Multivariate Estimations of Equilibrium Climate Sensitivity From Short Transient Warming Simulations

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Abstract One of the most used metrics to gauge the effects of climate change is the equilibrium climate sensitivity, defined as the long-term (equilibrium) temperature increase resulting from instantaneous doubling of atmospheric CO2. Since global climate models cannot be fully equilibrated in practice, extrapolation techniques are used to estimate the equilibrium state from transient warming simulations. Because of the abundance of climate feedbacks—spanning a wide range of temporal scales—it is hard to extract long-term behavior from short-time series; predominantly used techniques are only capable of detecting the single most dominant eigenmode, thus hampering their ability to give accurate long-term estimates. Here, we present an extension to those methods by incorporating data from multiple observables in a multicomponent linear regression model. This way, not only the dominant but also the next-dominant eigenmodes of the climate system are captured, leading to better long-term estimates from short, nonequilibrated time series.

Plain Language Summary Although it is clear that the atmospheric CO2 concentration influences the Earth's climate, it is difficult to quantify its long-term effects accurately. Scientific efforts in this direction focus on idealized experiments carried out in global climate models. In these experiments, atmospheric CO2 is (instantaneously) doubled, and the long-term temperature increase this causes is recorded. This resulting temperature increase is called the (equilibrium) climate sensitivity; accurately knowing its value helps to better quantify the effects of different emission scenarios on the future climate. However, it takes a very long time before all processes in a climate model are fully settled—especially in state-of-the-art, more and more detailed models—and, in practice, settling all is simply not feasible. Hence, climate sensitivity needs to be estimated from limited model data. This is particularly difficult as the climate system consists of many processes that behave on vastly different time scales. Here, we present a new estimation technique that is better capable of capturing the very slow processes than conventional techniques, and hence leads to a more accurate quantification of (equilibrium) climate sensitivity.

1. Introduction

The use of (equilibrium) climate sensitivity to assess the impact of changes in atmospheric CO2 dates back at least a century (Arrhenius, 1896; Lapenis, 1998). First, estimations of its value were made with rudimentary computations (Arrhenius, 1896; Charney et al., 1979); nowadays, improved knowledge of the climate system is used to infer climate sensitivity from observational data, proxy data, and global climate models (Knutti & Hegerl, 2008; Knutti et al., 2017; Lunt et al., 2010; Rohling et al., 2018; Von der Heydt et al., 2016). However, the reported values (still) vary much between studies (IPCC, 2013) and the current consensus is that climate sensitivity is between 2.3 and 4.7 K (5%-95% ranges, Sherwood et al., 2020). On top of that, recent results of the new generation of global climate models show even higher sensitivities, possibly due to better representation of cloud formation when using finer spatial grids (Andrews et al., 2019; Bacmeister et al., 2020; Bony et al., 2015; Duffy et al., 2003;Govindasamy et al., 2003; Haarsma et al., 2016; Zelinka et al., 2020). Still, even these state-of-the-art climate models report significantly different climate sensitivities (Flynn & Mauritsen, 2020; Forster et al., 2020; Zelinka et al., 2020); moreover, estimates for a single model tend to have large uncertainties further hampering accurate pinpointing of the climate sensitivity (Dai et al., 2020; Rugenstein et al., 2020).

For conceptual models and earth system models of intermediate complexity, it is possible to let a simulation run until the system is fully equilibrated (Holden et al., 2014). However, for more refined models, including...
contemporary and future state-of-the-art global climate models, this is not viable (Rugenstein et al., 2019); equilibrating those models simply takes too much computing power. With the current trend—and need—to build models with higher temporal and spatial resolutions (Duffy et al., 2003; Eyring et al., 2016; Govindasamy et al., 2003; Haarsma et al., 2016), this is not expected to resolve itself in the near future. Hence, the equilibrium climate sensitivity of these models is instead estimated by extrapolating transient warming simulations—way before these models have reached equilibrium (Dai et al., 2020; Knutti & Hegerl, 2008; Knutti et al., 2017; Rugenstein et al., 2020). There are several techniques to perform such extrapolation that use different physical and mathematical properties of the system to give sensible estimates for the true equilibrium climate sensitivity of a model (Dai et al., 2020; Geoffroy et al., 2013a; Gregory et al., 2004; Knutti & Hegerl, 2008; Proistosescu & Huybers, 2017).

