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Soft-collinear effects in prompt photon production

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We extend next-to-leading logarithmic threshold and joint resummation for prompt photon production to include leading collinear effects. The impact of these effects is assessed for both fixed-target and collider kinematics. We find them in general to be small but noticeable.

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I. INTRODUCTION

The perturbative QCD description of many observables measured at colliders is plagued by large corrections arising from soft and collinear parton emissions, even for fairly generic kinematical conditions. For example, near threshold, large logarithmic corrections remain [1,2] after cancellation of singular virtual and real gluon contributions, their large size being a result of the nearby threshold restricting the real gluons to be soft. In terms of a (Mellin) variable N, in terms of which threshold is approached by $N \to \infty$, such large threshold corrections take the form ($L = \ln N$),

$$\alpha_s^i \sum_j^{2i} a_{ij} L^j, \tag{1}$$

where the a_{ij} depend in general on the process. Another example [3–8] is when an identified part F_P of a final state has acquired small transverse momentum by soft recoil (Q_T) against the remaining, unmeasured part of the final state. Then the perturbative expression for the differential cross section with respect to p_T of F_P takes again the form of Eq. (1), but with different coefficients a_{ij} and with $L = \ln b$, b being the impact parameter Fourier conjugate to Q_T.

Such large logarithmic corrections can be brought under control by all-order resummation, and there is much literature demonstrating the viability, where applicable, of threshold, recoil as well as their joint resummation, for a wide variety of observables. It is interesting to try to extend all-order control to classes of large terms beyond the logarithmic corrections. One such new set consists of numerically large constants ("π^2 terms") originating from the same infrared-sensitive regions of those Feynman diagrams that also produce the logarithms [9–11].

Another important class of potentially large terms, of soft-collinear origin, can be represented as

$$\alpha_s^i \sum_j^{2i-1} d_{ij} \frac{\ln^j N}{N}. \tag{2}$$

Their phenomenological importance was first demonstrated in Ref. [12] in which the leading terms $j = 2i - 1$ were also summed to all orders for Higgs production and the Drell-Yan process. The assessment of these terms was made more meaningful in the context of a complete next-to-next-to-leading order (NNLO) [13–19] calculation, and a consistent next-to-next-to-leading logarithmic (NNLL) threshold-resummed result [20]. It is not yet clear how to sum next-to-leading terms in (2).

In this paper we examine the impact of the leading terms in (2) for a single-particle inclusive observable, the p_T spectrum of prompt photons produced in hadronic collisions. We do this in the context of both a threshold [21–25] and joint [26–29] resummed calculation for this spectrum.

The paper is organized as follows. In Sec. II we review briefly the threshold and joint-resummed prompt photon p_T distribution. In Sec. III we describe and motivate our extension to include the leading $\alpha_s^k \frac{\ln^{2k-1} N}{N}$ terms. In Sec. IV we assess the numerical impact of these corrections, and we conclude in Sec. V.

II. THRESHOLD AND JOINT RESUMMATION FOR PROMPT PHOTON PRODUCTION

We consider the inclusive p_T distribution of prompt photons produced in hadron-hadron collisions at center of mass (c.m.) energy $\sqrt{S}$

$$h_A(p_A) + h_B(p_B) \to \gamma(p_c) + X, \tag{3}$$

where $h_{A,B}$ refers to the two incoming hadrons and X to the unobserved part of the final state. The lowest order QCD processes producing the prompt photon at partonic c.m. energy $\sqrt{s}$ are

$$\begin{aligned} q(p_a) + \bar{q}(p_b) &\to \gamma(p_c) + g(p_d), \\ g(p_a) + q(p_b) &\to \gamma(p_c) + q(p_d). \end{aligned} \tag{4}$$

The distance to threshold is customarily measured by the variable $1 - x_T^2$, where $x_T^2 = 4p_T^2/S$. At the parton level this distance is given by $1 - \hat{x}_T^2 = 1 - 4p_T^2/s$. Below we review the result for the joint-resummed prompt photon p_T distribution. At the end of this section we recall how the threshold-resummed result may be derived from it.

The joint resummation formalism for prompt photon production [26,27] implements the notion that, in the presence of soft QCD radiation with summed transverse momentum $\mathbf{Q}_T$ of soft recoiling partons, the actual transverse momentum produced by the hard collision is not $\mathbf{p_T}$ but rather $\mathbf{p'_T} = \mathbf{p_T} - \mathbf{Q_T}/\mathbf{2}$. Stated more precisely: in the context of a refactorization analysis [27] one can identify a short-distance process at c.m. energy Q that produces a prompt photon with momentum $\mathbf{p'_T}$ in a recoiling frame. One defines accordingly $\tilde{x}_T^2 = 4p_T'^2/Q^2$. The extreme situation $Q_T = 2p_T$ in which all transverse momentum is produced through soft recoil leads to a singularity in the short-distance process, which we avoid by imposing an upper limit $\bar{\mu}$ on Q_T [26]. A recently proposed extension [28] of joint resummation avoids this singularity.

