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Numeral semantics

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Abstract

Words for numbers, numerals, are a special lexical class, halfway between natural and mathematical language. One would expect them to have a relatively straightforward semantics. However, during the last several decades, numerals proved to be a rich source of debate in linguistics, especially in semantics and pragmatics. The reason is that the study of numerals requires taking into account core issues such as plurality, quantification, implicature/exhaustivity, degree, modality, imprecision and cross-linguistic variation. In this article, we provide a thorough introduction to the issues connected to numeral semantics and pragmatics. We gradually develop analyses of meanings of numerals in natural language using a multitude of analytical tools. We evaluate the competing proposals in terms of empirical coverage and predictions.

1 | INTRODUCTION

This article provides a thorough introduction to issues connected to numeral semantics. As we show below, establishing an adequate analysis for the meaning of numerals is far from straightforward, still very much so after decades of discussion in the linguistic literature. The guiding question throughout this article will be what the compositional semantics of a numeral should be. As will become clear, however, numerals are used in several different environments yielding quite different meanings. Number words, it will turn out, correspond to a family of meanings, and the question will be how these meanings are related to one another.

As we will see, the way different uses of numerals correspond to different meanings is linked to what *kind* of contribution the numeral makes to the sentence. The question what numerals mean can therefore be operationalized by asking what *semantic type* we should give to them.[1] Thus phrased, the research question allows us to systematically explore the options. We will do this as follows. We start by discussing three prominent options: that numerals are quantifier-like in Section 2, that they are property-like in Section 3 and that they are entity-like in Section 4. Discussing these options will allow us to identify all the main empirical desiderata for numeral semantics, except for the issue of exhaustivity, which we discuss in Section 5. That discussion will bring issues of scope to the table, which will ultimately lead us to considering an analysis of numerals as degree quantifiers in Section 6.

2 | NUMERALS AS DETERMINERS

Numerals share an obvious resemblance to determiners like "*every*," "*some*," "*several*," "*most*," etc., in that they occur in a pre-nominal position.

(1) Every/some/several/most/twelve students came to the party.

The classical way of thinking of determiners is to see them as a particular kind of generalized quantifiers (Barwise & Cooper, 1981; Keenan & Stavi, 1986). In the tradition of Generalized Quantifier Theory (GQT), pre-nominal function words like those in (1) all receive interpretations as relations between sets: they return true if and only if the set denoted by their complement noun phrase stands in a particular relation to the set denoted by their second set denoting argument (in the case of [1], this is the verb phrase). Given this interpretation, determiners express meanings of type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$. For instance, "*every*" denotes the subset relation, and "*some*" expresses nonempty intersection.

(2) a. $[\![\text{every}]\!] = \lambda A.\lambda B.A \subseteq B$
b. $[\![\text{some}]\!] = \lambda A.\lambda B.A \cap B \neq \emptyset$

The type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ hosts a huge amount of meanings. Even if there were only four entities of type e, this would bring the class of $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ meanings to a staggering $2^{2^{4^2}} \approx 1.16 \times 10^{77}$. For this reason, it is an important research question within GQT *which* meanings of this type end up lexicalized in the languages of the world. To this end, formal semantics has tried to identify universal constraints on the class of determiners. Examples of such constraints include *conservativity* (Barwise & Cooper, 1981): relations expressed by determiners only ever concern the first argument set and the intersection between the first and second argument set. For (1), this means that to determine the truth-value of these sentences, you only need to look at the students and at the students who went to the party. Party guests that are not students are irrelevant.

A much discussed set of constraints aims to establish the intuition that the meanings of natural language determiners are not concerned with the actual content of the sets they combine with. To know whether "*every A B*" is true, it does not matter who is in A and B; it suffices to just know the cardinalities of A and of $A \cap B$.[2] For instance, we could redefine (2) into the equivalent statements in (3), using just cardinalities.

(3) a. $[\![\text{every}]\!] = \lambda A.\lambda B.|A| = |A \cap B|$
b. $[\![\text{some}]\!] = \lambda A.\lambda B.|A \cap B| \neq 0$

If we accept the constraints proposed by GQT, then determiners are cardinality operators. Given this, it would be a natural step to think that numerals are the ultimate example of a natural language determiner. Very clearly, the function of "*twelve*" in (1) is to convey the cardinality of the set of students who came to the party. As such, the semantics of numerals like "*twelve*" becomes very straightforward:

(4) $[\![\text{twelve}]\!] = \lambda A.\lambda B.|A \cap B| = 12$

It turns out that this is a pretty poor proposal for a semantics for numerals. We started with this proposal not only because of its initial natural appeal, but also because the reasons why this view fails will introduce a good deal of the observations that need to be accounted for. So let us go through all the reasons to dismiss a semantics as in (4).[3]

First of all, while numerals share with determiners that they can occur in a pre-nominal position, they differ in that they can co-occur with clear cases of determiners or articles. For instance,

(5) a. Every two houses come with one parking space.
b. The twelve students that came to the party had a nice time.

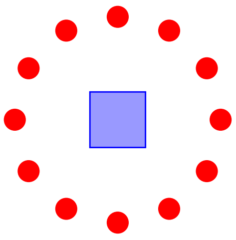
For examples like (5), an analysis of "*twelve*" as a determiner meaning of type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ makes no sense, since, given that analysis, it would be unclear how "*every*" and "*the*" contribute their meaning to the sentence.