The main problem with equilibrium estimations lies with the abundance of feedbacks present in the climate system (Von der Heydt et al., 2016, 2020). These feedbacks are quite diverse and span a wide range of spatial and temporal scales; these include, for example, the very fast Planck feedback, the slower ice-albedo feedback and the even slower ocean circulation feedbacks. Estimation techniques deal differently with this problem, for instance by incorporating multiple time scales directly in the estimation method (Proistosescu & Huybers, 2017), by explicit modeling of long-term (ocean) heat uptake (Geoffroy et al., 2013a) or more indirectly by ignoring initial fast warming behavior (Dai et al., 2020; Rugenstein et al., 2020).

The most predominantly used estimation technique is the one developed by Gregory et al. (2004). In this technique, the top-of-atmosphere radiative imbalance (ΔN) is fitted using a linear regression against the temperature increase (ΔT). However, recently, it has become clear that ΔN and ΔT do not always adhere to such linear relationship (Andrews et al., 2015; Armour, 2017; Knutti & Rugenstein, 2015). Typically, there is an initial fast warming which is followed by one or several slower additional (less substantial) warming processes. Hence, estimates made by this method depend heavily on the time period used in the regression and typically underestimate the equilibrium warming. Most of the times, this problem is largely circumvented by ignoring the first part of a simulation that contains the initial fast processes; the regression is then applied only on the last part of the simulation.

The thus ignored data does, however, still contain information about the dynamics of the system—even beyond the initial fast warming. The issue here is that this information cannot be extracted using a one-dimensional linear regression; that kind of fit will only ever recover the one process (i.e., one eigenmode of decay to equilibrium) that is most dominantly present on the time scale of the regression data. In this study, we present an extension to this technique that is capable of capturing multiple eigenmodes by incorporating additional observables into a multicomponent linear regression (abbreviated as MC-LR) model. Subsequently, we show the potential efficiency of this technique using both low-dimensional conceptual models and modern global climate models.

2. Method: A Multicomponent Linear Regression Model

In the linear regime of the decay to equilibrium, the evolution of any observable O (e.g., global mean temperature increase or top-of-atmosphere radiative imbalance) is given by the sum of exponentials (i.e., the eigenvalue decomposition), capturing the behavior on different time scales based on the different eigenmodes of the system. Specifically, denoting the equilibrium value of an observable by $O_*$, this evolution follows:

$$O(t) - O_* = \sum_j \beta_j^{(O)} e^{\lambda_j t}, \quad \sum_j \beta_j^{(O)} = O(0) - O_*$$

(1)

where $\lambda_j$ denote the eigenvalues and $\beta_j^{(O)}$ the contributions of each eigenmode to the evolution of the observable $O$.

If only one eigenmode would be present (or relevant, as other eigenmodes are exponentially small on the time scale of the data), the evolution of the global mean surface temperature increase $\Delta T$ and the top-of-atmosphere radiative imbalance $\Delta N$ can be combined into the linear relation
\[ \Delta N - \Delta N_e = \frac{\beta_{1[N]}}{\beta_{1[T]}} (\Delta T - \Delta T_e). \]  

(2)

Since \( \Delta N_e = 0 \), this readily gives rise to the commonly used regression model by Gregory et al. (2004),

\[ \Delta N = a \Delta T + f \]

(3)

where \( a := \frac{\beta_{1[N]}}{\beta_{1[T]}} \) and \( f := - a \Delta T_e \) are to be determined from the used regression data. In this case, the equilibrium warming is estimated by \( \Delta T_{e}^{\text{est}} := - \frac{1}{a} f \).

If multiple eigenmodes are relevant, there no longer is such linear relationship between \( \Delta N \) and \( \Delta T \) (as time \( t \) cannot be eliminated from the equations anymore) and this technique breaks down. It is, however, possible to extend the technique by taking additional observables into account: if \( M \) eigenmodes are relevant, one must use two sets of \( M \) observables, denoted here by \( \tilde{X} \) and \( \tilde{Y} \); using a similar procedure, the equations for their evolutions can be combined together (e.g., using basic matrix computations) to obtain the linear relation

\[ \tilde{Y} - \tilde{Y}_e = A (\tilde{X} - \tilde{X}_e). \]

(4)

where \( A \) is a \( M \times M \) matrix. If the set of observables in \( \tilde{Y} \) only contains observables that tend to zero in equilibrium (i.e., \( Y_e = 0 \)), this gives rise to a new multicomponent linear regression model

\[ \tilde{Y} = A \tilde{X} + \tilde{F}, \]

(5)

with \( A \) and \( \tilde{F} := -A \tilde{X}_e \) to be determined by the regression data. Here, equilibrium estimates are given by the vector \( \tilde{X}_{e}^{\text{est}} := -A^{-1} \tilde{F} \) and contain equilibrium estimates for all observables in \( \tilde{X} \).

