Axion inflation in type II string theory

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Inflationary models driven by a large number of axion fields are discussed in the context of type IIB compactifications with $\mathcal{N} = 1$ supersymmetry. The inflatons arise as the scalar modes of the R-R two-forms evaluated on vanishing two-cycles in the compact geometry. The vanishing cycles are resolved by small two-volumes or NS-NS B fields which sit together with the inflatons in the same supermultiplets. String world sheets wrapping the vanishing cycles correct the metric of the R-R inflatons. They can help to generate kinetic terms close to the Planck scale and a mass hierarchy between the axions and their nonaxionic partners during inflation. At small string coupling, D-brane corrections are subleading in the metric of the R-R inflatons. However, an axion potential can be generated by D1 instantons or gaugino condensates on D5-branes. Models with a sufficiently large number of axions admit regions of chaotic inflation which can stretch over the whole axion field range for potentials from gaugino condensates. These models could allow for a possibly detectable amount of gravitational waves with tensor to scalar ratio as high as $r < 0.14$.

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I. INTRODUCTION

Current observational cosmology allows us to test fundamental physics with a continuously improving precision. To evaluate and understand these data, theoretical models about the evolution of our universe are crucial. One promising paradigm receiving growing experimental support is cosmological inflation [1]. Inflation postulates a period of exponential expansion of the universe driven by scalar fields slowly rolling in an almost flat potential. This enormous growth stretches quantum fluctuations present in the early universe to currently observable astrophysical scales. The imprints of such a process can be found, for example, in the cosmic microwave background and the large scale structure of the universe [2–4].

In recent years much effort has focused on the realization of inflation within string theory [5,6]. In string theory there are potentially many scalar fields which could drive inflation and hence different opportunities to model inflation. Specific scenarios include realizations of Kähler moduli inflation [7,8], racetrack models [9] and the most intensively studied possibility of D-brane inflation [5,10]. However, as of today, it remains challenging to establish explicit scenarios in a controlled compactification without employing extreme fine-tuning to obtain a sufficiently flat potential [11]. Having found a realization of inflation reproducing the current cosmological observables, it is important to establish which models can incorporate possible future observations. For example, as argued in Ref. [12], many string scenarios do not allow for a high ratio r of gravitational waves produced in the early universe. The current experimental bound on gravitational waves is $r < 0.3$ [3], but future experiments, including Planck, BICEP and Spider [13], might allow the observation of r with a precision down to $r > 0.01$. It is thus desirable to study string embeddings of inflation which can incorporate an r observable in these experiments. Recent attempts to do that can be found, for example, in Refs. [14–18].

A possible scenario able to incorporate primordial gravitational waves was suggested by Dimopoulos, Kachru, McGreevy and Wacker [19]. The authors argue for an embedding of multifield inflation with a large number N of axion fields. Such models use an assistance effect studied in Refs. [20,21] ensuring that the small fraction $1/N$ controls the flatness of the potential. Indeed, generic compactifications of type II string theory on a six-dimensional manifold can admit 10^4, or more, axions from the NS-NS B field and the R-R form fields. An appropriate subset of N such axions was proposed to drive inflation in Ref. [19] and the authors termed these scenarios N-flation. For a sufficiently large N the scenarios can be interpreted as a realization of natural inflation [22,23]. If the inflatons can produce the desired amount of e-foldings already in the quadratic regime of the potential, the models admit chaotic inflation [24]. This implies that N-flation can yield a possibly observable signature of gravitational waves with $r < 0.14$ and thus distinguishes it from most other string realizations of inflation.

The aim of this paper is to study a specific realization of N-flation in type IIB string theory. The inflating axions correspond to the zero modes of the R-R forms in the compactification to four space-time dimensions. In compactifications preserving $\mathcal{N} = 2$ supersymmetry the R-R axions sit in the same supermultiplets as the Kähler structure moduli parametrizing the volumes of two-dimensional cycles in the internal space. Manifolds with a large number

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of nontrivial two-cycles are thus candidate backgrounds for N-flation. As will be shown, the density perturbations and slow roll parameters depend on the volume of the compact space and it has to be ensured that they do not become large with increasing N. We will thus argue that explicit examples always involve compact manifolds which admit many very small or vanishing cycles.

In the presence of small cycles, stringy effects become important and significantly alter the structure of the four-dimensional effective theory. In order to analyze these contributions we will concentrate on axions arising from the R-R two-form evaluated on vanishing two-cycles of the compact geometry. The standard example of a vanishing two-cycle is the resolved conifold [25]. In this case a conical singularity is resolved by a two-sphere supported by a geometric volume or an NS-NS B field. If this S^2 becomes smaller than the square string length, world sheets will start to wrap and significantly contribute to the metric of the R-R axions. Fortunately, in $\mathcal{N} = 2$ compactifications these corrections can be computed for the conifold and many other Calabi-Yau geometries [26,27] allowing the evaluation of the metric of the axions. We will illustrate this general fact on a toy model with N conifold singularities. For such examples it can be shown that the kinetic terms of the axions can be close to the Planck scale which is crucial to obtain inflation [28].

In addition to the fundamental strings also D-branes can become relevant in geometries with vanishing cycles [29]. In particular, D1 instantons can wrap the small cycles and correct the metric of the R-R axions. Such contributions appear with the exponential of the D1-instanton action which depends on the R-R two-form axions themselves. In contrast to the string world-sheet action the D-instanton action also contains a factor of the inverse string coupling. This implies that for small string coupling and finite volume or B field of the vanishing cycles the D1-instanton corrections are subleading in the axion metric. However, correction due to D-branes will be the leading contributions in the scalar potential and can induce the desired potential for the R-R axions.

To study the scalar potential for the axions and nonaxionic moduli we will focus on $\mathcal{N} = 1$ orientifold compactifications of type IIB string theory. Such compactifications have been studied intensively in the last years [30]. It was shown that $\mathcal{N} = 1$ potentials can be induced by background fluxes, D-brane instantons or gaugino condensates on space-time filling D-branes. The desired axion potentials can arise through nontrivial superpotentials from any of the three sources [30]. We will briefly discuss their properties in various orientifold compactifications: D1 superpotentials in type I [31], D1 dependences through the determinants in the D3-instanton superpotentials [32–34] and gaugino condensates on space-time filling D5-branes [30,35–37]. Remarkably, explicit computations of the nonperturbative superpotentials can often be performed in a dual flux picture where geometry dictates the form of the corrections [35–39].

In the final part of this work we will study the effective theory of orientifold compactifications with O3 and O7 planes in more detail. Using earlier results [40,41] we argue that the $\mathcal{N} = 2$ world-sheet corrections are inherited by the $\mathcal{N} = 1$ orientifold theory. In particular, they correct the $\mathcal{N} = 1$ Kähler potential and complex coordinates in a calculable way. This general fact can be applied to a simplistic compact toy model with $2N$ conifolds pairwise identified under the orientifold projection. Including a flux and D-instanton superpotential, we make first steps in establishing an effective theory with a large number of light axions and all other moduli stabilized. An explicit numerical evaluation indicates the presence of a nonsupersymmetric axion valley [42]. This ensures the desired mass hierarchy between the axions and their nonaxionic partners.

The paper is organized as follows. In Sec. II we review the N-flation scenario of [19] and discuss some of its cosmological implications. We recall that the kinetic terms of the axions, set by the axion decay constants, have to be large in order to obtain inflation. A discussion of axion decay constants in type IIB string theory is presented in Sec. III. The four-dimensional effective Lagrangian and the general form of the axion decay constants are studied in Sec. III A supplemented with Appendix A. In Sec. III B we argue that in compactifications with all cycles larger than string length, the axion decay constants typically become very small with an increasing number of axions. This implies that only compactification manifolds with small or vanishing cycles are candidate backgrounds to obtain N-flation. The quantum corrected axion decay constants for the resolved conifold and geometries with N resolved singularities are discussed in Secs. III C and III D. Nonperturbative D-brane effects can induce a scalar potential through a nonvanishing superpotential as discussed in Sec. IV. In Sec. IV A it is shown that such a superpotential can arise from D1-instanton corrections, while Sec. IV B discusses superpotentials originating from gaugino condensates on D5-branes. In the final part of this work, Sec. V, the embedding of N-flation into a concrete $\mathcal{N} = 1$ orientifold compactification is addressed. The general form of the $\mathcal{N} = 1$ data including the inherited $\mathcal{N} = 2$ perturbative and nonperturbative string world-sheet corrections is presented in Sec. V A and Appendix B. In Sec. V B, a toy model with N conifold pairs is used to illustrate that the string world-sheet corrections in the Kähler potential and the presence of a nonperturbative superpotential can ensure an effective theory with light axions. This indicates the possibility of N-flation in string theory.

II. REVIEW OF AXION N-FLATION

In this section we will review some basics about N-flation driven by a large number of axion fields follow-

ing [19,43]. The basic idea is that by increasing the set of inflaton fields an assistance effect can help to ensure that the slow roll conditions are met [20,21]. More precisely, one considers setups with inflaton fields $c^a, a = 1 \ldots N$, where each field feels the downward force of its own potential but is slowed down by the collective frictional force of all fields. To make this more explicit, let us consider a set of inflaton fields c^a with Lagrangian

$$\mathcal{L} = \tfrac{1}{2} f_{ab}^2 \partial_\mu c^a \partial^\mu c^b - V + \ldots. \tag{2.1}$$

To employ the assistance effect we will specify the setup further. We consider a scenario, where, at least approximately, the metric f_{ab}^2 is independent of c^a and can be diagonalized to have a diagonal $f_{aa}^2 = f_a^2$. In order to obtain canonically normalized kinetic terms in (2.1) we introduce the fields θ^a

$$c^a = \theta^a / f_a, \qquad a = 1 \ldots N. \tag{2.2}$$

Moreover, we also constrain the potential for the fields θ^a. We assume that in an effective description the potential $V \approx V_{\text{eff}}$ is given by

$$V_{\text{eff}}(\theta^a) = \sum_a V_a(\theta^a), \tag{2.3}$$

where each term $V_a(\theta^a)$ only depends on the ath inflation field θ^a. For a time-dependent evolution in a Friedmann-Robertson-Walker universe the equations of motion for the fields θ^a take the simple form

$$\ddot{\theta}^a + 3H\dot{\theta}^a + \partial_a V = 0, \qquad H^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}(\dot{\theta}^a)^2 + V\right). \tag{2.4}$$

The assistance effect is now apparent. The Hubble friction contains the whole potential of all fields θ^a, while the downward force $\partial_a V$ only yields a nonvanishing contribution from the ath potential term in (2.3). For a large number of fields this assistance can help to ensure slow roll for the inflaton fields θ^a.

In the following we will be more concrete and study an explicit potential in more detail. Our aim is to identify $c^a = \theta^a / f_a$ with axion fields in a string compactification. The axions c^a are periodic with period 2π such that the accessible field range is the intervals

$$-\pi < c^a \leq \pi, \qquad -f_a \pi < \theta^a \leq f_a \pi. \tag{2.5}$$

A potential term for the axions arises only through nonperturbative corrections to the four-dimensional effective theory as will be discussed in Sec. IV. This implies that the approximate effective potential (2.3) for the axion fields is of the form[1]

$$V_{\text{eff}}(\theta^a) = C + \sum_{a=1}^{N} \Lambda_a^4 (1 - \cos[\mu^a \theta^a / f_a]). \tag{2.6}$$

Let us introduce the different variables appearing in V_{eff}. The constants f_a arise through the redefinition (2.2) ensuring that θ^a have canonically normalized kinetic terms. f_a are the axion decay constants appearing in the metric (2.1). Nontrivial μ^a might already arise in the potential for c^a and thus do not appear due to the rescaling of the fields. In Sec. IV B it will be analyzed how nontrivial μ^a arise in string theory. The constants Λ_a set the scale of inflation and are typically determined by the vacuum expectation values of other fields in the full string compactification. The constant C is the value of the effective potential at the minimum where all $\theta^a = 0$. This cosmological constant C is, in accordance with current observations, very small and can be safely approximated to be zero for the following analysis. Figure 1 shows one of the periodic potentials of the sum (2.6).

Let us now discuss inflation driven by the scalar fields θ^a in the potential (2.6). To obtain slow roll inflation we need to satisfy the standard slow roll conditions [44,45]. We will first introduce the slow roll parameters for a separable potential V as in (2.3). In this case the slow roll parameters are given by

$$\epsilon = \frac{M_P^2}{2} \sum_a \left(\frac{V_{,a}}{V}\right)^2, \qquad \eta = M_P^2 \min_a \left(\frac{V_{,aa}}{V}\right), \tag{2.7}$$

where $V_{,a} \equiv \partial_{\theta^a} V$ and $V_{,aa} = \partial_{\theta^a}^2 V$. The slow roll conditions read $\epsilon < 1$ and $|\eta| < 1$ and define a multidimensional subspace in the fields θ^a where inflation takes place. In this inflationary region of the field space the Hubble friction (2.4) is well approximated by the potential $H = V/(3M_P^2)$ and the physical observables can be defined as a function of the potential V and its derivatives.

The number of e-foldings and the magnitude of scalar density perturbations during the slow roll epoch is given by

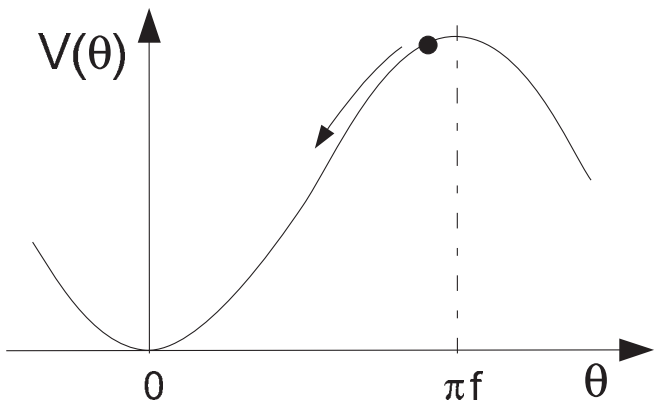

FIG. 1. Potential for one axion field θ with $\mu = 1$.