Second, the semantics of numerals is not always about the cardinality of the intersection of two sets. Consider the example in (6).

(6) Twelve apples can fit in this shoe box.

If we take the proposal in (4) and apply it to (6), then the meaning of this sentence would involve the intersection between two sets, namely the set of apples and the set of entities that can fit in the box. On the assumption that this is a pretty normal shoe box, this second set is huge. Many things fit in the box. In particular, bar perhaps a few extraordinarily big apples, each single apple in the world is such that it will fit in the box. The sentence in (6) is now predicted to mean that there are twelve apples that are small enough that they fit in the shoe box. But this does not seem to be the most salient reading of this sentence (even though it is probably an available one). On the most salient reading, the sentence does not involve counting entities that can fit in the box, but instead it involves quantifying over groups of apples. A good paraphrase is probably something like: *any normal group of twelve apples is such that this group fits in the shoe box.*

Clearly, this is not what a GQT determiner-style denotation can give us. There are two reasons for this. First of all, (6) involves some sort of generic quantification, but there is no room for that in a proposal along the lines of (4). Second, (6) involves groups of apples, while in meanings like (4) everything is based entirely on the number of atoms in the intersection set. This latter issue also becomes clear from more straightforward cases of collectivity. Consider the following picture, for instance, and the contrast between (7-a) and (7-b).



(7) a. In this picture, twelve dots surround the square.
b. $^{??}$In this picture, every dot surrounds the square.

Numerals like "*twelve*" support collective predication in the sense that (7-a) expresses the existence of a group of twelve dots such that that group has the property of surrounding the square. A similar reading is not (easily) available for (7-b). Instead, this sentence ends up expressing the problematic classical GQT meaning that each atomic dot has the property of surrounding the square.

The contrast in (8) points to a different way in which numerals can be shown not to (generally) express relations between sets. If the position "*two*" and "*every*" occur in in (8) were a position suitable for $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ operators, then we would come to expect that (8-b) could express the statement that apart from the Coen brothers there are not any other famous filmmakers from Minnesota. (8-b), in fact, is grammatically degraded.

(8) a. The Coen brothers are two famous filmmakers from Minnesota.
b. *The Coen brothers are every famous filmmaker from Minnesota.

The reader may not find this a very strong argument, however, given that (8-b) could be out for independent reasons. Still, even if this were the case, it will be hard to see the meaning of (8-a) as a statement about a relation between two sets: the set of famous Minnesota filmmakers and the set of entities identical to the Coen brothers. For instance, if "*two*" is to express that the cardinality of the intersection between these sets is two, then this wrongly predicts that (8-a) entails that the Coen brothers are the *only* famous filmmakers from Minnesota.

By far the clearest examples of occurrences of numerals that are not determiner-like are those in which the numeral does not occur pre-nominally. This is for instance the case in English statements about mathematics.

(9) a. Twelve is a Fibonacci number.
b. Twelve plus twelve is twenty four.

These are extreme examples, since one may question whether such sentences are even part of English competence. It could be that *mathematical English* is just an entirely separate linguistic entity, with a trivial lexical overlap. While we think this is not implausable for (9), it seems to us to be an untenable position for (8), given that such sentences can mix different meanings of numerals, as in (10).

(10) Twelve and fifteen are two Fibonacci numbers.

One may want to distinguish then between two kinds of occurrences of numerals: on the one hand those that express mathematical entities and on the other those that express information about quantity. We will not dismiss this option. However, it should be clear that the ideal scenario is one in which we will be able to clarify in what way all the occurrences of numerals are related.

In what follows, we review three main strands of thought in the literature on numeral semantics. Each of them is an implementation of an idea about what numerals mean, given that determiner-like semantics for numerals is not tenable.

3 | NUMERALS AS MODIFIERS

An influential idea concerning the semantics of numerals is that they are adjective-like (Bartsch, 1973; Chierchia, 1985; Hoeksema, 1983; Landman, 2003; Rothstein, 2013, 2017), not just in their syntax—their ability to co-occur with determiners/articles, for instance—but also in their meaning. On that view, a numeral expresses a cardinality property.[4] For instance:

(11) $[\![\text{twelve}]\!] = \lambda x.\#x = 12$

According to (11), "*twelve*" expresses the set of (plural) entities, each of which contains 12 atoms. (So, $\#\alpha$ returns the number of atoms of some plurality, just like $|\cdots|$ returns the cardinality of a set.) "*Twelve*" can combine with a noun, say, "*students*." To talk more precisely about the meaning of this combination, we need to make a digression about plurality.

A common noun like "*student*" denotes a set of atomic entities, such that each of those entities is a student. Atomic entities cannot have the property "twelve," only *plural* entities can. So, for the semantics of "*twelve*," as in (11), to work, a predicate like "*student*" has to be able to undergo pluralization to include plural entities in its denotation as well.[5]

We use * to mark the pluralization operation, following Link (1983). In particular, if a and b are two atomic entities in the denotation of predicate P, then in the denotation of $^*P$ there is a plural entity $a \sqcup b$, the plurality that consists of nothing but a and b, or, the *sum* of a and b.