The joint-resummed p_T distribution of prompt photons in hadronic collisions is written as

$$\begin{aligned}\frac{p_T^3 d\sigma^{(\text{resum})}_{AB\to\gamma+X}}{dp_T} &= \sum_{ij}\frac{p_T^4}{8\pi S^2}\int_C \frac{dN}{2\pi i} f_{i/A}(N,\mu_F) f_{j/B}(N,\mu_F)\\ &\times \int_0^1 d\tilde{x}_T^2 (\tilde{x}_T^2)^N \frac{|M_{ij}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} C^{(ij\to\gamma k)}(\alpha_s(\mu),\tilde{x}_T^2)\\ &\times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\Theta(\bar{\mu}-Q_T)\left(\frac{S}{4\mathbf{p}_T'}\right)^{N+1}\\ &\times \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \exp\left[E_{ij\to\gamma k}\left(N,b,\frac{4p_T^2}{\tilde{x}_T^2},\mu_F\right)\right].\end{aligned} \tag{5}$$

Let us explain each of the terms on the right-hand side of Eq. (5). The top line displays the moments of standard parton distribution functions, as well as the sum over initial state parton flavors. The next line contains the Mellin transform over the partonic scaling variable $\tilde{x}_T^2$ in the recoiling frame, the Born amplitudes, and the N- and b-independent hard virtual corrections summarized in $C^{(ij\to\gamma k)}$. The second to last line contains the integral over the recoil momentum of the soft partons, as well as a kinematic factor linking recoil and threshold effects. The last line contains the Sudakov exponentials from initial and final state partons, as well as soft wide-angle radiation in combined Mellin-impact parameter space.

As indicated in the last line of Eq. (5), large threshold and recoil logarithms, expressed through $\ln N$ and $\ln b$, can be resummed into an exponential form. The perturbative exponential moment dependence at next-to-leading logarithmic (NLL) accuracy is given by

$$\begin{aligned}E^{\text{PT}}_{ij\to\gamma k}(N,b,Q,\mu,\mu_F) &= E_i^{\text{PT}}(N,b,Q,\mu,\mu_F)\\ &+ E_j^{\text{PT}}(N,b,Q,\mu,\mu_F)\\ &+ F_k(N,Q,\mu) + G_{ijk}(N,\mu).\end{aligned} \tag{6}$$

Let us discuss each of these terms in turn. The initial state perturbative exponent reads, in integral form

$$\begin{aligned}E_i^{\text{PT}}(N,b,Q,\mu,\mu_F) &= -\int_{Q^2/\chi^2}^{Q^2}\frac{dk_T^2}{k_T^2}\left\{A_i(\alpha_s(k_T))\ln\left(\frac{Q}{\bar{N}k_T}\right)\right\}\\ &- 2\ln\bar{N}\int_{\mu_F^2}^{Q^2}\frac{dk_T^2}{k_T^2}A_i(\alpha_s(k_T)),\end{aligned} \tag{7}$$

where μ, μ_F are the renormalization and factorization scale, respectively. The function $\chi(N,b)$ defines the N- and b-dependent scale of soft gluons to be included in the resummation and is chosen as [30]

$$\chi(N,b) = \bar{b} + \frac{\bar{N}}{1+\frac{\eta\bar{b}}{\bar{N}}}, \tag{8}$$

where η is a suitably chosen constant and

$$\bar{N} = Ne^{\gamma_E}, \qquad \bar{b} = bQe^{\gamma_E}/2, \tag{9}$$

with γ_E the Euler constant. An older form used in [26]

$$\chi(N,b) = \bar{b} + \bar{N} \tag{10}$$

generates spurious subleading logarithms in Q_T [30]. We postpone elaborating on the integral in Eq. (7) to the next section. The final state jet exponent reads to NLL accuracy

$$F_k(N,Q,\mu) \equiv \frac{1}{\alpha_s(\mu)}f_k^{(0)}(\lambda) + f_k^{(1)}(\lambda,Q,\mu), \tag{11}$$

where

$$\lambda = b_0\alpha_s(\mu^2)\ln\bar{N}. \tag{12}$$

The exponent associated with wide-angle soft radiation is

$$G_{abc}(N) \equiv g_{abc}^{(1)}(\lambda). \tag{13}$$

The functions $f_k^{(0,1)}$ and $g_{ijk}^{(1)}(\lambda)$ as well as the functions $C^{(ij\to\gamma k)}$ [22,23] are listed in the appendix.

A nonperturbative term must be added to the perturbative exponent in Eq. (6) in order to regularize the limit in which Q_T is very small. As in Refs. [26,27] we take

$$E_{ij}^{\text{NP}} = -\tfrac{1}{2}g_{\text{NP}}b^2, \qquad ij = q\bar{q}, qg. \tag{14}$$

The threshold-resummed result can now be derived by simply neglecting $\mathbf{Q_T}$ in the factor $(S/[4|\mathbf{p}_T - \mathbf{Q}_T/2|^2])^{N+1}$ in Eq. (5). Then the $\mathbf{Q_T}$ integral sets $\mathbf{b}$ to zero everywhere, yielding the threshold-resummed result.

