{I}_G^\rho\Big\downarrow & & \Big\downarrow\mathrm{I}_G^{\rho'} \\ \mathrm{Model}_G\circ i_G(\rho) & \xrightarrow{\mathrm{Model}_G\circ i_G(g,T)} & \mathrm{Model}_G\circ i_G(\rho'). \end{array}$$

commutes, and that I_G^ρ is an isomorphism for every object ρ of $\widetilde{\mathrm{SympRep}}_G$. In other words the map $\mathrm{Model}_G(g,T)$ is given by

$$\mathrm{Model}_G(g,T)=\mathcal{F}_G(g,T)=\rho'_{g^{-1}}T:Y_\rho\to Y_{\rho'}$$

via the natural identifications $\mathrm{I}_G^\rho:V\xrightarrow{\cong}Y_\rho$ and $\mathrm{I}_G^{\rho'}:V'\xrightarrow{\cong}Y_{\rho'}$.

2. Proof of Theorem 1.8(i-vii) (Hamiltonian G-model functor)

For the proof of Theorem 1.8(i) we need the following. We denote by Ad and Ad^* the adjoint and coadjoint representations of G. We define the map

$$\mu^L:T^*G\to\mathfrak{g}^*,\quad \mu^L(a,a\varphi)=\mathrm{Ad}^*(a)\varphi.$$

This is a momentum map for the lifted left-translation action of G on T^*G. We denote by $\mathrm{pr}_1:T^*G\times V\to T^*G$ the canonical projection. Since left and right translations commute, μ^L is preserved by the lifted right translation action of H on T^*G. Hence the map $\mu^L\circ\mathrm{pr}_1$ descends to a map

$$\mu_\rho:Y_\rho\to\mathfrak{g}^*.$$

Proof of Theorem 1.8(i). The map μ_ρ is a momentum map for ψ_ρ. Hence ψ_ρ is a Hamiltonian action, and therefore Model_G is well-defined on objects, as claimed. □

For the proof of Theorem 1.8(ii) we need the following.

2.1. *Remark* (product of proper maps). Let X,Y,X',Y' be topological spaces, with Y and Y' Hausdorff. Let $f:X\to Y$ and $f':X'\to Y'$ be proper continuous maps. Then the Cartesian product map $f\times f':X\times X'\to Y\times Y'$ is proper. This follows from an elementary argument. (Hausdorffness ensures that every compact subset of $Y\times Y'$ is closed.)

Proof of Theorem 1.8(ii). Let (H,V,σ,ρ) and (H',V',σ',ρ') be objects of $\mathrm{SympRep}_{\leq G}$ and (g,T) a morphism between them. We denote by $\mathfrak{h}$ and $\mathfrak{h}'$ the Lie algebras of H and H'. By (1.1) we have $c_{g^{-1}}(H')=H$. It follows that $\mathrm{Ad}_{g^{-1}}(\mathfrak{h}')=\mathfrak{h}$. Hence $\mathrm{Ad}^*(g)=\mathrm{Ad}^*_{g^{-1}}$ induces a map from $\mathfrak{h}^*$ to $\mathfrak{h}'^*$, which we again denote by $\mathrm{Ad}^*(g)$. We have

$$\mathrm{Ad}^*(g)(\varphi)|\mathfrak{h}'=\mathrm{Ad}^*(g)(\varphi|\mathfrak{h}),\quad\forall(a,a\varphi)\in T^*G. \tag{2.2}$$

The map

$$\rho'\circ c_g:H\to\{\text{isomorphisms of }(V',\sigma')\}$$

is a Hamiltonian action with momentum map

$$c_g^*\circ\nu_{\rho'}=\mathrm{Ad}_g^*\circ\nu_{\rho'}:V'\to\mathfrak{h}^*,$$

where $\nu_{\rho'}$ is as in (1.4). By (1.2) ρ' leaves the image $T(V)$ invariant and T is a symplectic embedding that is equivariant w.r.t. ρ and $\rho'\circ c_g$. It follows that

$$\mathrm{Ad}_g^*\circ\nu_{\rho'}\circ T=\nu_\rho. \tag{2.3}$$

(Here we use that both sides vanish at $v = 0 \in V$.) For every $(a, a\varphi, v) \in T^*G \times V$ we have

$$\begin{aligned}\mu^D_{\rho'} \circ \left(R^{g^{-1}}_* \times T\right)(a, a\varphi, v) &= \mu^D_{\rho'}\big(ag^{-1}, ag^{-1}\operatorname{Ad}^*(g)(\varphi), Tv\big)\\ &= -\operatorname{Ad}^*(g)(\varphi)|\mathfrak{h}' + \nu_{\rho'}(Tv)\\ &= \operatorname{Ad}^*(g)\big(-\varphi|\mathfrak{h} + \nu_\rho(v)\big) \qquad \text{(using (2.2,2.3))}\\ &= \operatorname{Ad}^*(g) \circ \mu^D_\rho(a, a\varphi, v).\end{aligned}$$

The claimed inclusion (1.9) follows. We define

$$\widetilde{\Phi} := R^{g^{-1}}_* \times T : (\mu^D_\rho)^{-1}(0) \to (\mu^D_{\rho'})^{-1}(0).$$

Let $h \in H$. By (1.1) we have $h' := c_g(h) \in H'$. By (1.2) the map $\widetilde{\Phi}$ intertwines the diagonal action of h on $T^*G \times V$ with the diagonal action of h' on $T^*G \times V'$. It follows that the right hand side of (1.7) does not depend on the choice of the representative $(a, a\varphi, v)$, as claimed. We denote by

$$\Phi := \operatorname{Model}_G(g, T) : Y_\rho \to Y_{\rho'},$$

the map induced by $\widetilde{\Phi}$. We show that Φ is a morphism of $\operatorname{Ham}^{\text{ex}}_G$. The map $\widetilde{\Phi}$ is smooth, presymplectic, and equivariant w.r.t. the G-actions induced by the left translations on G. It follows that Φ is smooth, symplectic, and equivariant w.r.t. to the G-actions ψ_ρ and $\psi_{\rho'}$.