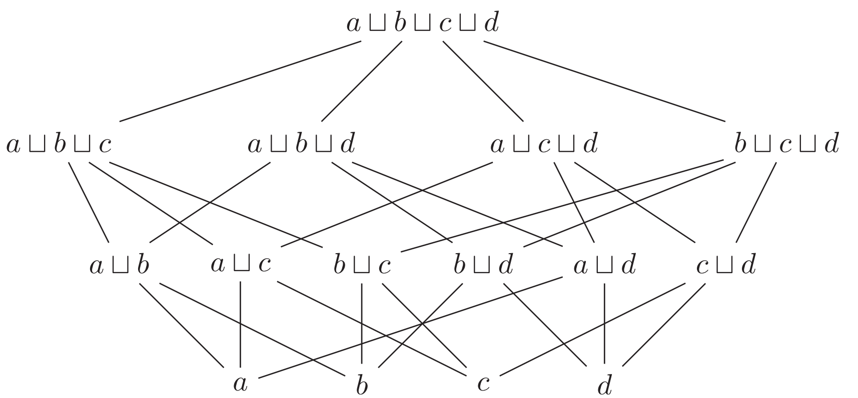


FIGURE 1 The set of pluralities $^*\{a, b, c, d\}$

In general, for any set of entities X, there exists an entity $\sqcup X$ whose parts are the elements of X as well as *their* parts, while nothing else is part of that individual. So, $\sqcup$\{john, mary\} is john$\sqcup$mary and $\sqcup$\{john $\sqcup$mary, sue, ann\} is john $\sqcup$mary $\sqcup$sue $\sqcup$ann.

We define * below in (12) (a definition commonly assumed in the literature, see Link, 1983; Landman, 1991, and see Nouwen, 2016 for an overview) and illustrate it in Figure 1. The arcs between the nodes correspond to *inclusion*, $\sqsubset$, when read from bottom to top.

(12) $^*Z = \{\sqcup X \mid X \subseteq Z \,\&\, X \neq \emptyset\}$.

Once we have plural individuals in this way, we can also express numerical properties. In Figure 1, there are four layers. The bottom layer is the layer of atoms, entities of cardinality 1. The layer above that has the pluralities of cardinality 2, and so forth. Structures based on pluralizations of larger sets will contain the layer of cardinality 12.

The meanings of "*twelve*" and "*students*" combine by means of what is often called *predicate modification* (Heim & Kratzer, 1998), which amounts simply to set intersection. This semantics of "*twelve students*" is completely parallel to an adjective-plus-noun combination such as "*American students*": the modifiers denote sets of groups of twelve and sets of groups of Americans, respectively.

$\lambda x.\#x = 12 \wedge {}^*\text{student}(x)$
- $\lambda x.\#x = 12$ — twelve
- $\lambda x.{}^*\text{student}(x)$ — students

$\lambda x.{}^*\text{American}(x) \wedge {}^*\text{student}(x)$
- $\lambda x.{}^*\text{American}(x)$ — American
- $\lambda x.{}^*\text{student}(x)$ — students

Just like "*American*" narrows down the set of student groups to arrive at the set of groups of American students, "*twelve*" narrows down the set of groups of students to groups of students of twelve.

The main advantage of a proposal like that in (11) is then that it does justice to the observation that numerals do not introduce quantificational force, but that they are dependent on external sources of quantification, for instance by determiners ("*every twelve students*", "*no American students*") or by covert operators. In (13-a), the numeral noun phrase ends up with existential force, just like any other bare plural in that position would gain existential force, as in (13-b).

(13) a. I have twelve students in my class.
b. I have American students in my class.

To account for such sentences, one could, for instance, adopt a silent existential operator (or, equivalently, a type-shift) $\boxminus$, as in (14).[6]

(14) $[\![\boxminus]\!] = \lambda P_{\langle e,\, t\rangle}.\lambda Q_{\langle e,\, t\rangle}.\exists x[P(x) \wedge Q(x)]$

The modificational view effortlessly deals with examples involving collective predication. Consider (7-a) once more.

(7-a) In this picture, twelve dots surround the square.

If we assume that this sentence contains covert existential force, then using the modificational view on numerals we get to a semantics like that in (15).

(15) $\exists x[\#x = 12 \wedge {}^{*}\text{dot}(x) \wedge \text{surround-the-square}(x)]$

The extension of "*surround the square*" will consist of groups surrounding the square. As is desirable, only if one of these groups is a group of 12 dots will (7-a) be predicted to be true.

Given that on the modificational view, numerals just create predicates, it should be possible to use numerals in predicative position. This is exactly what happened in example (8-a), which now receives the very straightforward analysis in (16) (where *cb* denotes the Coen brothers).

(8-a) The Coen brothers are two famous filmmakers from Minnesota.

(16) $(\lambda x.\#x = 2 \wedge {}^{*}\text{famous}(x) \wedge {}^{*}\text{filmmaker}(x) \wedge {}^{*}\text{from-Minnesota}(x))\,(cb) = \#cb = 2 \wedge {}^{*}\text{famous}(cb) \wedge {}^{*}\text{filmmaker}(cb) \wedge {}^{*}\text{from-Minnesota}(cb)$

What about mathematical uses of numerals? In principle, the modificational view can handle mathematical uses fine, as long as we are comfortable with thinking of numbers as classes of equinumerous groups. For instance, (9-a) is false, since the extension of "*being a Fibonacci number*" is that in (17).

(9-a) Twelve is a Fibonacci number.

