

# Enabling market participation of distributed energy resources through a coupled market design

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**Abstract:** In this study, a new market model is proposed to enable the smooth integration of distributed energy resources (DERs) in day-ahead (DA) and balancing (BL) markets. The DA market is a joint energy and reserve market. DERs participate in the local market operated by the distribution market operator (DMO) which is coordinated with the wholesale market. During local market clearing, the DMO considers the technical constraints of the distribution grid to keep the operation of the distribution grid within limits. Moreover, the DMO, through preliminary scheduling, estimates preliminary prices for energy, reserve and balancing services. These can be seen as offer prices by which the DMO will bid into the TMO DA and BL market. The uncertainty of renewable-based DERs is considered through the use of stochastic programming. The proposed market model is compared with a centralised market model. Results show that adding the distribution network constraints to the proposed market model is capable of alleviating overloading and appropriately accounting this effect in terms of increasing the operational cost. Moreover, it has been shown that the scalability of the proposed market model is higher compared to the currently applied centralised market models.

## Nomenclature

### Sets and indices

$g$	index of generations
$G_D$	set of distributed generations
$G_T$	set of transmission generations
$i(I)$	index(set) of sending nodes
$j(J)$	index(set) of receiving nodes
$l(L)$	index(set) of line
$L_D$	set of distribution lines
$L_T$	set of transmission lines
$N_D$	set of distribution nodes
$N_T$	set of transmission nodes
$N_{D-T}$	set of interface nodes in distribution system
$N_{T-D}$	set of interface nodes in transmission system
$s(S)$	index(set) of scenarios
$t(T)$	index(set) of time steps
$w(W)$	index(set) of wind farms

### Parameters

$b_i$	nodal susceptance (p.u.)
$B_l$	line shunt susceptance of transmission line $l$ (p.u.)
$O_{g,t}^{ED}$	energy offer price of generators at distribution system (€/MWh)
$O_{g,t}^{RD}$	reserve capacity offer price of generators at distribution system (€/MW)
$O_{g,t}^{ET}$	energy offer price of generators at transmission system (€/MWh)
$O_{g,t}^{RT}$	reserve capacity offer price of generators at transmission system (€/MW)
$G_i$	Distribution nodal admittance (p.u.)
$P_{i,t}^{load}/Q_{i,t}^{load}$	active/reactive power load demand (MW/MVAr)
$P_{w,t,s}^{Wact}$	actual wind farm power production (MW)
$P_w^{Wmax}$	installed wind farm power (MW)
$\overline{P}_{g,t}$	scheduled power of generator $g$ (MW)
$\overline{P}_{w,t}^{WDA}$	scheduled power of wind generator (MW)

$\overline{P}_{i,t}^{DT}$	scheduled power at interface node T-D (MW)
$\overline{P}_{i,t}^{DER}$	scheduled power of DERs at interface node T-D (MW)
$P_g^{gmax}/P_g^{gmin}$	maximum/minimum active power of generator $g$ (MW)
$Q_g^{gmax}/Q_g^{gmin}$	maximum/minimum reactive power of generator $g$ (MVAr)
$r_l$	resistance of a distribution line
$\overline{R}_{g,t}$	scheduled reserve capacity of generator $g$ (MW)
$\overline{R}_{i,t}^{DT}$	injected reserve capacity from TSO at node $i \in N_{T-D}$ (MW)
$S_{g,t}, S_{g,t,s}$	rated apparent power of generator $g$ (MVA)
$S_{l,t}$	rated apparent power of line $l$ (MVA)
$TC_l$	transmission line capacity
$V_{i,t}^{max}/V_{i,t}^{min}$	maximum/minimum voltage of bus $i \in N_D$
$x_l$	reactance of a distribution line
$\alpha_D$	coefficient for total reserve capacity requirement in distribution system
$\alpha_T$	coefficient for total reserve capacity requirement in transmission system
$\overline{\Delta P}_{i,t}^{DT}$	scheduled power at node $i \in N_{D-T}$ (MW)
$\overline{\Delta P}_{i,t}^{TD}$	scheduled power at node $i \in N_{T-D}$ (MW)
$\lambda_{t,s}^{DA}$	day-ahead market price (€/MWh)
$\pi_s$	scenario probability

### Decision variables

$f_{l,t,s}^p/f_{l,t,s}^q$	active/reactive power over line $l$ (MW/MVAr)
$I_{l,t}/I_{l,t,s}$	square current over line $l$ (A)
$P_{g,t}^{sch}/Q_{g,t}^{sch}$	scheduled active/reactive power output from generator at the distribution system (MW/MVAr)
$P_{g,t}/Q_{g,t}$	scheduled active/reactive power output from generator $g$ (MW/MVAr)
$P_{g,t,s}/Q_{g,t,s}$	active/reactive power output from generator $g$ after scenario realisation (MW/MVAr)
$P_{w,t}^{WDA}$	scheduled wind power (MW)

$P_{i,t}^{\text{TD}}/Q_{i,t}^{\text{TD}}$	real/reactive power injection in T-D interface node $i$ (MW/MVar)
$P_{i,t}^{\text{DT}}$	real power injection in interface node $i \in N_{D-T}$ (MW)
$P_{i,t}^{\text{DER}}$	real power injection from DER in the interface T-D node (MW)
$R_{g,t}$	scheduled reserve capacity of the generator $g$ (MW)
$R_{i,t}^{\text{DT}}$	aggregated reserve capacity at node $i \in N_{D-T}$ (MW)
$R_{i,t}^{\text{DER}}$	aggregated reserve capacity from DERs at T-D node (MW)
$V_{i,t}/V_{i,t,s}$	square bus voltage (p.u.) at node $i \in N_D$ after scenario realisation
$\Delta P_{g,t,s}$	schedule adjustment of generator $g$ in scenario $s$ and time $t$ (MW)
$\Delta P_{g,t}$	schedule adjustment of generator $g$ and time $t$ (MW)
$\Delta P_{i,t}^{\text{DT}}$	schedule adjustment of node $i \in N_{D-T}$ and time $t$ (MW)
$\Delta P_{i,t}^{\text{TD}}$	schedule adjustment of node $i \in N_{T-D}$ and time $t$ (MW)
$\Delta P_{i,t,s}^{\text{TD}}$	schedule adjustment of node $i \in N_{T-D}$ in scenario $s$ and time $t$ (MW)
$\Delta Q_{g,t,s}$	schedule reactive power adjustment of generator $g$ in scenario $s$ and time $t$ (MVar)
$\Delta_{w,t}$	power imbalance caused by wind uncertainty (MW)
$\lambda_{i,t}^{\text{ED}}$	DMO day-ahead energy market price (€/MWh)
$\lambda_{i,t}^{\text{RD}}$	DMO day-ahead reserve capacity market price (€/MW)
$\lambda_{i,t}^{\text{DBL}}$	DMO balancing energy market price (€/MWh)
$\theta_{i,t}$	transmission bus angle

## 1 Introduction

The number of distributed energy resources (DERs) such as distributed diesel generators, electric vehicles, storage systems, wind turbines, and solar PV is increasing. New studies show that in addition to supplying energy, DERs can also deliver ancillary services, e.g. reserve and balancing services, to the grid. In this way, DERs can contribute to the stability of the grid [1]. However, scheduling DERs remains an open question, especially from a market perspective. The electricity market is usually cleared at the national level by the wholesale market operator and consequently is designed for large generators which are connected to the transmission system. The existing market model is unable to deal with the participation of DERs in the wholesale market in the most efficient way, although many options have been examined to counteract the specific problem that these DERs pose for the current electricity markets. This has led to many different approaches to the electricity market design. Though the design of the electricity market differs across the globe, the objectives reliable and affordable electricity is the main driver to many of these markets [2]. For example, in [3], a competitive market for distributed generations is proposed to formalise their participation in the wholesale market. The way the electricity market is designed in practice can have a significant effect on the market prices and the price volatility [4]. The introduction of large renewable resources poses additional challenges for the market design and has led to specific changes in the market [5]. The proliferation of prosumers poses additional difficulties for the electricity markets and has therefore given rise to new approaches for the electricity market design [6]. In this paper, a new market design is proposed to efficiently integrate DREs in the electricity markets.

There are some issues that hamper the participation of DERs in the wholesale market. Firstly, there is usually a minimum bid-size requirement for participating generators in the wholesale market [7, 8]. In other words, the minimum bid size is there to keep the number of participating generators manageable; as DERs are small in size this limits their possibilities to participate in the market. Secondly, there are usually many DERs connected to the distribution network and managing small-size numerous generators for one wholesale market operator could be very complicated. For example, in [9], a central electricity market is proposed for the participation of all generators connected to the transmission and

distribution networks and a common optimisation is applied during the market clearing process. Although in this market structure, distribution, and transmission constraints are simultaneously taken into account, the market clearing involves a computationally heavy procedure, due to a large number of constraints that must be taken into account. Lastly, usually, the transaction fees for market players who want to participate in the wholesale market are quite high and not affordable for one small DER unit [10]. The changes in the energy system have led to renewed interest in the electricity market design.

One solution in order to cope with the aforementioned problems is aggregating DERs. Based on the definition in [11], the aggregator is ‘a market participant that combines multiple customer loads or generated electricity for sale, for purchase, or auction in any organised energy market.’ In [12, 13], the different values that aggregators bring in electric systems have been presented. Although in literature, aggregation seems a promising solution to facilitate the participation of DERs in the electricity market, however, aggregators do not take into account distribution network constraints as they have no incentive to do so. Also, aggregators do not usually have information about the geographical location of the DERs in the distribution network. Consequently, dispatching of DERs can lead to over/under voltages or congestion problems in the distribution grid. Some earlier publications have taken into account the grid constraints during aggregator’s optimisation approach, however, they are somehow complicated and time-consuming [14].