[1]The potential generally also contains cross coupling terms of the form $\cos[\mu^a \theta^a / f_a - \mu^b \theta^b / f_b]$ as discussed in Refs. [19,43]. These will be omitted in this section.

$$N_e = -\sum_a \int_{\theta_{\rm in}}^{\theta_{\rm fin}} \frac{V_a}{V_{,a}} \frac{d\theta^a}{M_P^2}, \qquad \left(\frac{\delta\rho}{\rho}\right)^2 = \frac{V}{75\pi^2 M_P^6} \sum_a \left(\frac{V_a}{V_{,a}}\right)^2. \tag{2.8}$$

N_e parametrizes the exponential growth of the universe during inflation. It has to be sufficiently large, $N_e \gtrsim 50$, to ensure that different parts of the early universe have been in causal contact. The density perturbations seen in the cosmic microwave background arise from the time 50–60 e-foldings before the end of inflation. For simplicity, we will take $\theta^a_{\rm in} \approx \theta_{\rm in}$ to be the starting point of 55 e-foldings of inflation and evaluate all cosmological observables around this point in field space. In accordance with current observations, we demand

$$N_e \approx 55, \qquad (\delta\rho/\rho)_{\theta_{\rm in}} \approx 2 \times 10^{-5}. \tag{2.9}$$

Similarly, the other cosmological observables can be defined for multifield scenarios. We will not review all the details here, but rather refer to the literature for a more exhaustive discussion [45]. There is, however, one more observable which we would like to introduce. We denote by $r = P_g/P_{\mathcal{R}}$ the relative magnitude of gravity waves P_g to density perturbations $P_{\mathcal{R}}$. Recalling that P_g and $P_{\mathcal{R}}$ are given by

$$P_g = \frac{2}{3\pi^2} \frac{V}{M_P^4}, \qquad P_{\mathcal{R}} = \frac{25}{4} \left(\frac{\delta\rho}{\rho}\right)^2, \tag{2.10}$$

the ratio r is seen to be

$$r = 8M_P^2 / \sum_a \left(\frac{V_a}{V_{,a}}\right)^2. \tag{2.11}$$

The current bounds on r demand that

$$(r)_{\theta_{\rm in}} < 0.3, \tag{2.12}$$

where r is evaluated close to 55 e-foldings before the end of inflation. However, future experiments might show that r is not much below this bound. As we will discuss momentarily, due to the assistance effects of the many axions, r can be close to this bound in models of axion N-flation. This provides one of the major motivations to study these scenarios, since typically r in string motivated models such as D-brane inflation [5] takes a rather small value [12].

In the remainder of this section we will evaluate the slow roll parameters ϵ, η as well as the number of e-foldings N_e and the ratio r for the potential (2.6). Clearly, using (2.7), (2.8), and (2.11) this is straightforward and can be done in full generality. However, to simplify the analysis and to make the results more transparent we will set all axion decay constants f_a and the coefficients μ^a to be approximately of the same size, and set $\mu^a/f_a \approx A$. We also assume that $\Lambda_a \approx \Lambda$. Now we evaluate the slow roll parameters (2.7) for $\theta^a \approx \theta$ by using the effective potential (2.6). Explicitly, we find

$$\epsilon = \frac{M_P^2}{2} \frac{A^2 \sin^2(A\theta)}{N(1-\cos(A\theta))^2}, \qquad \eta = M_P^2 \frac{A^2\cos(A\theta)}{N(1-\cos(A\theta))}. \tag{2.13}$$

An immediate conclusion is that ϵ as well as η are independent of the scale Λ, but crucially depend on the constants A and the number of axions N. The number of e-foldings and amount of scalar density perturbations (2.8) are given by

$$N_e = -\frac{2N}{M_P^2 A^2} \log\left[\frac{\cos(A\theta_{\rm in}/2)}{\cos(A\theta_{\rm fin}/2)}\right], \qquad \frac{\delta\rho}{\rho} = \frac{2\Lambda^2 N}{M_P^3} \frac{(1-\cos(A\theta_{\rm in}))^{3/2}}{A\sin(A\theta_{\rm in})}, \tag{2.14}$$

where $\theta_{\rm in}$ and $\theta_{\rm fin}$ are the starting and endpoint of inflation. Finally, we will derive the relative magnitude of gravity wave to density perturbations r. Inserting (2.13) into (2.11) one finds with the above simplifications

$$r = 16\epsilon = 8M_P^2 \frac{A^2\sin^2(A\theta_{\rm in})}{N(1-\cos(A\theta_{\rm in}))^2}. \tag{2.15}$$

From this expression we infer that r will be small if $\theta_{\rm in}$ is close to the maximum of the potential. However, for sufficiently large N and small A horizon crossing can take place away from the maximum. In such cases, r can be close to the current observational bound (2.12). This is particularly easy to see in the quadratic regime of the potential which we will study in the final part of this section.

For a subset of values N and f_a, μ^a a quadratic approximation of the potential (2.6) suffices for slow roll inflation. More precisely, we will concentrate on the regime

$$\mu^a\theta^a/f_a \approx A\theta^a < 1, \qquad V_{\rm eff}(\theta^a) \approx m^2 \sum_a (\theta^a)^2, \tag{2.16}$$

where $m^2 = \Lambda^4 A^2$ are the masses of the axions θ^a which are assumed to be similar in magnitude. In this regime we can expand the trigonometric functions in (2.14) and (2.15) to lowest order in the θ's. One obtains a model of chaotic inflation for which the assistance effect of the N inflatons allows effectively super-Planckian vacuum expectation values. To qualitatively discuss this regime we again assume $\theta^a \approx \theta = \alpha M_P$ and include appropriate factors of N in our analysis. In this limit (2.13) and (2.14) simplify to

$$\epsilon \approx \frac{2}{N\alpha^2}, \qquad \eta \approx \frac{2}{N\alpha^2}, \tag{2.17}$$

and

$$N_e \approx \frac{N\alpha_{\rm in}^2}{4}, \qquad \frac{\delta\rho}{\rho} \approx \frac{mN\alpha_{\rm in}^2}{M_P}, \qquad r \approx \frac{32}{N\alpha_{\rm in}^2}. \tag{2.18}$$

Using (2.9) we conclude that $N\alpha_{\rm in}^2 \approx 220$ and $m/M_P \approx 10^{-7}$. Evaluating r using these results gives $r \approx 0.14$, which is not significantly below the current bound (2.12).

In case we are able to realize axion inflation in a controlled string compactification it might be the ratio r which crucially distinguishes it from other scenarios such as the inflation of a small number of D-branes [12].

Let us note that in order to obtain slow roll in the quadratic regime (2.16) we have to ensure that N is very large and μ_a/f_a is small. In particular, it has to be guaranteed that the axions generate a sufficient number of e-foldings, $N_e \approx 55$, during the slow roll phase in the regime (2.16). However, for $\mu_a = 1$ a significant part of the e-foldings are generated close to the maximum of the cosine in (2.6), where the quadratic approximation is no longer valid. Including the constants μ_a in the discussion is more subtle, as we will discuss in more detail in Sec. IV B. The reason is that a change in μ_a does not alter the field range (2.5) of the canonically normalized axions. Nevertheless, the quadratic approximation (2.16) becomes valid for more values of θ^a if $\mu_a \leq 1$ is small. Eventually, small values of μ_a can push the maximum out of the field range of θ^a and even ensure that a quadratic approximation is valid for all values of θ^a in (2.5). Both in the quadratic regime as well as for the full potential, we observe that for large N and f_a the slow roll condition $\epsilon < 1$, $\eta < 1$ is easier to satisfy for many values of θ^a. We thus have to ensure that sufficiently large values for N and f_a are accessible in a string embedding.

In the next section we will start the discussion of axion inflation in type IIB string theory by evaluating the axion decay constants for different R-R and NS-NS axions. We will recall that in a controlled string compactification $f_a \lesssim M_P$ seems unavoidable. In order to nevertheless obtain a sufficiently long period of slow roll inflation, we have to ensure that both N and f_a are close to the accessible values.

III. AXION DECAY CONSTANTS IN IIB STRING THEORY

In the previous section we reviewed a simple inflationary scenario driven by the dynamics of a large number of axions. In order to embed such a model into string theory, we have to identify an appropriate set of axions θ^a arising in a compactification of string theory from ten to four space-time dimensions. As we have seen, the cosmological models crucially depend on the number of axions N and the value of the axion decay constants f_a. In order to guarantee an epoch of slow roll inflation we have to ensure that both N and f_a take sufficiently large values. In this section, we discuss the axion decay constants of type IIB compactifications on Calabi-Yau manifolds with $\mathcal{N} = 2$ supersymmetry. We will see that in string theory an implementation of axion decay constants close to the Planck scale ($f_a \lesssim M_P$), for a large number of axions, is in general hard to achieve. It appears that candidate scenarios can only arise away from the large volume limit, where the compact geometry admits small or vanishing cycles.

A. The four-dimensional axion Lagrangian

In this section we will fix our conventions and discuss the four-dimensional effective Lagrangian for the NS-NS and R-R axions obtained by compactifying type IIB string theory on a Calabi-Yau manifold Y. This will allow us to determine the axion decay constants for the various types of axions.

Let us begin by summarizing our conventions following [46,47]. The ten-dimensional gravitational coupling κ_{10} in the Einstein-Hilbert term is given by $\kappa_{10}^2 = g_s^2 \ell_s^8/4\pi$, where $\ell_s = 2\pi\sqrt{\alpha'}$ is the string length and g_s is the type II string coupling. The string coupling is normalized such that S duality acts as $g_s \rightarrow 1/g_s$. The four-dimensional Planck mass M_P is obtained by dimensionally reducing the Einstein-Hilbert term to four dimensions. Denoting by V_Y the volume of the internal manifold, $M_P = 2 \times 10^{18}$ GeV is given by

$$M_P^2 = \frac{4\pi V_Y}{g_s^2 \ell_s^8}, \qquad V_Y = R^6, \tag{3.1}$$

where R denotes the typical radius of the compactification space. It turns out to be convenient to introduce the dimensionless volume $\mathcal{V} = V_Y/\ell_s^6$. The ratio of string scale $m_s = 1/\ell_s$ to Planck scale is then given by

$$\frac{m_s}{M_P} = \frac{g_s}{\sqrt{4\pi\mathcal{V}}}. \tag{3.2}$$

This fraction is smaller than unity, since we will consider the volume $\mathcal{V}$ to be above string scale, i.e. $\mathcal{V} > 1$, and g_s to be at weak coupling. Following [47] we will normalize the NS-NS and R-R field strengths to have integral periods. This implies that the kinetic terms for the metric as well as the NS-NS and R-R fields B_2, C_p are obtained from the ten-dimensional string-frame action of the form

$$-\frac{2\pi}{\kappa_{10}^2}\int\Big[\frac{1}{2}e^{-2\phi}R*1 + \frac{1}{4}\frac{1}{\ell_s^4}e^{-2\phi}H_3\wedge *H_3 + \frac{1}{4}\frac{1}{\ell_s^{8-2p}}F_{p+1}\wedge *F_{p+1}\Big], \tag{3.3}$$

where H_3 and F_{p+1} are the field strengths of B_2 and C_p, respectively. In the expression (3.3) the field e^{ϕ} denotes the ten-dimensional dilaton with vacuum expectation value $g_s = e^{\langle\phi\rangle}$ being the string coupling constant. Finally, in accordance with (3.3), the coupling of the R-R forms to Dp-branes takes the form $2\pi\int C_p$. The tension of a Dp-brane is given by $T_p = 2\pi/(g_s\ell_s^{p+1})$.

In compactifications of type IIB string theory a large number of axions can only arise from the NS-NS and R-R two-forms C_2, B_2 and the R-R four-form C_4. More precisely, taking the internal six-dimensional space to be a Calabi-Yau manifold Y, the axions arise by expanding B_2, C_2 and C_4 in a basis of harmonic forms. Denoting by ω_A a basis of two-forms in $H^2(Y,\mathbb{Z})$ and by $\tilde{\omega}^A$ its dual basis of four-forms in $H^4(Y,\mathbb{Z})$ we can expand

$$B_2 = \frac{1}{2\pi} b^A \omega_A, \qquad C_2 = \frac{1}{2\pi} c^A \omega_A, \qquad C_4 = \frac{1}{2\pi} \rho_A \tilde{\omega}^A, \tag{3.4}$$

where the factors of $1/2\pi$ were included to ensure that the axions are 2π periodic. We also expand the Kähler form J of Y into the above integral basis

$$J = v^A \omega_A = (R^A/\ell_s)^2 \omega_A. \tag{3.5}$$

Here J is normalized to be dimensionless, while the $(R^A)^2$ are the dimensionfull volumes of two-cycles. Compactifying type IIB string theory on the Calabi-Yau manifold Y yields a four-dimensional $\mathcal{N} = 2$ supergravity theory. Indeed, the axion fields in (3.4) combine with the Kähler structure deformations into $\mathcal{N} = 2$ hypermultiplets with scalars (v^A, b^A, c^A, ρ_A). The effective four-dimensional action for these hypermultiplets was derived in [48]. In the following, we will only discuss some of the relevant terms and determine the leading axion decay constants. A more complete study of the action and the axion decay constants can be found in Appendix A.

As in Sec. II, the axion decay constants are determined from the kinetic terms of b^A, c^A and ρ_A. These are derived by dimensionally reducing the ten-dimensional action (3.3). The resulting four-dimensional Lagrangian for the axions takes the form

$$\mathcal{L} = -\frac{M_P^2}{2\pi}(G_{A\bar{B}} \partial_\mu b^A \partial^\mu b^B + e^{2\phi} G_{A\bar{B}} \partial_\mu c^A \partial^\mu c^B + e^{2\phi} \tilde{G}^{A\bar{B}} \partial_\mu \rho_A \partial^\mu \rho_B), \tag{3.6}$$

where $G_{A\bar{B}}$ and $\tilde{G}^{A\bar{B}}$ are the moduli space metrics. It is important to note that in the expression (3.6) we have left out additional terms which will not be relevant in the following but are of crucial importance for the effective action to have $\mathcal{N} = 2$ supersymmetry (see Appendix A). The leading axion decay constants are determined in terms of the metrics $G_{A\bar{B}}$ and $\tilde{G}^{A\bar{B}}$ as

$$B_2\text{:}\ \frac{f_{AB}^2}{M_P^2} = \frac{1}{\pi} G_{A\bar{B}}, \qquad C_2\text{:}\ \frac{f_{AB}^2}{M_P^2} = \frac{g_s^2}{\pi} G_{A\bar{B}}, \qquad C_4\text{:}\ \frac{f_{AB}^2}{M_P^2} = \frac{g_s^2}{\pi} \tilde{G}^{A\bar{B}}. \tag{3.7}$$

In these expressions we have taken the dilaton to be fixed to its vacuum value g_s. Clearly, in order to evaluate the typical size of the axion decay constants we will need the explicit form of the moduli space metrics appearing in (3.7).