III. INCLUDING LEADING $\ln N/N$ TERMS

The leading terms in Eq. (2) originate from both initial- and final-state radiation, and to resum them we will use two different methods upon which we elaborate in this section. There are moreover two classes of functions in momentum space at order α_s^j that generate the leading $\ln^{2j-1}N/N$ terms upon Mellin transformation. In terms of the variable z, $0 < z < 1$ which in the present case can represent either $\hat{x}_T^2$ or $\tilde{x}_T^2$, one of the two classes is formed by the singular plus distributions $[\ln^{2j-1}(1-z)/(1-z)]_+$, the other by the singular but integrable $\ln^{2j-1}(1-z)$. The $\ln N/N$ contributions from the former can be computed using the methods of [31] and can be found e.g. in Ref. [12]. The $\ln N/N$ contributions from the latter can be generated at any order in perturbation theory by a simple replacement in the resummed expression [see below in Eqs. (25)], expanding the resulting expression to the desired order, and keeping the leading term in Eq. (2). Roughly speaking, the replacement is equivalent to exchanging at order j one soft-collinear gluon (corresponding to one factor $\alpha_s \ln^2 N$) for a hard-collinear one (corresponding to a factor $\alpha_s \ln N/N$)

$$\alpha_s^k \ln^{2k} N \rightarrow \alpha_s^k \frac{\ln^{2k-1} N}{N}. \qquad (15)$$

This replacement is in fact easily included in the existing threshold resummation formulas. A preliminary study for prompt photon production was carried out in Ref. [32]. We employ this replacement method in fact for the final state related $\alpha_s^k \ln^{2k-1} N/N$ terms. It was pointed out in Refs. [30,33] that the initial state related $\alpha_s^k \ln^{2k-1} N/N$ terms could be generated in the context of joint resummation by extending evolution of parton densities to a soft scale. We will use this method as well, for the first time for a one-particle inclusive observable. We now discuss the initial and final state $\ln N/N$ contributions in turn.

A. Initial state

Our procedure for the initial state follows Refs. [30,33], where the joint resummation for electroweak or Higgs boson production at mass Q and transverse momentum Q_T was given. We recall the key points here. The integral form of the initial state NLL exponent (7) can be written as

$$E_i^{\rm PT}(N, b, Q, \mu, \mu_F) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left\{ A_i(\alpha_s(k_T)) \ln\left(\frac{Q}{k_T}\right) + B_i(\alpha_s(k_T)) \right\} + \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \times \{ -\ln\bar{N} A_i(\alpha_s(k_T)) - B_i(\alpha_s(k_T)) \}. \qquad (16)$$

The first term in this expression leads to

$$E_i^{\rm PT}(N, b, Q, \mu) = \frac{1}{\alpha_s(\mu)} h_i^{(0)}(\beta) + h_i^{(1)}(\beta, Q, \mu), \qquad (17)$$

where

$$\beta = b_0 \alpha_s(\mu) \ln(\chi). \qquad (18)$$

We recall that the χ depends on N and b through Eq. (8). The functions $h_i^{(0,1)}$ are listed in the appendix.

The second term represents flavor-conserving evolution to NLL accuracy [the integrand consists of the $\ln N$ and constant terms for the anomalous dimension matrix $\gamma_{i/j}(N)$ for $j = i$] from the hard scale μ_F to the soft scale Q/χ. One now performs the replacement [30,33]

$$-A_i(\alpha_s) \ln(\bar{N}) - B_i(\alpha_s) \rightarrow \gamma_{i/i}(N)(\alpha_s), \qquad (19)$$

that includes the leading, flavor-diagonal $\ln N/N$ effects generated by the k_T integral (the $1/N$ part of $\gamma_{i/i}$ combines with the $\ln N$ terms). In fact, one may go further and include the off-diagonal contributions via the replacement

$$\delta_{ig} \exp\left[\frac{-A_g^{(1)} \ln\bar{N} - B_g^{(1)}}{2\pi b_0} s(\beta)\right] f_{g/H}(N, \mu_F) \rightarrow \mathcal{E}_{ik}(N, Q/\chi, \mu_F) f_{k/H}(N, \mu_F), \qquad (20)$$

where $s(\beta) = \ln(1 - 2\beta)$ plus NLL corrections. As a result, we can replace in Eq. (5) the combination

$$f_{i/A}(\mu_F, N) f_{j/B}(\mu_F, N) \exp[E_i^{\rm PT}(N, b, Q, \mu, \mu_F) + E_j^{\rm PT}(N, b, Q, \mu, \mu_F)] \qquad (21)$$

by

$$\mathcal{C}_{i/A}(Q, b, N) \mathcal{C}_{j/B}(Q, b, N) \exp[E_i^{\rm PT}(N, b, \mu, Q) + E_j^{\rm PT}(N, b, \mu, Q)], \qquad (22)$$

where

$$\mathcal{C}_{i/H}(Q, b, N) = \sum_k \mathcal{E}_{ik}(N, Q/\chi, \mu_F) f_{k/H}(N, \mu_F). \qquad (23)$$

The matrix $\mathcal{E}$ implements evolution from scale μ_F to scale Q/χ and is normalized to be the unit matrix if these two scales are equal. Note that the dependence on μ_F cancels among the factors in Eq. (23).

B. Final state

Leading $\ln N/N$ effects arising from final state radiation can be derived from the jet functions [1,34] that enter threshold or joint-resummed expressions for observables having final state partons at lowest order. The integral form of the final state exponent F_k in Eq. (6) reads

$$\int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \left\{ \int_{(1-z)^2 Q^2}^{(1-z)Q^2} \frac{dq^2}{q^2} A_k(\alpha_s(q^2)) + B_k(\alpha_s((1-z)Q^2)) \right\}. \qquad (24)$$