Moreover, not only the aggregator but also the current wholesale market operator has only access to (simplified) information about the transmission network. The resources at both the distribution and transmission system bid into the wholesale market. In the market clearing process, the distribution network constraints are not taken into account and the distribution system operator (DSO) is not involved in the procurement of energy and balancing services from the resources in its network. The proposed market scheme in [15] is such an example where there is only one central electricity market. The independent system operator (ISO) as the market operator is the only entity which has access to all the resources at the distribution and transmission system to be used for the entire power system balancing. As the wholesale market operator does not have detailed information on the distribution system where DERs are injecting their power, there is a risk of violating the operational constraints of the distribution network.

Another alternative for the participation of DERs in the energy and balancing (BL) market is a local electricity market (LEM) where DERs’ services could be procured and the DSO (or a new entity) is involved as an active player [16]. The focus of this paper is on this solution. This paragraph presents some existing literature where the LEM has been studied either practically or theoretically. In [17], an analysis of different organisational models for flexibility management in the local market and the feasibility of large-scale deployment of these models in the European regulatory context is presented. Another example in which the concept of LEM has been developed is in [18], which shows a general description of local flexibility markets as a market-based management mechanism for aggregators and the needed interactions between all local market stakeholders. There are some European pilot projects where the idea of LEM has been extended. The De-Flex Market developed by the German Association of Energy Market Innovators (BNE) is an example which provides an instrument for the DSO to solve local capacity constraint using DER flexibility [19]. Another example is the FLECH market, a Danish project which tries to solve the congestion happened in the distribution system with a high number of DERs [20]. The following papers utilise the LEM form different aspects. In [21], a local energy market design is investigated to realise market-based control for the integration of PV generation and residential energy storage. In [22], the LEM is applied in which individual users can sell their excess electricity either to other users in their neighbourhood or to suppliers, based on a system of bidding. Finally, Luth *et al.* [23] study the application of LEM in the peer to peer trading for electricity storage systems. It investigates the value of prosumer batteries and the market features

by which the economic potential of end-user batteries can be extracted the most.

Based on the above-mentioned literature, there are several advantages in having an LEM for DERs. First form the DER's perspective, aside from facilitating the participation of DERs in electricity markets, the LEM is an opportunity where the flexibility and ancillary services (e.g. voltage regulation, reserve capacity etc.) form DERs can be fully extracted. Moreover, through the LEM, the issues regarding the scalability and integrating of numerous amounts of DERs into the distribution system, which is expected to happen in the near future, can be remarkably reduced. From the distribution system's perspective, the DSO would be the first beneficiary of flexibility and ancillary services which are now directly delivered to the distribution system through the LEM. Furthermore, the dependency of the distribution system on the transmission or higher grid systems would be reduced through the LEM and consequently, the resilience of the entire power system would be improved. Finally, from transmission system's perspective, besides eliminating the complexity of scheduling many DERs in one central market, the LEM can also act as a balance responsible party (BRP) helping the transmission system in balancing the supply and demand.

However, in the aforementioned literature, detailed and complete modelling of the distribution network security constraints during the market clearing process of their LEM scheme is absent. Note that, 'detailed modelling' of distribution network constraints refers to a full AC power flow representing the distribution grid, as the DC optimal power flow (DCOPF) cannot fully reflect the nature of the distribution network. The DCOPF does not consider voltage and reactive power, which are critical features in ensuring the transmission of real power in distribution system operations especially when subjected to voltage problems [24]. Therefore, this still remains as an open question how to develop an LEM to facilitate the participation of DERs into the electricity market while the distribution network constraints can still work in their secure and stable operational limits.

In addition to the introduction of an LEM in the distribution level, there should be coordination between the national market and the LEM [25]. For example, Verzijlbergh *et al.* [26] propose a local market in the distribution system which is operated by the DSO. The transmission system operator (TSO) is only the operator of the central market and cannot access DERs and utilise their balancing services for its own purposes. In this market model, the efficiency of resource allocation is relatively low and BRPs may face a higher balancing cost, as both the TSO and DSO have limited access to resources outside their jurisdiction area and there is no guarantee that the resources are used efficiently throughout the whole system. Additionally, if the local market is relatively small, it might not have enough resources to ensure adequate security of supply. Thus, coordination between the central market operator and the local market operator is essential to unlocking the full potential of DERs for the provision of flexibility and ancillary services to the benefit of the entire power system [27]. In [13, 28] a coordinated approach has been shown to be able to balance supply and demand system-wide while resolving voltage and congestion issues locally.

To overcome the aforementioned issues, this paper introduces a new market structure which allows full participation of DERs in the day-ahead (DA) and BL markets. The focus of this paper is on the short-term market operation. Although in this paper, the market model mostly follows European markets where the imbalance are dissolved through the BL market, however, the DA market is assumed to be a joint energy and reserve (JoEnRe) capacity market which is more seen in the US market designs [29]. Simultaneous arrangement of energy and reserve capacity gives the authority to market players to arbitrate between both on shorter notice and consequently increases the utilisation of their flexibility in the market. Therefore, JoEnRe capacity markets are being investigated in the current market organisation of the European markets [30], although currently in Germany, separate markets for scheduled energy, reserve capacity, and balancing energy exist [31]. Particularly, in the coupled market the need for the reserve capacity market in the DA market is more tangible, as there is a separate

market in the local and central markets and there is a limitation for the available resource in the balancing phase. At each of DA or BL markets, the local market operator first solves a preliminary scheduling problem where the total cost of the local market is minimised while the distribution network constraints are taken into account. Through this optimisation, the local market operator can estimate the local market price for the energy or balancing services and based on those prices, he will bid into the central market, on behalf of the DERs. In this market model, a stochastic programming approach is used to take the uncertainty of DERs such as wind turbines into account. A total cost comparison of a system with the proposed market model and the current market model is also conducted. In summary, the main research question which is going to be answered through this paper is whether the coupled market model is an efficient market design when large amount of DERs are present. This question is answered by comparing the total system cost for the coupled market and the centralised market models while including or ignoring the distribution network constraints.

The contribution of this paper is three-fold:

- A new market model is proposed for the participation of DERs in both DA and BL markets. Distribution network constraints are included during local market clearing process to keep the network within secure operating limits. Moreover, the proposed market model introduces an ancillary service market where DERs can trade their reserve capacity.
- A new coordination scheme between the wholesale and the local markets is introduced.
- The comparison between the proposed market model and the centralised market model shows that distribution network constraints might be violated when DERs participate in the wholesale market.

The remainder of this paper is organised as follows. In Section 2, the proposed coupled market model together with its mathematical formulations is presented. In Section 3, a version of the current centralised market model is described. In Section 4, the case studies and the results are shown. The conclusions on the proposed market model are drawn in Section 5.

## 2 Coupled market model

In this section, the coupled market model is explained thoroughly. First, in Section 2.1 Market organisation, the role and responsibility of each market player, distribution, and transmission market operators are described. In Section 2.2 TMO-DMO interaction, the interactions between the two market layers in the distribution and transmission level are described. In Section 2.3 Market timing, the sequence of steps is explained. Finally, in Section 2.4 Scalability, it is shown how the coupled market can deal with an increasing amount of DERs in distribution systems.

### 2.1 Market organisation

This section gives an overview of how the coupled market model works. Fig. 1 shows this market scheme. In this market model,

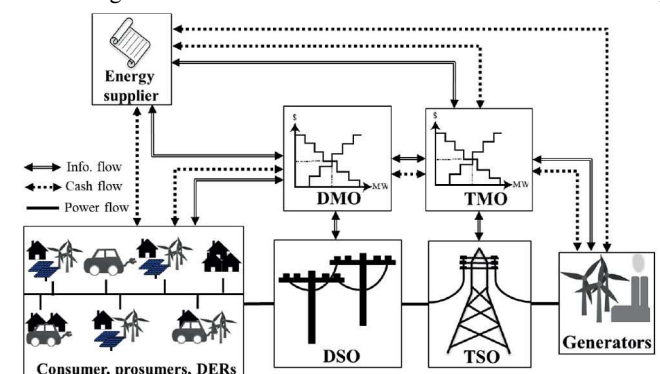


Fig. 1 Market organisation

there is one national market operated by the transmission market operator and there can be multiple local markets operated by distribution market operator (DMO). Below, the main market players and their roles, responsibilities, and interactions are explained in details.

- *Consumers, prosumers, and DERs:* Local resources including consumers, active loads, stochastic generators, e.g. wind turbines and dispatchable generators, e.g. electric vehicles which are connected to the distribution network. These local resources participate in the local market which is operated by the DMO. Those resources which are not participating in the local market may sign a long-term contract with an energy supplier who buys energy from bulk generators and sells it to local resources.

Moreover, the uncertainties of the power output of some local renewable-based generators such as wind turbines or PVs can be eliminated through this local market. In other words, the aggregated bid from all the local resources which will be sent to the central market by the DMO does not contain any uncertainties. Consequently, this can reduce the complexity of handling many uncertain renewable-based generators in a central market.

- *Distribution market operator:* As mentioned earlier, the DMO is the local market operator to whom DERs offer their energy and/or balancing services. The DMO can be considered as the distribution network equivalent of the market operator, which is responsible for managing the electricity market and scheduling power transfers to achieve the secure operation of the distribution network [32, 33]. The DMO should be able to perform load forecasting, be responsible for balancing the supply and demand locally, be able to receive offers from DERs, aggregate them and participate in the central market, and eventually be able to control the local market for various DERs. The DMO can be part of the DSO or an independent entity. In both cases, DMO and DSO should exchange information regarding the distribution network security condition and dispatching the local resources. The DMO has two objectives: first, to aggregate individual bids received from local consumers and DERs in its jurisdiction area to create an aggregated bid and hence, send the aggregated bid to the central market to participate in the national energy and BL markets; secondly, through the local market clearing disaggregate the assigned quantity by the central market operator to the local consumers and DERs.
- *Transmission Market Operator:* Similar to the DMO, the transmission market operator (TMO) is the equivalent market operator which manages the wholesale energy and BL markets where the DMO, energy suppliers, and bulk generators participate. The TMO can be considered as the power exchange in the European Electricity Market or the ISO in the United States. The TMO and the TSO should also exchange information regarding the security of the transmission network during market clearing processes.