The moduli space metrics in (3.7) are functions of the complexified Kähler structure deformations

$$t^A = -b^A + i v^A. \tag{3.8}$$

Because of the underlying $\mathcal{N} = 2$ supersymmetry all dependence of $G_{A\bar{B}}$ on t, $\bar{t}$ can be encoded by a holomorphic function $\mathcal{F}(t)$ known as the prepotential. In general, $\mathcal{F}$ contains a classical piece cubic in t as well as perturbative and nonperturbative string world-sheet corrections of order α'. The metric $G_{A\bar{B}} = \partial_{t^A} \partial_{\bar{t}^B} K$ is a Kähler metric with Kähler potential [49]

$$K(t, \bar{t}) = -\ln \mathcal{V}, \qquad \mathcal{V} = 2i(\mathcal{F} - \bar{\mathcal{F}}) - i(\partial_{t^A} \mathcal{F} + \partial_{\bar{t}^A} \bar{\mathcal{F}})(t^A - \bar{t}^A), \tag{3.9}$$

where $\mathcal{V}$ is the quantum corrected volume of the compact Calabi-Yau space Y. Up to a factor of $\mathcal{V}$ the metric $\tilde{G}^{A\bar{B}}$ is the inverse of $G_{A\bar{B}}$, i.e. $\tilde{G}^{A\bar{B}} = (\mathcal{V}/2)^{-2} G^{A\bar{B}}$. Having given the metrics in (3.7) in terms of a general prepotential $\mathcal{F}$, we are free to evaluate them at various points in the moduli space parametrized by t^A. In the next section we will do that in the large volume limit, while Secs. III C and III D are devoted to a study of the axion decay constants on more singular points in the moduli space.

B. Large volume compactifications

Let us now assume that we are in the strict large volume limit in the moduli space. This implies that all physical volumes of the two-cycles are larger than the square string length ℓ_s^2. All α' corrections are then suppressed and the prepotential can be approximated by

$$\mathcal{F}^{\text{class}} = -\frac{1}{3!} \mathcal{K}_{ABC} t^A t^B t^C, \qquad \mathcal{K}_{ABC} = \int_Y \omega_A \wedge \omega_B \wedge \omega_C, \tag{3.10}$$

which is the classical contribution depending on the triple intersections $\mathcal{K}_{ABC}$. In the large volume limit, $\mathcal{V}$ given in (3.9) is simply the geometrical volume of the Calabi-Yau manifold and reads[2]

$$\mathcal{V} = \frac{8}{3!} v^A v^B v^C \mathcal{K}_{ABC}. \tag{3.11}$$

The moduli space metrics in (3.6) can be expressed through $\mathcal{V}$ and its derivatives $\mathcal{V}_A = \partial_{v^A} \mathcal{V}$ and $\mathcal{V}_{AB} = \partial_{v^A} \partial_{v^B} \mathcal{V}$ as

$$G_{AB} = -\frac{1}{4} \frac{\mathcal{V}_{AB}}{\mathcal{V}} + \frac{1}{4} \frac{\mathcal{V}_A \mathcal{V}_B}{\mathcal{V}^2}, \qquad \tilde{G}^{AB} = -32 \frac{\mathcal{V}^{AB}}{\mathcal{V}} + 8 \frac{v^A v^B}{\mathcal{V}^2}, \tag{3.12}$$

where $\mathcal{V}^{AB}$ is the inverse of the matrix $\mathcal{V}_{AB}$. Using these explicit results for the large volume limit, we will be able to argue that the axion decay constants are sensitive to the number of axions. This will lead us to conclude that natural scenarios contain manifolds with small string-scale cycles.

[2]The volume is actually $\mathcal{V}/8$, but we will keep this normalization for simplicity.

Let us now argue that a scenario in the strict $\mathcal{N} = 2$ large volume limit excludes the possibility of slow roll. As introduced in (3.5) the v^A parametrize the string-scale size of the two-cycles in the Calabi-Yau space Y. They have to take values inside the Kähler cone in order to ensure that the physical volumes of two- and four-cycles are positive [50].[3] To make this more transparent, we note that, at least for toric-projective Calabi-Yau manifolds, we can choose a basis ω_A such that

$$0 < v^A < \infty \tag{3.13}$$

parametrizes the Kähler cone [50,51]. In this basis, the large volume limit is obtained when all $v^A > 1$ ensuring that the radii R^A in (3.5) satisfy $R^A > \ell_s$. Since the world-sheet instantons contribute with exponential suppression e^{-v^A} these α' corrections can be neglected in the above regime. A more detailed discussion of the correct choice of basis can be found in, for example, Ref. [27].

We are now in the position to evaluate the decay constants. For simplicity, we will assume that all cycles in this limit are approximately of the same size $v^A \approx (R/\ell_s)^2$. The axion decay constants (3.7) then take the form [47]

$$B_2\text{:}\ \frac{f_{AB}^2}{M_P^2} \cong \left(\frac{\ell_s}{R}\right)^4 \frac{x_{AB}}{2\pi}, \qquad C_2, C_4\text{:}\ \frac{f_{AB}^2}{M_P^2} \cong g_s^2 \left(\frac{\ell_s}{R}\right)^{2p} \frac{x_{AB}}{2\pi}, \tag{3.14}$$

where $p = 2$ for the axions of C_2 and $p = 4$ for the axions of C_4. The constants x_{AB} arise from the sum over intersection numbers $\mathcal{K}_{ABC}$ in the metrics G_{AB} and $\tilde{G}^{AB}$ contained in (3.7). In order to get a rough estimate of x_{AB} we note that in the parametrization (3.13) of the Kähler cone the intersections are necessarily positive, $\mathcal{K}_{ABC} \geq 0$ ensuring positivity of the total volume in the whole Kähler cone. Inspecting (3.12) we find that the axion decay constants generically depend on N, since we have as many axions (c^A, b^A, ρ_A) as volumes v^A. For axions of B_2, C_2 a rough estimate yields $x_{AB} \propto 1/N$, while for axions of C_4 we have $x_{AB} \propto 1/N^2$.[4] Even though this analysis is strictly only valid for Calabi-Yau manifolds with Kähler cones (3.13) one expects that more exotic examples will not significantly alter the conclusion that x_{AB} and hence the axion decay constants f_{AB} scale with N. This conclusion rules out large volume scenarios of axion N-flation since the N dependence of f_{AB} will compensate the assistance effect which was of essential importance in achieving slow roll inflation in Sec. II.

[3]Formally, these conditions translate to $\int_Y J \wedge J \wedge J > 0$, $\int_D J \wedge J > 0$ and $\int_C J > 0$ for all nontrivial divisors D and holomorphic curves C in Y.

[4]To illustrate this result let us concentrate on the first term in G_{AB} in (3.12). For this term we evaluate the quotient of $\mathcal{V}_{AB} = R^2 \sum_C \mathcal{K}_{ABC}$ and $\mathcal{V} = \frac{4}{3} R^6 \sum_{D,E,F} \mathcal{K}_{DEF}$. We conclude that for each A, B the sum in the numerator runs over significantly more values than the sum in the denominator.

In summary, we note that large volume scenarios make it hard, if not impossible, to implement slow roll inflation driven by a large number of axions. This is in accord with the findings of Ref. [18]. Clearly, a possible conclusion is to relax the requirement that all physical cycles are larger than string scale. As we will discuss in the next section, axions arising on vanishing cycles are natural candidates to realize N-flation. Such small cycles are not untypical in string theory. However, it is clear that in such scenarios additional stringy corrections of the axion decay constants will be of importance and have to be included.

C. Axions from vanishing cycles—The resolved conifold

In this section we discuss axions arising from vanishing cycles of a compact manifold Y. Since in type IIB string theory axions can arise from the two- and four-forms B_2, C_2 and C_4 a general discussion would include the physics of vanishing two- and four-cycles. However, in the following we will exclusively discuss vanishing two-cycles. The main reason for this restriction is the fact that corrections are significantly better understood for lower-dimensional cycles. For two-cycles stringy corrections arise from wrapping world sheets and D1-branes and have been studied intensively in the literature [26,27,52,53].

The standard example for a vanishing two-cycle is the resolved conifold. The conifold is a cone over $T^{1,1}$, which is topologically $S^3 \times S^2$. Its metric and harmonic two-form ω_2 are know explicitly and the axion decay constants for the R-R axion c of $C_2 = \frac{1}{2\pi} c \omega_2$ are computed directly [47]

$$\frac{f_{\text{cone}}^2}{M_P^2} \cong g_s^2 \frac{\ell_s^4 \Lambda^2}{R^6} \frac{x}{\pi}, \tag{3.15}$$

where Λ is the cutoff regularizing the infinite cone and x is a dimensionless number of order 1. In case we take $\Lambda \approx R$ the result coincides with (3.14). Note that this analysis is purely classical and follows directly from the geometry of the conifold. In the following we will discuss the case where the conifold singularity is resolved by an $S^2 \cong \mathbb{P}^1$. We will see that corrections to f_{cone} arise from string world sheets and D1 instantons wrapped around the S^2.

In order to discuss the resolved conifold we recall that in string theory the size of the resolution is parametrized by the integral over the Kähler form J and the NS-NS two-form B_2. Explicitly, introducing the complexified Kähler structure deformation t as

$$t = -b + iv = -\frac{1}{2\pi} \int_{S^2} B_2 - iJ, \tag{3.16}$$

the size of the S^2 is given by $|t|\ell_s^2$. The geometry becomes singular in the limit $|t| \to 0$, but remains resolved as long as the geometrical volume v or the NS-NS B field b are nonzero. In the following we will study the effects of a small resolution by $|t|$ in more detail.

Turning to the corrections to (3.15) we recall that the string world sheet couples to t such that the nonperturba-

tive α' corrections arise through the exponential e^{it}. As discussed in Sec. III A all α' corrections are conveniently encoded by the holomorphic prepotential $\mathcal{F}(t)$. This prepotential can be split as

$$\mathcal{F} = \mathcal{F}^{\text{class}} + \mathcal{F}^{\text{pert}} + \mathcal{F}^{\text{sing}}, \tag{3.17}$$

where $\mathcal{F}^{\text{class}}$ is a cubic classical term given in (3.10), and $\mathcal{F}^{\text{sing}}$ contains the nonperturbative α' corrections from string world sheets wrapping the vanishing cycle. The second term $\mathcal{F}^{\text{pert}}$ contains a perturbative α' correction as well as linear and quadratic terms in the Kähler moduli. For the discussion of the size of the axion decay constants we will ignore these contributions, even though the known $\mathcal{N} = 2$ corrections can be straightforwardly included. In the case of a conifold singularity the nonperturbative α' corrections can be summed up to give a closed expression for $\mathcal{F}^{\text{sing}}(t)$. The leading prepotential is thus given by [27,54,55]

$$\begin{aligned} \mathcal{F}^{\text{class}} &= \frac{i}{3!}(R/\ell_s)^6 + \frac{i}{2!}(R/\ell_s)^2 t^2 + \mathcal{O}(t^3), \\ \mathcal{F}^{\text{sing}} &= \frac{1}{2!} t^2 \log t + \mathcal{O}(t^2), \end{aligned} \tag{3.18}$$

for $|t| < 1$. Because of the fact that $|t|$ is small we have omitted further terms regular in t in $\mathcal{F}^{\text{sing}}$. Using the quantum corrected prepotential including the conifold corrections we compute the quantum volume (3.9) as

$$\begin{aligned} \mathcal{V} = &\frac{4}{3}(R/\ell_s)^6 - 4(R/\ell_s)^2 v^2 \\ &+ 2(|t|^2 \log|t| + v^2) + \mathcal{O}(t^0), \end{aligned} \tag{3.19}$$

where $v = (t - \bar{t})/2i$ is the volume of the vanishing S^2. It is important to note that for a small resolution $|t| < 1$ the logarithmic term in (3.19) becomes negative. This implies that both the second and third term in (3.19) lower the size of $\mathcal{V}$.

The general form of the leading axion decay constant for the R-R axion c was given in (3.7). It contains the second derivative of the Kähler potential (3.9) which has to be evaluated for the resolved conifold prepotential (3.18). One derives

$$\frac{f_{\text{cone}}^2}{M_P^2} \cong \frac{g_s^2}{\pi} \frac{2(R/\ell_s)^2 - 2\log|t| - 1}{\mathcal{V}} + \ldots, \tag{3.20}$$

where $\mathcal{V}$ is given in (3.19). As expected this expression correctly reproduces the classical piece (3.15). It also contains the leading singular term for $|t| < 1$. We note that, at least to leading singular order, the quantum volume in the denominator of (3.20) is reduced due to the α' corrections on the conifold singularity, while the expression in the numerator increases by a positive term proportional to $-\log|t|$. Hence, the size of axion decay constant f_{cone} does not necessarily decrease if the axion arises from a small cycle. This is a desired behavior. It implies that even in the presence of a large number of vanishing cycles the axion decay constants are not necessarily small. Therefore, we can circumvent a scaling of the decay constants with the number of axions as encountered in compactifications with all cycles of size $R > \ell_s$ as discussed in Sec. III B. It remains to discuss other corrections due to D1 instantons and to estimate the typical size of the axion decay constants f_{cone} for the vanishing cycle.

Inspecting (3.20) one would naively conclude that f_{cone} can be made very large by taking the limit $|t| \to 0$, i.e. by shrinking the geometrical size and also switching off the NS-NS B field. However, this will drive us into a regime where the effects of D1 instantons become important and correct the axion decay constants as well as the potential significantly. This would then imply that the effective action (2.1) with potential (2.6) for the axion fields will not be trustable. To make the coupling to D1 instantons more explicit we introduce the fields[5]

$$G = c - i\frac{|t|}{g_s} = \int_{S^2} (C_2 - C_0 B_2) - i \int_{S^2} e^{-\phi} |B_2 - iJ|, \tag{3.21}$$

where C_2 and B_2 are the R-R and NS-NS two-forms as above. The first integral in (3.21) corresponds to the Chern-Simons action on a D1 instanton, while the second term is the minimal coupling in the Born-Infeld action.[6] D1 instantons couple via the exponential

$$Z_{D1} = \exp(-iG) = \exp\left(-\frac{|t|}{g_s} - ic\right). \tag{3.22}$$

In order to make sure that there are no significant corrections to the axion decay constants as well as no higher harmonics proportional to Z_{D1} we have to guarantee that $|Z_{D1}| < 1$. This provides a lower bound on the size of $|t|/g_s$ and hence prevents us from taking $|t| \to 0$. It will be sufficient to take $|t|$ slightly larger than g_s. In order to get an estimate for the axion decay constants (3.20) we also take the typical radius R to be slightly larger than ℓ_s. We thus find

$$f_{\text{cone}} \lesssim M_P. \tag{3.23}$$

We thus conclude that the axion decay constants can be maximally of order Planck scale. This is not a new result, but rather supports the analysis of Ref. [28] carried out in various different string scenarios. Equation (3.23) implies that we have to make use of the assistance effect of many axions as discussed in Sec. II.

