

Limiting Cases of Modal Modification: Reply to Kosterec

Bjørn Jespersen*

Kosterec (2019) points out that my current theory of modal modifiers cannot deal satisfactorily with limiting cases. This note solves the problem. The form of the solution is to leave the existing theory as is and instead add a clause handling the limiting case which Kosterec brings up and another clause handling the limiting case at the other end of the spectrum.

My theory of modal modifiers, as set out in (2013), works well, as long as the argument property being modified is either (i) a purely contingent property or (ii) a contingent property with an essential core, provided the resulting modified property (*MF*) is not applied to an element of the essential core of *F*.¹ To stick with the original example of mine that Kosterec takes over, we treat this predication as a datum:

“Individual *a* is an alleged assassin”

Its analysis in Transparent Intensional Logic is this:

$$\lambda w \lambda t [[{}^0 \text{Alleged} {}^0 \text{Assassin}]_{wt} {}^0 a]$$

Types: *Alleged*/ $((\text{ot})_{\tau\omega} (\text{ot})_{\tau\omega})$; *Assassin*/ $(\text{ot})_{\tau\omega}$; *a*/ v ; *w*/ $*_1 \rightarrow_v \omega$; *t*/ $*_1 \rightarrow_v \tau$.

¹ See (Duží et al. 2010, §1.4.2.1) for the definitions of *purely contingent property* and *contingent property with an essential core*. See (*ibid.*) for notions and notation.

* Utrecht University and VŠB-Technical University of Ostrava
✍ Department of Philosophy and Religious Studies, Utrecht University, Janskerkhof 13, 3512 BL Utrecht, The Netherlands; Department of Computer Science, VŠB-Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava, Czech Republic
✉ b.t.f.jespersen@uu.nl



I claim that two conclusions are forthcoming. The first conclusion is that there is some property f which a is alleged to have:

$$\lambda w \lambda t [{}^0 \exists \lambda f [{}^0 \text{Alleged } f]_{wt} {}^0 a]]$$

Types: $f / *_{1 \rightarrow v} (\text{ot})_{\tau\omega}$; $\exists / (\text{o}(\text{o}(\text{ot})_{\tau\omega}))$.

This predication is non-trivial, because not all of us are being alleged to have some property or other.² The second conclusion is that maybe a is an assassin and maybe a is not an assassin:

$$\lambda w \lambda t [{}^0 \text{Alleged } {}^0 \text{Assassin}]_{wt} {}^0 a] \supset \\ [{}^0 \exists \lambda w' [{}^0 \exists \lambda t' [{}^0 \text{Assassin}_{w't'} {}^0 a]] \wedge {}^0 \exists \lambda w'' [{}^0 \exists \lambda t'' \neg [{}^0 \text{Assassin}_{w''t''} {}^0 a]]]$$

where $w' \neq w''$, $t' \neq t''$.

A dichotomy is induced over the domain of world/time pairs, such that in one half of the domain it is true that a is one of the assassins and in the other half it is false that a is among the assassins. The open question is which side of the fence a given world/time pair of evaluation comes down on. The logical behaviour that the modal modifier displays is that it oscillates, as it were, between being *subsective* and being *privative*. A subsective modifier has the effect that the modifier is eliminated and the original argument property is predicated of the individual in question. For instance, a skilful surgeon is a surgeon. A privative modifier has the effect that the predication of the privatively modified property is replaced by the boolean negation of the predication of the argument property.³ For instance, it is not the case that a fake banknote is a banknote.

The counterexample Kosterec levels against my theory is this predicate:

‘is an alleged discoverer of the highest prime number’

There is no highest prime number, hence nobody can instantiate the property of discovering the highest prime number, hence the left-hand conjunct

² See (Jespersen 2016) for the *general rule of left subsectivity*, which in (Duží et al. 2010, §4.4) was introduced under the name of *pseudo-detachment*.

³ – in the case of *single* privation, that is. In the case of *iterated* privation, privative modifiers are replaced by the general privative modifier *Non*. See (Jespersen et al. 2017).