(17) $\{\lambda x.\#x = 0, \lambda x.\#x = 1, \lambda x.\#x = 2, \lambda x.\#x = 3, \lambda x.\#x = 5, \lambda x.\#x = 8, \lambda x.\#x = 13, \lambda x.\#x = 21, \ldots\}$

This is, in fact, very close to Frege's conception of numbers (Frege, 1884).

4 | NUMERALS AS NUMBER-DENOTING WORDS

One may feel, however, that numerals should denote numbers, *whatever* it may be that numbers are. It seems intuitive that "*twelve*"—at least in some uses—simply means whatever concept we associate to the number "12":

(18) $[\![\text{twelve}]\!] = 12$

We will assume that the meaning in (18) belongs to the semantic domain of degrees D_d. Degrees are like entities of type *e*, except that their domain is ordered. That is, numbers like "12" are similar to heights, weights, degrees of tiredness, etc., in that they are part of fixed orders. Just like the height of the Dom tower in Utrecht exceeds the height of either of the two authors of this article, 12 exceeds 9. Entities of type *e* do not come with such a natural fixed ordering. (See Kennedy [2007] for discussion of degrees and degree semantics.)

On such a view, at least two uses of numerals should be distinguished. Those in which this simple numerical concept is conveyed, as in, for instance, (9-a) and those in which the numeral occurs in a pre-nominal position and conveys information about group cardinality.

(9-a) Twelve is a Fibonacci number.

Sentences like (9-a) are straightforward under (18), they express a simple set membership statement: number twelve belongs to the set of Fibonacci numbers. The extension of "*being a*

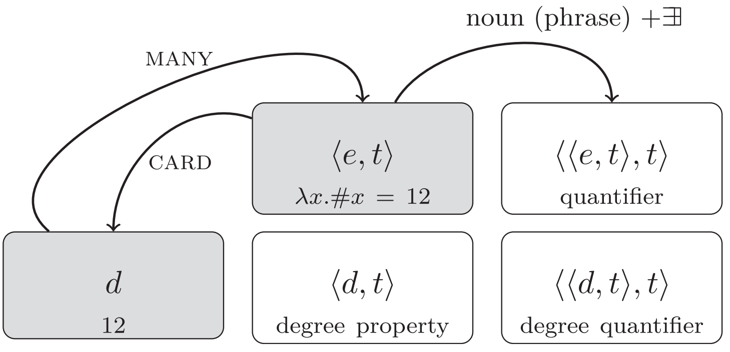


FIGURE 2 Schematic overview of the type landscape for numeral semantics. The shaded cells indicate the two possible basic meanings discussed so far

Fibonacci number" would then simply be a set of numbers:

(19) {0, 1, 2, 3, 5, 8, 13, 21, 34, ...}

The meanings of other expressions of mathematical language would be similarly simple:

(20) a. $[\![\text{plus}]\!] = \lambda d\lambda d'.d + d'$
b. $[\![\text{times}]\!] = \lambda d\lambda d'.d \times d'$

Beyond unquestionably mathematical discourse, seeing numerals as number-denoting expressions provides a natural understanding, for example, of their role as differential expressions in comparative constructions, where they seem to contribute a similarly arithmetic meaning:

(21) a. $[\![\text{There are more A's than B's}]\!] = |A| > |B|$
b. $[\![\text{There are two more A's than B's}]\!] = |A| \geq |B| + 2$
c. $[\![\text{There are three times more A's than B's}]\!] = |A| \geq |B| \times 3$

When a numeral appears in a pre-nominal position, the number semantics in (18) is not enough for the combination of the numeral and the noun to work compositionally—types d and $\langle e, t\rangle$ cannot combine in a straightforward way. There is a way to connect them, however. On quite a number of approaches to numeral semantics, noun phrases combine with numerals intermediated by a silent counting operator, often represented as MANY.[7]

(22) $[\![\text{MANY}]\!] = \lambda d\lambda x.\#x = d$

On this proposal, "*twelve students*" has the structure [[twelve MANY] students]. The combination of "*twelve*" and MANY has exactly the same semantics as "*twelve*" has lexically under the modifier view. Let us subscript "*twelve*" with m for "modificational" and with n for "number-denoting":

(23) $[\![\text{twelve}_n \text{ MANY}]\!] = [\![\text{twelve}_m]\!] = \lambda x.\#x = 12$

Due to the equivalence in (23), the number view can successfully account for the same range of constructions as the modifier view. The number view probably gives a more straightforward account of mathematical constructions, but has to introduce an additional element (structurally or as a type-shift) for pre-nominal cases.

As a further illustration of interpretational equivalence of the modifier and the number view, one can introduce an element that would be dual to MANY and map the modificational meaning of the numeral onto the number meaning. Let us call it CARD:

(24) $[\![\text{CARD}]\!] = \lambda P \iota d.\ \forall x[P(x) \rightarrow \ \#x = d]$

The effects of this operator applied to the modifier interpretation of "*twelve*" is as follows:

(25) $[\![\text{CARD}]\!]([\![\text{twelve}_m]\!]) = [\![\text{twelve}_n]\!] = 12$

We have defined CARD for illustration of the equivalence of the modifier and number views—in practice, CARD is not used in existing analyses of numeral constructions. Our purpose is to show that there are several ways of thinking of numeral semantics that, given the availability of correspondences between meanings of different types, end up with equivalent empirical coverage. The ensuing theoretical landscape is depicted as a landscape of types in Figure 2.