2.4. **Claim.** *The maps T and Φ are proper.*

Proof of Claim 2.4. The map $T : V \to V'$ is linear symplectic and hence injective. Since V is finite-dimensional, it follows that

$$\sup_{0 \neq v \in V} \frac{\|v\|}{\|Tv\|'} < \infty,$$

where $\|\cdot\|, \|\cdot\|'$ are arbitrary norms on V, V'. This implies that T is proper, as claimed.

We denote by

$$\pi_\rho : (\mu^D_\rho)^{-1}(0) \to Y_\rho = (\mu^D_\rho)^{-1}(0)/\psi^D_\rho \tag{2.5}$$

the canonical projection. Let $K' \subseteq Y_{\rho'}$ be a compact subset. Since $\Phi \circ \pi_\rho = \pi_{\rho'} \circ \widetilde{\Phi}$, we have

$$\pi_\rho^{-1} \circ \Phi^{-1}(K') = \widetilde{\Phi}^{-1} \circ \pi_{\rho'}^{-1}(K'). \tag{2.6}$$

The projection $\pi_{\rho'}$ is proper, since H' is compact. It follows that $\pi_{\rho'}^{-1}(K')$ is compact. The map $R^{g^{-1}}_* : T^*G \to T^*G$ is proper, since it is invertible with continuous inverse. Using Remark 2.1 and properness of T, it follows that the Cartesian product map $R^{g^{-1}}_* \times T : T^*G \times V \to T^*G \times V'$ is proper. Since this map restricts to $\widetilde{\Phi}$ on $(\mu^D_\rho)^{-1}(0)$, it follows that $\widetilde{\Phi}$ is proper. Since $\pi_{\rho'}^{-1}(K')$ is compact, it follows that the right hand side of (2.6) is compact, hence also the left hand side. Since π_ρ maps this set to $\Phi^{-1}(K')$, it follows that $\Phi^{-1}(K')$ is compact. This proves Claim 2.4. □

Using Claim 2.4, it follows that Φ is a G-equivariant proper symplectic embedding, i.e., a morphism of $\operatorname{Ham}^{\text{ex}}_G$. This proves that the map Model_G is well-defined on morphisms. This completes the proof of Theorem 1.8(ii). □

Proof of Theorem 1.8(iii). It follows from a straight forward argument that Model_G maps the unit morphisms to unit morphisms and intertwines the compositions. Hence it is a covariant functor. This proves Theorem 1.8(iii). □

For the proof of Theorem 1.8(iv) we need the following. Let G be a group, X a set, ψ an action of G on X, and $x \in X$. We denote by

$$G_x := \mathrm{Stab}_x^\psi := \{g \in G \,|\, \psi_g(x) = x\}$$

the stabilizer of x under ψ.

2.7. *Remark.* Let G be a Lie group, (ρ, H) an object of $\mathrm{SympRep}_{\leq G}$, and $y = [a, a\varphi, v] \in Y_\rho$. Then

$$G_y = \{c_a(h) \,|\, h \in H : \rho_h v = v\}.$$

2.8. **Lemma.** *Let G be a topological (finite-dimensional) manifold with a continuous group structure, N, N' compact submanifolds of G, and $g \in G$, such that*

$$c_g(N) \subseteq N', \tag{2.9}$$

and N' is conjugate to some subset of N. Then we have

$$c_g(N) = N'.$$

In the proof of this lemma we will use the following.

2.10. *Remark* (invariance of domain). Let M and N be topological manifolds of the same finite dimension, without boundary. Then every continuous injective map from M to N is open. In the case $M = N = \mathbb{R}^n$ this is the statement of the Invariance of Domain Theorem, see [Hat02, Theorem 2B.3, p. 172]. The general situation can be reduced to this case.

Proof of Lemma 2.8. We choose $g' \in G$, such that

$$c_{g'}(N') \subseteq N, \tag{2.11}$$

and define $\psi := c_{g'g} : G \to G$. We have

$$\psi(N) = c_{g'} \circ c_g(N) \subseteq N.$$

Let A be a connected component of N. Since N is a submanifold of G, the set A is open in N. The map ψ is bijective and continuous. Hence by Remark 2.10 the restriction $\psi : N \to N$ is open. Thus $\psi(A)$ is open in N.

Since A is a connected component of N, it is closed in N. Since N is compact, it follows that A is compact. Therefore, $\psi(A)$ is compact and hence a closed subset of N. It follows that $\psi(A)$ is a connected component of N. Hence the map

$$\{\text{connected component of } N\} \ni A \mapsto \psi(A) \in \{\text{connected component of } N\} \tag{2.12}$$

is well-defined. This map is injective. Since N is compact, the number of its connected components is finite. It follows that the map (2.12) is surjective. It follows that $N \subseteq \psi(N)$, and therefore, $c_{g'}^{-1}(N) \subseteq c_g(N)$. By (2.11) we have $N' \subseteq c_{g'}^{-1}(N)$. It follows that $N' \subseteq c_g(N)$. Combining this with (2.9), it follows that $c_g(N) = N'$. This proves Lemma 2.8. □

Let G be a Lie group, (M, ω, ψ) a symplectic G-action, and $x \in M$.

Remark. The isotropy representation of ψ at x is by definition the map

$$\rho^{\psi,x} : \mathrm{Stab}^\psi_x \times T_xM \to T_xM, \quad (g,v) \mapsto d\psi_g(x)v.$$

This is a symplectic representation of the isotropy group Stab^ψ_x.

In order to define the symplectic quotient representation of ψ at x, we need the following remarks.

2.13. *Remarks* (symplectic quotient representation). (i) Let G be a Lie group, (M,ψ) a G-action on a manifold, and $x \in M$. We denote by $\mathfrak{g}$ the Lie algebra of G and by

(2.14) $$L_x := L^\psi_x : \mathfrak{g} \to T_xM$$

the infinitesimal action at x. The equality

$$d\psi_g(x)(\mathrm{im}L_x) = \mathrm{im}L_{\psi_g(x)}$$

holds.