The method by Gregory et al. (2004) is a special example of this regression model, where \( M = 1 \), \( \tilde{X} = \Delta T \) and \( \tilde{Y} = \Delta N \). Here, this model is extended by adding one or two observables to the data vectors \( \tilde{X} \) and \( \tilde{Y} \) (i.e., \( M = 2 \) or \( M = 3 \), lining up with previous studies by Caldeira and Myhrvold (2013), Tsutsui (2017), and Proistosescu and Huybers (2017)). Specifically, the mean global effective top-of-atmosphere short-wave albedo \( \alpha \) and long-wave emissivity \( \varepsilon \) are considered as additional observables. Their values are added to the data vector \( \tilde{X} \) and the values of their (numerical) time-derivatives—that tend to zero in equilibrium—to \( \tilde{Y} \).

The fits in this study are all made using standard least squares regression.

A different and much more extensive take on the rationale behind the technique can be found in supporting information Text S1.

3. Results: Conceptual Models

First, we present the results on a variant of the conceptual Budyko-Sellers energy balance model for global mean surface temperature (Budyko, 1969; Sellers, 1969). This model has been extended such that albedo and emissivity are no longer instantaneous processes, but will settle slowly over time. Moreover, white noise has been added to simulate climate variability. Thus, a three-component stochastic ordinary differential equation is created, which has been simulated in MATLAB with a Euler-Maruyama scheme. A more extensive description of the model can be found in supporting information Text S2.

Output of this model has been analyzed using the previously described MC-LR technique with the use of some or all of the observables. The resulting estimates for the equilibrium climate increase \( \Delta T_{e}^{\text{est}}(t) \) are given in Figure 1 for simulation runs with moderate noise (figures for other noise levels can be found in Figures S3 and S4). These estimates are given as functions of model time: the value for time \( t \) indicates the estimate is made with model output up to time \( t \) only. To evaluate the various estimation techniques and
track their accuracy depending on the amount of data used, the remaining relative error is computed: the maximum in relative error of the estimates occurring after the current time (i.e., when more data points are used). This gives a better impression of the kind of error to expect when using data up to time $t$. Mathematically, the remaining error is defined as

$$
e_{\text{rel}}(t) = \max_{s > t} \frac{\Delta T_{\text{est}}^s - \Delta T_*}{\Delta T_{\text{est}}^s (s)}$$

where $\Delta T_*$ is the true equilibrium warming (determined numerically via Newton’s method).

For this kind of low-dimensional models, it is clear that the multicomponent linear regression leads to better estimations of the real equilibrium warming than conventional techniques (Figure 1). Although the estimations for very short time series are not very accurate, estimations for slightly longer time series quickly pick up and are much better compared to the linear “Gregory” fit (Figure 1a), because also the longer time dynamics are taken into account (and are accurately fitted; see supporting information Text S2 and Figure S5). It takes some tens of (arbitrary) time units for the new estimates to get within 0.1 K of the actual equilibrium value, whereas hundreds of time units are needed for the conventional technique (Figure 1b). Moreover, it also seems that the MC-LR technique still works reliably in case of noise.

4. Results: LongRunMIP Models

The MC-LR technique has also been tested on more detailed global climate models. Specifically, data are taken from abrupt $4 \times CO_2$ forcing experiments of models participating in LongRunMIP, a model intercomparison project that focuses on millennia-long simulation runs (Rugenstein et al., 2019). Because of these long time series, a relative accurate value for the true equilibrium temperature can be determined, which is needed to adequately assess the performance of the estimation techniques.
For these climate models, global data on near-surface atmospheric temperature ($T =$ “tas”) and top-of-atmosphere radiative fluxes (incoming short-wave, “rsdt,” outgoing short-wave, “rsut,” and outgoing long-wave, “rlut”) has been downloaded from the LongRunMIP data server (Rugenstein et al., 2019). These data sets have been used to compute top-of-atmosphere radiative imbalance ($N =$ “rsdt” – “rsut” – “rlut”), effective short-wave albedo ($\alpha =$ “rsut”/“rsdt”) and effective long-wave emissivity ($\varepsilon = rlut/(tas)^4$; where $\sigma$ is the Stefan–Boltzmann constant). Initial, nonforced values were defined as means of piControl runs and changes $\Delta T$, $\Delta N$, $\Delta \alpha$, and $\Delta \varepsilon$ were computed from the abrupt $4 \times CO_2$ forcing runs. The real equilibrium warming $\Delta T$ for these models was estimated from the last warming of the forcing experiments, following the approach taken in Rugenstein et al. (2020). A more detailed description of these procedures, including minor practical variants, can be found in supporting information Text S3.