To include leading $\ln N/N$ dependence in $F_k(N, Q, \mu)$ we make the replacement [12,20,32]

$$\frac{z^{N-1}-1}{1-z}A_i^{(1)} \to \left[\frac{z^{N-1}-1}{1-z} - p_i z^{N-1}\right]A_q^{(1)} + \mathcal{O}\left(\frac{1}{N^2}\right), \tag{25}$$

where $p_q = 1$, $p_g = 2$. The extra terms can be cast in a more convenient form. Using

$$z^{N-1} = \frac{z^{N-1} - 1 - (z^N - 1)}{1-z} \tag{26}$$

and the replacement (accurate to NLL)

$$z^{N-1} - 1 \to -\theta\left(1 - z - \frac{1}{\bar{N}}\right) \tag{27}$$

one finds

$$F_k(N, Q, \mu) = \frac{1}{\alpha_s(\mu)} f_k^{(0)}(\lambda) + f_k^{(1)}(\lambda, Q, \mu) + f_k'(\lambda, \alpha_s) + O(\alpha_s(\alpha_s \ln N)^n), \tag{28}$$

where the extra terms f_k' that include the leading $\ln N/N$ terms due to final-state radiation read

$$f_q' = \frac{A_q^{(1)}}{2\pi b_0} \exp\left(-\frac{\lambda}{\alpha_s b_0}\right)[\ln(1-2\lambda) - \ln(1-\lambda)], \tag{29}$$

$$f_g' = \frac{3A_g^{(1)}}{2\pi b_0} \exp\left(-\frac{\lambda}{\alpha_s b_0}\right)[\ln(1-2\lambda) - \ln(1-\lambda)]. \tag{30}$$

There is no leading $\ln N/N$ contribution arising from wide-angle soft radiation.

As a result, we finally arrive at the following equation for the joint-resummed prompt photon hadroproduction p_T spectrum in which leading soft-collinear effects are included:

$$\begin{aligned}\frac{p_T^3 d\sigma_{AB\to\gamma}^{(\mathrm{resum})}}{dp_T} &= \frac{p_T^4}{8\pi S^2}\sum_{ij}\int_C \frac{dN}{2\pi i}\int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{Q}_T}\int\frac{d^2\mathbf{Q}_T}{(2\pi)^2}\theta(\bar{\mu} - |\mathbf{Q}_T|)\int_0^1 d\tilde{x}_T^2(\tilde{x}_T^2)^N \frac{|M_{ij}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} C^{(ij\to\gamma k)}(\alpha_s(\mu), \tilde{x}_T^2)\\ &\times\left(\frac{S}{4|\mathbf{p}_T - \mathbf{Q}_T/2|^2}\right)^{N+1}\mathcal{C}_{i/A}(Q, b, N)\mathcal{C}_{j/B}(Q, b, N)\exp[E_i^{\mathrm{PT}}(N, b, \mu, Q) + E_j^{\mathrm{PT}}(N, b, \mu, Q)]\\ &\times\exp\left[\frac{1}{\alpha_s(\mu)}f_k^{(0)}(\lambda) + f_k^{(1)}(\lambda, Q, \mu) + f_k'(\lambda, \alpha_s) + g_{ijk}^{(1)}(\lambda)\right].\end{aligned} \tag{31}$$

As before, the corresponding threshold result may be obtained by neglecting $-\mathbf{Q}_T/2$ in the last factor on the second line.

IV. RESULTS

Here we study numerically the inclusion of the $\ln N/N$ terms for the case of prompt photon production for two kinematic conditions: those of $p\bar{p}$ collisions at the Tevatron at $\sqrt{S} = 1.96$ TeV [35,36], and those of the pN collisions in the E706 [37] fixed-target experiment with $E_{\mathrm{beam}} = 530$ GeV. Our main aim is to assess the effect of such terms in relevant kinematic conditions, rather than provide optimized and realistic theoretical calculations for comparison with data (see Ref. [38] for a recent study). For instance, we do not include contributions from fragmentation processes, which recently have been addressed in Ref. [39] and shown to be significant. Our assessments mainly consist of comparing the same calculation with and without $\ln N/N$ terms.

Our default choices for various input parameters are as follows. We use the GRV parton density set [40], corresponding to $\alpha_s(M_Z) = 0.114$, with the evolution code of Ref. [41], changing flavor number at $\mu = m_c(1.4\ \mathrm{GeV})$ and $m_b(4.5\ \mathrm{GeV})$. We choose the factorization and renormalization scale equal to p_T, and the nonperturbative parameter g_{NP} in Eq. (14) equal to 1 GeV^2. For the parameter χ we use the expression in Eq. (8), following [30], with $\eta = 1/4$.[1] For our joint-resummed results, we chose for Tevatron (E706) kinematics the cutoff $\bar{\mu}$ in Eq. (5) equal to 15(5) GeV. Regarding logarithmic accuracy, and unless otherwise stated, we refer to LL when using only $h_a^{(0)}$ in Eq. (17), $f_k^{(0)}$ in Eq. (28), and $\bar{C}^{(ij\to\gamma k)} = 1$ for the processes in (4); we refer to NLL when also including $h_a^{(1)}$ and $f_k^{(1)}$ and the virtual corrections in (A10). For the evolution from scale μ_F to Q/χ in Eq. (20) we use the full NLO anomalous dimension in all cases.

Starting with Tevatron kinematics we compare in Figs. 1–3 results at LL and NLL accuracy, with and without the leading $\ln N/N$ contribution for joint resummation. For clarity we have here included the constant corrections in (A10) also for the LL case. Figure 1 shows

[1]Choosing $\eta = 1$ does not substantially modify results, but choosing the form in Eq. (10), which generates spurious subleading recoil logs [30], does lead to significant changes at larger p_T.