[5]We slightly abuse the notation of (3.4) in defining c to also contain the lower R-R form scalars.

[6]In $\mathcal{N} = 1$ compactifications either the real or imaginary part of t will control the size $|t|$ in the definition of G. In Sec. V we show that for O3/O7 orientifolds the B field b survives in G, while in O5 orientifolds the volume v appears in G [40].

Before turning to the multiaxion case let us summarize our strategy. As discussed in this section, we will allow for *two* different scales in the problem. On the one hand, the radius R sets the size of the large cycles, while on the other hand a small parameter $|t|$ sets the size of the vanishing cycles. In the following we will always take

$$g_s \lesssim 1, \qquad 1 \lesssim |t|/g_s, \qquad 1 \lesssim (R/\ell_s)^2. \tag{3.24}$$

This choice of hierarchy implies that we can neglect world-sheet and D1-instanton corrections on the large cycles of radius R. On the small cycles of size $|t|$ the D1 instantons do not significantly correct the metric and hence the axion decay constants. However, world-sheet instantons are relevant on these cycles and have to be taken into account. Fortunately, in $\mathcal{N} = 2$ setups such as the ones discussed in Secs. III C and III D, these are encoded by the prepotential and are thus, at least in principle, computable by standard techniques such as mirror symmetry.

D. Axions from vanishing cycles—The multiaxion scenario

Having discussed the qualitative feature of a single axion from a vanishing cycle, we will now turn to the multiaxion case. Recall that due to (3.23) we always obtain sub-Planckian axion decay constants, such that in order to implement slow roll inflation as in Sec. II a scenario with a number of N axions seem unavoidable.

In order to introduce setups with many vanishing cycles, we divide the two-cycles of the compact space Y into two sets. Let us denote by $\mathcal{C}_i \in H_2(Y)$, $i = 1 \ldots N$ a set of two-cycles, which can be made small without forcing the overall volume of the Calabi-Yau manifold to vanish. The remaining cycles in $H_2(Y)$ will be denoted by $\mathcal{C}_I$. The two-forms dual to $\mathcal{C}_A = (\mathcal{C}_i, \mathcal{C}_I)$ are denoted by

$$\omega_A = (\omega_i, \omega_I), \qquad i = 1 \ldots N, \qquad I = 1 \ldots h^{(1,1)} - N, \tag{3.25}$$

where $h^{(1,1)}$ is the dimension of the second cohomology group $H^{(1,1)}$ of the Calabi-Yau manifold. The triple intersections $\mathcal{K}_{ABC}$ defined in (3.10) are evaluated using this basis. As in Sec. III C we will introduce two scales and assume that all large cycles are of approximate radius $R \gtrsim \ell_s$ and all small cycles are of size $|t|$, i.e. we demand

$$v^I \approx (R/\ell_s)^2 \gtrsim 1, \qquad |t^i| \approx |t| < 1. \tag{3.26}$$

As an estimate of complicated final expressions we will express them as functions of R and $|t|$ as well as the number N of axions only.

The leading classical axion decay constants for the R-R axions c^i have been derived in Sec. III B. They are obtained by taking the derivatives of the classical volume (3.11). The resulting classical metric G_{ij} given in (3.12) is inserted into the general expression (3.7) for the decay constants. As in the example of the resolved conifold discussed in Sec. III C it will be crucial to evaluate the quantum corrections to this classical result. To make this explicit, this would force us to specify the type of singularities we are considering and thus requires further information about the geometry of the setup. In this paper we will only discuss a simple toy model of N conifolds and it would be desirable to extend this analysis in future work. However, the general way to proceed should be as follows. In the split (3.17) of the prepotential $\mathcal{F}$ the contribution $\mathcal{F}^{\text{sing}}$ will only depend on the small moduli t^i and should be derived for a given geometry. This singular geometry can be considered to be local or noncompact and later embedded into a compact space with cutoff given by the radius R of the large cycles. Given a prepotential (3.17) the leading axion decay constants are straightforwardly computed by evaluating

$$\frac{f_{ij}^2}{M_P^2} = \frac{g_s^2}{\pi} G_{i\bar{j}}, \tag{3.27}$$

where $G_{i\bar{j}} = \partial_{t^i}\partial_{\bar{t}^j} K$ is the Kähler metric to the Kähler potential (3.9).

Let us now exemplify a multiaxion scenario with a toy model admitting N resolved conifold singularities. To derive the axion decay constants we investigate the prepotential

$$\mathcal{F} = \frac{i}{3!}(R/\ell_s)^6 + \frac{i}{2}\sum_{i=1}^{N}(t^i)^2[(R/\ell_s)^2 - \log t^i] + \ldots, \tag{3.28}$$

which is the straightforward generalization of the single conifold prepotential (3.18). It can be viewed as describing a toy model of N conifold singularities in a compact Calabi-Yau manifold with simple intersection numbers. We have assumed that the nontrivial intersection numbers of the large cycles of radius R with the vanishing cycles of size $|t^i|$ are given by $\mathcal{K}_{Iij} \approx -\delta_{ij}$, while all other intersections of the large cycles are of order one $\mathcal{K}_{IJK} \approx \mathcal{O}(1)$ or vanish. The quantum corrected volume (3.9) is computed to be

$$\begin{aligned}\mathcal{V} &= \frac{4}{3}(R/\ell_s)^6 - 4(R/\ell_s)^2\sum_i (v^i)^2 \\ &\quad + 2\sum_i (|t^i|^2 \log|t^i| + (v^i)^2) + \ldots \\ &\approx \frac{4}{3}(R/\ell_s)^6 - 4N(R/\ell_s)^2 v^2 \\ &\quad + 2N(|t|^2\log|t| + v^2) + \ldots,\end{aligned} \tag{3.29}$$

where in the second line we have used (3.26). This expression should be compared to the case of a single conifold (3.19). The axion decay constants are computed using the general expression (3.27). Inserting the prepotential (3.28) we find that the leading contributions to $f_{ij}^2 = \delta_{ij} f_i^2$ are diagonal with

$$\frac{f_i^2}{M_P^2} = \frac{g_s^2}{\pi}\frac{2(R/\ell_s)^2 - 2\log|t| - 1}{\mathcal{V}} + \ldots. \tag{3.30}$$

We can now draw the conclusion already indicated in the previous section. Even though we are dealing with a setup with a possibly large number N of small cycles, the axion decay constants can be close to the Planck scale for appropriate values of R, t and g_s.

To illustrate (3.30) let us consider a numerical example. Let us assume that we have 100 large cycles of volume $(R/\ell_s)^2 \approx 1.44$ and to each there are coupling 8 different small cycles of volume $|t| \approx 0.31$ with coupling $(R/\ell_s)^2 \times (t^i)^2$ in (3.28). This gives $N = 800$ axions. At string coupling $g_s = 0.25$ one evaluates

$$f_i \approx 0.1 M_P \tag{3.31}$$

at a string-frame volume $\mathcal{V} \approx 18.6$ and Einstein-frame volume $V_E = g_s^{-3/2}\mathcal{V} \approx 148$ in units of ℓ_s. We also evaluate

$$\frac{|t|}{g_s} \approx 1.25, \qquad \frac{(R/\ell_s)^2}{g_s} \approx 5.7. \tag{3.32}$$

To end this section let us also comment on the corrections due to D-brane instantons to the axion decay constants f_{ij}. In particular, D1 instantons will introduce a dependence of f_{ij} on the axion fields c^i themselves. This is clear from the fact that D1 instantons on a cycle $\mathcal{C}_i$ couple to the complex coordinates G^a defined as[7]

$$G^i = c^i - i\frac{|t^i|}{g_s} = \int_{\mathcal{C}_i}(C_2 - C_0 B_2) - i\int_{\mathcal{C}_i} e^{-\phi}|B_2 - iJ|, \tag{3.33}$$

which is the obvious generalization of (3.21). The corrections are proportional to the exponential of the G^a as in (3.22). In complete analogy to the discussion in Sec. III C we want to make sure that these D-brane corrections are parametrically small and can be neglected in the analysis of the axion decay constants. Therefore, we demand that each $|t^i|/g_s$ is larger than unity, such that $|Z_{D1}| \ll 1$. In other words, we will demand that each $t^i \approx t$ satisfies the constraint (3.24). Again, we have two scales parametrizing our models; one parameter R setting the size of the large cycles, and another much smaller parameter $|t|$ associated to the stringy volume of the vanishing cycles.

IV. AXION POTENTIALS IN TYPE IIB STRING THEORY

In the previous section we studied the kinetic terms of R-R axions for type IIB Calabi-Yau compactifications. Most of our analysis focused on axions c^a, $a = 1 \ldots N$ arising from the two-form C_2 integrated over vanishing cycles in the Calabi-Yau manifold. In this section we will discuss the nonperturbative effects which lead to the generation of a nonvanishing scalar potential for the fields c^a. We will identify sources which generate an axion potential of the form

$$V_{\text{eff}}(\theta^a) = C + \sum_{a=1}^{N}\Lambda_a^4(1 - \cos[\mu^a\theta^a/f_a]), \tag{4.1}$$

which was already given in (2.6). In this effective potential, the θ^a are the canonically normalized axions (2.2) and f_a are the diagonalized axion decay constants.

To make the discussion of axion potentials more explicit we will further break the supersymmetry of the $\mathcal{N} = 2$ scenarios of Sec. III to $\mathcal{N} = 1$. This can be done by including background fluxes, space-time filling D-branes or orientifold planes [30]. If these additional sources are appropriately chosen they only preserve half of the eight supercharges in four dimensions. It is well-known, that the effective action of the resulting four-dimensional supergravity theory can be described in terms of a set of $\mathcal{N} = 1$ characteristic functions. The kinetic terms for a set of chiral multiplets containing the complex scalars M^n are encoded by the metric $K_{n\bar{m}} = \partial_{M^n}\partial_{\bar{M}^m}K$, which is the second derivative of a Kähler potential K. Supersymmetry also implies that the effective scalar potential can always be written as

$$V = e^{K/M_P^2}(K^{n\bar{m}}D_n W\overline{D_m W} - 3|W|^2/M_P^2) + (\mathrm{Re}f)^{kl}D_k D_l. \tag{4.2}$$

Here W is the superpotential holomorphic in M^I and $D_n W = \partial_{M^n}W + (\partial_{M^n}K)W$ denotes its Kähler covariant derivative. The second term in (4.2) arises if some of the scalars are gauged. It depends on the nontrivial D terms D_k as well as the real part of the gauge-kinetic coupling function f_{kl}. In the following we will discuss sources for a superpotential W which inserted into the potential (4.2) induces an axion potential of the form (4.1).

A. Corrections due to D1 instantons

In this section we will discuss a first set of corrections which can induce a nontrivial axion potential of the form (4.1). More precisely, we will study an effective potential arising from D1 instantons wrapped around small cycles in the Calabi-Yau manifold.

In order to study the effects of D instantons we will focus entirely on the superpotential W. Recently, much effort has focused on the study of D3 instantons wrapped on four-cycles in O3/O7 orientifold and F-theory compactifications as reviewed, for example, in Ref. [30]. These instantons can, under certain well-known conditions [56], induce a superpotential of the form

$$W_{\text{D3}} = \sum_\alpha A_\alpha e^{in_\alpha^\beta T_\beta}. \tag{4.3}$$

[7]We slightly abuse the notation of (3.4) in defining c^i to also contain the lower R-R form scalars.

In this expression T_β are the moduli containing the Kähler structure deformations v^β defined in (3.5). In (4.3) the functions A_α generically depend on other fields in the spectrum, while $n_\alpha{}^\beta$ is a constant matrix parametrizing the wrapping numbers of the D3 instantons. In the work of KKLT [57] the corrections (4.3) have been used to stabilize the moduli T_α at large volume $v^\alpha > 1$. Moreover, in specific setups the corrections (4.3), together with a superpotential due to background fluxes, are shown to be sufficient to stabilize all moduli in the model [58]. Typically, such examples have no R-R two-form scalars in the spectrum, since these have been projected out in the $\mathcal{N} = 2$ to $\mathcal{N} = 1$ reduction. In order to realize the scenarios of Secs. III C and III D we thus have to focus on a more general set of examples which contain R-R two-forms in the spectrum.

Examples admitting R-R two-form axions have been studied, for example, in Refs. [34,59]. In such O3/O7 orientifolds the fields G^a defined in (3.33) arise as chiral multiplets in the spectrum. Based on the earlier works [32,33], it has been argued in [34] that a dependence on G^a can appear through the prefactors A_α in (4.3). More precisely, this dependence arises through generalized theta functions $\Theta_\alpha(\tau, G)$ each being a power series in the exponentials $e^{i\tau}$ and e^{-iG^a}. The superpotential (4.3) can be written as

$$W_{\rm D3} = \sum_\alpha \hat{A}_\alpha \Theta_\alpha(\tau, G) e^{i n_\alpha^\beta T_\beta}, \tag{4.4}$$

with new coefficient functions $\hat{A}_\alpha$ independent of τ, G^a.[8] For our purposes it will be sufficient to consider the first few terms in $\Theta(\tau, G)$. For example, consider a setup with one field T coupling to a set of moduli G^1, G^2, etc. Since $\Theta(\tau, G)$ generically starts with a constant, a candidate superpotential is of the form

$$W_{\rm D3} = e^{iT} + e^{i\tau}[e^{-iG^1} + e^{-iG^2} + \ldots]e^{iT}, \tag{4.5}$$

where the prefactors were taken to be of order one. Note that due to the presence of the exponential factor involving T_β the G^a dependence through $\Theta_\alpha(\tau, G)$ would be subleading if contributions entirely due to D1 instantons are present. This is the case since $\mathrm{Im}T_\alpha > 1$ parametrizes large volumes of four-cycles in a self-consistent analysis. However, as can be seen by analyzing the F theory underlying the $O3/O7$ orientifold, there is no calibration which can support D1 instantons alone [32–34].