(from being an alleged assassin to being an assassin) is false, hence the conjunction is false, hence the inference is invalid.

When confronted with impossibilities, the strategy pursued by Transparent Intensional Logic is not to usher in impossible worlds as additional points of evaluation. Instead we introduce constructions of conditions that could not possibly be satisfied (see Duží et al. 2020). What we need here is, first of all, a construction of the impossible property of being a discoverer of the highest prime:

$$\lambda w \lambda t [\lambda x [{}^0 Discover_{wt} x [{}^0 \lambda y [{}^0 Prime y] \wedge {}^0 \forall \lambda z [{}^0 Prime z] \supset [{}^0 \geq y z]]]]]$$

Types: $x/*_1 \rightarrow_v \iota$; $y, z/*_1 \rightarrow_v \tau$; $Discover/(o\iota^*_n)_{\tau\omega}$; $Prime/(o\tau)$; $\iota/(\tau(o\tau))$; $\forall/(o(o\tau))$; $\geq/(o\tau\tau)$.

The analysis of “*a* is an alleged discoverer of the highest prime” is:

$$\lambda w \lambda t [{}^0 Alleged \lambda w \lambda t [\lambda x [{}^0 Discover_{wt} x [{}^0 \lambda y [{}^0 Prime y] \wedge {}^0 \forall \lambda z [{}^0 Prime z] \supset [{}^0 \geq y z]]]]]_{wt} {}^0 a]$$

How do we eliminate *Alleged*? By invoking the fact that at no world/time pair is *a*, or anyone else, someone with the property of discovering the highest prime.

We are going to define the property *X*, which is an analytic property of ι -properties, namely the property of being necessarily uninstantiated (‘empty’). Thus, its functional arguments being $F_i \in X$, *Alleged* modifies impossible empirical conditions.⁴ First of all, we define \emptyset^1 as the set of empty ι -sets, whose respective characteristic functions do not return the truth-value 1 for any argument, i.e., they either return 0 or are undefined:

$${}^0 \emptyset^1 =_{df} \lambda e [{}^0 \forall \lambda x \neg [{}^0 True^* {}^0 [e x]]]$$

Types: $e/*_1 \rightarrow (o\iota)$; $\emptyset^1/(o(o\iota))$; $=/(o(o(o\iota))(o(o\iota)))$; $True^*/(o^*_n)$: the set of such constructions as v -construct 1 for every valuation v .

⁴ I should stress that the addition to the theory of modal modifiers I have offered here still does not extend to purely arithmetical cases as expressed by predicates like ‘is an alleged proof of the continuum hypothesis’. What is already clear, though, is that, *Proof* being of type (o^*_n) , namely, a set of hyperpropositions, *Alleged* as denoted in ‘is an alleged proof’ must be of type $((o^*_n) (o^*_n))$.

Now define X as follows:

$${}^0X = {}'_{df} \lambda f [{}^0\forall \lambda w [{}^0\forall \lambda t [{}^0\exists' f_w]]]$$

Types: $X / (\mathbf{o}(\mathbf{o}\iota)_{\tau\omega})$; $f / {}^*_{i \rightarrow v} (\mathbf{o}\iota)_{\tau\omega}$; $= / (\mathbf{o}(\mathbf{o}(\mathbf{o}\iota)_{\tau\omega})(\mathbf{o}(\mathbf{o}\iota)_{\tau\omega}))$.

A parallel definition of X , to be deployed below, is this one:

$${}^0X = {}'_{df} \lambda f [{}^0\forall \lambda w [{}^0\forall \lambda t [{}^0\forall \lambda x \neg [{}^0True_{wt} \lambda w \lambda t [f_{wt} x]]]]]$$

Type: $True / (\mathbf{o}\mathbf{o}\iota)_{\tau\omega}$: the empirical property of truth-conditions/ $\mathbf{o}\iota_{\tau\omega}$ of being satisfied (i.e., returning 1) at a given world/time pair of evaluation.

