Journal of Mathematical Biology (2020) 81:905–906
https://doi.org/10.1007/s00285-020-01506-w

Mathematical Biology

CORRECTION

Correction to: Finite dimensional state representation of physiologically structured populations

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Correction to: Journal of Mathematical Biology (2020) 80:205–273. https://doi.org/10.1007/s00285-019-01454-0

“In the original publication of the article, the Subsection 2.1.2 was published incorrectly. The corrected Subsection 2.1.2. is given below”.

2.1.2 The mathematical question

Our starting point thus are models that can be represented as in the following diagram.

$$Y \xrightarrow{U_E^c(t,s)} Y \xrightarrow{O(E(t))} \mathbb{R}^r \qquad \text{(A)}$$

Here Y is the p-state space and $\mathbb{R}^r$ the output space. E is the time course of the environment and $U_E^c(t,s)$ the (positive) linear state transition map with s, t the initial

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The original article can be found online at https://doi.org/10.1007/s00285-019-01454-0.

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and final time. (The upper index c here refers to the mathematical construction of the p-state, explained in Section 3, through the cumulation of subsequent generations.) Finally $O(E(t))$ is the linear output map. The mathematical question then is under which conditions on the model ingredients it is possible to extend diagram (A) (for all E, t, s) to the following diagram.

$$\begin{tikzcd}
Y \arrow[r, "U_E^c(t,s)"] \arrow[d, "P"'] & Y \arrow[d, "P"] \arrow[dr, "O(E(t))"] & \\
& & \mathbb{R}^r \\
\mathbb{R}^k \arrow[r, "\Phi_E(t,s)"'] & \mathbb{R}^k \arrow[ur, "Q(E(t))"'] &
\end{tikzcd} \tag{B}$$

Here P is a linear map, $\Phi_E(t, s)$ a linear state transition map (which should be differentiable with respect to t) and $Q(E(t))$ a linear output map. The dynamics of the output cannot be generated by an ODE when the space spanned by the output vectors at a given time is not finite dimensional. Hence ODE reducibility implies that there exists an r such that the outputs at a given time can be represented by $\mathbb{R}^r$. (Below we drop the time arguments to diminish clutter, except in statements that make sense only for each value of the argument separately, or when we need to refer to those arguments.) Moreover, the biological interpretation dictates that

$$O(E)m = \langle m, \Gamma(E)\rangle := \int_\Omega \Gamma(E)(x)m(dx),$$

where m is the p-state and the components of the vector $\Gamma(E)$ functions $\gamma_i(E)\colon \Omega \to \mathbb{R}$.

Thanks to the linearity of $U_E^c(t, s)$ and $O(E(t))$ we can without loss of generality assume $P, \Phi_E(t, s)$ and $Q(E(t))$ to be linear. Moreover, ODE reducibility requires that P can be written as $Pm = \langle m, \Psi\rangle$ with $\Psi = (\psi_1, \ldots, \psi_k)^{\mathrm{T}}$, $\psi_i : \Omega \to \mathbb{R}$, where the ψ_i should be sufficiently smooth to allow

$$dN/dt = K(E)N, \text{ with } N := Pm \text{ and } K(E(t)) := d\Phi_E(t,s)/dt|_{s=t}. \tag{2.1}$$

(The last expression comes from combining $d\Phi_E(t, s)/dt = K(E(t))\Phi_E(t, s)$ and $\Phi_E(t, t) = I$.) Finally, we should have $O(E) = Q(E)P$, and therefore $\Gamma(E) = Q(E)\Psi$, implying that the output weight functions should be similarly smooth. (The precise degree of smoothness needed depends on the other model ingredients in a manner that is revealed by the TEST described in Sect. 2.1.4.)

In addition Fig. 2 on p. 230 should be removed, ‘the diagram in Fig. 2’ three lines above Eq. (4.11) changed into ‘Diagram (B)’, and ‘Fig. 2’ on the 3rd line of p. 271 into ‘Diagram (B)’.