This is not to say that there are no ways of distinguishing the modifier and the number view. Since we probably want to have both d-type and $\langle e, t\rangle$-type meanings for numerals, the question becomes which shift is the more natural one: CARD, which assumes degrees can be reinterpreted as properties of entities, or MANY, which assumes that the pre-nominal position may contain a degree slot.

Here, we present a potential argument in favor of a type d slot in structures containing pre-nominal numerals. To do so, we turn from bare numerals to modified numerals, such as "*at most three*". It would be safe to assume that such expressions are not number-denoting—there is no single number such that "*at most three*" refers to it. So, "*at most three*" cannot be assigned type d—the type expected of the first argument of MANY. Instead, modified numerals can be analyzed as quantifiers over degrees, type $\langle dt, t\rangle$ (see Kennedy, 2015). The type mismatch can then be resolved by the same means as with quantifiers over individuals, say, "*every book*." One way of doing this is via movement—Quantifier Raising (Heim & Kratzer, 1998).

As potential evidence for QR with modified numerals, consider the Dutch verb "*hoeven*", which exhibits NPI properties. As illustrated by (26), the grammaticality of a sentence with "*hoeven*" depends on the presence of a negative element that has "*hoeven*" in its scope:

(26) a. *Jan hoeft te scoren — DUTCH
Jan has to score
b. Niemand hoeft te scoren
nobody has to score

The (Dutch equivalent of) downward-monotone "*at most three*" in the complement of the NPI verb licenses it:

(27) Jan hoeft maximaal drie boeken te lezen — DUTCH
Jan has maximally three books to read

If the modified numeral is a degree quantifier that undergoes QR, with the landing cite higher in the structure than the position occupied by "*hoeven*," the latter's licensing requirements are met.

This is an argument in favor of the availability of a type d slot in numeral structures. Given this, it would be natural to think that, accordingly, numerals are of that type. An interesting fact about scope and licensing of "*hoeven*" is at odds with this, however. As (28) shows, "*hoeven*" is licensed even by a bare numeral in its complement, if this numeral is "*zero*":

(28) Jan hoeft nul boeken te lezen DUTCH
Jan has zero books to read

Does this mean that bare numerals are also quantifiers that take scope? We turn to this possibility below.

5 | EXHAUSTIVITY AND SCOPE

In the previous sections, we have built two analyses of numeral semantics that, for the basic pre-nominal cases, give equivalent results:

(29) $[\![\text{Twelve students came to the party}]\!] =$
$\exists x[\#x = 12 \wedge {}^*\text{student}(x) \wedge {}^*\text{came-to-the-party}(x)]$

Notice that this is an *at least* (lower-bound) reading of the sentence. The existence of the group of twelve students that came to the party is compatible with, and in fact entailed by, there being a group of thirteen students that came to the party.[8]

However, numerals systematically get both *at least* and *exactly* readings. One illustration of this ambiguity is the fact that in a situation in which John took (exactly) eleven biscuits, (30) can have both a positive and a negative answer:

(30) **Q:** Did John take ten biscuits?
(31) **A:** Yes, he took eleven.
A: No, he took eleven.

One of the much-debated questions in the literature on numeral semantics is how these readings are related to each other. One prominent option is to derive the *exactly* reading from the *at least* one. This can be done with a pragmatic mechanism of Gricean scalar implicature, or its counterpart that is more embedded into grammar—the exhaustivity operator EXH that attaches to a propositional node. (See Spector [2013] in this journal for an overview of the relevant discussion that leads to such a view).

The idea of an exhaustivity operator is that it denies stronger alternatives to the proposition it is attached to:

(32) $[\![\text{EXH } S]\!] = 1$ iff
$[\![S]\!] = 1$ and for any stronger alternative S' to S: $[\![S']\!] = 0$

An example of EXH in action when numerals are not involved could be any noncardinality scale, say, a scale of temperature:

(33) $[\![\text{EXH The soup is warm}]\!] = 1$ iff
The soup is warm $\wedge$ The soup is not hot

With numerals, it gives the *exactly* interpretation (again, see Spector [2013] for details and discussion):

(34) $[\![$EXH Twelve students came to the party$]\!] =$
$\exists x[\#x = 12 \wedge {}^*\text{student}(x) \wedge {}^*\text{came-to-the-party}(x)] \wedge$
$\neg\exists x[\#x > 12 \wedge {}^*\text{student}(x) \wedge {}^*\text{came-to-the-party}(x)]$

This shows that the modifier or number view on numeral semantics suffices to account for both the *at least* and the *exactly* reading, on the assumption of the availability of an exhaustivity operator. A view like this, however, also makes quite specific further predictions about readings. In the presence of more than one propositional node, there are different potential attachment sites for EXH, giving rise to potential ambiguity. Consider (35), for instance:

(35) You are allowed to eat two biscuits

There are two propositional nodes for EXH to attach to. One within the scope of the modal predicate and one taking scope over the modal. This leads to two distinct readings:

(36) a. $\Diamond[\exists x[\#x = 2 \wedge {}^*\text{biscuit}(x) \wedge {}^*\text{eat}(y, x)] \wedge$ allow > EXH
$\neg\exists x[\#x > 2 \wedge {}^*\text{biscuit}(x) \wedge {}^*\text{eat}(y, x)]\!]$
b. $\Diamond\exists x[\#x = 2 \wedge {}^*\text{biscuit}(x) \wedge {}^*\text{eat}(y, x)] \wedge$ EXH > allow
$\neg\Diamond\exists x[\#x > 2 \wedge {}^*\text{biscuit}(x) \wedge {}^*\text{eat}(y, x)]$

This is, so far, a good prediction. The most prominent readings of sentences like (35) is the one in (36-b), where the exhaustivity operator takes scope at the matrix level: you are allowed to eat two biscuits and you are not allowed to eat more. The reading in (36-a) is rather weak, since it merely asserts that taking exactly two biscuits is allowed, without saying anything about other quantities. This reading is therefore more likely to surface in situations where maximizing informativity is not required, for instance, when we reformulate (35) as a question.