Where $F \in X$, the elimination of *Alleged* proceeds as follows:

$${}^0\forall \lambda w [{}^0\forall \lambda t [{}^0\forall \lambda x [[{}^0Alleged {}^0F]_{wt} x] \supset \neg [{}^0True_{wt} \lambda w \lambda t [{}^0F_{wt} x]]]]]$$

This clause is the solution to the problem presented by the first limiting case, which Kosterec has brought up. Of course, X has a mirror-image, Y , which is the analytic property of ι -properties of being necessarily instantiated ('trivial'):

$${}^0Y = {}'_{df} \lambda f [{}^0\forall \lambda w [{}^0\forall \lambda t [{}^0\forall \lambda x [f_{wt} x]]]]]$$

The elimination of *Alleged* now proceeds as follows, for any $G \in Y$:

$${}^0\forall \lambda w [{}^0\forall \lambda t [{}^0\forall \lambda x [[{}^0Alleged {}^0G]_{wt} x \supset [{}^0G_{wt} x]]]]]$$

The addition of the first clause to my theory of modal modification departs from the observation that nobody and nothing could possibly instantiate any property F when $F \in X$ and F has been modified by *Alleged*. Remember that the definition of modal modifiers applicable to contingent empirical properties (i.e., (*i*), (*ii*)) embodies a bifurcation, and that the way the cookie happens to crumble determines whether the alleged property is true of the individual in question. Modal modifiers applicable to necessarily uninstantiated properties are importantly different, in that the predication can go only one way: the alleged property must fail to be true of the individual in question. Therefore, my account of this second category of modal modifiers aligns them formally with privative modifiers. The difference between the two, though, is that the source of privation is not the modifier (*Alleged* versus *Fake*), but the argument property itself (*being a discoverer of the highest prime*, *being a married bachelor* versus *being a banknote*). At the other end of the spectrum, when $G \in Y$ and G has been modified by

Alleged, everyone and everything must instantiate *G*, thus aligning this third category of modal modifiers with (trivial) subsective modifiers. The difference between the two is that the source of triviality is, likewise, not the modifier (though *Genuine*, as in *being a genuine diamond*, adds or detracts nothing), but the argument property itself (e.g., *being as tall as one is*).

Acknowledgments

I am indebted to Marie Duží for valuable comments.

Funding

This research was funded by the Grant Agency of the Czech Republic (GACR) project GA18-23891S *Hyperintensional Reasoning over Natural Language Texts*.

References

- Duží, Marie, Glavaničová, Daniela, and Jespersen, Bjørn. 2020. "Impossible Individuals as Necessarily Empty Individual Concepts." In *Logic in High Definition: Developing Fine-Grained Semantics, Trends in Logic*, edited by Alessandro Giordani and Jacek Malinowski, forthcoming.
- Duží, Marie, Jespersen, Bjørn, and Materna, Pavel. 2010. *Procedural Semantics for Hyperintensional Logic. Foundations and Applications of Transparent Intensional Logic*, LEUS, vol. 17, Heidelberg et al.: Springer-Verlag.
<https://doi.org/10.1007/978-90-481-8812-3>
- Jespersen, Bjørn. 2016. "Left Subsectivity: How to Infer That a Round Peg is Round." *dialectica* 70 (4): 531–47. <https://doi.org/10.1111/1746-8361.12159>
- Jespersen, Bjørn, Carrara, Massimiliano, and Duží, Marie. 2017. "Iterated Privation and Positive Predication." *Journal of Applied Logic* 25 (suppl.): 48–71.
<https://doi.org/10.1016/j.jal.2017.12.004>
- Jespersen, Bjørn and Primiero, Giuseppe. 2013. "Alleged Assassins: Realist and Constructivist Semantics for Modal Modifiers." In *Lecture Notes in Computer Science*, vol. 7758, edited by Guram Bezhanishvili, Sebastian Löbner, Vincenzo Marra, and Frank Richter, 94–114. https://doi.org/10.1007/978-3-642-36976-6_8
- Kostelec, Miloš. 2019. "Contradiction of Modal Modification." *Organon F* 26 (2): 298–300. <https://doi.org/10.31577/orgf.2019.26207>