Let us now turn to potentials for the R-R two-forms scalars entirely generated by effects of D1 instantons. These couple to the complex coordinates G^a defined in (3.33). In analogy to the discussion of D3 instantons one expects these to generate a superpotential of the form

$$W_{\rm D1} = \sum_a B_a e^{-i m_a^b G^a}. \tag{4.6}$$

This form is not surprising since nonperturbative corrections are necessarily weighted by the exponential of the action of the corresponding instanton. Hence, as in (3.22) D1 instanton contributions on vanishing cycles contain an exponential factor of G^a. A study of explicit examples will reveal if the coefficient functions B_a are indeed nonvanishing for the D-brane configurations under consideration.

A detailed investigation of the superpotential (4.6) has been carried out for type I string theory on a Calabi-Yau manifold in [31]. Type I strings can be viewed as an orientifold setup with O9 planes and D9-branes. In this case the conditions for a nonvanishing superpotential (4.6) are known explicitly. One expects that the analysis of [31] can be extended to the other orientifolds such that a superpotential (4.6) will be generated for part or all of the moduli G^a. For example, explicit computations of such a D1 superpotential including multi-instanton contributions have been recently carried out in [39].

B. Gaugino condensates on D5-branes

In the following we comment on an interesting further source, which induces the desired axion potentials. More precisely, let us consider space-time filling D5-branes which wrap the small cycles in the compact Calabi-Yau manifold. Such branes can be supersymmetric and consistently included in orientifold setups with O5 planes. In the presence of appropriate orientifold planes the tadpoles can be canceled yielding a stable configuration [30]. Alternatively, one might try to include pairs of D5 and anti–D5-branes wrapping two-cycles in the same homology class [61]. Even though such configurations are only metastable and will break supersymmetry, they can be sufficiently long-lived to accommodate our universe.

For concreteness, let us first focus on a stack of M D5-branes wrapping a small S^2 resolving the conifold singularity as in Sec. III C. The gauge theory on the space-time filling D5-branes is a pure $U(M)$ Yang-Mills theory with $\mathcal{N} = 1$ supersymmetry. At low energies this gauge theory will be strongly coupled leading to gaugino condensation. The gauge coupling is given by the complex field $G = c - |t|/g_s$ already defined in (3.21). The effective theory for the gaugino condensate S admits a superpotential of Veneziano-Yankielowicz form

$$W_{\rm VY} = -iGS + \frac{1}{2\pi i} MS(\log(S/\Lambda_0^3) - 1), \tag{4.7}$$

where Λ_0 is the cutoff scale. The extrema of $W_{\rm VY}$ correspond to the vacua of the gauge theory. Eliminating S the effective superpotential is of the form

$$W_{\rm D5} = \Lambda_0^3 e^{-i(G/M)}. \tag{4.8}$$

[8] An explicit compactification, where such a superpotential is induced, can be found in Ref. [60].

The superpotential has a dependence on the field G similar to the one arising from D1 instantons (4.6). However, there is a crucial factor of $1/M$ appearing in the exponential. In the axion potential (4.1) we thus identify $\mu = 1/M$ where μ is appearing in the cosine multiplying the canonically normalized axion.

It is important to note that such a result has to be treated with care. The factor $1/M$ should not spoil the 2π periodicity of the axions. This can be achieved by replacing the potential (4.1) with

$$V_{\text{eff}} = C + \sum_a \Lambda_a^4 \min_{n^a \in \mathbb{Z}} \left(1 - \cos\left[\frac{(\theta^a/f_a) + 2\pi n^a}{M^a}\right]\right), \quad c \tag{4.9}$$

for gaugino condensates of an $U(M^a)$ gauge theory on the ath stack of D5-branes. That such a form is indeed obtained by analyzing the confining gauge theories using string duality was explicitly shown in Ref. [38]. The form (4.9) implies that the field range of the canonically normalized axions θ^a will remain limited by the value of the axion decay constants alone. Considering c^a and θ^a in the intervals (2.5) the potential (4.9) is again identical to (4.1) with $\mu^a = 1/M^a$. Remarkably, independent of the value of the axion decay constants the factors $1/M^a$ can push the smooth maximum of the cosine out of the accessible field range of θ^i (compare Fig. 1 with Fig. 2). For $M^a > 3$ the potential becomes approximately quadratic in the complete field range (2.5) of θ^i and we can apply Eqs. (2.17) and (2.18) for all values of θ^i.

Since this seems an interesting opportunity to built explicit models for chaotic inflation, let us briefly comment on the computation of (4.9) in Refs. [38,61]. The authors consider stacks of several D5 (and anti–D5-branes) in the same homology class on a small two-cycle. This configuration can be pushed through a conifold transition. In this process the branes get replaced by three-form fluxes on small three-spheres S^3 with complex structure modulus S replacing the gaugino bilinear [35]. In the dual picture the superpotential and scalar potential are entirely determined by the local geometry. It can be given in terms of the dual prepotential $\hat{\mathcal{F}}(S)$ as

$$W = \int G_3 \wedge \Omega(S) = -iGS + M\partial_S \hat{\mathcal{F}}(S), \tag{4.10}$$

where G_3 is the complex NS-NS and R-R three-form flux. In (4.10) the prepotential takes the form $\hat{\mathcal{F}} = -\frac{1}{2\pi i} S^2 \log(S/\Lambda_0^3) + f_{\text{geom}}(S)$, with $f_{\text{geom}}(S)$ encoding further corrections to (4.7) given by the local geometry. Eliminating S in the local vacuum allows explicitly computing (4.9) and determining Λ_a [38]. Note that in order to fully embed these local results into a global scenario further work remains to be done. However, combining available techniques for geometric transitions with the analysis of the axion decay constants and scales of Sec. III provides interesting possibilities to establish explicit scenarios.

Before moving on to the $\mathcal{N} = 1$ orientifold scenarios, let us stress the points we still need to address in order to make our scenarios consistent. Having analyzed the axion decay constants encoding the kinetic terms and the scalar potential due to nonperturbative D-brane effects, it remains to discuss two important issues:

(a) It has to be ensured that all nonaxionic moduli are stabilized at the desired scales.

(b) It has to be true that the axions are the only fields which are relevant during inflation. In particular, the nonaxionic scalar partners in the supermultiplets of the axions should not interfere with their dynamics during inflation.

Both of these requirements are hard to address in general and are not necessarily satisfied in many string compactifications [18]. First steps in establishing a consistent scenario for a class of $\mathcal{N} = 1$ orientifold compactifications will be made in the next section.

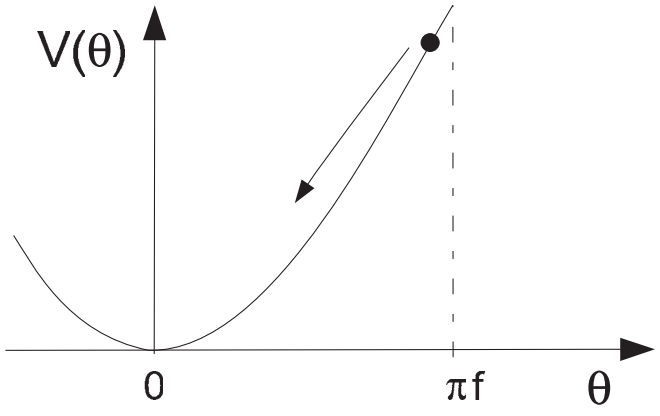

FIG. 2. Potential for one axion field θ with $M > 3$.

V. AXION INFLATION IN $\mathcal{N} = 1$ ORIENTIFOLDS

In the following we embed the scenario outlined in previous sections into an $\mathcal{N} = 1$ orientifold compactification. We begin with the study of the four-dimensional effective $\mathcal{N} = 1$ theory obtained by compactifying type IIB string theory on a Calabi-Yau orientifold with O3/O7 planes in Sec. V A. The $\mathcal{N} = 1$ data are expressed as a function of the underlying $\mathcal{N} = 2$ prepotential and hence inherit the α' corrections discussed in Sec. III. This will be illustrated for a toy model with N conifolds in Sec. V B. Combined with the superpotentials of Sec. IV an approximate evaluation of the potential allows us to comment on axion inflation in these $\mathcal{N} = 1$ scenarios.

A. The four-dimensional $\mathcal{N} = 1$ effective theory

Let us discuss the $\mathcal{N} = 1$ effective theory in more detail. As above, the compact Calabi-Yau manifold is

denoted by Y. In order to define the orientifold projection we demand that Y admits a holomorphic, isometric involution σ. The full orientifold action is given by

$$\mathcal{O} = (-)^{F_L}\Omega_p\sigma^*, \qquad \sigma^*J = J, \qquad \sigma^*\Omega = -\Omega, \tag{5.1}$$

where Ω_p is the world-sheet parity, F_L is the left moving fermion number and σ^* is the pullback of σ acting on forms. In (5.1) we have also displayed the action of σ on the Kähler form J and holomorphic three-form Ω of Y. J is invariant due to the fact that σ is holomorphic and isometric. The negative sign for the action on Ω implies that we are considering scenarios with $O3$ and $O7$ planes.

In order to determine the $\mathcal{N} = 1$ spectrum we note that σ splits the cohomologies into positive and negative eigenspaces. In particular, the cohomology of two-forms splits as

$$H^{(1,1)} = H^{(1,1)}_+ \oplus H^{(1,1)}_-, \tag{5.2}$$

with dimensions $h^{(1,1)}_+$ and $h^{(1,1)}_-$. Accordingly, the basis of $H^{(1,1)}$ can be split into $\omega_A = (\omega_\alpha, \omega_a)$ with ω_α being a basis of the positive eigenspace in (5.2) and ω_a being a basis in the negative eigenspace of (5.2). Note that the Kähler form J lies in the positive eigenspace of (5.2) as indicated in (5.1). In contrast, the NS-NS B field has to transform as $\sigma^*B_2 = -B_2$ in order to remain in the spectrum and thus lies in the negative eigenspace of (5.2). Hence, in the basis $\omega_A = (\omega_\alpha, \omega_a)$ we have to expand

$$\begin{aligned} J &= v^\alpha\omega_\alpha, \qquad & B_2 &= b^a\omega_a, \\ t^a &= -b^a, \qquad & t^\alpha &= iv^\alpha, \end{aligned} \tag{5.3}$$

where $t^A = (t^\alpha, t^a)$ are the complexified Kähler structure deformations introduced in (3.8). We find that the negative two-cycles dual to ω_a have vanishing geometric volume $v^a = 0$, but are supported by a nontrivial B field. The positive two-cycles dual to ω_α have geometric volume v^α and vanishing B field.

The four-dimensional $\mathcal{N} = 1$ effective theory obtained by compactifying on the Calabi-Yau orientifold Y/σ was derived in [40,41]. It was shown in [40] that the scalar modes arising from B_2 as well as C_2, C_4 combine together with the Kähler structure deformations into chiral $\mathcal{N} = 1$ multiplets. The complex scalars in the chiral multiplets are the dilaton-axion $\tau = C_0 + ie^{-\phi}$, the purely axionic fields G^a already given in (3.33) and the complexified Kähler moduli T_α. Explicitly, we define

$$G^a = c^a + ie^{-\phi}\,\mathrm{Re}t^a, \qquad T_\alpha = -\rho_\alpha + ie^{-\phi}\,\mathrm{Re}\mathcal{F}_\alpha, \tag{5.4}$$

where $\mathrm{Re}t^a = -b^a$ and the axions c^a and ρ_α are defined as

$$\begin{aligned} c^a &= \frac{1}{2\pi}\int_{\mathcal{C}_a} C_2 - C_0B_2, \\ \rho_\alpha &= \frac{1}{2\pi}\int_{\tilde{\mathcal{C}}^\alpha} C_4 - B_2\wedge C_2 + \frac{1}{2}C_0B_2\wedge B_2. \end{aligned} \tag{5.5}$$

In (5.5) the cycles $\mathcal{C}_a \in H^-_2$ are dual to ω_a and $\tilde{\mathcal{C}}^\alpha \in H^+_4$ are four-cycles transforming with a positive sign under σ^*. At the end of Sec. III D, the fields G^a have been identified as the correct couplings to D1 instantons for two-cycles of vanishing geometrical volume. The coordinates T_α provide the correct couplings to D3 instantons wrapping four-cycles in Y.

Because of the $\mathcal{N} = 1$ supersymmetry of the four-dimensional effective action the metric on the field space spanned by τ, G^a and T_α is necessarily Kähler. This implies that it takes the form $G_{I\bar{J}} = \partial_{M^I}\partial_{\bar{M}^J}K_\mathrm{q}$, where $M^I = (\tau, G^a, T_\alpha)$ are the complex coordinates and $K_\mathrm{q}(M, \bar{M})$ is the Kähler potential. The Kähler potential for the complex scalars M^I is shown to be [34,41]

$$\begin{aligned} K_\mathrm{q}(\tau, G, T) = -2\ln[ie^{-2\phi}(2(\mathcal{F} - \bar{\mathcal{F}}) \\ - (\mathcal{F}_A + \bar{\mathcal{F}}_A)(t^A - \bar{t}^A))]. \end{aligned} \tag{5.6}$$

It is important to note that despite the obvious similarity to (3.9) the Kähler potential K_q has a more complicated functional dependence on its complex coordinates M^I. For a general prepotential $\mathcal{F}$ it is impossible to explicitly write K_q as the function of τ, G^a, T_α. This is due to the fact that one would need to express $e^{-2\phi}\mathcal{V}$ as a function of $e^{-\phi}\mathrm{Re}t^a$ and $e^{-\phi}\mathrm{Re}\mathcal{F}_\alpha$ appearing in the $\mathcal{N} = 1$ coordinates (5.4). This functional dependence is nonpolynomial and can only be determined explicitly in very specific examples.[9] Nevertheless, one can derive the Kähler metric by using the underlying $\mathcal{N} = 2$ special geometry [41] or the work of Hitchin [62] as done in [63]. Also the derivatives of the Kähler potential (5.6) are known as a function of the prepotential [41]. The Kähler metric and its inverse as well as the first derivatives of the Kähler potential are summarized in Appendix B.