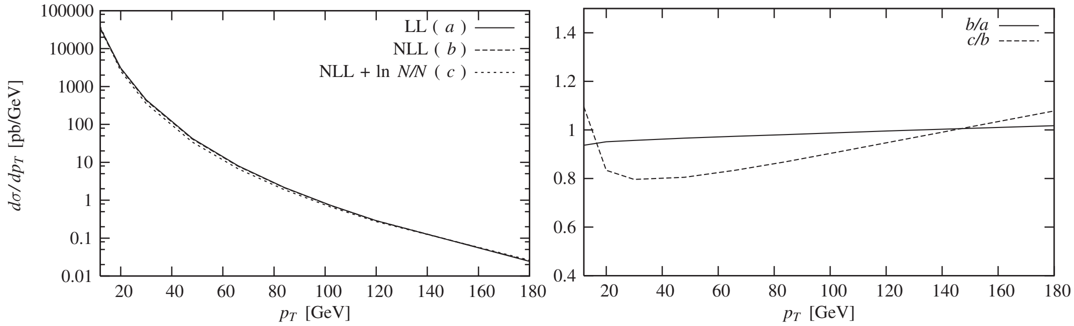

FIG. 1. $\ln N/N$ contributions for Tevatron kinematics. Left pane: LL without $\ln N/N$ (a, solid line), NLL without $\ln N/N$ (b, dashed line), NLL with $\ln N/N$ (c, short dashed line). Right pane: ratio of NLL to LL (solid line), ratio of NLL with $\ln N/N$ to NLL without (dashed line).

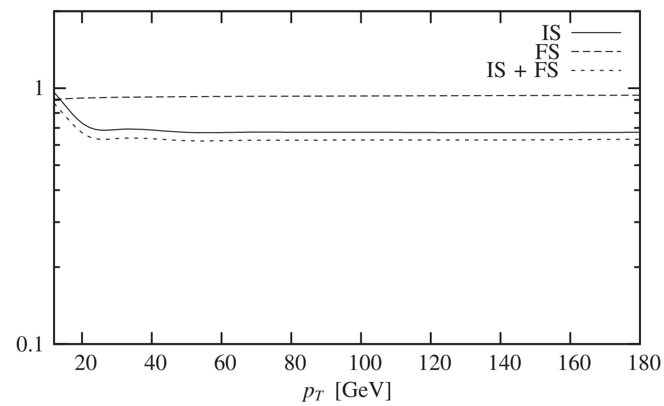

FIG. 2. $\ln N/N$ effects for $q\bar{q}$ channel at LL, Tevatron kinematics. Ratio to LL without $\ln N/N$ of initial-state (solid line) and final-state (dashed line) effects, and both (short dashed line).

that the effect of the leading $\ln N/N$ is appreciable when compared to the effect of passing from LL to NLL, the latter difference being almost negligible. Inclusion of $\ln N/N$ effects leads to noticeable suppression for most of the p_T range and to enhancement at very small and very large p_T. To better understand the origin of these $\ln N/N$

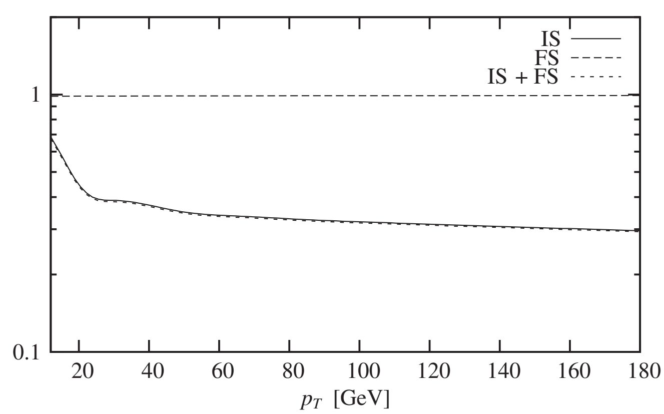

FIG. 3. $\ln N/N$ effects for qg channel at LL, Tevatron kinematics. Labels as in Fig. 2.

suppressed contributions, we examine in Figs. 2 and 3 for each channel in (4) the contributions from the initial and final states. We plot these contributions for the LL cross sections only to facilitate interpretation. To help understand the results, we can expand the perturbative exponent in Eq. (6) to lowest order in α_s, keeping only the $\ln^2 N$ and $\ln N/N$ terms

$$q\,\bar{q}:\quad : \frac{\alpha_s}{\pi}\ln^2 N\left(2A_q^{(1)} - \frac{1}{2}A_g^{(1)}\right) + \frac{\alpha_s}{\pi}\frac{\ln N}{N}\left(2A_q^{(1)} - \frac{3}{2}A_g^{(1)}\right), \tag{32}$$

$$\begin{aligned} qg:\quad : &\frac{\alpha_s}{\pi}\ln^2 N\left(A_q^{(1)} + A_g^{(1)} - \frac{1}{2}A_q^{(1)}\right) \\ &+ \frac{\alpha_s}{\pi}\frac{\ln N}{N}\left(A_q^{(1)} + 3A_g^{(1)} - \frac{1}{2}A_q^{(1)}\right). \end{aligned} \tag{33}$$

The expressions suggest that the initial state $\ln N/N$ terms enhance the cross section for the $q\bar{q}$ and, in particular, the qg channels, while the final state $\ln N/N$ terms suppress it, again by an amount that depends on the channel. The net result turns out to be suppression in the former channel and enhancement in the latter. These qualitative aspects are indeed borne out if we use the same method to compute initial state $\ln N/N$ effects as we did for the final state in Sec. III B.[2] In the present case, however, the net $\ln N/N$ effect in both channels is suppression, indicating that the nondiagonal terms in the evolution matrix give a sizeable negative contribution. Note that for Tevatron kinematics, when combining channels, the qg channel dominates at low p_T, because the required momentum fractions are not too large. At large p_T, where parton momentum fractions are larger, the valence-quark dominated $q\bar{q}$ channel takes over.