For the TSO, the DSO and energy suppliers, this paper assumes the same roles and responsibilities as explained in Universal Smart Energy Framework (USEF) [34].

In summary, reducing the complexity of direct scheduling of DERs in the wholesale market, solving scalability issues, improving grid resilience by reducing the dependency on the TSO are among the beneficial functions that the DMO-operated LEM can provide to the power system [35, 36].

## 2.2 TMO-DMO interaction

One of the main challenges of the LEM is TMO-DMO interaction. In general, there are two types of TMO-DMO interactions regarding the power flow over the interface transformer between transmission and distribution grids: (a) unidirectional and (b) bidirectional power flow. Distribution systems for many years have been designed based on the assumption unidirectional power flow [37]. In this case, the power can only flow from HV to MV level. In contrast, in the bidirectional power flow, the power can be

imported or exported from or to the local market. Therefore, in case those DERs are not enough to fulfil the load of the distribution level, the energy from the transmission level will be imported. Therefore, there is always a connection between distribution and transmission networks and in case of lack or excess of power in the distribution network, it can be imported or exported from the higher level.

Not only the power flow but also the information flow between the DMO and the TMO, can be divided into the unidirectional and bidirectional information flow. In the unidirectional information flow, there is a static limit for the power flow over the interface transformer. This means, during the market clearing process at a local or at the wholesale level, there is no exchange of information between the TMO and the DMO regarding the interface power flow. On the contrary, in the bidirectional information flow, there is a dynamic limit for the power flow over the interface transformer which means there is an exchange of information between the TMO and the DMO during the market clearing processes. The focus of this work is on the bidirectional flow both for power and information. This way of interacting between the TMO and the DMO is the reason for calling this model a ‘coupled TMO-DMO market’.

Fig. 2 describes the concept of bidirectional information flow between the TMO and the DMO. First, the DMO through preliminary scheduling solves an optimisation problem where the total system cost within its jurisdiction area is minimised. In this optimisation, the distribution network constraints are taken into account. The results of this preliminary scheduling are the local market prices (depending on the market, either DA energy and reserve or BL market prices) and an initial limit for the power flow over the HV/MV transformer ( $P_{T-D}^1$ ). These results will be sent to the TMO central market where the DMO will participate as a market party. In the next step, the central market is cleared by the TMO and a final value for the power flow over the HV/MV transformer will be sent back to the DMO ( $P_{T-D}^2$ ). Finally, the DMO clears the local (DA or BL) market.

## 2.3 Market timing

In Fig. 3, the time sequence of the coupled TMO-DMO market model is shown. The three steps of the bidirectional power flow shown in Fig. 2 happen in both the DA and BL markets, as it is shown in Fig. 3. In total there are six steps in Fig. 3. Step 1 is preliminary scheduling for the local DA-JoEnRe market. The reasons for introducing a reserve capacity market and including this as part of a JoEnRe capacity market in this paper are as follows. Firstly, in the coupled market, the TSO relies on the DMO market for the BL market. As the TSO does not have control over DERs and the distribution grid, there is a chance that in the BL market, there will be a lack of resources. To avoid this situation, the reserve market should be created to guarantee that there will be enough energy available for the balancing phase. The provision of

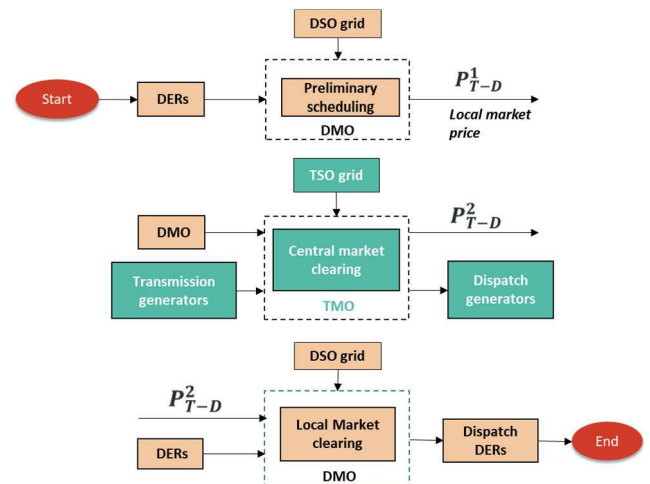
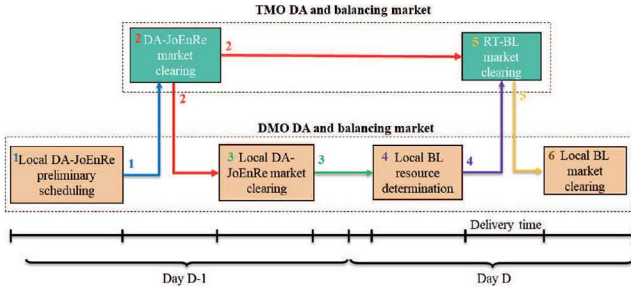


Fig. 2 TMO-DMO bidirectional interaction





**Fig. 3** Coupled TMO-DMO DA and BL markets model

reserve capacity is remunerated with a reservation price (€/MW) for reserving the capacity. Secondly, the European regulators are paying more attention to the reserve market and the simultaneous alignment of energy provision and reserve capacity as a more efficient market design [30]. Currently, a simultaneous market clearing for energy and reserve market does not happen in European electricity markets and it more belongs to US electricity markets as explained in [29].

Step 1 happens in D-1, the day before the delivery time and at a time, before the clearing time of the DA wholesale market. Through this step, the DMO solves an optimisation problem for determining the preliminary local DA energy and reserve capacity market prices and an initial limit for the power flow over the HV/MV transformer. Note that in this step the local market has not been yet cleared but instead, the DMO obtain information by with can later bid into the TMO DA market (in Step 2).

Step 2 is the TMO DA market and clears in D-1 with the time resolution of 1 h and in a 24-hour time horizon. Same as the local market, the DA market is the JoEnRe capacity market. The results for this step are the scheduled power of bulk generators which will be sent to the central BL market (Step 5) and a final value of the power flow over the HV/MV transformer which will be sent back to the DMO.

Right after clearing the DA wholesale market, the local DA-JoEnRe market is cleared by the DMO in Step 3. The results of this market will be sent to the local BL market in Step 4.

The procedure in the BL market is similar to that of the DA market. The difference is the duration of the scheduling interval which is 15 min for the BL market. Step 4 which happens in D, the day of the delivery time, is preliminary scheduling for the local BL market and happens near real-time. Through this step, the local BL market price and an initial value for the power flow over HV/MV transformer are estimated. In Step 5 (real-time), the TMO clears the central real-time BL market according to the scheduled energy and reserve of market players. The TMO will send back the final value for the HV/MV transformer power to the DMO. Finally, in Step 6, the local BL market is cleared by the DMO, based on the updated interface power flow from Step 5 and the DER scheduled energy and reserve from Step 3.

Note that, in this paper, it is assumed that only the market entities which have been accepted in the reserve market are allowed to participate in the BL market where the actual activation of the energy will happen. The procured balancing energy is limited to the scheduled reserve in the reserve capacity market. However, the generators which have been chosen in the reserve capacity market, will not necessarily have to deliver energy in the balancing phase. Therefore, the BL market is needed to make sure that at any time the balance between generation and demand in the system is achieved in the most economically efficient manner.

A combination of reserve capacity market and BL market can be for example found in Germany [31]. However, there is the difference between reserve capacity and BL markets in Germany and the ones in this paper. In Germany, in the reserve capacity markets, the TSOs procure reserve capacity via one-sided auctions some time ahead of its contingent use. The contracted capacity is called in real-time as required to balance the system when a difference between the planned energy schedule and the required load arises in real-time. In other words, there is not a separate

market for real-time balancing and whoever was called has to deliver energy in the real-time balancing. The option-like character of reserve capacity is reflected in the two-part pricing. The provision of reserve capacity is remunerated with a reservation price (€/MW) for reserving the capacity, and a reserve energy price (€/MWh) is paid for exercising the reserve option to generate the required energy in real-time.

However, in our paper, there are separate markets for balancing and reserve capacity. Therefore, there are separate prices for the capacity reservation (€/MW) and for balancing energy (€/MWh) for each generator.

## 2.4 Scalability

It has been claimed earlier that one of the advantages of this model is regarding the scalability. This section tries to prove this assertion. Since in any optimisation problem dealing with a problem with integer variables are more difficult rather than a linear problem, scalability issue is highly correlated with the scalability of the unit commitment problem. Assume that  $n$  is the total number of DERs at the distribution network and  $m$  is the number of DMOs with equal sizes. Therefore, one DMO has  $n/m$  DERs in its jurisdiction area. We consider only the DA market. In a centralised market model where all the DERs are dispatched in the wholesale market, the total number of DERs unit commitments is  $2^n$ . However, as there are three steps in the coupled market model, the total combination for DER's unit commitments is the summation of the combinations in Step 1, Step 2, and Step 3. As there are  $m$  DMOs at the distribution network, the total combination of DER's commitment for all DMOs is calculated as:  $\{m \cdot 2^{n/m} + m \cdot 2^{n/m} + 2^m\}$  and if  $m \ll n$ , this term is equivalent to  $2^{n/m}$  which is smaller than  $2^n$ , the total combination in the centralised model. It means, if the number of DERs rise in the distribution network, the total combination of DER's commitment in the centralised market model raises with a faster rate than in the coupled market model. Therefore, the scalability in the coupled market compared with the centralised market is higher.

## 2.5 Mathematical formulations of the coupled market

In this section, the mathematical formulation related to each of the six steps of the coupled market model, are described in detail.

