What history's most overqualified calculus student tells us about liberal arts mathematics

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In the spring of 1672, Gottfried Wilhelm Leibniz arrived in Paris. History remembers him as one of the foremost philosophers and mathematicians of his century. But such fame was still decades away for the baby-faced Leibniz who stepped off the coach in Paris for the first time on that chilly March evening. He had completed a doctorate in law at a remarkably young age and was sent to Paris as a junior diplomat. But Leibniz had greater ambitions in mind than the treaty negotiations he was sent to facilitate.

Paris, at that time, was an exciting place for a voracious scholar and philosopher such as Leibniz, who would later wish that he had 'twenty heads'[1] to pursue all the thinking and study he wanted to do. The Paris intelligentsia, meanwhile, were delighted to have such a passionate conversationalist enliven their dinner parties with his bold and original ideas.

Leibniz felt right at home and would have stayed for life if he could. But times were changing. French politics soured. Foreigners were driven out of the country and the once sparkling Paris intellectual milieu lost a number of its brightest stars. Leibniz had to return to Germany. For the rest of his life, he did his mathematical and scientific work in his spare time when he wasn't too busy with his day job as a courtier at the House of Hanover.

But during those beautiful few years in Paris, Leibniz formed friendships that would last a lifetime. Christiaan Huygens – the world's foremost mathematician and a key member of the Paris Academy of Sciences – had taken Leibniz under his wing. Under the tutelage of this maestro, Leibniz got obsessed with mathematics and was quickly becoming one of the field's most creative minds. His frantic notes from the Paris years show the first fledgling steps of the calculus – mistakes and all – that hundreds of thousands of students retrace in modern classrooms every year.[2]

With the expulsion of Protestants from France, Huygens withdrew to his family mansion in the Netherlands. The building – Hofwijck – is still there. If you take the train from Utrecht to The Hague you get a good look at it on your left-hand side as you approach the city. But Huygens did not retire to feed the ducks in his estate gardens. Though old and frail at this point, he kept up with the latest mathematics. This meant learning the new calculus developed by his former protégé Leibniz. The student had become the master, as the saying goes. But perhaps more intriguingly, the master had become the student.

What a treat of history this is. Reading the correspondence between Huygens and Leibniz during those years, we get to see learning in action. We get to see how the calculus is taught by its inventor, and how a sage mathematician of the highest credentials goes about learning it. We get to see the former director of scientific research at the Academy of Sciences take a seat in the front row of Calculus I, pencils sharpened and notebook in hand. It's a naked view of calculus genesis, unique in history.

Huygens proved a feisty pupil. He was not the kind of student who merely copies out the formulas and asks for help on the homework problems he got stuck on. Even proofs and demonstrations did not impress him much. What he demanded most of all is motivation. He wanted new mathematics to be thoroughly justified, not in the narrow sense of being logically correct, but in the broader sense of being a worthy human enterprise.

A fundamental concept of the calculus is the derivative. It corresponds to the speed at which things are changing, or the steepness of tangent lines. This is tangible enough, and Huygens soon understood it. But the way derivatives are calculated in calculus suggests the possibility of iterating the procedure: to take the derivative of the derivative. Calculating such 'second derivatives' is a natural thing to do from the point of view of the inner logic of the way the calculus deals with formulas and relationships. But has the calculus thereby got lost in a formalistic indulgence – a game with formulas – out of touch with concrete meanings such as speeds and slopes? Or are second derivatives actually good for something?

That is what Huygens wanted to know before bothering to study second derivatives. He writes to Leibniz:

> I still do not understand anything about *ddx* [that is to say, second derivatives], and I would like to know if you have encountered any important problems where they should be used, so that this gives me desire to study them.[3]

Tell me why I would want to study second derivatives, Huygens demands. Not the formal rules for working with them, and the proofs thereof, and artificial problems specifically invented for them. No, not that. Any mathematician can make up such mathematics ad infinitum. A new mathematical theory must prove itself not by solving its own internal problems, but by proving itself on a worthy, honest-to-god problem recognized in advance.

Leibniz understands this well, and replies:

> As for the *ddx*, I have often needed them; they are to the dx, as the conatus to heaviness or the centrifugal solicitations are to the speed. Bernoulli employed them for the curves of sails. And I have used them for the movement of the stars.[4]

In other words, we don't care about second derivatives because the symbolism suggested we could do derivatives once over. We care about them because they are the right way to tackle mathematically a rich range of fascinating and important phenomena. Do you want to understand the shape of a sail bowed by the wind? Do you want to describe how planets move around the sun? Then you want to understand second derivatives.

These examples were agreeable to Huygens. Not because he thought mathematics must be 'applied' – he had done plenty of *l'art pour l'art* pure mathematics himself – but because nature has excellent mathematical taste. Any mathematical theory can show off on technical pseudo-problems specifically designed to be solvable by that method. But it's one thing to chew on teething toys, and another to cut the mustard with the real deal.

There was no burning need to find the equation for the shape of a sail. But this application shows that second derivatives have nature's endorsement, as it were. That's a good letter of recommendation, and one that is not easily forged. Thus a real-world problem is a good acid test of the worth of a new idea. As Huygens explains:

> I have often considered that the curves which nature frequently presents to our view, and which she herself describes, so to speak, all possess very remarkable properties. Such curves merit, in my opinion, that one selects them for study, but not those curves newly made up solely for using the geometrical calculus upon them.[5]

Leibniz agrees: 'You are right, Sir, to not approve if one amuses oneself researching curves invented for pleasure'.[6]

If only modern calculus books lived by the same rule! Flip to the problem section at the end of any chapter in any standard calculus textbook and you will find a thousand problems 'made up solely for using the calculus upon them' – exactly what Huygens condemns. Perhaps it should give us pause for thought when both the inventor of the calculus and its most able student ever are in complete agreement that our way of writing textbooks is stupid.