Turning to E706 kinematics we perform the same studies as we did for the Tevatron. The results are shown in Figs. 4–6. We observe an overall enhancement due to the

[2]The net result in the $q\bar{q}$ is actually still enhancement, because the contribution of the $f'_{q,g}$ functions in (29) and (30) is very small.

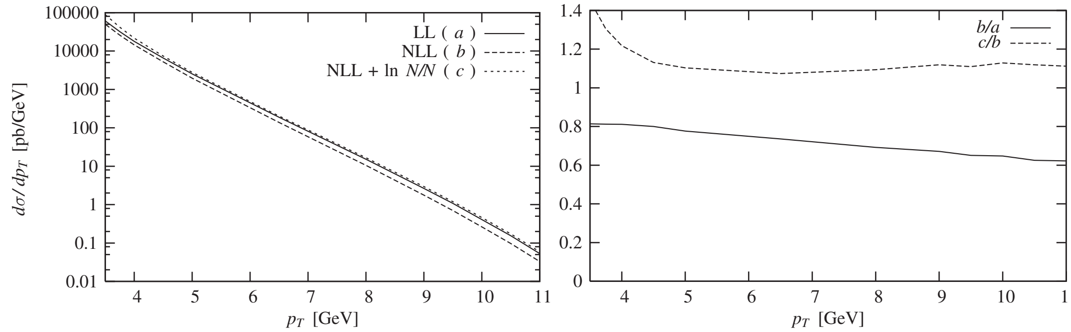

FIG. 4. lnN/N contributions for E706 kinematics. Labels as in Fig. 1.

lnN/N effects, somewhat smaller than the change from LL to NLL. Both effects are more pronounced than for the Tevatron. This is due both to a larger value of α_s as well as being closer to threshold in this fixed-target kinematical regime. Examining the effects per channel in Figs. 5 and 6, we now see a noticeable enhancement from the initial state lnN/N effects in the $q\bar{q}$ channel, but still suppression in the qg channel. Clearly the nondiagonal terms in the evolution matrix play a significant role for the E706 case as well.

Next, we examine the differences between threshold and joint resummation. In Fig. 7 we compare resummed results directly by showing the ratios with respect to the joint-resummed p_T distribution without lnN/N terms. We see for Tevatron kinematics that the threshold resummed dominates the joint resummed at large p_T, while at low p_T the converse is true. For the E706 case the threshold-resummed results are entirely below the joint-resummed ones. The threshold-resummed curves are shown sepa-

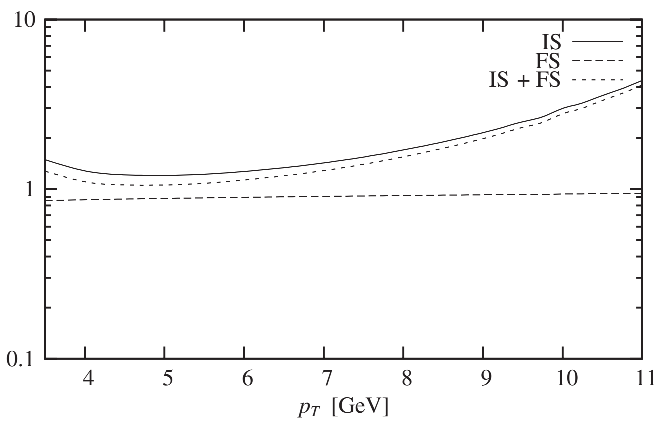

FIG. 5. lnN/N effects for $q\bar{q}$ channel at LL, E706 kinematics. Labels as in Fig. 2.

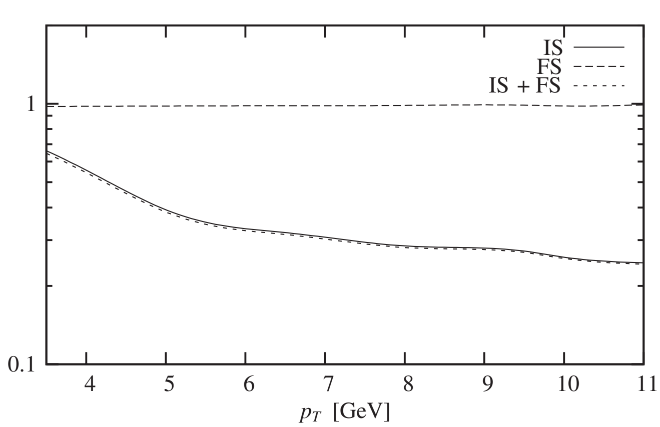

FIG. 6. lnN/N effects for qg channel at LL, E706 kinematics. Labels as in Fig. 2.