First, some assumptions which are applied in the formulations need to be clarified.

### 2.5.1 Assumptions:

- Demands at both transmission and distribution systems are inelastic.
- DERs are discerned in stochastic generators and dispatchable generators. In this work, the stochastic generator is a wind generator. Dispatchable generators can participate in DA energy and reserve capacity markets and the BL market. However, stochastic generators can only participate in the DA energy market.
- Wind forecast errors are assumed to be the only cause for real-time system imbalances. These possible imbalances are represented via a set of scenarios. Following German markets [38], the wind power producer does not endure the balancing responsibility since its electricity production and injection on the grid is made without any special obligation automatically. Therefore, the imbalance costs are generally paid by the system operator then spread over network users.
- The reserve capacity market presented in this paper ensures that enough balancing energies will be available in the BL market. The minimum requirement of the reserve in the local or central market is determined in a way that it includes a safety factor which does not lead to network disturbances. Moreover, during the actual deployments of the reserve in the BL market, the network constraints in both local and central markets are taken into accounts.

- Upward and downward reserve capacities is considered symmetrical and follow a single pricing mechanism which is explained in [39].

**2.5.2 Step 1: local DA JoEnRe capacity (preliminary scheduling):** As mentioned above, the DMO first aggregates all the bids and offers from DERs by solving a preliminary scheduling problem where the objective function is minimising the total cost of energy and reserve capacity. The objective function is shown in the following equation:

$$\begin{aligned} \text{Minimise} \quad & \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^{\text{ED}} \cdot P_{g,t} + O_{g,t}^{\text{RD}} \cdot R_{g,t}) \\ & + \sum_{t \in T} \sum_{i \in N_{D-T}} \sum_{s \in S} (\pi_s \cdot \lambda_{t,s}^{\text{DA}} \cdot P_{i,t}^{\text{TD}}) \end{aligned} \quad (1)$$

Equation (1) minimises the total cost of generation and reserve capacity of DERs in the local market plus the expected cost of buying/selling energy and reserve from/to the TMO market. The cost of this energy is basically the DA price in the wholesale market. This price has to be estimated by the DMO and is therefore based on a set of scenarios and their associated probabilities of occurrence.

As mentioned earlier, in this step, the technical constraints of the distribution network are considered. The distribution network is represented through a second-order cone programming relaxation, which is tight for radial distribution networks [40]. Given a distribution node  $i \in N_D$ ,  $j$  refers to its unique ancestor.

$$(\theta_{i,l,t}): V_{i,t} = V_{j,t} + 2(r_l \cdot f_{l,t}^p + x_l \cdot f_{l,t}^q) - I_{l,t} \cdot (r_l^2 + x_l^2), \quad \forall i \in N_D, l \in L_D, t \quad (2)$$

$$\begin{aligned} (\lambda_{i,t}^{\text{ED}}): \quad & \sum_{l=(j,i)} (f_{l,t}^p - I_{l,t} \cdot r_l) + \sum_{g \in G_D \in i} P_{g,t} + \sum_{w \in i} P_{w,t}^{\text{WDA}} \\ & = P_{i,t}^{\text{load}} + \sum_{l=(i,j)} f_{l,t}^p + G_i \cdot V_{i,t}, \quad \forall i \in N_D, t \end{aligned} \quad (3)$$

$$\begin{aligned} (\lambda_{i,t}^{\text{ED0}}): \quad & \sum_{l=(i,j)} f_{l,t}^p - \sum_{l=(j,i)} (f_{l,t}^p - I_{l,t} \cdot r_l) - P_{i,t}^{\text{TD}} \\ & + G_i \cdot V_{i,t} = 0, \quad \forall i \in N_{D-T}, t \end{aligned} \quad (4)$$

$$\begin{aligned} (\mu_{i,t}): \quad & \sum_{l=(j,i)} (f_{l,t}^q - I_{l,t} \cdot x_l) + \sum_{(g \in G_D) \in i} Q_{g,t} + Q_{i,t}^{\text{TD}} \\ & = Q_{i,t}^{\text{load}} + \sum_{l=(i,j)} f_{l,t}^q - b_i \cdot V_{i,t}, \quad \forall i \in N_D, t \end{aligned} \quad (5)$$

$$\begin{aligned} (\mu_{i,t}^0): \quad & \sum_{l=(j,i)} (f_{l,t}^q - I_{l,t} \cdot x_l) Q_{i,t}^{\text{TD}} \\ & = \sum_{l=(i,j)} f_{l,t}^q - b_i \cdot V_{i,t}, \quad \forall i \in N_{D-T}, t \end{aligned} \quad (6)$$

$$(\lambda_t^{\text{RD}}): \sum_{g \in G_D} R_{g,t} \geq \alpha_D \cdot \left( \sum_{g \in G_D} P_g^{\text{gmax}} + \sum_w P_{w,t}^{\text{WDA}} \right), \quad \forall t \quad (7)$$

$$(\varphi_{g,t}^+): P_{g,t} + R_{g,t} \leq P_g^{\text{gmax}}, \quad \forall g \in G_D, t \quad (8)$$

$$(\varphi_{g,t}^-): P_{g,t} - R_{g,t} \geq P_g^{\text{gmin}}, \quad \forall g \in G_D, t \quad (9)$$

$$\begin{aligned} (\xi_{l,t}): \quad & (f_{l,t}^p)^2 + (f_{l,t}^q)^2 \leq V_{i,t} \cdot I_{l,t}, \\ & \forall i \in N_D, l = (i, j) \in L_D, t \end{aligned} \quad (10)$$

$$\begin{aligned} (\zeta_{l,t}): \quad & (f_{l,t}^p)^2 + (f_{l,t}^q)^2 \leq S_{l,t}^2, \\ & \forall i \in N_D, l = (i, j) \in L_D, t \end{aligned} \quad (11)$$

$$\begin{aligned} (\theta_{i,t}): \quad & (P_{i,t}^{\text{TD}})^2 + (Q_{i,t}^{\text{TD}})^2 \leq V_{i,t} \cdot I_{l,t}, \\ & \forall i \in N_{D-T}, l = (i, j) \in L_D, t \end{aligned} \quad (12)$$

$$(\phi_{g,t}): (P_{g,t})^2 + (Q_{g,t})^2 \leq S_{g,t}^2, \quad \forall g \in G_D, t \quad (13)$$

$$(\sigma_{i,t}^+, \sigma_{i,t}^-): V_i^{\text{min}} \leq V_{i,t} \leq V_i^{\text{max}}, \quad \forall i \in N_D, t \quad (14)$$

$$(\delta_{g,t}^+, \delta_{g,t}^-): Q_i^{\text{gmin}} \leq Q_{g,t} \leq Q_i^{\text{gmax}}, \quad \forall g \in G_D, t \quad (15)$$

$$(\beta_{g,t}^+, \beta_{g,t}^-): P_g^{\text{gmin}} \leq P_{g,t} \leq P_g^{\text{gmax}}, \quad \forall g \in G_D, t \quad (16)$$

$$(\eta_{w,t}^+, \eta_{w,t}^-): 0 \leq P_{w,t}^{\text{WDA}} \leq P_{w,t}^{\text{Wmax}}, \quad \forall w \in W, t \quad (17)$$

Constraint (2) accounts for the voltage difference which is induced by the power flow over a line. Constraints (3) and (4) are active power balance equations of the distribution system. Constraints (5) and (6) are reactive power balance equations. In (7) the minimum amount of total reserve capacity procured from DERs is considered as an  $\alpha_D$  ratio of the total generation capacity in the distribution network and includes a safety factor to ensure the security of the distribution network. This constraint guarantees that a certain amount of the total installed capacity from dispatchable generators is available for the balancing purpose. Later in the BL market (Step 4 and Step 6) and during the deployment of reserve capacities, distribution network constraints are included. Constraints (8) and (9) are limits for the reserve capacity of generators. Constraints (10) and (12) show the relation between voltage and current and active and reactive power flow over a line, and are the conic equations of the distribution grid. Constraint (11) imposes the congestion limit for the distribution lines. Constraint (13) is related to the generation capability curves and is linearised by the method explained in [41]. Constraints (14)–(16) impose limits on the involved decision variables.

Through this optimisation process, the local market price for energy and reserve will be determined. The energy price ( $\lambda_{i,t}^{\text{ED}}$ ) is the Lagrange multiplier of (4) at the interface node between the TSO and DSO, which can be determined by deriving the Karush–Kuhn–Tucker (KKT) conditions of the above convex optimisation problem. Moreover, the Lagrangian multiplier of (7) symbolised with  $\lambda_{i,t}^{\text{RD}}$  is the price for the reserve capacity in the distribution system. Appendix provides the KKT conditions for the constraints of this step.

The power injected at the interface node ( $\widetilde{P}_{i,t}^{\text{TD}}$ ), total reserve capacity of DERs ( $\widetilde{R}_{g,t}$ ), energy price ( $\lambda_{i,t}^{\text{ED}}$ ) and a reserve price ( $\lambda_{i,t}^{\text{RD}}$ ) are outputs of this step. Based on  $\lambda_{i,t}^{\text{ED}}$ ,  $\lambda_{i,t}^{\text{RD}}$ , and  $\widetilde{P}_{i,t}^{\text{TD}}$  the DMO participates in the DA market in Step 2.

**2.5.3 Step 2: TMO DA JoEnRe capacity market clearing:** In this step, the wholesale DA-JoEnRe capacity market (DA-JoEnRe market clearing) is cleared by the TMO. The DMO and transmission generators participate in this market. The objective of this market is maximising the social welfare, however, since in this paper the demand is considered to be inelastic, the social welfare is equivalent to minimise the total generation cost. More explanations for calculating the social welfare can be found in [42].

$$\begin{aligned} \text{Minimise} \quad & \sum_{t \in T} \sum_{g \in G_T} (O_{g,t}^{\text{ET}} \cdot P_{g,t} + O_{g,t}^{\text{RT}} \cdot R_{g,t}) \\ & + \sum_{t \in T} \sum_{i \in N_{T-D}} (\lambda_{i,t}^{\text{ED}} \cdot P_{i,t}^{\text{DT}} + \lambda_{i,t}^{\text{RD}} \cdot R_{i,t}^{\text{DT}}) \end{aligned} \quad (18)$$

The first line in (18) consists of the cost of energy and reserve capacity procured by the transmission generators. The second line accounts for the total costs of energy and reserve capacity procured by DERs.