Before moving on to the $\mathcal{N} = 1$ axion decay constants let us stress again that K_q still contains the α' corrections inherited from the underlying $\mathcal{N} = 2$ theory. It does not, however, depend on the axions C_0, c^a, ρ_α. This can be traced back to the fact that no corrections due to D-branes are included in the expression (5.6), which is in accord with the discussion in Secs. III C and III D. Clearly, in addition to the inherited $\mathcal{N} = 2$ corrections one expects further $\mathcal{N} = 1$ corrections to appear. However, it seems unlikely that these will cancel the $\mathcal{N} = 2$ effects. We thus have some confidence that the appearance of large axion decay constants is not restricted to the $\mathcal{N} = 2$ setups outlined in Sec. III.

[9]This is equivalent to the problem of solving the attractor equations for $\mathcal{N} = 2$ black holes.

Let us discuss the axion decay constants for the R-R two-form axions c^a in (5.4) in more detail. Using $K_{\rm q}$ these are simply given by the Kähler metric

$$\frac{f_{ab}^2}{M_P^2} = 2\partial_{G^a}\partial_{\bar{G}^b}K_{\rm q}. \tag{5.7}$$

As a consistency check we can derive f_{ab} for the classical prepotential (3.10). This was done in Ref. [40] (Appendix C.1) and reproduces the leading classical result discussed in Sec. III B. In the special cases such as when $h_+^{(1,1)} = 1$, i.e. in case there is only one $T_1 \equiv T$, the computation of the classical $K_{\rm q}$ simplifies significantly and (5.6) reduces to

$$K_{\rm q} = -\ln i(\tau-\bar{\tau}) - 3\ln i\Big(T-\bar{T}-\frac{C_{ab}(G-\bar{G})^a(G-\bar{G})^b}{2(\tau-\bar{\tau})}\Big), \tag{5.8}$$

where C_{ab} is a positive definite integral matrix given by the triple intersection $C_{ab} = -\int \omega\wedge\omega_a\wedge\omega_b$, with $\omega \in H_+^{(1,1)}$ corresponding to the one modulus T. It is now straightforward to compute f_{ab} and compare the result with the $\mathcal{N}=2$ counterpart of Sec. III B.

Recall that we found in Secs. III C and III D that in order to obtain large axion decay constants we need to move close to singular points in the moduli space. In the vicinity of a singularity the α' corrections in the Kähler potential $K_{\rm q}$ become important and have to be included.

B. A simplistic N conifold toy model

In this last part of Sec. V we will study an orientifold scenario of a simplified toy model with $2N$ resolved conifold singularities. The aim of this section is to illustrate that the $\mathcal{N}=1$ effective theory can be evaluated explicitly including the simple logarithmic $\mathcal{N}=2$ corrections near the singularities. This allows the illustration of some of the features necessary to model N-flation. However, the reader should not consider this as an explicit construction, but rather as support of our general arguments that the outlined scenarios provide a promising possibility to model axion inflation.

Let us start with a compact geometry with a set of $2N$ conifold singularities resolved by two-spheres S_i^2, S_{-i}^2, where $i=1\ldots N$. As in Sec. III D, the blown-up spheres are supported by a geometric volume and NS-NS B field. We assume that there is an orientifold projection mapping[10]

$$\sigma S_i^2 = S_{-i}^2, \qquad i=1\ldots N. \tag{5.9}$$

The two-forms associated to the two-cycles (S_i^2, S_{-i}^2) are denoted by $(\tilde{\omega}_i, \tilde{\omega}_{-i})$. To obtain invariant and anti-invariant forms these are combined as

$$\omega_i^+ = \tilde{\omega}_i + \tilde{\omega}_{-i}, \qquad \omega_i^- = \tilde{\omega}_i - \tilde{\omega}_{-i}, \tag{5.10}$$

with $\omega_i^\pm$ transforming with a plus or minus sign under the orientifold involution σ^*. The corresponding special coordinates are likewise given by

$$t_+^i = \tfrac{1}{2}(t^i + t^{-i}) = i v^i, \qquad t_-^i = \tfrac{1}{2}(t^i - t^{-i}) = -b^i, \qquad t_{\rm R} = i(R/\ell_s)^2, \tag{5.11}$$

where t^i, t^{-i} are the complexified Kähler moduli corresponding to the resolved conifolds and $t_{\rm R}$ parametrizes the larger cycles of radius R. Note that in (5.11) we have already applied the orientifold constraint (5.3) to express $t_\pm^i$ through the B field and Kähler form alone. In other words, the orientifold projection enforces that the volumes of S^i and S^{-i} are identical, $v^i = v^{-i}$, while the B fields have the opposite sign, $b^i = -b^{-i}$.

In order to derive the metric on the $\mathcal{N}=1$ field space, we will consider the simple prepotential (3.28) written as

$$\mathcal{F} = -\frac{1}{3!}t_{\rm R}^3 + \frac{1}{2!}\sum_{i=-N}^{N}(t^i)^2[t_{\rm R} - i\log t^i], \tag{5.12}$$

where the sum does not include the term with $i=0$. Using the coordinate transformation (5.11), $\mathcal{F}$ can be expressed in terms of $t_\pm^i$, $t_{\rm R}$, and hence v^i, b^i and R, as

$$\mathcal{F} = -\frac{1}{3!}t_{\rm R}^3 + t_{\rm R}\sum_{i=1}^{N}[(t_+^i)^2 + (t_-^i)^2] - \frac{i}{2}\sum_{i=1}^{N}(t_+^i + t_-^i)^2 \times \log(t_+^i + t_-^i) - \frac{i}{2}\sum_{i=1}^{N}(t_+^i - t_-^i)^2\log(t_+^i - t_-^i). \tag{5.13}$$

This explicit ansatz for $\mathcal{F}$ should be understood as our main simplification in the study of these $\mathcal{N}=1$ orientifold models with resolved conifold singularities. In general, there will be further perturbative and nonperturbative corrections to the $\mathcal{N}=1$ Kähler potential depending on b^i, v^i and R which are only suppressed in certain regimes of the moduli space. In particular, the prepotential (5.13) contains contributions from two rather extreme regimes: the large volume corrections in R, and the logarithmic corrections in v^i, b^i due to the singularity. However, we cannot make R extremely large, nor v^i, b^i extremely small as we have already discussed in Sec. III. Let us note that for a given Calabi-Yau geometry the $\mathcal{N}=2$ prepotential can be computed much more explicitly than (5.13). However, also additional perturbative $\mathcal{N}=1$ corrections might alter the precise form of the effective theory. These perturbative corrections are believed not to depend on the axions. Hence, in case they do not cancel the $\mathcal{N}=2$ effects, an effective theory with light axions might still be accessible. Therefore, even though the following analysis appears

[10]The local geometry of this setup is similar to the T dual of the O5 orientifold geometries discussed, for example, in Ref. [39].

rather explicit, its results should be interpreted with care and further study will be required to gain a solid picture.

For a given prepotential the $\mathcal{N}=1$ Kähler coordinates are computed using (5.4). Clearly, the complex dilaton τ and the coordinates

$$G^i = c^i + ie^{-\phi}\,\mathrm{Re}t^i_- = c^i - ie^{-\phi}b^i \tag{5.14}$$

do not depend on the form of the prepotential. In our setup, we denote by T_R the coordinates corresponding to the larger cycles of radius R. Here R is not an index, but we will later work in a toy model with several T_R of the same size and include appropriate factors to label this degeneracy. The definition of T_R depends on the prepotential (5.13) in a rather simple way, since we only kept the classical terms in $\mathcal{F}$. However, the Kähler coordinates T_i associated with v^i contain the terms $\mathrm{Re}(\partial\mathcal{F}/\partial t^i_+)$ and hence will receive α' corrections form the logarithmic corrections. Using the prepotential (5.13) in (5.4) we derive

$$\begin{aligned} T_\mathrm{R} &= -\rho_\mathrm{R} + ie^{-\phi}\Big(\frac{1}{2}(R/\ell_s)^4 - \sum_j[(v^j)^2 - (b^j)^2]\Big),\\ T_i &= -\rho_i + ie^{-\phi}(-2(R/\ell_s)^2 v^i + v^i\\ &\quad + [-2b^i\arg(t^i) + 2v^i\log|t^i|]), \end{aligned} \tag{5.15}$$

where we abbreviated $t^i = -b^i + iv^i$. Finally, the Kähler potential K_q can be evaluated using (5.13). This is straightforward, since the quantum corrected volume (3.29) only depends on v^i and the absolute value of t^i. The B field b^i appears only quadratic and the sign flip of b^i in the last two terms of (5.13) only yields a factor of 2. The expression for K_q is simply

$$\begin{aligned} K_\mathrm{q} = -2\log\Big[e^{-2\phi}\Big(&\frac{4}{3}(R/\ell_s)^6 - 8(R/\ell_s)^2\sum_i(v^i)^2\\ &+ 4\sum_i(|t^i|^2\log|t^i| + (v^i)^2)\Big)\Big]. \end{aligned} \tag{5.16}$$

The combination in the brackets is precisely $e^{-2\phi}\mathcal{V}$, where $\mathcal{V}$ is the quantum volume of Y, and should be compared to the earlier expression (3.29) in Sec. III D. Expressing K_q as a function of the $\mathcal{N}=1$ Kähler coordinates τ, G^i, T_i and T_R is considerably harder. In order to do that we would have to solve $\tau = C_0 + ie^{-\phi}$, as well as Eqs. (5.14) and (5.15) for $e^{-\phi}$, R, v^i, b^i and insert the result into (5.16). Fortunately, we will not need an the explicit expression for K_q as a function of the $\mathcal{N}=1$ coordinates.

At least in principle, we are now in the position to compute the $\mathcal{N}=1$ scalar potential (4.2) induced by a superpotential due to background fluxes and nonperturbative effects. A nonperturbative superpotential of the form (4.4) and (4.5) can potentially stabilize the moduli T_R, T_i and G^i. The flux superpotential depends on the complex dilaton τ and the complex structure moduli. Altogether, the superpotential is a function of all bulk moduli of the $\mathcal{N}=1$ effective theory and one expects that the scalar potential will admit minima in which all fields can settle to their vacuum values. First the heavier fields will roll quickly down to their minima, while only later the light degrees of freedom will follow. If the axions are indeed the lightest fields and the other fields are fixed to the desired values, the effective theory could allow the N-flation scenario of Sec. II.

Given the $\mathcal{N}=1$ characteristic data it is still not straightforward to explicitly check if our setup admits an epoch of axion inflation. The reason for this is of a technical nature. First, the general expressions for the derivatives of the Kähler potential summarized in Appendix B are complicated functions of the prepotential (5.13). In this work, we will simplify the computations and evaluate the potential using the leading results (3.7) for the axion decay constants and hence the Kähler metric.[11] Second, since the fields τ, G^i, T_i and T_R mix both in the Kähler potential as well as in the superpotential it is hard to determine an effective theory for any subset of fields. In order to nevertheless proceed, we will have to assume, similar to the approach of KKLT [57], that we can stabilize the fields in two steps. In a first step, the dilaton and complex structure moduli are stabilized using background fluxes. Later on, within the effective theory for the fields T_R, T_i and G^i, we will address the stabilization of the remaining bulk fields in an uplifted nonsupersymmetric vacuum. One expects that this will give at least a qualitative picture of the N conifold toy model.

Let us begin by utilizing the flux superpotential to fix the complex structure moduli z^k and the dilaton τ such that

$$D_{z^k}W = 0, \qquad D_\tau W = 0. \tag{5.17}$$

As already mentioned above, our setup is so complicated that the second condition in (5.17) will receive corrections through the derivative of the Kähler potential (B3). However, we will assume that we can choose the fluxes such that these corrections are subleading and we can fix τ nevertheless to an appropriate value. The effective potential for the remaining fields is obtained by inserting (5.17) into the general expression (4.2). Setting $S^A = (G^i, T_\mathrm{R}, T_i)$ one finds

$$V_\mathrm{eff} = e^K(K^{S^A\bar{S}^B}D_{S^A}W\overline{D_{S^B}W} - 3|W|^2) + V_\mathrm{up}. \tag{5.18}$$

As in the scenarios of Refs. [57,64,65], we will add an uplift term of the form $V_\mathrm{up} = \kappa/\mathcal{V}^\alpha$ to $V(S)$, where κ is a tunable constants and α is a rational number of order unity depending on the source of the uplifting energy. By fine-tuning κ, the positive contribution V_up can be utilized to lift a vacuum at negative vacuum energy to a very small positive cosmological constant. Typically, this uplifting does not significantly change the location of the vacua

[11] For a small number of fields it can be checked numerically that this is a reasonable approximation. It becomes more accurate with increasing volume $\mathcal{V}$.

and the local shape of the potential close to the lifted minimum. Therefore, we will set the uplifting term in (5.18) to be zero in order to simplify our analysis. Clearly, in order to make contact to Sec. II it eventually has to be included and appropriately fine-tuned such that the cosmological constant C in (2.6) is negligible during inflation.

Following Ref. [57] and using (4.4) and (4.5), the effective instanton superpotential in (5.18) takes the form

$$W = W_0 + \sum_{j=1}^{N} e^{iT_j} + e^{iT_{\rm R}} + e^{-1/g_s} \sum_{j=1}^{N} e^{-iG^j} e^{iT_{\rm R}}, \quad (5.19)$$

where W_0 is a constant depending on the values of the background fluxes. We observe that only the last term in (5.19) depends on the R-R two-form axions. At small string coupling and $\mathrm{Im}T_{\rm R} > 1$ this term is strongly suppressed in comparison to the other contributions in W. Since the masses of the fields are weighted by the instanton action appearing in the superpotential, this simple observation leads us to expect that the masses of the fields in G^i might be lower than the masses of $T_{\rm R}$, T_i. Moreover, in the absence of the G^i dependent term in W, the axions $\mathrm{Re}G^i = c^i$ are actually massless. However, due to the b^i dependent corrections to the Kähler potential the fields $\mathrm{Im}G^i = -b^i/g_s$ can acquire a mass even if they do not appear in the superpotential. As we will see below, they can already be stabilized to their vacuum in the absence of the G^i dependent term in W.