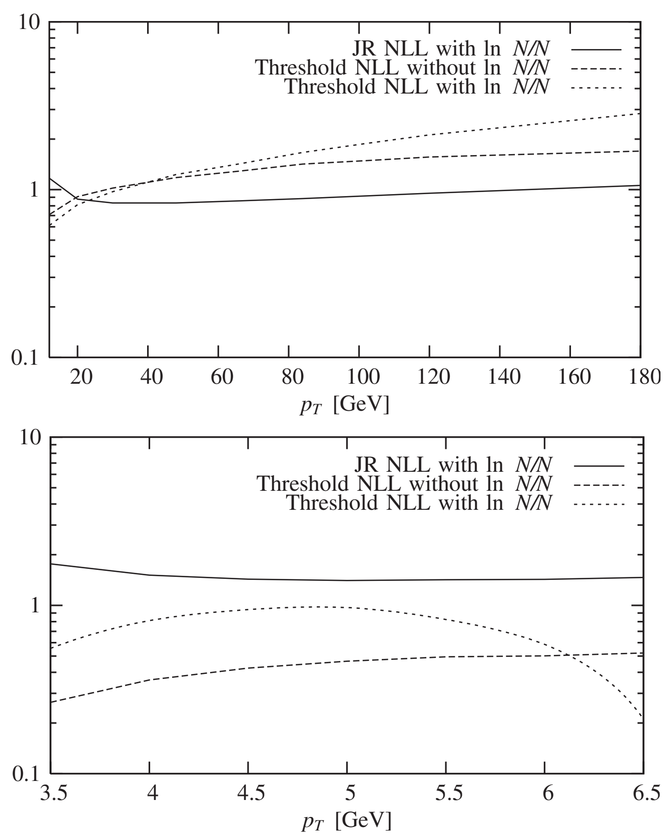

FIG. 7. Comparison of joint resummation and threshold resummation effects, ratios to NLL without lnN/N for Tevatron (top pane) and E706 (bottom pane).

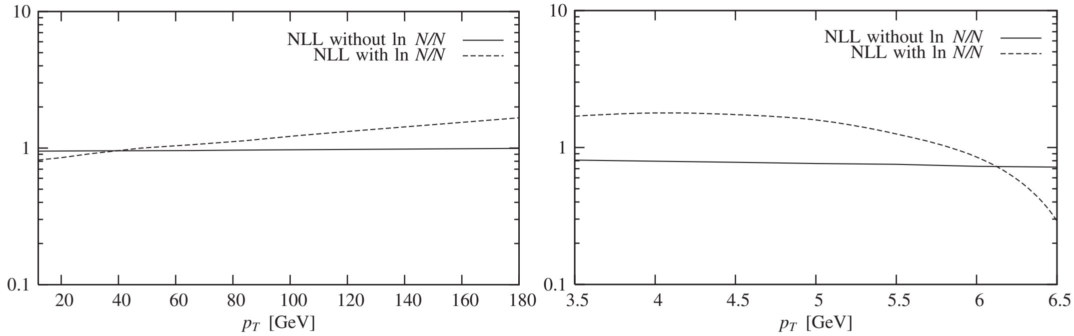

FIG. 8. $\ln N/N$ effects in threshold resummation, for Tevatron (left pane) and E706 (right pane). Labels as in Fig. 1 right pane.

rately in Fig. 8, which is analogous to the rightmost panels in Figs. 1 and 4. For Tevatron kinematics the inclusion of $\ln N/N$ terms in threshold resummation leads, as for joint resummation, from suppression at small p_T to enhancement at larger p_T, but more noticeably. For E706 kinematics, different from the joint resummation case, the enhancement at small p_T turns to suppression just below $p_T = 6$ GeV. The cross section even becomes negative beyond 6.5 GeV, which is due to the fact that the nearness of the threshold drives the scale Q/χ in Eq. (20) effectively below the starting scale of the PDF evolution.

V. CONCLUSIONS

We have examined the effects of including terms of the form

$$\alpha_s^i \sum_j^{2i-1} d_{ij} \frac{\ln^j N}{N} \tag{34}$$

in joint-resummed and threshold-resummed prompt photon p_T distributions at both collider and fixed-target kinematics, at leading accuracy ($j = i$). The complete structure of subleading terms of the form (34) is still unknown. Note that we have not considered the fragmentation component of the prompt photon production cross section in our analysis.[3]

To the extent that terms of the form (34) arise from initial-state radiation effects, we used the method of Refs. [30,33] to include them, now in a single-particle inclusive cross section. Those arising from final state emission we included by extending the jet function to leading $\ln N/N$ accuracy. Numerically we found the combined $\ln N/N$ terms to be comparable to NLL corrections and dependent on kinematics either enhancing or suppressing. The final-state $\ln N/N$ contributions were particularly small, while in the initial state the effects of nonleading $1/N$ effects are appreciable, depending again on channel and kinematics. The flavor nondiagonal terms in the evolution matrix were found to be numerically significant, and the main source of discrepancy with expectations based on simple approximations. We conclude that, because the effects, though small, are nonnegligible, understanding the structure of $\ln N/N$ terms better is a worthwhile pursuit.