Modern students may well sympathize with Huygens again when he makes a similar point regarding exponential expressions such as e to the power x:

> I must confess that the nature of that sort of supertranscendental curves, in which the unknowns enter the exponent, seems to me so obscure that I would not think about introducing them into geometry unless you could indicate some notable usefulness of them.[7]

Leibniz shows him how such expressions can solve certain problems, but Huygens is still not impressed: 'I do not see that this expression is a great help for that. I knew the curve already for a long time'.[8] The moral, once again, is: first show me what your technical thing can do, or else I have no reason to study it. And if I can do the same thing by other means then you have still failed.

So Huygens was a critical student who questioned and scrutinized everything. It is in the nature of the liberal arts to embrace and encourage little Huygenses whenever we find them in our classrooms. As another leading mathematician, Felix Klein, later put it:

> Apart from the majority of enthusiastic students there are always a few students who are not entirely satisfied, who criticize and question. These are the ones dearest to my heart. For I see in them what I consider to be the true goal of all teaching: independent thought.[9]

Yet conventional teaching runs the risk of alienating such students by neglecting precisely the kinds of questions that Huygens wanted addressed before deciding whether the new mathematics was worthy of his time. Let us make room in our educational system for the Huygenses of today, who want their passion for learning to stem from inner conviction rather than the passive acceptance of someone else's teaching.

Mathematics has been embraced as a core pillar of a liberal education since antiquity, in no small part due to the spirit of independence embodied by Huygens. In heart and soul, mathematics is still the same creative, free-thinking, curiosity-driven field that the liberal arts fell in love with all those years ago. But it's a marriage on the rocks. In many educational settings, mathematics has grown apart from liberal arts ideals, and not for nothing.

The famous inscription above the gates of Plato's Academy – 'Let no one enter here who is ignorant of geometry' – was no token curricular breadth requirement. Plato wasn't pulling any punches when he set out the central role that mathematics would play in his vision of an ideal republic. Future rulers should study advanced mathematics for ten years, between the ages of twenty and thirty, Plato demanded. No society has yet implemented this ambitious plan. Nevertheless, it is striking that a modern higher education in mathematics, from college through a PhD, corresponds quite closely to Plato's timeline.

Ten adult years is perhaps a reasonable part of a human life to spend on education and apprenticeship. That much has remained the same since Plato's time. But meanwhile the sum total of all mathematical knowledge has grown enormously. What Plato's students had ten years to learn, we would have to master in a single afternoon if we wanted to have equal time to cover all other parts of mathematics.

Yet the desire to somehow cover 'everything' remains. That is also what Plato envisioned. He didn't mean that people should take some maths classes to pick up applicable skills that would help them get ahead in tomorrow's knowledge-based economy. No, a proper mathematical education, according to Plato, encompasses all branches of mathematics and leads to 'a unified vision of their kinship'.[10] But that was easy to say back then, well before the birth of even leading Greek mathematicians such as Archimedes and Euclid, let alone the thousands of years' worth of mathematics contributed by later civilizations.

Such forces have pushed mathematics to be taught in ever more illiberal ways. No subject is as cumulative as mathematics, and the field has had to embrace a top-down, dictatorial modus operandi to keep up with an ever-growing body of material. Instead of dealing with topics A, B, C, the mathematician extracts an abstract core X common to them all and makes this alone mathematics. The efficiency of this approach is undeniable. But what is gained in power is lost in meaning. X grew out of A, B, C, and all its motivation and purpose lies in those roots. Yet those are precisely the ties that must be severed to enjoy the gain in efficiency. Thus, paradoxically, mathematics depends in an essential way on ignoring purpose and specifically avoiding the natural way of arriving at the ideas one is trying to learn.

This is precisely what we saw above with the second derivative. The concept of the second derivative is natural in contexts such as planetary motion, falling bodies, and maximally inflated sails. But it saves a lot of time to ignore the particulars of those contexts and teach only the abstract idea. This is an intellectual betrayal of sorts, since it covers up the actual path that led to these ideas.

Teaching only the abstracted X, only second derivatives for example, is to detach mathematical ideas from the contexts that give them life and meaning. In some ways it is indeed more efficient to study uprooted wildflowers under the fluorescent lights of a lab instead of hiking through mud and rain to see it grow in its natural habitat. Yet, to quote Felix Klein again, the romantics among us still strive to make mathematics education more akin to 'delightful and instructive walks through forests, fields, and gardens [...] without digging up the most profitable plants to replant them in prepared soil according to the principles of rational agronomy'.[11]

For the training of a technocratic workforce, the efficiency gained by putting romance and questioning aside may be a worthwhile bargain. But there should be a place in the world also for those who refuse this pact with the devil, and value their independence of mind higher than any promise of power. History vindicates such rebellious souls and suggests how to teach them. A liberal arts college is the place to keep alive the reflective approach sacrificed due to efficiency demands in other programmes.

Reinstating more organic historical paths to the ideas of mathematics enriches it with meaning, critical reflection, and contextual interconnections. Pursuing these dimensions does not mean departing from core mathematics for the sake of a liberal arts perspective; rather, it means remaining true to the spirit of the very pioneers of the subject, such as Huygens and Leibniz.

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