(37) Am I allowed to eat two biscuits?

On its most salient reading, (37) does not ask whether it is the case that the maximum number of biscuits one is allowed to eat is two. Rather, it asks for permission to eat (exactly) two biscuits.

In principle, then, the ambiguity displayed in (36) is desirable. Unfortunately, the way things are set up right now—a lower-bounded semantics via the number or modifier route with a free scoping exhaustivity operator—over-generates. Consider (38):

(38) Some students answered three of the questions correctly.

This sentence has one reading in which the numeral is construed doubly bounded: there are some students such that they answered exactly three of the questions correctly. To derive this reading, the exhaustivity operator would need to be in the scope of the subject quantifier. If this operator has the freedom to attach at any propositional node, however, then we would also expect a much stronger reading, namely:

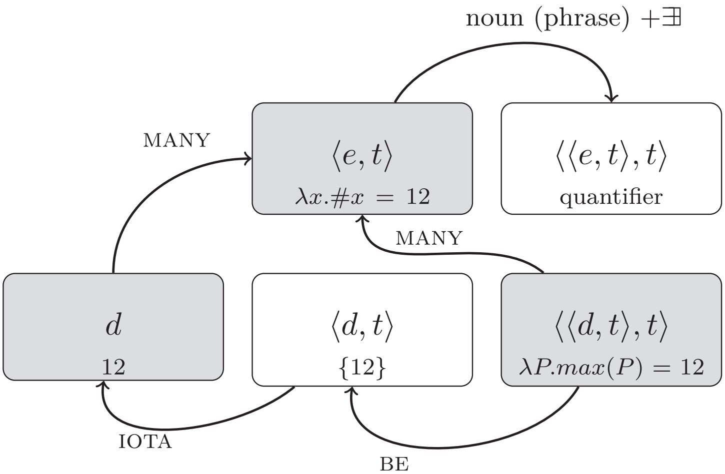


FIGURE 3 Schematic overview of the type landscape for numeral semantics. The shaded cells indicate the three possible basic meanings discussed so far

(39) $\exists x \exists y[\ {}^{*}\text{student}(x) \wedge \#y = 3 \wedge {}^{*}\text{question}(y) \wedge {}^{*}\text{answer.correctly}(x, y)]$
$\wedge \neg\exists x \exists y[\ {}^{*}\text{student}(x) \wedge \#y > 3 \wedge {}^{*}\text{question}(y) \wedge {}^{*}\text{answer.correctly}(x, y)]$

This says there are students that answered three questions correctly, but no student answered more than three questions. Such a strong reading is unavailable for (38).[9]

Why would there be a difference between sentences with a modal existential quantifier, (35), and sentences with a nominal existential quantifier, (38)? In fact, such contrasts are quite familiar from the literature on degree constructions. Heim (2000) showed that degree phrases can scope over intensional operators, but not over nominal ones.

(40) Rod A is 5 cm long. Rod B is allowed to be exactly 1 cm longer than that.
(41) Rod A is 5 cm long. Some rods are exactly 1 cm longer than that.

While (40) has a strong reading, saying that Rod B is not allowed to be longer than 6 cm, no such strong reading exists for (41). That is, (41) cannot be intended to convey that no rod is longer than 6 cm.

Data like (40)/(41) are usually interpreted as displaying a constraint on degree constructions, known in the subsequent literature as the Heim-Kennedy generalization:[10]

(42) The Heim-Kennedy generalization: degree operators cannot move to take scope over nominal quantifiers.

It is beyond the scope of this article to discuss the fine details of this generalization. (See Nouwen and Dotlačil (2017) for recent discussion on differences in degree-related scope between nominals and modals.) For the current purposes, it suffices to notice that (42) raises the question why we would observe a degree-oriented constraint like this for numerals. On neither the number nor the modifier view do the interpretations involve any kind of scope-taking degree operators. The only scope-taking operator is EXH, which clearly does not target degree.

This leads us to an alternative to the number/modifier view, namely one in which numerals *are* degree quantifiers.

6 | NUMERALS AS DEGREE QUANTIFIERS

The strong reading of examples like (35) lead Kennedy (2015) to propose a semantics for numerals that is both doubly bounded and quantificational.

(35) You are allowed to eat two biscuits.

On Kennedy's account, *twelve* denotes a set of degree properties, namely those properties whose maximal value is 12:

(43) $[\![\text{twelve}]\!] = \lambda P.max(P) = 12$

To see how this works for (35), assume the following (simplified) logical form:[11]

(44) allow [you eat $[\![$ ∃ [$\text{two}_{\langle\langle d,\ t\rangle,\ t\rangle}$ $\text{MANY}_{\langle d,\ \langle e,\ t\rangle\rangle}$] biscuits $]\!]$]

To resolve the type clash, *two* needs to move (QR), leaving a *d* type trace, as illustrated in (45), which leads to the semantics in (46).