(ii) Let (V,σ) be a symplectic vector space and $W \subseteq V$ a linear space. We denote by

$$W^\sigma := \big\{ v \in V \,\big|\, \sigma(v,w) = 0,\, \forall w \in W \big\}$$

the symplectic complement of W. Let (M,ω,ψ) be a symplectic G-action and $x \in M$. The form ω_x induces a linear symplectic form $\overline{\omega}_x$ on the quotient space

(2.15) $$V^\psi_x := (\mathrm{im}L_x)^{\omega_x} / \big(\mathrm{im}L_x \cap (\mathrm{im}L_x)^{\omega_x}\big).$$

It follows from (i) that $d\psi_g(x)\left((\mathrm{im}L_x)^{\omega_x}\right) = (\mathrm{im}L_{\psi_g(x)})^{\omega_{\psi_g(x)}}$. Therefore, using (i) again, $d\psi_g(x)$ induces a map

(2.16) $$V^\psi_x \to V^\psi_{\psi_g(x)}.$$

This map is a linear symplectic isomorphism w.r.t. $\overline{\omega}_x$ and $\overline{\omega}_{\psi_g(x)}$.

We define the *symplectic quotient representation of* ψ *at* x to be the map

(2.17) $$\overline{\rho}^{\psi,x} : \mathrm{Stab}^\psi_x \times V^\psi_x \to V^\psi_x,$$

where $\overline{\rho}^{\psi,x}(g,\cdot)$ is given by the map (2.16). By Remark 2.13(ii) this is a well-defined symplectic representation of Stab^ψ_x on the linear symplectic quotient V^ψ_x of $(\mathrm{im}L_x)^{\omega_x}$.[5]

2.18. *Remarks* (equivariant symplectomorphism, symplectic quotient representations). Let G be a Lie group, $(M,\omega,\psi), (M',\omega',\psi')$ symplectic G-actions, $\Phi : M \to M'$ a G-equivariant symplectomorphism, $x \in M$, and $x' := \Phi(x)$.

(i) Since Φ is G-equivariant and injective, we have

$$\mathrm{Stab}^\psi_x = \mathrm{Stab}^{\psi'}_{x'}.$$

Furthermore, we have $d\Phi(x)L^\psi_x = L^{\psi'}_{x'}$, and therefore, $d\Phi(x)\big(\mathrm{im}L^\psi_x\big) = \mathrm{im}L^{\psi'}_{x'}$. Since Φ is symplectic, it follows that $d\Phi(x)$ induces a map

$$V^\psi_x \to V^{\psi'}_{x'}.$$

This map is a linear symplectic isomorphism that intertwines $\overline{\rho}^{\psi,x}$ and $\overline{\rho}^{\psi',x'}$.

(ii) If ψ' is Hamiltonian with momentum map μ' then $\mu' \circ \Phi$ is a momentum map for ψ.

[5]In the literature $\overline{\rho}^{\psi,x}$ is called "symplectic slice representation". This terminology seems misleading, since $\overline{\rho}^{\psi,x}$ does not involve any choice of a local slice.

2.19. **Lemma** (symplectic quotient representation for model action)**.** *Let G be a compact Lie group and (H,V,σ,ρ) an object of* $\mathrm{SympRep}_{\leq G}$*. We denote*

$$\big(Y_\rho,\omega_\rho,\psi_\rho\big) := \mathrm{Model}_G(H,\rho).$$

Let $y\in Y_\rho$ be a point for which $\mu_\rho(y)$ is central and $\mathrm{Stab}_y^{\psi_\rho}=c_a(H)$*, for some representative $(a,a\varphi,v)$ of y. Then $\rho=(H,\rho)$ is isomorphic to $\overline{\rho}^{\psi_\rho,y}$.*

Remark. The subgroup $c_a(H)$ does not depend on the choice of the representative $(a,a\varphi,v)$ of y.

In the proof of this lemma we will use the following.

2.20. *Remark* (momentum map)*.* Let (M,ω,ψ) be a Hamiltonian G-action, μ a momentum map for ψ, and $x\in M$. Then

$$\ker d\mu(x) = (\mathrm{im} L_x^\psi)^{\omega_x}.$$

Proof of Lemma 2.19. We choose a representative $\widetilde{y}:=(a,a\varphi,v)$ of y. We define

$$\iota_{a,\varphi}: V\to T^*G\times V,\quad \iota_{a,\varphi}(w):=(a,a\varphi,w).$$

2.21. **Claim.**

$$\mathrm{im}\big(d\iota_{a,\varphi}(v)\big)\subseteq \ker d\mu_\rho^D(\widetilde{y}). \tag{2.22}$$

Proof of Claim 2.21. By our hypothesis $\mu_\rho(y)=\mu^L(\widetilde{y})=\mathrm{Ad}^*(a)\varphi$ is a central element of $\mathfrak{g}^*$. Hence, for every $g\in G$, we have

$$\mathrm{Ad}^*(a)\varphi = \mathrm{Ad}^*(c_a(g))\,\mathrm{Ad}^*(a)\varphi = \mathrm{Ad}^*(a)\,\mathrm{Ad}^*(g)\varphi,$$

and therefore $\varphi=\mathrm{Ad}^*(g)\varphi$. Hence φ is a central element of $\mathfrak{g}^*$. For every $h\in H$, we have

$$\begin{aligned}
[a,a\varphi,v] &= y\\
&= \psi_\rho(c_a(h),y) \qquad (\text{since } \mathrm{Stab}_y^{\psi_\rho}=c_a(H))\\
&= \big[c_a(h)a, c_a(h)a\varphi, v\big]\\
&= \big[ah, a\varphi h, v\big] \qquad (\text{using that } \varphi \text{ is central})\\
&= \big[a,a\varphi,\rho_h v\big],
\end{aligned}$$

and therefore $\rho_h v = v$. Hence v is a fixed point of ρ. It follows that $d\nu_\rho(v)=0$. Since $\mu_\rho^D(a,a\varphi,w)=-\varphi|\mathfrak{h}+\nu_\rho(w)$, it follows that

$$d(\mu_\rho^D\circ\iota_{a,\varphi})(v)=d\nu_\rho(v)=0.$$

The inclusion (2.22) follows. This proves Claim 2.21. □

We define π_ρ as in (2.5), and

$$A_{\widetilde{y}}^\rho := d\pi_\rho(\widetilde{y})d\iota_{a,\varphi}(v): V\to T_yY_\rho.$$

By Claim 2.21 this map is well-defined.