In the transmission network the error in the DC power flow is less than in the distribution network, therefore a DC power flow can be used to model the transmission network constraints:

$$f_{l,t}^p = B_l(\theta_{i,t} - \theta_{j,t}), \quad \forall (i, j) \in l, l \in L_T, t \quad (19)$$

$$-TC_l \leq f_{l,t}^p \leq TC_l, \quad \forall l \in L_T, t \quad (20)$$

$$\sum_{g \in G_T} P_{g,t} + P_{i,t}^{DT} + \sum_{(j,i) \in l} f_{l,t}^p = P_{i,t}^{load} + \sum_{(i,j) \in l} f_{l,t}^p, \quad \forall i \in N_T, l \in L_T, t \quad (21)$$

$$R_{i,t}^{DT} + \sum_{g \in G_T} R_{g,t} \geq \alpha_T \cdot \sum_{g \in G_T} P_g^{gmax}, \quad \forall i \in N_{T-D}, t \quad (22)$$

$$P_{g,t} + R_{g,t} \leq P_g^{gmax}, \quad \forall g \in G_T, t \quad (23)$$

$$P_{g,t} - R_{g,t} \geq P_g^{gmin}, \quad \forall g \in G_T, t \quad (24)$$

$$P_g^{gmin} \leq P_{g,t} \leq P_g^{gmax}, \quad \forall g \in G_T, t \quad (25)$$

$$0 \leq P_{i,t}^{DT} \leq \widetilde{P}_{i,t}^{TD}, \quad \forall i \in N_{T-D}, t \quad (26)$$

$$0 \leq R_{i,t}^{DT} \leq \sum_{g \in G_D} \widetilde{R}_{g,t}, \quad \forall i \in N_{T-D}, t \quad (27)$$

Constraint (19) considers the power flow over a transmission line and (20) imposes a limit on this power flow to the transmission line capacity. In (21), the power balance equation is shown. Constraint (22) is the required reserve capacity in the transmission level which is a ratio of the totalled generation directly connected to the transmission network. Constraints (23) and (24) correspond to limits for the reserve capacity procured from generators in the transmission grid. Constraints (25) and (26) impose limits for the energy from transmission generators and the DMO, respectively. Equation (27) limits the reserve capacity from the DMO.

After clearing this market, the DMO is informed about the allocated power flow over the HV/MV transformer ( $\widetilde{P}_{i,t}^{DT}$ ) and the required reserve capacity from DERs ( $\widetilde{R}_{i,t}^{DT}$ ).

**2.5.4 Step 3: local DA-JoEnRe capacity market clearing:** In this step, the DMO clears the DA-JoEnRe capacity market (DA-JoEnRe market clearing). This local DA market is cleared by solving the optimisation problem defined by constraints (28)–(30).

$$\text{Minimise} \quad \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^{ED} \cdot P_{g,t} + O_{g,t}^{RD} \cdot R_{g,t}) \quad (28)$$

s. t.:

$$\begin{aligned} \sum_{l=(j,i)} (f_{l,t}^p - I_{l,t,s} \cdot r_l) + \sum_{g \in G_D} P_{g,t} + \sum_{w \in i} P_{w,t}^{WDA} - \widetilde{P}_{i,t}^{DT} \\ = P_{i,t}^{load} + \sum_{l=(i,j)} f_{l,t}^p + G_i \cdot V_{i,t}, \quad \forall i \in N_D, t \end{aligned} \quad (29)$$

$$\sum_{g \in G_D} R_{g,t} \geq \widetilde{R}_{i,t}^{DT}, \quad \forall i \in N_{D-T}, t \quad (30)$$

and (2), (8), (9), (10), (11), (13), (14), (15), (16), (17)

The objective function in (28) is minimising the total generation cost of DERs in the distribution system. The constraints in this step are mostly similar to Step 1. The differences are in the power balance equation in (29) and the required reserve capacity in (30).

In (29),  $\widetilde{P}_{i,t}^{DT}$  is a parameter symbolised the power flow injected at the interface node of the distribution system. The outputs of this step, regarding the reserve capacity ( $\widetilde{R}_{g,t}$ ) and the scheduled energy of dispatchable generators ( $\widetilde{P}_{g,t}$ ), are inputs for the BL market which is explained further.

**2.5.5 Step 4: local balancing resource determination:** In this step, the DMO estimates the local BL market price by which he participates in the central real-time BL market. As mentioned earlier, the source of uncertainties in this work is the power from wind turbines which is underlined through a set of scenarios. Therefore, this optimisation is modelled through stochastic programming where the objective function is minimising the expected balancing service cost at the distribution network. A similar approach to [43] is applied.

$$\begin{aligned} \text{Minimise} \quad & \sum_t \sum_s \sum_{g \in G_D} \pi_s \cdot (O_{g,t}^{ED} \cdot \Delta P_{g,t,s}) \\ & + \sum_t \sum_s \sum_{i \in N_{D-T}} \lambda_{i,t,s}^{BL} \cdot \Delta P_{i,t,s}^{TD} \end{aligned} \quad (31)$$

The objective function in (31) consists of the cost of balancing services procured by DERs and by the transmission grid. Similar to Step 1, the price for the balancing service from the transmission grid is considered as the wholesale BL market price. The distribution network constraints are also taken into account in this step.

$$\begin{aligned} \sum_{l=(j,i)} (f_{l,t,s}^p - I_{l,t,s} \cdot r_l) + \sum_{g \in i} (\widetilde{P}_{g,t} + \Delta P_{g,t,s}) + \sum_{w \in i} P_{w,t,s}^{Wact} \\ + \Delta P_{i,t,s}^{TD} = P_{i,t,s}^{load} + \sum_{l=(i,j)} f_{l,t,s}^p + G_i \cdot V_{i,t,s}, \quad \forall i \in N_D, t, s \end{aligned} \quad (32)$$

$$\begin{aligned} \sum_{l=(j,i)} (f_{l,t,s}^q - I_{l,t,s} \cdot x_l) + \sum_{g \in i} \Delta Q_{g,t,s} = Q_{i,t,s}^{load} + \sum_{l=(i,j)} f_{l,t,s}^q \\ - b_i \cdot V_{i,t,s}, \quad \forall i \in N_D, t, s \end{aligned} \quad (33)$$

$$\widetilde{P}_{g,t} + \Delta P_{g,t,s} \leq P_g^{gmax}, \quad \forall g \in G_D, t, s \quad (34)$$

$$\widetilde{P}_{g,t} + \Delta P_{g,t,s} \geq P_g^{gmin}, \quad \forall g \in G_D, t, s \quad (35)$$

$$(\widetilde{P}_{g,t} + \Delta P_{g,t,s})^2 + (\Delta Q_{g,t,s})^2 \leq S_{g,t,s}^2, \quad \forall g \in G_D, t, s \quad (36)$$

$$-\widetilde{R}_{g,t} \leq \Delta P_{g,t,s} \leq \widetilde{R}_{g,t}, \quad \forall g \in G_D, t, s \quad (37)$$

$$(2), (10), (11), (13), (14), (15), (16) \quad (38)$$

Similar to Step 1, (32) and (33) are active and reactive power balance equations, respectively. However, in the (32), the actual wind power and the scheduled energy of DERs are new terms added to the equation. Equations (34) and (35) are limits for the procured balancing services. Finally, the local BL market price is  $\lambda_{i,t}^{DBL}$ , the Lagrangian multiplier of (32). With this price, the DMO will participate in the TMO balancing the market in Step 5.

**2.5.6 Step 5: TMO real-time BL market clearing:** In this step, the TMO clears the real-time central BL market. Generators connected to the transmission grid and the DMO with aggregated bids from DERs participate in this market. The objective function is as follows:

$$\text{Minimise} \quad \sum_t \sum_{g \in G_T} O_{g,t}^{ET} \cdot \Delta P_{g,t} + \sum_t \sum_{i \in N_{T-D}} \lambda_{i,t}^{DBL} \cdot \Delta P_{i,t}^{DT} \quad (39)$$

In (39), the cost of deployed balancing services from all the resources is taken into account. The first term is related to the cost of balancing services from the transmission generators. In the second term,  $\lambda_{i,t}^{DBL}$  is the price of balancing services from aggregated DERs by DMO. The network constraints of the transmission system are enforced as

$$\sum_{g \in i} (\overline{P}_{g,t} + \Delta P_{g,t}) + (\overline{P}_{i,t}^{DT} + \Delta P_{i,t}^{DT} + \Delta_{w,t}) + \sum_{(j,i) \in 1} f_{l,t}^p \quad (40)$$

$$= P_{i,t}^{\text{load}} + \sum_{(i,j) \in 1} f_{l,t}^p \quad \forall i \in N_T, l \in L_T, t$$

$$\overline{P}_{g,t} + \Delta P_{g,t} \leq P_{g,t}^{\text{gmax}}, \quad \forall g \in G_T, t \quad (41)$$

$$\overline{P}_{g,t} + \Delta P_{g,t} \geq P_{g,t}^{\text{gmin}}, \quad \forall g \in G_T, t \quad (42)$$

$$\Delta_{w,t} = \overline{P}_{w,t}^{WDA} - P_{w,t,s_r}^{\text{Wact}}, \quad \forall w \in N_D, t \quad (43)$$

$$-\overline{R}_{g,t} \leq \Delta P_{g,t} \leq \overline{R}_{g,t}, \quad \forall g \in G_T, t \quad (44)$$

$$-\overline{R}_{i,t}^{DT} \leq \Delta P_{i,t}^{DT} \leq \overline{R}_{i,t}^{DT}, \quad \forall i \in N_{T-D}, t \quad (45)$$

$$(19), (20), (25) \quad (46)$$

Note that in (40), the power imbalance  $\Delta_{w,t}$  has been added to the power balance equation. In (43),  $\Delta_{w,t}$  is defined as the difference between the actual wind power and the scheduled wind in the DA market. In (43),  $P_{w,t,s_r}^{\text{Wact}}$  is the random scenario chosen for the actual wind power.