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APPENDIX

Here we list the exponents used in Sec. III. The initial-state exponents (17) involve

$$h_a^{(0)}(\beta) = \frac{A_a^{(1)}}{2\pi b_0^2}[2\beta + \ln(1-2\beta)], \tag{A1}$$

$$\begin{aligned} h_a^{(1)}(\beta, Q, \mu) = {} & \frac{A_a^{(1)} b_1}{2\pi b_0^3}\left[\frac{1}{2}\ln^2(1-2\beta) + \frac{2\beta + \ln(1-2\beta)}{1-2\beta}\right] \\ & + \frac{B_a^{(1)}}{\pi b_0}\ln(1-2\beta) + \frac{1}{2\pi b_0}\left[A_a^{(1)}\ln\left(\frac{Q^2}{\mu^2}\right)\right. \\ & \left. - \frac{A_a^{(2)}}{\pi b_0}\right]\left[\frac{2\beta}{1-2\beta} + \ln(1-2\beta)\right]. \end{aligned} \tag{A2}$$

Here

$$A_a^{(1)} = C_a, \qquad A_a^{(2)} = \frac{1}{2}C_a\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9}T_R N_F\right], \tag{A3}$$

where $C_q = C_F$ and $C_g = C_A$. Also

$$B_q^{(1)} = -\tfrac{3}{4}C_F, \qquad B_g^{(1)} = -\pi b_0. \tag{A4}$$

[3] To do so would require inclusion of more partonic subprocesses, each containing a sum over color structures for the wide-angle soft radiation component, as well as photon fragmentation functions [39]. Presumably, soft-collinear effects for the fragmentation component of prompt photon production could be included in a way analogous to what we did in the present paper for the initial state: via adjustment of the resummed part, and evolution of the fragmentation functions.

The final-state exponents (6) involve the functions

$$f_a^{(1)} = -\frac{A_a^{(1)}}{2\pi b_0 \lambda}[(1-2\lambda)\ln(1-2\lambda) - 2(1-\lambda)\ln(1-\lambda)], \tag{A5}$$

$$\begin{aligned} f_a^{(2)} = &-\frac{A_a^{(1)} b_1}{2\pi b_0^3}\Big[\ln(1-2\lambda) - 2\ln(1-\lambda) + \frac{1}{2}\ln^2(1-2\lambda) - \ln^2(1-\lambda)\Big] + \frac{B_a^{(1)}}{\pi b_0}\ln(1-\lambda) \\ &- \frac{A_a^{(1)}\gamma_E}{\pi b_0}[\ln(1-\lambda) - \ln(1-2\lambda)] \\ &- \frac{A_a^{(2)}}{2\pi^2 b_0^2}[2\ln(1-\lambda) - \ln(1-2\lambda)] \\ &+ \frac{A_a^{(1)}}{2\pi b_0}[2\ln(1-\lambda) - \ln(1-2\lambda)]\ln\frac{Q^2}{\mu^2}. \end{aligned} \tag{A6}$$

The wide-angle soft radiation exponents (13) are

$$\begin{aligned} g_{q\bar{q}g}^{(1)}(\lambda) &= -\frac{C_A}{\pi b_0}\ln(1-2\lambda)\ln 2, \\ g_{qgq}^{(1)}(\lambda) &= -\frac{C_F}{\pi b_0}\ln(1-2\lambda)\ln 2. \end{aligned} \tag{A7}$$

In these equations

$$\begin{aligned} b_0 &= \frac{11C_A - 4T_R N_F}{12\pi}, \\ b_1 &= \frac{17C_A^2 - 10C_A T_R N_F - 6C_F T_R N_F}{24\pi^2}. \end{aligned} \tag{A8}$$

where $T_R = 1/2$. These expressions are obtained by expanding the perturbative functions $A_a(\alpha_s)$, $B_d(\alpha_s)$, and $D_{ab\to d\gamma}$ in powers of α_s,

$$A_a(\alpha_s) = \frac{\alpha_s}{\pi}A_a^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_a^{(2)} + O(\alpha_s^3) \tag{A9}$$

and so on.

Finally, the explicit forms of $C^{(ij\to\gamma k)}$ [22,23] are

$$C^{q\bar{q}\to\gamma g} = 1 + \frac{\alpha_s}{\pi}\Big[-\frac{1}{2}(2C_F - C_A)\ln 2 + \frac{1}{2}K - K_q + 2\zeta(2)\Big(2C_F - \frac{1}{2}C_A\Big) \tag{A10}$$

$$+ \frac{5}{4}\Big(2C_F - \frac{1}{2}C_A\Big)\ln^2 2 + \frac{3}{2}C_F(-\ln 2) - \pi b_0 \ln\frac{2p_T^2}{\mu^2}\Big], \tag{A11}$$

$$C^{qg\to\gamma q} = 1 + \frac{\alpha_s}{\pi}\Big[-\frac{1}{10}(C_F - 2C_A)\ln 2 - \frac{1}{2}K_q + \frac{\zeta(2)}{10}(2C_F + 19C_A) \tag{A12}$$

$$+ \frac{1}{2}C_F\ln^2 2 + \frac{3}{4}(C_F + \pi b_0)(-\ln 2) - \pi b_0 \ln\frac{2p_T^2}{\mu^2}\Big], \tag{A13}$$

where

$$K = C_A\Big(\frac{67}{18} - \frac{\pi^2}{6}\Big) - \frac{10}{9}T_R N_F \qquad K_q = \Big(\frac{7}{2} - \frac{\pi^2}{6}\Big)C_F. \tag{A14}$$

We note that there is no factorization scale dependence in $h_a^{(1)}$ and the coefficient functions in Eq. (A10) because of complete evolution from scale μ_F to Q/χ in Eqs. (21)–(23)

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