2.23. **Claim.** *The pair $\big(a,A_{\widetilde{y}}^\rho\big)$ is a morphism from ρ to $\rho^{\psi_\rho,y}$ (the isotropy representation of ψ_ρ at y).*

Proof of Claim 2.23. The map $\iota_{a,\varphi}$ is a symplectic embedding. It follows that $A^\rho_{\widetilde{y}}$ is linear symplectic. We denote by $\psi^L : G \times T^*G \times V \to T^*G \times V$ the action induced by the left-translation on G. Let $h \in H$. For all $w \in V$, we have

$$\begin{aligned}\iota_{a,\varphi} \circ \rho_h(w) &= \big(a, a\varphi, \rho_h w\big)\\ &= \big(ah, a\varphi h, w\big)\\ &= (\psi^L_\rho)_{c_a(h)} \circ \iota_{a,\varphi}(w) \qquad \text{(using that } \varphi \text{ is central)}\end{aligned}$$

Using that ρ_h is linear, it follows that

$$\begin{aligned}d\iota_{a,\varphi}(v)\rho_h &= d\iota_{a,\varphi}(v)d\rho_h(v)\\ &= d\big((\psi^L_\rho)_{c_a(h)}\big)(\widetilde{y})d\iota_{a,\varphi}(v).\end{aligned}$$

Since $\pi_\rho \circ (\psi^L_\rho)_g = (\psi_\rho)_g \circ \pi_\rho$, it follows that

$$\begin{aligned}A^\rho_{\widetilde{y}}\rho_h &= d\pi_\rho(\widetilde{y})d\iota_{a,\varphi}(v)\rho_h\\ &= d(\psi_\rho)_{c_a(h)}(y)d\pi_\rho(\widetilde{y})d\iota_{a,\varphi}(v)\\ &= d(\psi_\rho)_{c_a(h)}(y)A^\rho_{\widetilde{y}}.\end{aligned}$$

The statement of Claim 2.23 follows. □

Let $y \in Y_\rho$. Recall that

$$L_y = L^{\psi_\rho}_y : \mathfrak{g} \to T_yY_\rho$$

denotes the infinitesimal ψ_ρ-action.

2.24. **Claim.**

(2.25) $$\mathrm{im}L_y \textit{ is isotropic,}$$

(2.26) $$\mathrm{im}A^\rho_{\widetilde{y}} \subseteq \big(\mathrm{im}L_y\big)^{(\omega_\rho)_y}.$$

Proof of Claim 2.24. (2.25): Our hypothesis that $\mu_\rho(y)$ is central implies that

$$\mathrm{im}L_y \subseteq \ker d\mu_\rho(y).$$

By Remark 2.20 we have

(2.27) $$\ker d\mu_\rho(y) = (\mathrm{im}L_y)^{(\omega_\rho)_y}.$$

Statement (2.25) follows.

(2.26): Since $\mu_\rho \circ \pi_\rho = \mu^L \circ \mathrm{pr}_1$, we have

$$\begin{aligned}d\mu_\rho(y)A^\rho_{\widetilde{y}} &= d\mu_\rho(y)d\pi_\rho(\widetilde{y})d\iota_{a,\varphi}(v)\\ &= d\big(\mu_\rho \circ \pi_\rho \circ \iota_{a,\varphi}\big)(v)\\ &= d\big(\mu^L \circ \mathrm{pr}_1 \circ \iota_{a,\varphi}\big)(v)\\ &= 0.\end{aligned}$$

Here in the last step we used that the map $\mathrm{pr}_1 \circ \iota_{a,\varphi}$ is constantly equal to $(a, a\varphi)$. It follows that

$$\mathrm{im}A^\rho_{\widetilde{y}} \subseteq \ker d\mu_\rho(y).$$

Using (2.27), the claimed inclusion (2.26) follows. This completes the proof of Claim 2.24. □

By part (2.25) of this claim there is a canonical projection

$$\mathrm{pr}^\rho_y : \big(\mathrm{im} L_y\big)^{(\omega_\rho)_y} \to V^{\psi_\rho}_y = \big(\mathrm{im} L_y\big)^{(\omega_\rho)_y} / \mathrm{im} L_y.$$

By part (2.26) the restriction

$$\mathrm{pr}^\rho_y \left| \mathrm{im} A^\rho_{\widetilde{y}} \right.$$

is well-defined. It follows from Claim 2.23 and the equality $\mathrm{Stab}^{\psi_\rho}_y = c_a(H)$ that $\mathrm{im} A^\rho_{\widetilde{y}}$ is invariant under $\rho^{\psi_\rho, y}$.

2.28. **Claim.** *The pair* $\left(e, \mathrm{pr}^\rho_y \left| \mathrm{im} A^\rho_{\widetilde{y}}\right.\right)$ *is an isomorphism between the restriction of* $\rho^{\psi_\rho, y}$ *to* $\mathrm{im} A^\rho_{\widetilde{y}}$ *and* $\overline{\rho}^{\psi_\rho, y}$*.*

Proof of Claim 2.28. The projection pr^ρ_y is presymplectic. Since $\mathrm{im} A^\rho_{\widetilde{y}}$ is symplectic, the restriction $\mathrm{pr}^\rho_y \left| \mathrm{im} A^\rho_{\widetilde{y}}\right.$ is linear symplectic and therefore injective. We have

$$\begin{aligned}
\dim\left(V^{\psi_\rho}_y = \big(\mathrm{im} L_y\big)^{(\omega_\rho)_y} / \mathrm{im} L_y\right) &= \dim(Y_\rho) - 2 \dim \mathrm{im} L_y \\
&= \dim(T^*G \times V) - 2 \dim H - 2 \dim \mathrm{im} L_y \\
&= 2 \dim G + \dim V - 2 \dim H - 2 \dim G + 2 \dim \mathrm{Stab}^{\psi_\rho}_y \\
&= \dim V \qquad (\text{since } \mathrm{Stab}^{\psi_\rho}_y = c_a(H)) \\
&= \dim \mathrm{im} A^\rho_{\widetilde{y}} \qquad (\text{since } A^\rho_{\widetilde{y}} \text{ is linear symplectic, hence injective}) \\
&= \dim\left(\mathrm{pr}^\rho_y \left(\mathrm{im} A^\rho_{\widetilde{y}}\right)\right) \qquad (\text{since } \mathrm{pr}^\rho_y \left| \mathrm{im} A^\rho_{\widetilde{y}}\right. \text{ is injective}).
\end{aligned}$$