The results of this market, which will be passed on to the DMO, is  $\Delta P_{i,t}^{DT}$  indicating the deployed energy from transmission to the distribution system.

**2.5.7 Step 6: DMO BL market clearing:** This is the final step where DMO clears the local BL market. The objective function is minimising the balancing service cost deployed by DERs

$$\text{Minimise} \quad \sum_{t \in T} \sum_{g \in G_D} \{O_{g,t}^{\text{ED}} \cdot \Delta P_{g,t}\} \quad (47)$$

s.t.

$$\sum_{l=(j,i)} (f_{l,t}^p - I_{l,t} \cdot r_l) + \sum_{g \in G_D \in i} (\overline{P}_{g,t} + \Delta P_{g,t}) + \sum_{w \in i} P_{w,t,s_r}^{\text{Wact}} \quad (48)$$

$$= \overline{\Delta P}_{i,t}^{DT} + P_{i,t}^{\text{load}} + \sum_{l=(i,j)} f_{l,t}^p + G_i \cdot V_{i,t}, \quad \forall i \in N_D, t$$

$$\overline{P}_{g,t} + \Delta P_{g,t} \leq P_{g,t}^{\text{gmax}}, \quad \forall g \in G_D, t \quad (49)$$

$$\overline{P}_{g,t} + \Delta P_{g,t} \geq P_{g,t}^{\text{gmin}}, \quad \forall g \in G_D, t \quad (50)$$

$$-\overline{R}_{g,t} \leq \Delta P_{g,t} \leq \overline{R}_{g,t}, \quad \forall g \in G_D, t \quad (51)$$

$$(2), (10), (11), (13), (14), (15), (16) \quad (52)$$

In the power balance equation (48),  $\overline{\Delta P}_{i,t}^{DT}$  is the scheduled adjustment from transmission level to the distribution level which has been calculated at Step 5 and  $P_{w,t,s_r}^{\text{Wact}}$  is a random scenario chosen for the actual wind power. Equations (49)–(51) are limits enforced for the schedule adjustment of DERs. The rest of the equations in (52) are similar with the network constraints in Step 1.

### 3 Centralised market (benchmark)

A scheme consisting of centralised DA and BL markets is considered as the benchmark which has the most compatible with the current electricity market regulation. In this model, there are no DMO-operated local markets, and distribution network constraints are not taken into account. DERs are considered to be connected at the interface node of the transmission network and the TMO operates both DA and BL markets for all DERs and generators in the transmission system. Therefore, the centralised market model merely consists of Step 2 and Step 5 in the coupled market model which is further explained in details.

#### 3.1 DA-JoEnRe capacity market

The DA joint market of the centralised model is quite similar to Step 2 in the coupled model. The difference is that, for the objective function in the centralised model,  $\lambda_{i,t}^{\text{ED}}$  and  $\lambda_{i,t}^{\text{RD}}$  in (18) is equal to zero. Moreover,  $P_{g,t}$  and  $R_{g,t}$  represent energy and reserve for all generators including DERs and generators at the transmission system, respectively. Therefore, the objective function and constraints are as follows:

$$\text{Minimise} \quad \sum_{t \in T} \sum_{g \in (G_T \cup G_D)} \{O_{g,t}^{\text{ET}} \cdot P_{g,t} + O_{g,t}^{\text{RT}} \cdot R_{g,t}\} \quad (53)$$

s.t.:

$$\sum_{g \in i} P_{g,t} + \sum_{w \in i} P_{w,t}^{WDA} + \sum_{(j,i) \in 1} f_{l,t}^p = P_{i,t}^{\text{load}} + \sum_{(i,j) \in 1} f_{l,t}^p, \quad \forall i \in N_T/N_{T-D}, l \in L_T, t \quad (54)$$

$$\sum_{g \in N_T} R_{g,t} \geq \alpha_T \cdot \sum_{g \in N_T} P_{g,t}^{\text{gmax}}, \quad \forall t \quad (55)$$

$$P_{g,t} + R_{g,t} \leq P_{g,t}^{\text{gmax}}, \quad \forall g \in (G_T \cup G_D), t \quad (56)$$

$$P_{g,t} - R_{g,t} \geq P_{g,t}^{\text{gmin}}, \quad \forall g \in (G_T \cup G_D), t \quad (57)$$

$$0 \leq P_{w,t}^{WDA} \leq P_{w,t}^{\text{Wmax}}, \quad \forall w \in W, t \quad (58)$$

$$(19), (20), (25) \quad (59)$$

The network constraints are quite similar to the constraints in Step 2 of the coupled market model. The only difference is in the power balance equation (54) where  $P_{i,t}^{\text{load}}$  belongs to loads of transmission and distribution networks.

#### 3.2 BL market

BL market of the centralised model is cleared in a similar way with Step 5 of the coupled market model. However, in the objective function,  $\lambda_{i,t}^{\text{DBL}}$  in (39) is equal to zero.  $\Delta P_{g,t,s}$  represents the power adjustment (upward or downward) for all generators including DERs and transmission system generators.

$$\text{Minimise} \quad \sum_{t \in T} \sum_{g \in (G_T \cup G_D)} \{O_{g,t}^{\text{ET}} \cdot \Delta P_{g,t}\} \quad (60)$$

s.t.:

$$\sum_{g \in i} (\overline{P}_{g,t} + \Delta P_{g,t}) + (\overline{P}_{i,t}^{DT} + \Delta P_{i,t}^{DT} + \Delta_{w,t}) + \sum_{(j,i) \in 1} f_{l,t}^p \quad (61)$$

$$= P_{i,t}^{\text{load}} + \sum_{(i,j) \in 1} f_{l,t}^p \quad \forall i \in N_T/N_{T-D}, l \in L_T, t$$

$$\overline{P}_{g,t} + \Delta P_{g,t} \leq P_{g,t}^{\text{gmax}}, \quad \forall g \in (G_T \cup G_D), t \quad (62)$$

$$\overline{P}_{g,t} + \Delta P_{g,t} \geq P_{g,t}^{\text{gmin}}, \quad \forall g \in (G_T \cup G_D), t \quad (63)$$

$$\Delta P_{g,t} \leq \overline{R}_{g,t}, \quad \forall g \in (G_T \cup G_D), t \quad (64)$$

$$\Delta_{w,t} = \overline{P}_{w,t}^{WDA} - P_{w,t,s_r}^{\text{Wact}}, \quad \forall w \in W, t \quad (65)$$

$$(19), (20), (25) \quad (66)$$

Again,  $P_{w,t,s_r}^{\text{Wact}}$  is a random scenario of actual wind power. Network constraints are similar with constraints in Step 5, however, in (61),  $P_{i,t}^{\text{load}}$  additionally includes loads connected to the distribution network.



**Table 1** Data for transmission generators

Gens. bus no.	$P_g^{\max}$ , MW	$P_g^{\max}$ , MW	$O_{g,t}^{ET}$ , €/MWh	$O_{g,t}^{RT}$ , €/MW
1	152	30,4	90.58	50
2	152	30,4	90.58	50
7	300	75	130.63	70
13	591	206,85	130.27	70
15	60	12	210	120
15	155	54,25	60.75	40
16	155	54,25	60.75	40
18	400	400	30.39	20
21	400	400	30.39	20
23	310	108,5	60.75	40
23	350	140	70.03	50

**Table 2** Data for distribution generators

Gens. bus no.	$P_g^{\max}$ , MW	$O_{g,t}^{ED}$ , €/MWh	$O_{g,t}^{RD}$ , €/MW
3	5	25	12
4	5	20	10
5	5	15	7.5
17	5	30	15
19	5	22	12
26	5	22	12
29	5	18	9
31	5	18	9

**Table 3** Market results (k€) for the coupled market (Case 1) versus centralised market

		Coupled market		Centralised market
		Local market	TMO market	market
system	DA	0.445 (Step 3)	37.445 (Step 2)	36.280
cost	BL	0.213 (Step 6)	36.757 (Step 5)	35.949