In the following we numerically investigate our simple toy model in more detail. Our goal is to show that the included string world-sheet corrections alter the vacuum structure of the theory and can result in a theory of light axions. This should be viewed as a qualitative result, since the precise values presented in the following are only valid with our assumptions and an appropriate fine-tuning. For our numerical example, we will tune the fluxes such that $g_s = 0.16$ and $W_0 = -0.12$. Moreover, we will consider a setup of 100 larger cycles of identical radius R. We assume that there are 8 different resolved conifold singularities coupling to each of these cycles. The orientifold projection should identify these pairwise, such that we have 400 conifold pairs and $N = 400$ associated axions from the R-R two-form. In order to get a qualitative picture of the resulting potential we will restrict our study to a model with effectively four real fields: the radius R of the larger cycles and the three fields associated to each conifold pair

$$v \approx v^i, \qquad b \approx b^i, \qquad c \approx c^i, \quad (5.20)$$

where v, b parametrize the resolving volumes and B fields and c are the R-R two-form axions.[12] As in Sec. III D, it is important to also carefully keep track of factors labeling the degeneracy of the large cycles and the associated conifold pairs.

In the computation of the effective potential (5.18) we use the leading results for the Kähler metric (3.7), the superpotential (5.19) as well as the derivatives of the Kähler potential (B3). Explicitly, one computes

$$\begin{aligned} K_{T_{\rm R}} &= 4ig_s(R/\ell_s)^2\mathcal{V}^{-1}, \qquad K_{T_j} = 4ig_s v\mathcal{V}^{-1}, \\ K_{G^j} &= -4ig_s(-2(R/\ell_s)^2 b + b + [2v\arg(-b+iv) \\ &\quad + b\log(b^2+v^2)])\mathcal{V}^{-1}. \end{aligned} \quad (5.21)$$

In a next step the effective potential $V(R, v, b, c)$ can be minimized numerically. This yields an anti–de Sitter minimum at $\langle R\rangle = 1.208$, $\langle v\rangle = -0.1225$, $\langle b\rangle = 0.2564$ and $\langle c\rangle = 0$, which we have plotted in Appendix C, Figs. 5–7. Inserting these vacuum values for R, v, b and c into the Kähler coordinates (5.14) and (5.15), this minimum corresponds to[13]

$$\begin{aligned} \langle \mathrm{Im}T_{\rm R}\rangle = 7.924, \qquad \langle \mathrm{Im}T_j\rangle = 12.039, \\ \langle G^j\rangle = 0 - 1.603i. \end{aligned} \quad (5.22)$$

We observe that these vacuum values are in the field range consistent with the assumption that D-brane effects are subleading in the Kähler potential. The respective D1- and D3-instanton actions are sufficiently suppressed by $-\mathrm{Im}G^j$ and $\mathrm{Im}T_{\rm R}$, $\mathrm{Im}T_j$. It is also straightforward to compute the string-frame volume in (5.16) at this minimum, $\mathcal{V} = 205.6$. This is not extremely small, which implies that also the axion decay constant of each R-R two-form axion c is not expected to be extremely close to the Planck scale. Explicitly, the axion decay constants are computed by inserting the vacuum values (5.22) into Eq. (3.7) for C_2 yielding

$$f_{C_2} = 0.02 M_P. \quad (5.23)$$

Clearly, this is too small to match the cosmological data. However, f_{C_2} can made larger by fine-tuning the value of g_s, W_0 and considering a more sophisticated geometry. For the following qualitative analysis the precise value of f_{C_2} will not be directly relevant.

Let us now focus on the coordinates G^j containing the R-R two-form axions c. In order to get a picture of the axion potential, we first have to express $V_{\rm eff}(b, c, v, R)$ as a function of the $\mathcal{N} = 1$ Kähler coordinates T_i, $T_{\rm R}$ and G^i such that $V_{\rm eff}(T_i, T_{\rm R}, G^j)$. This can be only done numerically, since the coordinate definitions (5.14) and (5.15) are highly nonlinear. In a next step we fix T_R and T_i to their vacuum values (5.22) and plot the scalar potential as a function of the axion $\mathrm{Re}G^i = c$ and its nonaxionic partner $\mathrm{Im}G^i = -b/g_s$. The result is shown in Fig. 3. The poten-

[12] It is straightforward to also include the R-R four-form axions, but we will omit them for simplicity.

[13] Recall that we have 100 coordinates $T_{\rm R}$ with each 4 associated conifold pairs. This implies that the sums in the definition (5.15) of each $T_{\rm R}$ runs from $j = 1 \ldots 4$.

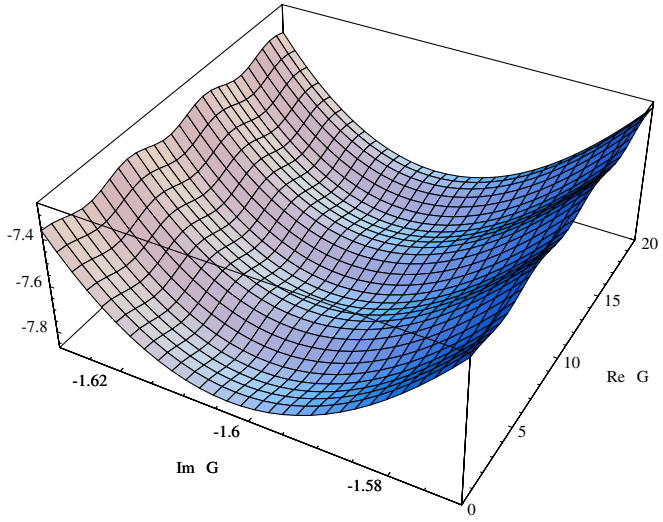

FIG. 3 (color online). Axion valley potential $V_{\rm eff}(G)$ (multiplied by 10^{11}) for axion ReG and its nonaxionic partner ImG, at fixed $\langle T_{\rm R}\rangle$ and $\langle T_i\rangle$.

tial takes the form of a valley. It is steep in the direction of the nonaxionic field b, but shallow in the direction of the axion c. This is very similar to the form of the axion valley used by Kallosh in Ref. [42] to model natural inflation in supergravity. In order to check that there is indeed a mass hierarchy between the axions ReG^i and their nonaxionic partners ImG^i, we can switch off the superpotential corrections depending on G^i by setting the last term in (5.19) to zero. The potential $V_{\rm eff}(G)$ at the vacuum values (5.22) for T_R and T_i is plotted in Fig. 4. We conclude that the minimum stabilization of ImG arises though corrections in the Kähler potential. The mass hierarchy between the axion and its nonaxionic partner is crucial for inflation. It ensures

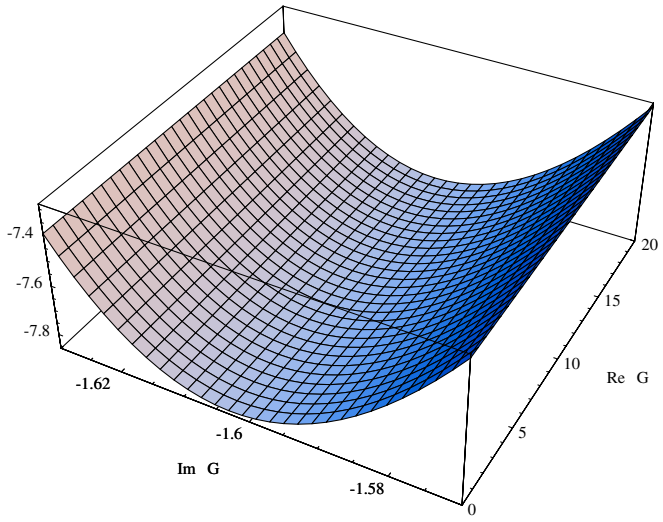

FIG. 4 (color online). Axion valley potential $V_{\rm eff}(G)$ in the absence of a superpotential depending on G. The axion direction ReG is flat, while ImG still has a minimum.

that first the nonaxionic field settles to its minimum leaving an effective theory for the light axion discussed in Sec. II. Note that this hierarchy is supported by the world-sheet corrections to the quantum volume $\mathcal{V}$.

Let us end this section by computing the axion potential near the minimum (5.22). For the canonically normalized R-R two-form axions $\theta^i = f_{C_2} c^i$ one finds

$$V_{\rm eff}(\theta^i)/M_P^4 = -7.86\cdot 10^{-11} + \sum_{i=1}^{N} 5.07\cdot 10^{-16} \times (1-\cos(50\theta^i/M_P)), \tag{5.24}$$

where we have taken all $N = 400$ axions to be of similar size $\theta \approx \theta^i$. The effective potential (5.24) is precisely of the form (2.6). However, note that our model was not sufficiently fine-tuned to obtain a pseudorealistic cosmology. First, the cosmological constant in (5.24) is still negative and has to be lifted to a small positive value by including the uplifting term $V_{\rm up}$ in the effective potential (5.18). As in Ref. [57], this additional term is independent of all R-R axions and will not change the location of the minimum significantly. Thus our discussion of the mass hierarchy still remains valid in the uplifted vacuum. Second, the axion decay constants (5.23) are still too small and we have to further improve our setup to find the desired cosmology. In any case, as already stressed above, the given form of the effective theory should only be interpreted qualitatively, since we worked with the simple prepotential (5.13) and applied several approximations in deriving $V_{\rm eff}$. Our analysis indicates that there might exist compactifications with a large number of axions as the lightest bulk moduli. A more intensive study of explicit models will reveal whether these scenarios are indeed suitable to obtain axion N-flation.

VI. CONCLUSIONS

In this paper we discussed the possibility of a string theoretical embedding of cosmological inflation driven by a large number of axions. Such scenarios use the fact that in the dimensional reduction of string theory to four spacetime dimensions a vast number of scalar fields arise in the low energy theory. In particular, we considered type IIB string theory on a compact Calabi-Yau manifold with many nontrivial two-cycles. To each of these two-cycles an axion from the R-R two-form and an axion from the R-R four-form can be associated. The effective theory for the axionic fields will sensitively depend on the values of the geometric moduli, i.e. the volume of the two-cycles, as well as the NS-NS B field. We have argued that a possible realization of axion inflation might only exist in special corners in the landscape of vacua. In these regimes various stringy effects become relevant and have to be included.

In recent years, $\mathcal{N} = 1$ type IIB compactifications with all volumes stabilized at scales much larger than string scale have been investigated intensively [30]. For vacua in

this regime of the field space, stringy corrections, such as wrapping stings and D-branes, play a subleading role in the derivation of the kinetic terms of the axions. Nonperturbative D-brane effects can induce a potential exponentially suppressed by the large volume of the cycles times the inverse of a small string coupling. We have argued that such scenarios are not suitable for axion inflation. They naturally admit very small axion decay constants which become even smaller if the number of axions increases. Moreover, due to the strong exponential suppression, the scale of inflation is typically too low for semirealistic scenarios. This conclusion led us to the consideration of manifolds with small or vanishing cycles.

In compactifications with cycles smaller than string scale the caveats of the large volume compactifications can be avoided. To obtain such geometries, we considered resolutions of singularities supported by a volume and an NS-NS B field. The standard example of such a blown-up singularity is the resolved conifold. In the vicinity of small cycles, new corrections will become relevant and alter the effective theory. We have discussed a subset of such corrections in $\mathcal{N} = 2$ compactifications of type IIB string theory on a Calabi-Yau manifold. Including the leading singular nonperturbative string world-sheet contributions, we have argued that the axion decay constants can take values close to the Planck scale also for scenarios with many axions. In $\mathcal{N} = 1$ compactifications further perturbative and nonperturbative corrections can become relevant and might alter the structure of the effective four-dimensional theory. However, in case these do not cancel the $\mathcal{N} = 2$ effects, large axion decay constants will remain accessible also in these less supersymmetric scenarios.

In addition to being close to the Planck scale, N-flation also requires the axion decay constants to be independent of the axions themselves. Corrections depending on the R-R axions are believed to arise only from nonperturbative D-brane effects. If the small cycles remain to be of finite size and we work at sufficiently small string coupling, D-brane corrections are subleading in the axion decay constants, since the instanton action is larger by a factor of the inverse string coupling. This will also be the case for vacua in an $\mathcal{N} = 1$ supergravity theory. In a consistent analysis, the vacuum values of the $\mathcal{N} = 1$ moduli fields have to be inside an appropriate field range to ensure that D-brane instantons are subleading in the Kähler potential encoding the kinetic terms of the scalar fields.

One of the remaining complications in realizing N-flation is to ensure that there indeed exists an effective theory for axions with appropriate masses during inflation. These masses have to be lower than the masses of their nonaxionic partner and other bulk moduli fields, but still sufficiently large to match the observed density perturbations during inflation. Again, this forces us to work away from the large volume regime, where at least some of the moduli masses are suppressed by large instanton actions. A careful investigation of the effective potential for the moduli is necessary to study the vacua of the theory. In this work we briefly discussed effective $\mathcal{N} = 1$ potentials arising from D instantons and gaugino condensates on space-time filling branes. We pointed out that for potentials induced by large rank gaugino condensates on D5-branes, the quadratic region of the axion potential can stretch over the entire accessible field range. In practice, D1 corrections arising through the prefactors of the D3 instantons are of particular interest. If these are the leading axion-dependent contributions to the $\mathcal{N} = 1$ superpotential, a mass hierarchy can be ensured due to the suppression by both the D1- and D3-instanton action.

To investigate the properties of the effective theory more explicitly, we discussed the embedding of axion inflation into $\mathcal{N} = 1$ Calabi-Yau orientifold compactifications with O3 and O7 planes. We showed that the $\mathcal{N} = 1$ characteristic data remain calculable even in the case that the nonperturbative $\mathcal{N} = 2$ string world-sheet corrections are included. Utilizing a flux superpotential together with a superpotential from nonperturbative D-brane effects a potential for all bulk moduli fields is generated. We illustrated that a theory of light axions could exist, if the axion dependence is suppressed in the superpotential and the nonaxionic partners of the axions in the $\mathcal{N} = 1$ chiral multiplet are stabilized due to the string world-sheet corrections to the Kähler potential. In an optimistic scenario, one can hope that such an effective theory of a large number of relatively light axions from the R-R forms will also survive further perturbative corrections.

Even though an explicit embedding of N-flation into string theory still remains to be constructed, the scenarios outlined and studied in this work might provide a promising route to achieve this goal. Likely, such an embedding will not solve intrinsic issues related to the fine-tuning of initial conditions in chaotic and natural inflation with many inflatons. However, it might provide a way to accommodate possible future observations of primordial gravitational waves in a string theoretic model.

