EMBODIED INSTRUMENTATION: REIFICATION OF SENSORIMOTOR ACTIVITY INTO A MATHEMATICAL ARTIFACT

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The poster proposes a theoretically grounded vision on the position of mathematical artifacts in a learning trajectory. Based on instrumental, radical embodied, and culture-historical approaches, we propose a new design principle for computer-based design sequences: reification of enacted coordinations*. A design study for trigonometry learning is described.*

Keywords: instrumental genesis, embodied design, reification, trigonometry, design research

THEORETICAL FRAMEWORK

New dynamic technology leads to a great increase in the amount of tools designed to enhance students' mathematical reasoning. However, it is still unclear in which cases new tools are capable to actually enhance conceptual understanding. The research aim of this poster is to elaborate on a theoretically grounded vision on the position of a new artifact in a learning trajectory. This vision is inspired by radical embodied, instrumental and culture-historical approaches to the use and development of the artifacts. From an instrumental perspective, the difficulty that students might encounter by outsourcing part of their mathematical activity to an artifact comes from the *dialectics between pragmatic and epistemic value* of the instrumented action *schemes* (Artigue, 2002). In the instrumental genesis process, students need to understand the mathematical meaning behind the appropriated techniques. From a radical embodied perspective we match *a scheme* with the notions of a sensory-motor coordination. We consider conceptual understanding as a coordination of mathematical actions with a variety of artifacts, in consonance with Radford's definition of knowledge "as an ensemble of culturally and historically constituted embodied processes of reflection and action" (Radford, 2013, p. 10). As knowledge itself understood as to be reified (or, metaphorically speaking, crystallized) labor, we recognize cultural artifacts as the reified traces of historically preceding activities. In our educational designs we emulate this historical process by provoking students to disclose a sine function as a depiction of coordination between a sine and an arc length of the unit circle. From the elaborated vision, instrumental actions with an artifact (e.g. sine graph) contribute to mathematical understanding if this mathematical artifact emerges for a student as a reification of her own sensory-motor coordinations.

DESIGN RESEARCH: FROM AN EMBODIED EXPERIENCE TO THE ARTIFACTS

Our work elaborates the design research presented by Rosa Alberto at CERME 2019 (Drijvers, in press) and suggests the series of interactive activities within the embodied action-based design genre (Abrahamson, 2014) that stimulate students to connect a unit circle and a sine graph. The iterative design cycles were recorded on video, audio and by an eye-tracker. The design attempts led us to a new design principle, namely *reification of the previously enacted coordinations* in a following mathematical artifact. Here we present the final sequence that was designed with the guidance of this principle: a sine graph is reified from a student's sensory-motor coordination and further embedded into technological instrument.

Phase 1: coordination of the length of an arc on the unit circle and the length of the segment on the *x*-axis. The task was posed as a motor problem in which students moved one point along a

unit circle and another one along a straight line. Color feedback would signal the moments when two passed distances were equal. At first the student would reflect on their coordination as "moving hands with the same speed". Then additional visual cues would push them from a "same-speed" strategy, towards keeping the length of an arc on the circle equal to a line segment. The phase finishes by manipulating an artifact that automatically shows a point on the *x*-axis that corresponds to the point on the unit circle. In this way, the enacted coordination was reified in the artifact.

Phase 2: coordination of the heights of a point on the unit circle and a point on the Cartesian plane. In this phase the point on the Cartesian plane was automatically adjusted horizontally according to the point on the unit circle, as reification of the coordination from the previous phase. At first the students manipulated one point on the unit circle, and the vertical position of another point on the Cartesian plane, thus directly coordinating the heights of two points. The direct enactment made it very easy to notice the relation. Further, they manipulated only one point on the Cartesian plain with horizontal and vertical degrees of freedom, while keeping it at the same height with the automatically adjusted point on the unit circle. This solution allowed them to focus on the enacted trajectory, that they finally drew, thus reifying their previous enactments.

Phase 3: coordination of two coordinations. In this phase students were invited to coordinate the movement on the unit circle and the horizontal and vertical movements of a point on the Cartesian plain. Finally, the students drew a sine graph as they combined two coordinations from the previous phases. This target artifact appeared as reification of coordinated enactment of two previously established coordinations. The students were able to explain how the sine graph was constructed and how might be used, so we claim the artifact emerged within the system of instrumental actions.

CONCLUSIONS

An initial design principle of embodied design required a continuous color feedback on the spatially articulated prospectively mathematical sensory-motor actions (Abrahamson, 2014). In a few iterative design cycles we supplemented it by another, instrument-oriented principle: *a previously enacted coordination is reified in the mathematical artifact* for the next phase. Later the coordination that was fixed in the artifact might be released and enacted again for the further coordination with another feature. This approach can be seen as an embodied alternative to the classical instrumental orchestration of a pre-given artifact. Further research is needed to understand if an artifact, as it is reified by a student, is properly and solidly coordinated with mathematical actions, thus conceptually understood and can be involved into student's instrumental activity.

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