It follows that $V^{\psi_\rho}_y = \mathrm{pr}^\rho_y \left(\mathrm{im} A^\rho_{\widetilde{y}}\right)$, hence $\mathrm{pr}^\rho_y \left| \mathrm{im} A^\rho_{\widetilde{y}}\right.$ is surjective. Hence this map is a linear symplectic isomorphism. It is $\mathrm{Stab}^{\psi_\rho}_y$-equivariant. The statement of Claim 2.28 follows. ☐

It follows from Claims 2.23 and 2.28 that ρ and $\overline{\rho}^{\psi_\rho, y}$ are isomorphic. This proves Lemma 2.19. ☐

Proof of Theorem 1.8(iv). Let (H, ρ) and (H', ρ') be two objects of $\mathrm{SympRep}_{\leq G}$ whose images under Model_G are isomorphic. We choose an isomorphism Φ between these images. We define

$$y := [e, 0, 0] \in Y_\rho, \quad [a', a'\varphi', v'] := y' := \Phi(y).$$

By Remark 2.7 we have

$$\mathrm{Stab}^{\psi_\rho}_y = H, \tag{2.29}$$

$$\mathrm{Stab}^{\psi_{\rho'}}_{y'} \subseteq c_{a'}(H').$$

Since Φ is G-equivariant, we have

$$\mathrm{Stab}^{\psi_\rho}_y = \mathrm{Stab}^{\psi_{\rho'}}_{y'}. \tag{2.30}$$

It follows that $H \subseteq c_{a'}(H')$. By considering Φ^{-1}, an analogous argument shows that H' is conjugate to a subgroup of H. Since G is compact and H and H' are closed, these subgroups are compact. Therefore, applying Lemma 2.8, it follows that

$$H = c_{a'}(H'). \tag{2.31}$$

Since $\mu_\rho(y) = \mu_\rho^L(e,0,0) = 0$ and $\mathrm{Stab}_y^{\psi_\rho} = H$, the hypotheses of Lemma 2.19 are satisfied. Applying this lemma, it follows that ρ is isomorphic to $\overline{\rho}^{\psi_\rho,y}$.

By Remark 2.18(i) $\overline{\rho}^{\psi_\rho,y}$ is isomorphic to $\overline{\rho}^{\psi_{\rho'},y'}$.

2.32. **Claim.** *$\overline{\rho}^{\psi_{\rho'},y'}$ is isomorphic to ρ'.*

Proof of Claim 2.32. By (2.30,2.29,2.31) we have $\mathrm{Stab}_{y'}^{\psi_{\rho'}} = c_{a'}(H')$. By Remark 2.18(ii) the map $\mu_{\rho'} \circ \Phi$ is a momentum map for ψ_ρ. Since G is connected, the same holds for Y_ρ. It follows that $\mu_{\rho'} \circ \Phi - \mu_\rho$ is constantly equal to a central element of $\mathfrak{g}^*$. At y this map attains the value

$$\mu_{\rho'}(y') - \mu_\rho(y) = \mu_{\rho'}(y') - 0,$$

which is thus a central element of $\mathfrak{g}^*$. Hence the hypotheses of Lemma 2.19 are satisfied. Applying this lemma, the statement of Claim 2.32 follows. □

Combining this claim with what we already showed, it follows that ρ is isomorphic to ρ'.

Hence Model_G induces an injective map between the sets of isomorphism classes. This proves Theorem 1.8(iv). □

Proof of Theorem 1.8(v,vi,vii). (v) follows from a straight-forward argument.

(vi): Let ρ and ρ' be objects of $\mathrm{SympRep}_{\leq G}$, such that ρ' is momentum proper and there exists a morphism (g,T) from ρ to ρ'. Let $Q \subseteq \mathfrak{h}$ be compact. Equality (2.3) implies that

$$\nu_\rho^{-1}(Q) = (\nu_{\rho'} \circ T)^{-1}\left(\mathrm{Ad}^*(g)(Q)\right). \tag{2.33}$$

The set $\mathrm{Ad}^*(g)(Q)$ is compact. By hypothesis $\nu_{\rho'}$ is proper, and by Claim 2.4 the same holds for T. It follows that $\nu_{\rho'} \circ T$ is proper, and therefore, using (2.33), the set $\nu_\rho^{-1}(Q)$ is compact. Hence ν_ρ is proper, i.e., ρ is momentum proper, as claimed.

Let now (M,ω,ψ) and (M',ω',ψ') be objects of $\mathrm{Ham}_G^{\mathrm{ex}}$, such that ψ' is momentum proper and there exists a morphism Φ from ψ to ψ'. We choose a momentum map μ' for ψ'. By definition, Φ is a proper G-equivariant symplectic embedding. It follows that $\mu' \circ \Phi$ is a proper momentum map for ψ. Hence ψ is momentum proper. This proves (vi).

(vii): We prove "$\Rightarrow$": Assume that (H,ρ) is momentum proper, i.e., that ν_ρ is proper. Let $K \subseteq \mathfrak{g}^*$ be compact. We denote by $i : H \to G$ the inclusion and by $i^* : \mathfrak{g}^* \to \mathfrak{h}^*$ the induced map. We define

$$A := \big\{(a, a\varphi, v) \in T^*G \times V \,\big|\, \mathrm{Ad}^*(a)(\varphi) \in K,\ i^*\varphi = \nu_\rho(v)\big\} \subseteq (\mu_\rho^D)^{-1}(0). \tag{2.34}$$

We denote by $L_* : G \times T^*G \to T^*G$ the map induced by left translation. A is a closed subset of

$$B := L_*\big(G \times \mathrm{Ad}^*(G)(K)\big) \times \nu_\rho^{-1}\big(i^*\,\mathrm{Ad}^*(G)(K)\big).$$

Since G is compact and Ad^* is continuous, the set $\mathrm{Ad}^*(G)(K)$ is compact. Since i^* is continuous and ν_ρ is proper, it follows that $\nu_\rho^{-1}\big(i^*\,\mathrm{Ad}^*(G)(K)\big)$ is compact. Using that L_* is continuous, it follows that B is compact. It follows that A is compact, and hence $\mu_\rho^{-1}(K) = \pi_\rho(A)$ is compact. Hence μ_ρ is proper. This proves "$\Rightarrow$".