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APPENDIX A: AXIONS IN $\mathcal{N} = 2$ CALABI-YAU COMPACTIFICATIONS

In this appendix we recall the effective action of type IIB string theory compactified on a Calabi-Yau manifold Y. Our analysis will not necessarily take place in the large volume limit, such that stringy corrections have to be taken into account. Clearly, we will not be able to incorporate all of these corrections into our analysis. However, for small string coupling g_s there is a regime in parameter space in which we have control over the effective theory as discussed in Secs. III C and III D. In particular, contributions due to D-branes in the type IIB theory are subleading for small g_s. Nevertheless, the four-dimensional physics can be corrected by contributions from string world sheets. In an $\mathcal{N} = 2$ compactification such α' corrections are encoded by a holomorphic prepotential $\mathcal{F}$ which can be expanded around the desired point in the moduli space. In general, the prepotential $\mathcal{F}$ is a function of the complexified Kähler structure deformations t^A defined in (3.8). We also introduce homogenous coordinates $X^{\hat{A}} = (X^0, X^A)$ and write

$$F(X^{\hat{A}}) \equiv (X^0)^2 \mathcal{F}(t^A), \qquad t^A = X^A/X^0. \tag{A1}$$

All four-dimensional $\mathcal{N} = 2$ data will be given as a function of the homogenous prepotential $F(X)$ and hence as a function of $\mathcal{F}(t)$.

Recall that compactifying type II string theory on a Calabi-Yau manifolds leads to an $\mathcal{N} = 2$ supergravity theory with $h^{(1,1)} + 1$ hypermultiplets. The scalars in these hypermultiplets are the complex scalars t^A given in (3.8), the lowest modes of the ten-dimensional dilaton and type IIB R-R forms C_0, C_2, C_4. More explicitly, this includes the universal hypermultiplet (ϕ, C_0, ρ_0, c^0), where ρ_0, c^0 are the duals of the four-dimensional two-form part in B_2, C_2. In addition there are $h^{(1,1)}$ hypermultiplets (t^A, ρ_A, c^A) with

$$\begin{aligned} c^A &= \frac{1}{2\pi}\int_{\mathcal{C}_A} C_2 - C_0 B_2, \\ \rho_A &= \frac{1}{2\pi}\int_{\tilde{\mathcal{C}}^A} C_4 - B_2 \wedge C_2 + \frac{1}{2} C_0 B_2 \wedge B_2, \end{aligned} \tag{A2}$$

where $\mathcal{C}_A$ and $\tilde{\mathcal{C}}^A$ are harmonic two- and four-cycles of Y. Note that we are slightly abusing the notation compared to (3.4), since c^A, ρ_A now also include corrections due to lower R-R forms. We combine (ρ_0, c^0) and (ρ_A, c^A) by writing ($\rho_{\hat{A}}$, $c^{\hat{A}}$) with $\hat{A} = 0 \ldots h^{(1,1)}$. Having identified the $\mathcal{N} = 2$ hypermultiplets we turn to their effective action and moduli space metric. From a Kaluza-Klein reduction one derives the effective Lagrangian [48]

$$\begin{aligned} \mathcal{L}^{(4)} = {} & (\partial D)^2 + G_{A\bar{B}} \partial t^A \partial \bar{t}^B + \tfrac{1}{4} e^{4D} (dC_0 \\ & - (\rho_{\hat{A}} \partial c^{\hat{A}} - c^{\hat{A}} d\rho_{\hat{A}}))^2 - \tfrac{1}{2} e^{2D} \mathrm{Im}\mathcal{M}_{\hat{A}\hat{B}} \partial c^{\hat{A}} \partial c^{\hat{B}} \\ & - \tfrac{1}{2} e^{2D} (\mathrm{Im}\mathcal{M})^{-1\hat{A}\hat{B}} (\partial \rho_{\hat{A}} - \mathrm{Re}\mathcal{M}_{\hat{A}\hat{C}} \partial c^{\hat{C}}) \\ & \times (\partial \rho_{\hat{B}} - \mathrm{Re}\mathcal{M}_{\hat{B}\hat{D}} \partial c^{\hat{D}}), \end{aligned} \tag{A3}$$

where $G_{A\bar{B}} = \partial_{t^A} \partial_{\bar{t}^B} K$ is the metric on the space of complexified Kähler structure deformations t^A and given in terms of the Kähler potential (3.9). The complex coupling matrix $\mathcal{M}_{\hat{K}\hat{L}}$ appearing in (A3) depends on t^A, $\bar{t}^A$ and is defined as

$$\mathcal{M}_{\hat{A}\hat{B}} = \bar{F}_{\hat{A}\hat{B}} + 2i \frac{(\mathrm{Im}F_{\hat{A}\hat{C}}) X^{\hat{C}} (\mathrm{Im}F_{\hat{B}\hat{D}}) X^{\hat{D}}}{X^{\hat{E}} (\mathrm{Im}F_{\hat{E}\hat{F}}) X^{\hat{F}}}, \tag{A4}$$

where $F_{\hat{A}\hat{B}} = \partial_{X^{\hat{A}}} \partial_{X^{\hat{B}}} F$. Finally, the Lagrangian (A3) contains the four-dimensional dilaton D defined in terms of the ten-dimensional dilaton ϕ according to

$$e^D = e^{\phi} \mathcal{V}^{-(1/2)}, \tag{A5}$$

where $\mathcal{V}(t, \bar{t})$ is given in (3.9).

Using the Lagrangian (A3) it is straightforward to read off the axion decay constants for the axions b^A, c^A and ρ_A. For the NS-NS B-field axions b^A we find

$$B_2: \ \frac{f_{AB}^2}{M_P^2} = \frac{1}{\pi} G_{A\bar{B}}, \tag{A6}$$

just as given in (3.7). The expression for the R-R two- and four-form axions c^A, ρ_A are more complicated:

$$\begin{aligned} C_2: \ \frac{f_{AB}^2}{M_P^2} = {} & -\frac{g_s^2}{2\pi \mathcal{V}} (\mathrm{Im}\mathcal{M}_{AB} \\ & + \mathrm{Re}\mathcal{M}_{A\hat{C}} (\mathrm{Im}\mathcal{M})^{-1\hat{C}\hat{D}} \mathrm{Re}\mathcal{M}_{\hat{D}B}), \end{aligned} \tag{A7}$$

$$C_4: \ \frac{f_{AB}^2}{M_P^2} = -\frac{g_s^2}{2\pi \mathcal{V}} (\mathrm{Im}\mathcal{M})^{-1AB}. \tag{A8}$$

These expressions appear to be different from the ones used in the main text (3.7). However, due to the underlying special geometry we can use

$$-2\mathcal{V}^{-1} \mathrm{Im}\mathcal{M}_{AB} = G_{A\bar{B}} + \ldots. \tag{A9}$$

In other words, the metric $G_{A\bar{B}}$ is the leading contribution to these axion decay constants and (A7) reduces to (3.7). Of course, the study of the axion decay constants performed in the main text can be repeated with the general expressions (A7).

APPENDIX B: THE $\mathcal{N} = 1$ KÄHLER METRIC

In this appendix we summarize some useful formulas allowing the derivation of the derivatives of the Kähler potential K_{q} given in (5.6). Our summary will follow Ref. [41]. Let us recall, for completeness, that K_{q} takes the form

$$K_{\rm q}(\tau, G, T) = -2\ln[ie^{-2\phi}(2(\mathcal{F} - \bar{\mathcal{F}}) - (\mathcal{F}_A + \bar{\mathcal{F}}_A)(t^A - \bar{t}^A))], \tag{B1}$$

with complex coordinates

$$\tau = C_0 + ie^{-\phi}, \qquad G^a = c^a + ie^{-\phi}\,{\rm Re}t^a, \qquad T_\alpha = -\rho_\alpha + ie^{-\phi}\,{\rm Re}\mathcal{F}_\alpha. \tag{B2}$$

In the equations (B1) and (B2), we have denoted by $\mathcal{F}_A = (\mathcal{F}_\alpha, \mathcal{F}_a)$ the derivatives of a general prepotential $\mathcal{F}$ with respect to the special coordinates t^α, t^a. Note that t^α and t^a are associated to the positive and negative two-cycles in the eigenspace of the orientifold projection (5.2).

With our conventions, the first derivatives of the Kähler potential (B1) are given by

$$K_\tau = -4ie^{-\phi}\,{\rm Im}(2\mathcal{F} - t^A\mathcal{F}_A)e^{K_{\rm q}/2}, \qquad K_{G^i} = -4ie^{-\phi}\,{\rm Im}\mathcal{F}_a e^{K_{\rm q}/2}, \qquad K_{T_\alpha} = 4ie^{-\phi}\,{\rm Im}t^\alpha e^{K_{\rm q}/2}. \tag{B3}$$

We note that the Kähler coordinates (B2) are functions of the real parts ${\rm Re}t^a$, ${\rm Re}\mathcal{F}_\alpha$, while the first derivatives of the Kähler potential are the imaginary part of $\mathcal{F}$ and its derivatives.

The Kähler metric and its inverse can be also expressed as functions of a prepotential $\mathcal{F}$. Let us denote $N^{\hat{a}} = (\tau, G^a)$, where the complex dilaton τ is identified with N^0. One has

$$\begin{aligned} K_{T_\alpha\bar{T}_\beta} &= -2e^{2D}({\rm Im}\mathcal{M})^{-1\alpha\beta}, \\ K_{T_\alpha\bar{N}^{\hat{a}}} &= 2e^{2D}({\rm Im}\mathcal{M})^{-1\alpha\beta}{\rm Re}\mathcal{M}_{\beta\hat{a}}, \\ K_{N^{\hat{a}}\bar{N}^{\hat{b}}} &= -2e^{2D}({\rm Im}\mathcal{M}_{\hat{a}\,\hat{b}} + {\rm Re}\mathcal{M}_{\hat{a}\alpha}({\rm Im}\mathcal{M})^{-1\alpha\beta}{\rm Re}\mathcal{M}_{\beta\hat{b}}), \end{aligned} \tag{B4}$$

with inverse

$$\begin{aligned} K^{T_\alpha\bar{T}_\beta} &= -\tfrac{1}{2}e^{-2D}({\rm Im}\mathcal{M}_{\alpha\beta} + {\rm Re}\mathcal{M}_{\alpha\hat{a}}({\rm Im}\mathcal{M})^{-1\hat{a}\,\hat{b}}{\rm Re}\mathcal{M}_{\hat{b}\beta}), \\ K^{N^{\hat{a}}\bar{N}^{\hat{b}}} &= -\tfrac{1}{2}e^{-2D}({\rm Im}\mathcal{M})^{-1\hat{a}\,\hat{b}}, \\ K_{T_\alpha\bar{N}^{\hat{a}}} &= -\tfrac{1}{2}e^{-2D}({\rm Im}\mathcal{M})^{-1\hat{a}\,\hat{b}}{\rm Re}\mathcal{M}_{\hat{b}\alpha}. \end{aligned} \tag{B5}$$

In these expressions $\mathcal{M}_{\hat{A}\,\hat{B}}$ is the complex matrix defined in (A4) and e^D is the four-dimensional dilaton given in (A5).

APPENDIX C: PLOTS OF MINIMUM

This appendix contains the plots of the minimum for the toy model with $N = 400$ conifold pairs discussed in Sec. V B (Figs. 5–7). The effective theory was evaluated using the prepotential (5.13) at $g_s = 0.16$, $W_0 = -0.12$. The numerically determined minimum is found at $\langle R\rangle = 1.208$, $\langle v\rangle = -0.1225$, $\langle b\rangle = 0.2564$ and $\langle c\rangle = 0$.

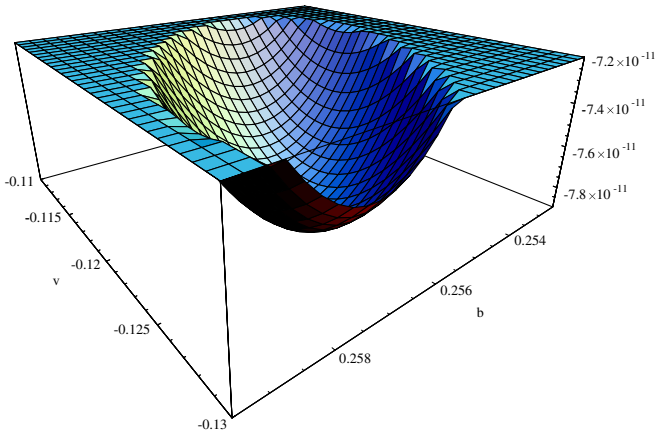

FIG. 5 (color online). Potential as function of conifold volume v and B field b. R, c are fixed to minimum.

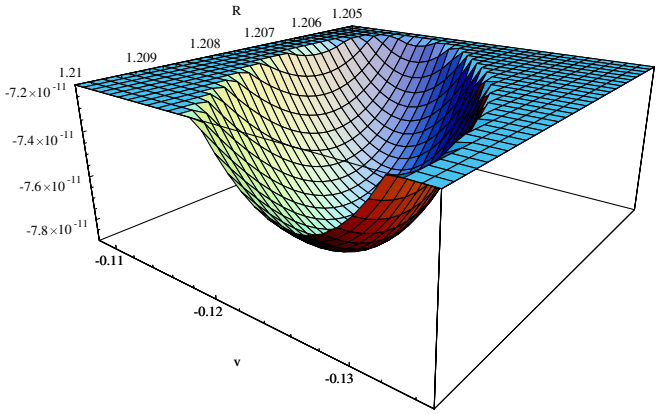

FIG. 6 (color online). Potential as function of radius R and conifold volume v. b, c are fixed to minimum.

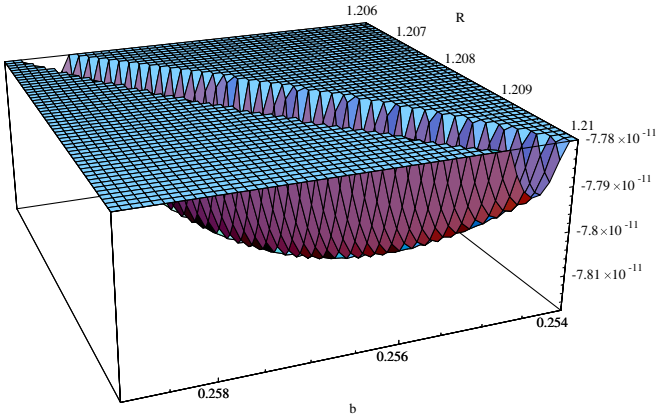

FIG. 7 (color online). Potential as function of radius R and conifold B field b. v, c are fixed to minimum.

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