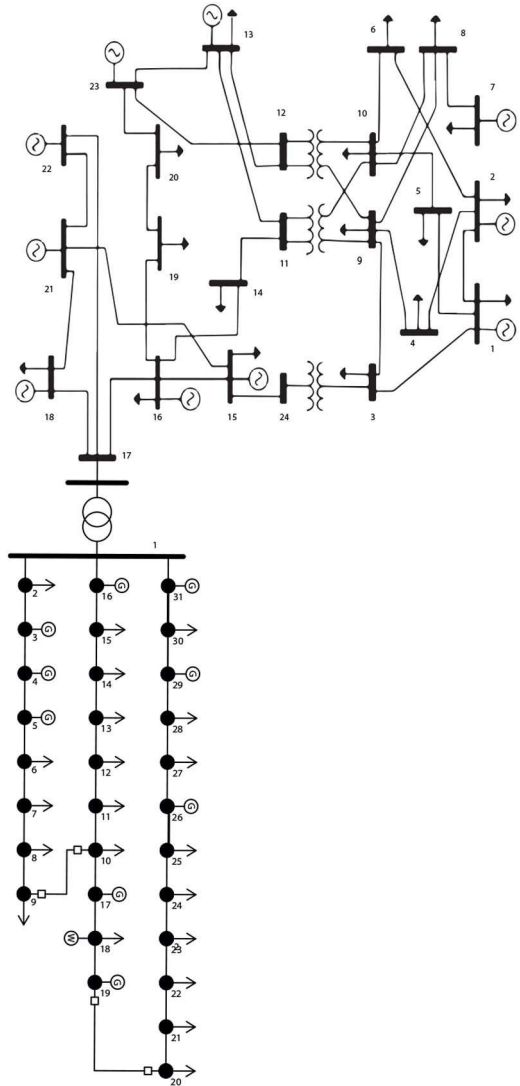
## 4 Case study and results

### 4.1 Case study

The proposed coupled TMO-DMO market model is tested using a radial 30-bus medium voltage Dutch distribution system and the IEEE-24 bus transmission system [44]. The data for the offer prices of distributed generators are from [45]. Tables 1 and 2, summarise the data for generators at transmission and distribution grids, respectively. A wind turbine which is the source of uncertainty in the system is located at bus number 18 (at the end of the feeder) of the distribution grid. The DA and imbalance market prices are obtained from the Belgian TSO Elia [46]. The residential loads in the distribution system are generated with the method described in [47]. For the industrial loads, the data from the NEDU profiles [48] has been used. To generate the scenarios an autoregressive integrated moving average (ARIMA) time series modelling approach is applied and subsequently, a k-means clustering algorithm is applied to obtain a set of 75 scenarios of wind power generation, DA and imbalance market prices [49]. The time resolution of the DA market is one hour with a time horizon of 24-h and for the BL market is 15 min. Parameters  $\alpha_D$  and  $\alpha_T$  are considered as 30% of the total installed generation at the distribution and transmission system, respectively. The mathematical models are formulated in the General Algebraic Modelling System and solved with the solvers CPLEX and MOSEK.

### 4.2 Results and discussions

To understand the effect of considering the distribution network constraints during the market clearing on the total cost and security

**Fig. 4** Transmission and distribution systems case studies

of the system, the market results of the two cases are studied. Fig. 4. Case (1) is the TMO-DMO coupled market model with the distribution network constraints forced to be inactive. Case (1) with no network constraints has been performed to have a fair comparison between the centralised and the coupled market (since in the centralised market model which is compatible with the current market structure, the distribution network constraints are not considered). This resembles the case which the distribution network constraints will not be disturbed in any situation. To show that the coupled market model is indeed capable of respecting the network constraints, Case (2) has been performed where the distribution network constraint (as described in Section 2.4) has been added.

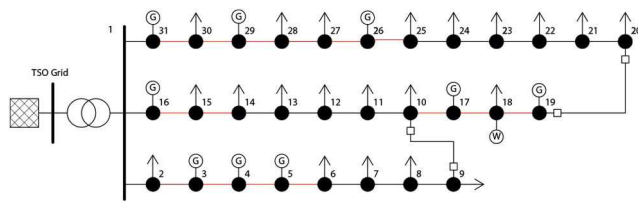
Table 3 shows the total system costs (k€) of the coupled market model in Case (1) and the centralised market model, both with inactive distribution network constraints. In the table, it is indicated for each number to belong to which step of Sections 2.5.2–2.5.7. In the coupled market model, the total system cost is the cost of the TMO market. As can be seen from the table, the total system cost in the coupled market model with inactive distribution network constraints is slightly higher than that of the centralised market model. This low-cost difference shows that the coupled market model can theoretically be operated with little additional cost compared to the current centralised model. However, the reason for this cost difference is related to the DMO's overestimation or underestimation of the wholesale market price. To explain this, we consider only the DA market and the objective function in (1). If the DMO overestimates the wholesale market price, the more expensive DERs can be dispatched in the local market while the DMO could have imported cheaper energy from the wholesale

**Table 4** Market results (k€) for the coupled market (Case 1) versus centralised market

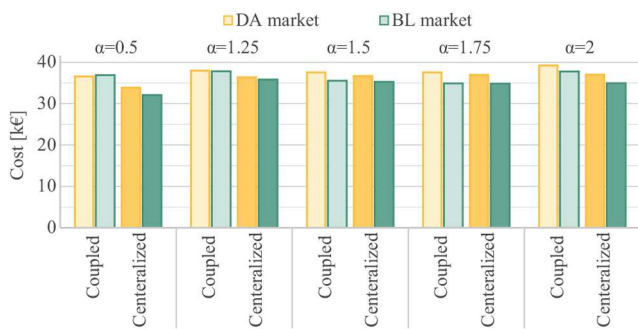
		Coupled market		Centralised market
		Local market	TMO market	
system cost	DA market	5.163	60.195	58.042
	BL market	2.582	37.512	25.520

**Table 5** Market results (k€) for the coupled market (Case 2) versus centralised market

		Coupled market		Centralised market
		Local market	TMO market	
system cost	DA market	2.143 (Step 3)	47.379 (Step 2)	36.280
	BL market	1.120 (Step 6)	45.608 (Step 5)	35.949



**Fig. 5** Overloading in the distribution system when grid constraints are not taken into account



**Fig. 6** Increasing DERs in coupled versus centralised market,  $\alpha$ : a ratio indicating the total shares of DERs compared with the maximum peak load

market instead. In contrast, if the DMO underestimates the wholesale market price, the cheaper DERs will not be dispatched and the DMO has to buy more expensive energy from the wholesale market.

A similar simulation for Case (1) is run but with new data for DA and BL market prices which are more compatible with the reality. For the TMO market, the DA and BL market prices of the Dutch system available in ENTSO-e transparency platform is used [50]. As Table 4 shows, with this set of new data, the coupled market with inactive distribution network constraint is still cost-comparable with the centralised market.

Table 5 shows the total system costs of the coupled market model in Case (2). In Case (2), the distribution network constraints have been activated in the coupled market model. The table shows that the additional cost of taking into account the distribution network constraints to the local market clearing process in the coupled market model will increase the cost of the system significantly, to 131% compared to Case (1) where the distribution network constraints are inactive. Due to the increase of constraints in Case (2) which limits the network, the cheaper DERs cannot be used all the time. Therefore, the lower cost of Case (1) can only be obtained if the distribution network is reinforced.

Fig. 5 shows the single line diagram of the distribution network after dispatching the DERs in Case (1). The red lines in the figure indicate which feeders are overloaded during at least a single time

instance over the 24-h horizon. In Case (2), the feeders are never overloaded as the implementation of the distribution network constraints in the local market clearing are preventing the dispatch of generators which will induce overloading.

Note that, in this paper, there is only a single DMO market connected to the large transmission network. Therefore, the redispatch cost will be negligible in Case (1), as the relative size of generators in the distribution network is far smaller than in the transmission network. However, in the future case with many coupled markets from many distribution grids, the redispatch costs can become significant. For future research, it can be interesting to take into account redispatch costs when examining the performance of a market with one TMO and many DMOs.

Finally, a sensitivity analysis has been done to study the effect of the increasing share of DERs on the system. Fig. 6 shows a comparison between the system cost in the coupled and centralised market models when the share of DERs is increasing in the system. The coupled market model here refers to Case (1) which has been explained above.  $\alpha$  is a ratio indicating the total shares of DERs compared with the maximum peak load in the system. For example,  $\alpha = 0.75$  and  $\alpha = 1.25$  stand for the situations where the total DERs in the system are 75 and 125% of the maximum peak load, respectively. As the figure shows, by increasing the share of DERs in the system still the coupled market and the centralised market can stay cost-comparable. However by increasing the share of DERs to higher value (double the maximum peak load), the double of maximum peak load, the difference between coupled and centralised market is higher. Also in the case where the DER's share decrease to a lower value, half of the maximum peak load, a larger difference between the system cost in the coupled and centralised market is observed.

## 5 Conclusions

In this paper, a coupled TMO-DMO market model is proposed. This coupled market model allows DERs to participate in the DA and BL markets. We introduce a joint local market for energy and reserve capacity, operated by the DMO. The DMO acts as a (non-profit) aggregator, collecting the bids from DERs and participating in the wholesale DA and BL markets on their behalf. The distribution network constraints are included in the local market clearing process. The uncertainties in the wind power output, DA and system imbalance prices are implemented through stochastic programming with a set of scenarios. The coupled market model has been compared with the currently applied centralised market model. The results show that the coupled market model without the distribution network constraints is cost-comparable with the centralised market model even in the situations where the share of DERs in the system has been increased. To strengthen this conclusion, the result with different BL market cost is included to show the system cost in the coupled market with inactive distribution network constraints is comparable with that of the centralised market. Moreover, the couple market seems to be a more scalable model compared with the current market. However, it may happen that the distribution network constraints are violated and assets in the distribution network become overloaded. Adding the distribution network constraints to the coupled market model will alleviate the overloading while raising the system cost. However, to have a more accurate overview about the costs, it is needed to compare the increase of the system cost due to adding the distribution network constraint in the market clearing process and the cost of the blackout and electricity cut-off in the system due to the overloading happened by neglecting the distribution network constraints.