(45) $\text{two}_{\langle\langle d,\ t\rangle,\ t\rangle}$ λd [allow [you eat $[\![$ ∃ [d $\text{MANY}_{\langle d,\ \langle e,\ t\rangle\rangle}$] biscuits $]\!]$]
(46) $max(\ \lambda d.\Diamond\exists x[\#x = d \wedge {}^*\text{biscuit}(x) \wedge {}^*\text{eat}(y, x)]\ ) = 2$

The denotation in (46) is equivalent to (36-b)—two is the highest number of biscuits that you are allowed to eat, the permission does not extend to three or any other higher number of biscuits. The reading with lower EXH in (36-a) would be captured under the degree quantifier analysis by the lower QR landing site for the numeral.

Treating numerals as degree quantifiers derives all the necessary *exactly* readings. Importantly, it bans the problematic *exactly* readings that are wrongly predicted to be available under other theories.

An important property of this analysis is that it does not derive *exactly* readings from the *at least* ones, but produces them directly. *At least* readings are still available under the degree quantifier view, however: they can be derived from *exactly* readings via a type-shift—more precisely, a succession of type-shifts that, step-wise, turn a quantifier into a predicate and then into a term. In the domain of individuals, these type-shifts have been introduced by Partee (1987) under the names of BE and IOTA, respectively:

(47) a. $\text{BE} = \lambda Q.\lambda x.Q(\{x\})$
b. $\text{IOTA} = \lambda P.\iota x.P(x)$

Applied in succession, BE and IOTA turn a quantifier over individuals into an individual-denoting expression—for example, they take all properties of Mary as input and return Mary as output.

(48) $\text{IOTA}(\text{BE}(\lambda P.P(\text{Mary}))) = \text{Mary}$

In exactly the same way, Kennedy (2015) derives a type *d* numeral denotation from the $\langle dt,\ t\rangle$ one. If $[\![\text{twelve}]\!]$ is the set of intervals that end in 12, then $\text{BE}([\![\text{twelve}]\!])$ is the set of degrees that each interval in $[\![\text{twelve}]\!]$ shares—that is, {12}:

(49) BE($[\![$twelve$]\!]$) = {12}

IOTA can be then applied to (49) to give us the number that the singleton in (49) contains:

(50) IOTA(BE($[\![$twelve$]\!]$)) = 12

IOTA(BE($[\![$twelve$]\!]$)), a degree-denoting expression, can be interpreted in situ, in exactly the same way as under the number view described above. Again, as in the number view, the result will be an *at least* reading.

The degree quantifier view has roughly the same empirical bite as the number view and the modifier view, all three being related to one another by type-shifting and operator paths, as illustrated in the updated type landscape in Figure 3.

However, there are two differences between the degree quantifier view and the number/modifier view that we would like to sum up again. First, for Kennedy (2015) but not others, numerals are quantifiers that undergo QR as a means of scope-taking. This has the benefit of explaining why we observe the Heim-Kennedy generalization for bare numerals. Second, according to Kennedy (2015), the *at least* readings are derived from the *exactly* ones, not the other way around.

Whether this property of the analysis makes desirable predictions is less clear. For an argument against the *exactly* analysis for numerals based on the polarity profile of the numeral "*zero*" see Bylinina and Nouwen (2018). In short, under the degree quantifier analysis, "*zero*" gives rise to a meaning that is indistinguishable from that of "*no*." As such, one comes to predict that "*zero*" and "*no*" also license negative polarity items to the same degree. This is, however, not the case empirically, compare (51-a) and (51-b) (Bylinina & Nouwen, 2018; Zeijlstra, 2007):

(51) a. No students have visited me in years.
b. *Zero students have visited me in years.

As Bylinina and Nouwen argue in detail, this pattern can be made sense of under the view that the basic meaning of "*zero*" (and, consequently, other numerals) is an *at least* meaning, with an additional derivational step that turns it into an *exactly* reading.

This does not mean that the degree quantifier analysis has to be abandoned altogether. Rather, this can be a reason to explore the connection between the two properties of this analysis that we pointed out above: the quantificational nature of the numeral meaning and its *exactly* property. In principle, they are independent from each other. One can keep the quantifier semantics for numerals, but make it an *at least* semantics. (52) is a quantificational meaning of the numeral "*twelve*." Unlike its *exactly* counterpart, the meaning in (52) is a set of intervals that include 12 (not necessarily end in 12):

(52) $[\![$twelve$]\!] = \lambda P.P(12)$

The numeral interpreted as in (52) cannot be interpreted in situ—it has to QR due to the type mismatch, as in Kennedy (2015). However, after QR, it will produce an *at least* reading—if it is not strengthened by other means.

The strengthening cannot occur by a freely inserted propositional EXH operator—this would undermine the explanation of why we observe Heim-Kennedy effects with bare numerals. Nothing prevents the strengthening from happening as quantificational modification, however. If there is an operator available that combines with the *at least* numeral quantifier and turns it into an *exactly* numeral quantifier, one can keep the explanation of the Heim-Kennedy effects observed with numerals and preserve the desirable direction of the derivational relation between *at least* and *exactly* readings. Let us call this operator MAX:

(53) $[\![\text{MAX}]\!] = \lambda D_{\langle\langle d,\, t\rangle,\, t\rangle}.\lambda P.max(P) \in \cap D$

MAX in (53) takes a set of intervals and passes on only those of them that end in the number that all the input intervals share.