Figure 2. Gregory plot of $\Delta N$ as function of $\Delta T$ for a 5,900 years abrupt CO$_2$ quadrupling experiment in the CESM 1.0.4 model along with results of various common equilibrium estimation methods and the here introduced multicomponent linear regression (MC-LR) method when used on data up to model year 300. In the plot, red dots denote all data points (the later in the run, the smaller in size). The blue line shows the linear “Gregory” fit when all data from years 1 to 300 are used and the yellow line the Gregory fit when the first 20 years are ignored. The green line shows the three exponent fit (Proistosescu & Huybers, 2017). The cyan line indicates a fit to the EBM-$\varepsilon$ model that includes ocean heat uptake (Geoffroy et al., 2013a). The magenta line visualizes the newly introduced multicomponent linear regression that, in this case, utilizes both albedo and emissivity (for this visualization only—and not for any of the fits in this study—averaged data from the experiment are used). The stars (⋆) are the estimated equilibrium warming values from the different methods. Finally, dotted and dashed black lines indicate linear Gregory fits for the first and last part of the simulation that can be used for comparison—and that show how the various estimation methods capture dynamics on multiple time scales.
With the use of the model output, various techniques have been used to estimate equilibrium warming for all models. In Figure 2, a Gregory (ΔT, ΔN)-plot is given along with results of commonly used estimation techniques for one of the models (CESM 1.0.4) when applied on data up to model year 300. This illustrates the capabilities of the various techniques in capturing the behavior of the model system over different time scales. Clearly, the classical Gregory method mainly captures initial fast warming from the data. Hence, it is common practice to ignore an arbitrary number of years from the start of the simulation run—that show the initial fast warming—in a Gregory fit (Dai et al., 2020; Rugenstein et al., 2020). That technique has also been tested here, where the initial 20 years have been excluded. In contrast, the multicomponent linear regression technique does not rely on such arbitrary choices for data selection and outperforms both of these classical methods. Certainly, there also exist other alternative estimation techniques that aim to extract long-term behavior from short simulation runs (of which two have been added to Figure 2). However, these often amount to fitting an explicit low-dimensional model to transient simulations (e.g., Geoffroy et al., 2013a) and/or a nonlinear regression (e.g., Proistosescu & Huybers, 2017). The proposed MC-LR method does neither—and furthermore seems to perform similar or better than the mentioned other methods.

The results for other time frames are shown in Figure 3. Here, as before, estimates ΔT(t) are functions of time, which only use data up to a given time t for the estimation, and remaining relative errors have been computed as well. These results show that the MC-LR method also performs better on other time frames; in particular, when data for more than 150 years is being used, a multicomponent linear regression that utilizes both albedo and emissivity leads to better estimates compared to the classical Gregory methods. Especially on a century time scale this leads to significant improvements. Detailed results for all models can be found in Figures S8–S18.

To further disseminate the results and to assess the effectiveness over the range of models, in Figure 4 the remaining errors are given for all considered models at given times t = 150 years (CMIP protocol, Eyring et al., 2016), t = 300 years and t = 500 years. These results indicate that the MC-LR method can lead to more accurate equilibrium warming estimates. This new approach also better captures the long-term dynamics than the classical Gregory method when used on all data (with the HadGEM2 model for t = 150 years being the exception, where performance is similar). Moreover, the MC-LR method also tends to outperform the
Gregory method that ignores the first 20 years of data when $t > 150$ years. For $t = 150$ years, results vary much per model. This is closely related to the difference in model behavior: if dynamics happen on two dominant time scales, and the Gregory plot has an inflection point around (the arbitrarily chosen) year 20, this Gregory method works well (see e.g., the model MPI-ESM 1.1); otherwise, the MC-LR method will (eventually) outperform it. It is expected that this kind of result also holds for other variants of the Gregory method that ignore more years. A more in-depth discussion per model is included in supporting information Text S3.5.