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## 6 Appendix

This appendix includes the KKT conditions of constraints (1)–(17) in Step 1. The Lagrangian multipliers are mentioned in parentheses next to each constraint. First, the Lagrangian is driven as below

$$\begin{aligned}
&= \sum_i \sum_g (O_{g,t}^{\text{ED}} \cdot P_{g,t} + O_{g,t}^{\text{RD}} \cdot R_{g,t}) \\
&+ \sum_i \sum_s \sum_t (\pi_s \cdot \lambda_{t,s}^{\text{DA}} \cdot P_{i,t}^{\text{TD}}) \\
&+ \sum_i \sum_l \sum_{l=(i,j)} \theta_{i,l,t} \cdot (V_{i,t} - V_{j,t} - 2(r_1 \cdot f_{l,t}^p - x_1 \cdot f_{l,t}^q) \\
&+ I_{l,t} \cdot (r_1^2 + x_1^2)) \\
&+ \sum_i \sum_t \lambda_{i,t}^{\text{ED}} \cdot \left( \sum_{l=(j,i)} (f_{l,t}^p - I_{l,t} \cdot r_1) + \sum_{g \in i} P_{g,t} \right. \\
&\left. + \sum_{w \in i} P_{w,t}^{\text{WDA}} - P_{i,t}^{\text{load}} - \sum_{l=(i,j)} f_{l,t}^p - G_i \cdot V_{i,t} \right) \\
&+ \sum_i \sum_t \lambda_{i,t}^{\text{ED0}} \cdot \left( \sum_{l=(j,i)} (f_{l,t}^p - I_{l,t} \cdot r_1) + P_{i,t}^{\text{TD}} \right. \\
&\left. - \sum_{l=(i,j)} f_{l,t}^p - G_i \cdot V_{i,t} \right) \\
&+ \sum_i \sum_t \mu_{i,t} \cdot \left( \sum_{l=(j,i)} (f_{l,t}^q - I_{l,t} \cdot x_1) + \sum_{g \in i} Q_{g,t} - Q_{i,t}^{\text{load}} \right. \\
&\left. - \sum_{l=(i,j)} f_{l,t}^q - b_i \cdot V_{i,t} \right) \\
&+ \sum_i \sum_t \mu_{i,t}^0 \cdot \left( \sum_{l=(j,i)} (f_{l,t}^q - I_{l,t} \cdot x_1) + Q_{i,t}^{\text{TD}} - \sum_{l=(i,j)} f_{l,t}^q - b_i \cdot V_{i,t} \right. \\
&+ \sum_i \sum_g \lambda_{i,t}^{\text{RD}} \cdot (-R_{g,t} + \alpha_D \cdot P_g^{\text{gmax}}) \\
&+ \sum_i \sum_g \varphi_{g,t}^+ \cdot (P_{g,t} + R_{g,t} - P_g^{\text{gmax}}) \\
&+ \sum_i \sum_g \varphi_{g,t}^- \cdot (P_g^{\text{gmin}} - P_{g,t} + R_{g,t}) \\
&+ \sum_i \sum_{l=(i,j)} \xi_{l,t} \cdot \left( (f_{l,t}^p)^2 + (f_{l,t}^q)^2 - \sum_i V_{i,t} \cdot I_{l,t} \right) \\
&+ \sum_i \sum_t \zeta_{l,t} \cdot \left( (f_{l,t}^p)^2 + (f_{l,t}^q)^2 - S_l^2 \right) \\
&+ \sum_i \sum_t \vartheta_{i,t} \cdot \left( (P_{i,t}^{\text{TD}})^2 + (Q_{i,t}^{\text{TD}})^2 - \sum_{l=(i,j)} V_{i,t} \cdot I_{l,t} \right) \\
&+ \sum_i \sum_g \phi_{g,t} \cdot (P_{g,t}^2 + Q_{g,t}^2 - S_g^2) \\
&+ \sum_i \sum_t \sigma_{i,t}^+ \cdot (V_{i,t} - V_i^{\text{max}}) \\
&+ \sum_i \sum_t \sigma_{i,t}^- \cdot (V_i^{\text{min}} - V_{i,t}) \\
&+ \sum_i \sum_g \delta_{g,t}^+ \cdot (Q_{g,t} - Q_g^{\text{gmax}}) \\
&+ \sum_i \sum_g \delta_{g,t}^- \cdot (Q_g^{\text{gmin}} - Q_{g,t}) \\
&+ \sum_i \sum_g \beta_{g,t}^+ \cdot (P_{g,t} - P_g^{\text{gmax}}) \\
&+ \sum_i \sum_g \beta_{g,t}^- \cdot (P_g^{\text{gmin}} - P_{g,t}) \\
&+ \sum_i \sum_w \eta_{w,t}^+ \cdot (P_{w,t}^{\text{WDA}} - P_{w,t}^{\text{Wmax}}) \\
&+ \sum_i \sum_w \eta_{w,t}^- \cdot (0 - P_{w,t}^{\text{WDA}})
\end{aligned} \tag{67}$$

Besides the primary conditions which are constraints (2)–(17), the stationarity condition of the Lagrangian function with respect to variables are as follows:

$$(P_{g,t}): O_{g,t}^{\text{ED}} + \lambda_{i,t}^{\text{ED}} + \varphi_{g,t}^+ - \varphi_{g,t}^- + \beta_{g,t}^+ - \beta_{g,t}^- = 0 \tag{68}$$

$$(P_{i,t}^{\text{TD}}): \sum_s (\pi \cdot \lambda_{t,s}^{\text{DA}}) + \lambda_{i,t}^{\text{ED0}} + 2 \cdot \vartheta_{i,t} \cdot P_{i,t}^{\text{TD}} = 0 \tag{69}$$

$$(R_{g,t}): O_{g,t}^{\text{RD}} - \lambda_{i,t}^{\text{RD}} + \varphi_{g,t}^+ + \varphi_{g,t}^- = 0 \tag{70}$$

$$(Q_{g,t}): \mu_{i,t} + \delta_{g,t}^+ - \delta_{g,t}^- = 0 \tag{71}$$

$$(Q_{i,t}^{\text{TD}}): \mu_{i,t}^0 + 2 \cdot \vartheta_{i,t} \cdot Q_{i,t}^{\text{TD}} = 0 \tag{72}$$

$$(f_{l,t}^p): -2 \cdot r_1 \cdot \theta_{i,l,t} + \lambda_{i,t}^{\text{ED}} - \lambda_{j,t}^{\text{ED}} + \lambda_{i,t}^{\text{ED0}} - \lambda_{j,t}^{\text{ED0}} \\ + 2 \cdot \xi \cdot f_{l,t}^p + 2 \cdot \zeta \cdot f_{l,t}^p = 0 \tag{73}$$

$$(f_{l,t}^q): -2 \cdot x_1 \cdot \theta_{i,l,t} + \mu_{i,t} - \mu_{j,t} + \mu_{i,t}^0 - \mu_{j,t}^0 \\ + 2 \cdot \xi \cdot f_{l,t}^q + 2 \cdot \zeta \cdot f_{l,t}^q = 0 \tag{74}$$

$$(V_{i,t}): \theta_{i,l,t} - \theta_{j,l,t} - G_i \cdot \lambda_{i,t}^{\text{ED}} - G_i \cdot \lambda_{i,t}^{\text{ED0}} - \mu_{i,t} \cdot b_i \\ - \mu_{i,t}^0 \cdot b_i - \xi_{l,t} \cdot I_{l,t} - \vartheta_{i,t} \cdot I_{l,t} + \sigma_{i,t}^+ - \sigma_{i,t}^- = 0 \tag{75}$$

$$(I_{l,t}): (r_1^2 + x_1^2) \cdot \theta_{i,l,t} - r_1 \cdot \lambda_{i,t}^{\text{ED}} - r_1 \cdot \lambda_{i,t}^{\text{ED0}} - \mu_{i,t} \\ - \mu_{i,t}^0 - \xi \cdot V_{i,t} - \vartheta_{i,t} \cdot V_{i,t} = 0 \tag{76}$$

Finally, the complementary conditions are as follows:

$$0 \leq \lambda_{i,t}^{\text{RD}} \perp \left( -\sum_g R_{g,t} + \alpha_D \cdot \sum_g P_g^{\text{gmax}} \right) \tag{77}$$

$$0 \leq \varphi_{g,t}^+ \perp (P_{g,t} + R_{g,t} - P_g^{\text{gmax}}) \tag{78}$$

$$0 \leq \varphi_{g,t}^- \perp (P_{g,t} - R_{g,t} - P_g^{\text{gmin}}) \tag{79}$$

$$0 \leq \xi_{l,t} \perp (f_{l,t}^p)^2 + (f_{l,t}^q)^2 - V_{i,t} \cdot I_{l,t} \tag{80}$$

$$0 \leq \zeta_{l,t} \perp (f_{l,t}^p)^2 + (f_{l,t}^q)^2 - S_l^2 \tag{81}$$

$$0 \leq \vartheta_{i,t} \perp (P_{i,t}^{\text{TD}})^2 + (Q_{i,t}^{\text{TD}})^2 - V_{i,t} \cdot I_{l,t} \tag{82}$$

$$0 \leq \phi_{g,t} \perp (P_{g,t}^2 + Q_{g,t}^2 - S_g^2) \tag{83}$$

$$0 \leq \sigma_{i,t}^+ \perp (V_{i,t} - V_i^{\text{max}}) \tag{84}$$

$$0 \leq \sigma_{i,t}^- \perp (V_i^{\text{min}} - V_{i,t}) \tag{85}$$

$$0 \leq \delta_{g,t}^+ \perp (Q_{g,t} - Q_g^{\text{gmax}}) \tag{86}$$

$$0 \leq \delta_{g,t}^- \perp (Q_g^{\text{gmin}} - Q_{g,t}) \tag{87}$$

$$0 \leq \beta_{g,t}^+ \perp (P_{g,t} - P_g^{\text{gmax}}) \tag{88}$$

$$0 \leq \beta_{g,t}^- \perp (P_g^{\text{gmin}} - P_{g,t}) \tag{89}$$

$$0 \leq \eta_{w,t}^+ \perp (P_{w,t}^{\text{WDA}} - P_{w,t}^{\text{Wmax}}) \tag{90}$$

$$0 \leq \eta_{w,t}^- \perp (0 - P_{w,t}^{\text{WDA}}) \tag{91}$$

The KKT conditions (73) and (74) connect the real and reactive power price at the node  $i$  to the price of its ancestor, respectively. To replace  $\theta_{i,l,t}$  and  $\xi_{l,t}$  in (73), an additional KKT condition (76) is derived. Therefore, it can be concluded that the distribution locational marginal price (DLMP) at node  $i$  is a function of the active power price at the ancestor node  $j$ , the reactive power price at the current and ancestor node, and the price of congestion over the line attached to  $i$  and  $j$ .