"$\Leftarrow$": Assume that μ_ρ is proper. Let $Q \subseteq \mathfrak{h}^*$ be compact. We choose a compact set $K \subseteq \mathfrak{g}^*$ such that $i^*(K) = Q$. (We may e.g. choose a linear complement $W \subseteq \mathfrak{g}^*$ of $\ker i^*$ and define

$K := (i^*)^{-1}(Q) \cap W$.) Since H is compact, the map $\pi_\rho : (\mu_\rho^D)^{-1}(0) \to Y_\rho$ is proper. It follows that $\mu_\rho \circ \pi_\rho$ is proper. Hence the set

$$\big(\mu_\rho \circ \pi_\rho\big)^{-1}(K)$$

is compact. This set agrees with A, defined as in (2.34). We denote by $\mathrm{pr}_2 : T^*G \times V \to V$ the canonical projection. The set

$$C := \big\{(e, \varphi, v) \,\big|\, \varphi \in K,\, i^*\varphi = \nu_\rho(v)\big\}$$

is a closed subset of A, hence compact. It follows that $\mathrm{pr}_2(C)$ is compact. Since $i^*(K) = Q$, we have $\mathrm{pr}_2(C) = \nu_\rho^{-1}(Q)$. It follows that ν_ρ is proper. This proves "$\Leftarrow$", and completes the proof of (vii) and therefore of Theorem 1.8 (except for (viii), which is proved in [KZ18, 1.5. Theorem]). □

3. Proof of Corollary 1.14 (classification of critical Hamiltonian actions)

The well-definedness part of Corollary 1.14 follows from the next lemma.

3.1. **Lemma.** *(i) The functor T^*_G maps objects of $\mathrm{Act}_G^{\mathrm{trans}}$ to objects of $\mathrm{Ham}_G^{\mathrm{crit}}$.*
(ii) It maps morphisms of $\mathrm{Act}_G^{\mathrm{trans}}$ to morphisms of $\mathrm{Ham}_G^{\mathrm{crit}}$.

Proof of Lemma 3.1. **(i):** Let (Q, θ) be an object of $\mathrm{Act}_G^{\mathrm{trans}}$. Then $T^*_G(Q, \theta)$ is an object of $\mathrm{Ham}_G^{\mathrm{ex}}$. To see that is an object of $\mathrm{Ham}_G^{\mathrm{ex,prop}}$, we denote by $\mathfrak{g}$ the Lie algebra of G. We choose a Finsler norm $\|\cdot\|$ on T^*Q and a norm $|\cdot|$ on $\mathfrak{g}^*$. We define L_q^θ as in (2.14) and

$$\mu : T^*Q \to \mathfrak{g}^*, \quad \mu(q, p) := pL_q^\theta. \tag{3.2}$$

This is a momentum map for the lifted G-action θ_*. Since θ is transitive, the map L_q^θ is surjective. Therefore, an elementary argument using (3.2) and that Q is compact, shows that

$$\sup\big\{\|p\| \,\big|\, (q,p) \in T^*Q :\, |\mu(q,p)| \le C\big\} < \infty, \quad \forall C \in \mathbb{R}.$$

It follows that μ is proper. Therefore, $T^*_G(Q, \theta)$ is an object of $\mathrm{Ham}_G^{\mathrm{ex,prop}}$. Since Q is closed and T^*Q deformation retracts onto Q, it follows that $T^*_G(Q, \theta)$ is an object of $\mathrm{Ham}_G^{\mathrm{crit}}$. This proves (i).

(ii): Let $f : Q \to Q'$ be a morphisms of $\mathrm{Act}_G^{\mathrm{trans}}$, i.e., a G-equivariant diffeomorphism. The induced map $f_* : T^*Q \to T^*Q'$ is a G-equivariant symplectomorphism, hence a morphism of $\mathrm{Ham}_G^{\mathrm{ex}}$, and therefore of $\mathrm{Ham}_G^{\mathrm{crit}}$. This proves (ii) and therefore Lemma 3.1. □

By Lemma 3.1 the restriction

$$T^*_G : \mathrm{Act}_G^{\mathrm{trans}} \to \mathrm{Ham}_G^{\mathrm{crit}}$$

is well-defined. The Chain Rule implies that it is functorial. In order to show that the map (1.15) is a bijection, we need the following lemma. We define $\mathrm{Sub}_G^{\mathrm{cl}}$ to be the category whose objects are the closed subgroups of G and whose morphisms between H and H' are those elements g of G, satisfying $c_g(H) = H'$.[6] We define the functor

$$G/ : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Act}_G^{\mathrm{trans}}$$

[6] The composition of morphisms is given by the composition in G.

as follows. It maps an object H to the quotient G/H, equipped with the canonical left G-action. Let (H, H', g) be a morphism of $\mathrm{Sub}_G^{\mathrm{cl}}$. We denote by $\mathrm{pr}_H : G \to G/H$ the canonical projection and define $G/(H, H', g) : G/H \to G/H'$ to be the unique map satisfying

$$G/(H, H', g) \circ \mathrm{pr}_H = \mathrm{pr}_{H'} \circ R^{g^{-1}}. \tag{3.3}$$

This is a well-defined morphism of $\mathrm{Act}_G^{\mathrm{trans}}$. This construction is functorial. This defines the functor $G/$.

We define the functor

$$i_G : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{SympRep}_{\leq G}, \quad i_G(H) := \big(H, \{0\}, 0, 0\big),\ i_G(g) := (g, 0).$$

3.4. **Lemma.** *The target-restricted functor* $\mathrm{Model}_G \circ i_G : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Ham}_G^{\mathrm{crit}}$ *is well-defined and naturally isomorphic to* $T_G^* \circ G/ : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Ham}_G^{\mathrm{crit}}$.