5. Discussion

In this study, we have introduced a new equilibrium climate sensitivity estimation technique—the multicomponent linear regression (MC-LR)—that better captures the long-term behavior compared to conventional techniques. This MC-LR method has one prime rationale: a perturbed climate system evolves according to a linear system (given that the radiative perturbation is small). This linear evolution is recovered through the multicomponent linear regression (i.e., regression to $Y = AX + \hat{F}$). Although, here, only data from one transient simulation is used in the fits, data from multiple runs (with the same radiative forcing) can also be put together—possibly leading to even better estimates. As the goal of the method is to recover the eigenmodes in the linear regime of the system, such combination of runs seems extremely beneficial if runs follow the evolution of different eigenmodes. Indeed, it seems plausible—and an interesting direction for further research—that a small ensemble of short runs, each with a different perturbation of the initial system state, will better estimate the coefficients of the fitted linear system (i.e., $A$ and $\hat{F}$) without compromising in terms of total computing power.

Figure 4. Remaining relative error for various estimation techniques when used on model output up to $t = 150y$ (lightly shaded), $t = 300y$ (moderately shaded), and $t = 500y$ (strongly shaded). Here, only the best MC-LR method is depicted for each model (because of differences in model dynamics, which observables yield the best estimates differs per model). A complete list for all variants of the estimation techniques can be found in Figure S6, and a scatter plot of the results in this figure can be found in Figure S5.
The most difficult—and the most important—aspect of the MC-LR method is the choice of the observables used in the regression data. It is key that this data well-represents the different eigenmodes of the system. If too few are used, not all eigenmodes are found; if too many (or redundant ones) are used, estimates become unusable (as data becomes linearly dependent, which causes the fitted matrix $A$ to become near singular). In this study, we have focused on the use of (effective top-of-atmosphere) albedo and/or emissivity—observables that can be computed from data sets that are already normally used for climate sensitivity (Gregory et al., 2004, 2013a; Proistosescu & Huybers, 2017). However, the use of other, more curated observables might—and probably will—work better. For instance, the very long-term ocean dynamics might be better represented in data on ocean heat uptake (Geoffroy et al., 2013a, 2013b; Li et al., 2013; Raper et al., 2002).

It also seems natural to capture the known climate feedbacks, e.g., surface albedo, water vapor, and lapse rate (Von der Heydt et al., 2020). One should beware though that all these (feedback) processes together combine to the system’s eigenmodes in nonstraightforward ways. For example, summing feedbacks—like is commonly done in climate literature—only makes sense in systems that only have one component; in systems with multiple components, processes, and eigenmodes are not linked directly like this. Nevertheless, a careful inclusion of these feedbacks might lead to even better estimates and may further shorten the needed length of simulation runs.

The method described in this study does not only lead to better estimates for the equilibrium climate sensitivity, but can also be used to develop extensions of climate sensitivity, by incorporating other observables. Regression of the multicomponent model $\mathbf{X} = A \mathbf{F} + \tilde{F}$ leads to equilibrium estimates for all the observables in $\tilde{X}$ as $\tilde{X}_{est} = -A^{-1}F$. This estimate can be seen as a multivariate metric for climate sensitivity, in contrast to classical univariate metrics that focus only on changes in global temperature. Such multivariate metrics can better describe and quantify the changes that occur to the climate system due to changes in radiative forcing. In fact, many—if not all—climate subsystems and ecosystems do not depend critically on the global mean surface temperature, but on other observables such as the amount of precipitation or ocean heat transport (Lenton et al., 2008; Rockström et al., 2009; Scheffer et al., 2009). Estimating those directly—rather than considering them enslaved to the global mean surface temperature—will possibly lead to better projections for those (sub)systems.

Accurate estimations of equilibrium climate sensitivity are hard to come by, mostly due to the lengthy computation times needed to fully equilibrate modern global climate models. Going forward, it seems the more and more realistic state-of-the-art models will only take longer and longer to equilibrate (even considering developments in computer hardware). In particular, for high-resolution simulations with ultrafine numerical grids such equilibration runs are just not a practical option. For these kinds of simulations, it is vital to have extrapolation techniques that only need relatively short transient simulations to estimate the system’s long-term behavior. Once fully developed, such methods—the one introduced in this study being a first step toward them—can help to design the kind, amount, and length of the experiments performed with these high-resolution models, indicating an optimum between accurate (multivariate) climate sensitivity estimation and computing time.

**Data Availability Statement**

Simulation data from models in LongRunMIP can be accessed on data.iac.ethz.ch/longrunmip/. More information and details of the simulations can be found on longrunmip.org and in Rugenstein et al. (2019).

**References**