When combined with a quantificational non-upper bound "*twelve*," as in (52), MAX returns the quantificational upper bound "*twelve*," equivalent to Kennedy's (2015) "*twelve*":

(54) $[\![\text{MAX twelve}]\!] = \lambda P.max(P) \in \{12\}$

The view we sketched here fills in an empty slot in the available types of numeral semantic analyses in the literature, by assuming that the semantic type of numerals is independent of the derivational relation between *at least* and *exactly* readings.

7 | CONCLUSION

We introduced three main views on the semantics of bare numerals: the number view, the modifier view and the degree quantifier view. We pointed out how these analyses are related to each other via type-shifts or operators, but also identified points of important differences between such analyses that lead to differing empirical predictions. We showed ways to test such predictions.

One important aspect our review underlines is why numerals are such an interesting topic of enquiry. As we illustrated above, the question we posed—what is the semantics of numerals—cannot be seen independent of a whole range of research questions central to semantics and pragmatics: What are the sources of quantificational force in sentences? What is semantic plurality? How are scope shifts constrained? What is the source of exhaustification?

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ENDNOTES

[1]For the uninitiated, in formal linguistic semantics, meanings are distinguished in terms of types. The meaning of a sentence is a proposition, something that yields a truth-value given some state of affairs. A truth-value is type *t*. A verb phrase expresses a property, a function that takes an entity and returns true for entities that have the property in question and returns false otherwise. This is another way of saying that verb phrases express sets of entities. If we assume that the subject expresses an entity, type *e*, then the verb phrase will denote a function

of e to t, which we will write as $\langle e, t\rangle$. Using basic types like e and t, we can build arbitrarily complex semantic objects, like properties ($\langle e, t\rangle$), binary relations ($\langle e, \langle e, t\rangle\rangle$), quantifiers ($\langle\langle e, t\rangle, t\rangle$, see below), etc. Functions are standardly written in λ-notation, so that $\lambda\alpha.\beta$ is a function that maps any object α to β.

[2]For the discussion of potential constraints on quantifier meanings in natural language, such as monotonicity, extensionality, isomorphism closure, and conservativity, see Van Benthem (1984) and subsequent literature.

[3]Similarly, there are arguments against a GQT analysis of "*many*" and other "quantity words" (Rett, 2018).

[4]A related approach that views numerals as modifiers can be found in Ionin and Matushansky (2006). The goal there is not just to account for simple numerals like *four* or *eight*, but also for complex ones, like *four hundred*.

[5]Note that we are interested in *semantic* plurality. Whether and how it corresponds to morphosyntactic plurality is outside the scope of this article. In English-like languages, numerals more often than not require the noun to be marked for plural, which makes numeral "one" an odd exception that requires explanation. In other languages, however—one example being Turkish—nouns are systematically morphologically singular in combination with numerals. For a defense of a syntactic agreement view on number marking on nouns with numerals, see, for example, Krifka (2003) and Ionin and Matushansky (2006). Semantically speaking, both the nominal and the VP predicate in sentences with numerals need to be plural for all of the theories we discuss here to work.

[6]Parallel to this, one may want to also have an operator introducing generic quantification. See Buccola (2017) for intricacies and difficulties with such a move.

[7]There are many variations on the definition in (22). Hackl (2000) made MANY a parametrized existential determiner. As we have seen, however, it is desirable to sever quantificational force from the interpretation of the numeral. Krifka (1989) proposed that nouns come with an argument slot that addresses measurement or cardinality information. The MANY in (22) is closest to Buccola and Spector (2016) and Bylinina and Nouwen (2018).

[8]The sentence in (29) illustrates lower-boundedness with distributive predicates. The situation is different when the predicate is collective. The existence of a group of thirteen students who lifted a piano together does not guarantee the existence of a smaller group that did the same.

(i)$[\![$Twelve students lifted the piano together$]\!]$ =
$\exists x[\#x = 12 \wedge {}^*\text{student}(x) \wedge {}^*\text{lifted-the-piano-together}(x)]$
$\Leftarrow\exists x[\#x = 13 \wedge {}^*\text{student}(x) \wedge {}^*\text{lifted-the-piano-together}(x)]$

Still, these collective cases are in a sense lower-bounded. A sentence like "*twelve students lifted the piano together*" does not exclude the possibility that also there was a different group of students, one of more than twelve, that lifted the piano together, too. For a detailed discussion of maximality in relation to distributivity and collectivity see Buccola and Spector (2016). We will be looking primarily at distributive contexts.

[9]Questions have been raised about the correctness of this observation and a proper empirical study is required to clarify the facts. At the same time, there are reasons to think that the alleged lack of stronger readings in this configuration is not specific to numerals. In (i), the expected inference "No kid did all of the homework" is also not clearly there (Benjamin Spector, p.c.):

(i) Some kids did some of the homework.

If the status of the numeral example (39) is the same as that of (i), an independent explanation might be more correct. That is, the theoretical consequences of the observation in (38), of course, are conditioned by its empirical accuracy.

[10]However, see Beck (2012) for an alternative view in which the contrast in question is linked to properties of whatever occupies the differential position, in this case *exactly*.

[11]Kennedy (2015), in fact, assumes that the MANY operator contains existential quantification. Here, we adopt a more flexible approach, as per the discussion above.

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