Proof of Lemma 3.4. By Corollary 1.11 the functor $\mathrm{Model}_G \circ i_G$ takes values in $\mathrm{Ham}_G^{\mathrm{ex,prop}}$. Let H be an object of $\mathrm{Sub}_G^{\mathrm{cl}}$. The manifold part of $\mathrm{Model}_G \circ i_G(H)$ is homotopy equivalent to the closed manifold G/H and has dimension $2(\dim G - \dim H)$. Therefore, $\mathrm{Model}_G \circ i_G(H)$ is critical. Hence $\mathrm{Model}_G \circ i_G$ takes values in $\mathrm{Ham}_G^{\mathrm{crit}}$, as claimed.

Let $H \in \mathrm{Sub}_G^{\mathrm{cl}}$. We define $\mu_{H,\rho}^D, Y_{H,\rho}$ as in (1.5,1.6) and denote by $\pi_{H,\rho} : (\mu_{H,\rho}^D)^{-1}(0) \to Y_{H,\rho}$ the canonical projection. We canonically identify $Y_{H,0}$ with the symplectic quotient of the Hamiltonian H-action on T^*G induced by the right H-action on G. We define the map

$$\Phi_H : T^*(G/H) \to Y_{H,0}, \quad \Phi_H(\overline{q}, \overline{p}) := \pi_{H,0}\big(q, \overline{p} d\, \mathrm{pr}_H(q)\big), \tag{3.5}$$

where $q \in \overline{q}$ is an arbitrary representative. This map is a symplectomorphism, see [AM78, 4.3.3 Theorem]. The map Φ_H is G-equivariant, and therefore an isomorphism of $\mathrm{Ham}_G^{\mathrm{crit}}$.

3.6. **Claim.** *The map* $H \mapsto \Phi_H$ *is a natural isomorphism between the functors* $T_G^* \circ G/$ *and* $\mathrm{Model}_G \circ i_G$.

Proof of Claim 3.6: Let (H, H', g) be a morphism of $\mathrm{Sub}_G^{\mathrm{cl}}$. We show that

$$\Phi_{H'} \circ \big((T_G^* \circ G/)(H, H', g)\big) = \big(\mathrm{Model}_G \circ i_G\big)(H, H', g) \circ \Phi_H. \tag{3.7}$$

Let $(\overline{q}, \overline{p}) \in T^*(G/H)$. We choose a representative $q \in \overline{q}$ and denote $q' := R^{g^{-1}}(q)$, $\overline{q}' := \mathrm{pr}_{H'}(q')$, and $\overline{\varphi} := G/(H, H', g)$. We have

$$\begin{aligned} (T_G^* \circ G/)(H, H', g)(\overline{q}, \overline{p}) &= \big(\overline{\varphi}(\overline{q}), \overline{p} d\overline{\varphi}(\overline{q})^{-1}\big), \\ \overline{\varphi}(\overline{q}) &= \overline{q}' \qquad \text{(using (3.3))}, \end{aligned}$$

and therefore,

$$\begin{aligned} \Phi_{H'} \circ \big((T_G^* \circ G/)(H, H', g)\big)(\overline{q}, \overline{p}) &= \pi_{H',0}\big(q', \overline{p} d\overline{\varphi}(\overline{q})^{-1} d\, \mathrm{pr}_{H'}(q')\big) \qquad \text{(using (3.5))} \\ &= \pi_{H',0}\big(q', \overline{p} d\, \mathrm{pr}_H(q) dR^{g^{-1}}(q)^{-1}\big)\ \text{(using (3.3), Chain Rule)} \\ &= \mathrm{Model}_G(g, 0) \circ \pi_{H,0}\big(q, \overline{p} d\, \mathrm{pr}_H(q)\big) \qquad \text{(using (1.7))} \\ &= \big(\mathrm{Model}_G \circ i_G\big)(H, H', g) \circ \Phi_H(\overline{q}, \overline{p}) \qquad \text{(using (3.5))}. \end{aligned}$$

Hence equality (3.7) holds. This proves Claim 3.6 and therefore Lemma 3.4. □

Proof of Corollary 1.14. We show that the functor $\mathrm{Model}_G \circ i_G$ is essentially bijective. By Corollary 1.11 the inverse of the map (1.12) is well-defined. The image of the set of isomorphism classes of $\mathrm{Ham}_G^{\mathrm{crit}}$ under this inverse map is contained in the image of the map between isomorphism classes induced by i_G. This follows from the fact that the manifold part of $\mathrm{Model}_G\big(H, V, \sigma, \rho\big)$ is homotopy equivalent to the closed manifold G/H and has dimension $2(\dim G - \dim H) + \dim V$. It follows that $\mathrm{Model}_G \circ i_G : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Ham}_G^{\mathrm{crit}}$ is essentially surjective, i.e., it induces a surjective map between the sets of isomorphism classes.

Since i_G is essentially injective, Corollary 1.11 implies that the functor $\mathrm{Model}_G \circ i_G : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Ham}_G^{\mathrm{crit}}$ is also essentially injective, and therefore essentially bijective, as claimed.

Using Lemma 3.4, it follows that $T_G^* \circ G/ : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Ham}_G^{\mathrm{crit}}$ is essentially bijective. Therefore, $T_G^* : \mathrm{Act}_G^{\mathrm{trans}} \to \mathrm{Ham}_G^{\mathrm{crit}}$ is essentially surjective. The functor $G/ : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Act}_G^{\mathrm{trans}}$ is essentially surjective. This follows from the orbit-stabilizer theorem for G-actions on manifolds. Since $T_G^* \circ G/ : \mathrm{Sub}_G^{\mathrm{cl}} \to \mathrm{Ham}_G^{\mathrm{crit}}$ is essentially injective, it follows that $T_G^* : \mathrm{Act}_G^{\mathrm{trans}} \to \mathrm{Ham}_G^{\mathrm{crit}}$ is essentially injective, and therefore essentially bijective. This means that the map (1.15) is bijective. This proves Corollary 1.14. □

4. Inverse of the classifying map

The next result states that the inverse of the classifying map (1.12) is induced by assigning to a Hamiltonian action its symplectic quotient representation at any suitable point. To state the result, let G be a group, X a set, ψ a G-action on X, and $x \in X$. Recall that Stab_x^ψ denotes the stabilizer of ψ at x. We call x *ψ-maximal* iff for every $y \in X$, Stab_x^ψ contains some conjugate of Stab_y^ψ.

Let G be a compact and connected Lie group, (M, ω) a symplectic manifold, ψ a symplectic G-action on M, and $x \in M$. Recall that $\overline{\rho}^{\psi,x}$ denotes the symplectic quotient representation of ψ at x, see (2.17). The latter is a symplectic representation of Stab_x^ψ. Hence the pair $\big(\mathrm{Stab}_x^\psi, \overline{\rho}^{\psi,x}\big)$ is an object of $\mathrm{SympRep}_{\leq G}$.

Assume now that ψ is Hamiltonian. We call x *ψ-central* iff $\mu(x)$ is a central value of $\mathfrak{g}^*$ for every momentum map μ for ψ. (If M is connected then equivalently, *there exists* such a μ.)

4.1. **Proposition.** *Assume that ψ is an object of $\mathrm{Ham}_G^{\mathrm{ex,prop}}$.*

(i) There exists a ψ-maximal and -central point.

(ii) Let ψ and ψ' be isomorphic objects of $\mathrm{Ham}_G^{\mathrm{ex,prop}}$, x be ψ-maximal and -central point, and x' be a ψ'-maximal and -central point. Then $\overline{\rho}^{\psi,x}$ and $\overline{\rho}^{\psi',x'}$ are isomorphic.

(iii) The inverse map of (1.12) is given by

$$\big\{\textit{isomorphism class of } \mathrm{Ham}_G^{\mathrm{ex,prop}}\big\} \to \big\{\textit{isomorphism class of } \mathrm{SympRep}_{\leq G}^{\mathrm{prop}}\big\},$$
$$\Psi \mapsto [\overline{\rho}^{\psi,x}], \tag{4.2}$$

where ψ is an arbitrary representative of Ψ, and x is an arbitrary ψ-maximal and -central point.

Remark. It follows from (i,ii) that the map (4.2) is well-defined.

In the proof of Proposition 4.1 we will use the following.

4.3. *Remark.* Let ρ be an object of $\mathrm{SympRep}_{\leq G}$ and $y \in Y_\rho$ a ψ_ρ-maximal and -central point. Then ρ and $\overline{\rho}^{\psi_\rho,y}$ are isomorphic. To see this, we write $y =: [a, a\varphi, v]$. By Remark 2.7 we have

$\mathrm{Stab}_y^{\psi_\rho} \subseteq c_a(H)$. Since y is ψ_ρ-maximal, $\mathrm{Stab}_y^{\psi_\rho}$ contains some conjugate of $\mathrm{Stab}_{[e,0,0]}^{\psi_\rho} = H$. Using Lemma 2.8, it follows that

$$\mathrm{Stab}_y^{\psi_\rho} = c_a(H).$$

Using that y is ψ_ρ-central, the hypotheses of Lemma 2.19 are therefore satisfied. Applying this lemma, it follows that ρ and $\overline{\rho}^{\psi_\rho, y}$ are isomorphic, as claimed.

Proof of Proposition 4.1. (i): Consider first the **case** in which there exists $\rho \in \mathrm{SympRep}_{\leq G}^{\mathrm{prop}}$, such that $\psi = \mathrm{Model}_G(\rho)$. By Remark 2.7 the point $[e, 0, 0]$ is ψ-maximal. Since $\mu_\rho([e, 0, 0]) = 0$, this point is also ψ-central. This proves the statement in the special case.

The general situation can be reduced to this case, by using Theorem 1.8(viii) (essential surjectivity), the fact that stabilizers are preserved under equivariant injections, and Remark 2.18(ii). This proves (i).

(ii): Consider first the **case** in which there exists an isomorphism from ψ to ψ' that maps x to x'. Then it follows from Remark 2.18(i) that $\overline{\rho}^{\psi,x}$ and $\overline{\rho}^{\psi',x'}$ are isomorphic.

In the general situation, using Theorem 1.8(viii) and what we just proved, we may assume w.l.o.g. that $\psi = \psi' = \psi_\rho = \mathrm{Model}_G(H, \rho)$ for some object ρ of $\mathrm{SympRep}_{\leq G}$. By Remark 4.3 we have $\overline{\rho}^{\psi_\rho, x} \cong \rho \cong \overline{\rho}^{\psi_\rho, x'}$. This proves (ii).

(iii): Remark 4.3 implies that (4.2) is a left-inverse for (1.12). Since (1.12) is surjective, it follows that (4.2) is also a right-inverse. This proves (iii) and completes the proof of Proposition 4.1. □

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References

[Abo12] Mohammed Abouzaid, *Framed bordism and Lagrangian embeddings of exotic spheres*, Ann. of Math. (2) **175** (2012), no. 1, 71–185. MR 2874640

[AM78] Ralph Abraham and Jerrold E. Marsden, *Foundations of mechanics*, Benjamin/Cummings Publishing Co., Inc., Advanced Book Program, Reading, Mass., 1978, Second edition, revised and enlarged, With the assistance of Tudor Raţiu and Richard Cushman. MR 515141

[EKS16] Tobias Ekholm, Thomas Kragh, and Ivan Smith, *Lagrangian exotic spheres*, J. Topol. Anal. **8** (2016), no. 3, 375–397. MR 3509566

[ES16] Tobias Ekholm and Ivan Smith, *Exact Lagrangian immersions with a single double point*, J. Amer. Math. Soc. **29** (2016), no. 1, 1–59. MR 3402693

[FPP18] Alessio Figalli, Joseph Palmer, and Álvaro Pelayo, *Symplectic G-capacities and integrable systems*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **18** (2018), no. 1, 65–103. MR 3783784

[Hat02] Allen Hatcher, *Algebraic topology*, Cambridge University Press, Cambridge, 2002. MR 1867354

[KZ18] Yael Karshon and Fabian Ziltener, *Hamiltonian group actions on exact symplectic manifolds with proper momentum maps are standard*, Trans. Amer. Math. Soc. **370** (2018), no. 2, 1409–1428. MR 3729506

[MS17] Dusa McDuff and Dietmar Salamon, *Introduction to symplectic topology*, third ed., Oxford Graduate Texts in Mathematics, Oxford University Press, Oxford, 2017. MR 3674984

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