

Disagreement about logic from a pluralist perspective

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Abstract Logical pluralism is commonly described as the view that there is more than one correct logic. It has been claimed that, in order for that view to be interesting, there has to be at least a potential for rivalry between the correct logics. This paper offers a detailed assessment of this suggestion. I argue that an interesting version of logical pluralism is hard, if not impossible, to achieve. I first outline an intuitive understanding of the notions of rivalry and correctness. I then discuss a natural account of rivalry in terms of disagreement about validity claims and the argument from meaning variance that has been raised against it. I explore a more refined picture of the meaning of validity claims that makes use of the character-content distinction of classical two dimensional semantics. There are three ways in which pluralists can use that framework to argue for the view that different logics may be rivals but could nevertheless be equally correct. I argue that none of them is convincing.

 $\textbf{Keywords} \ \, \text{Logical pluralism} \cdot \text{Disagreement} \cdot \text{Rivalry} \cdot \text{Context-sensitivity} \cdot \\ \text{Validity}$

1 Introduction

Logics can be applied for a variety of purposes. They can be used for such diverse tasks as managing databases, simulating intelligent behaviour, analysing the grammatical structure of natural languages, or simplifying electronic circuits. It is



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therefore not surprising that there are many different logics. But even if we focus on more traditional applications like the study of valid arguments, there still are quite a number of different logics, each offering a theory of validity. This is still not very surprising. It is quite common, after all, that there are different theories about a domain subject to systematic study. A slightly more surprising thesis would be that more than one of those theories are correct. Call that the *plurality thesis*. Exactly how surprising the plurality thesis really is depends on how the theories in question relate to one another. This paper is concerned with those relations between different logics. More specifically, it is concerned with the question of whether an undoubtedly interesting version of the plurality thesis can be defended: a pluralism which claims that there are "at least two *opposing*, but equally correct, answers to the question of whether a single argument is valid" (Russell 2008, 609, emphasis added). Call this the *interesting plurality thesis*.

In order to answer that question, we first need to get a grip on the notions of opposition (or rivalry) and of correctness (Sect. 2). I sketch a natural approach to rivalry between logics that focuses on the semantic conception of disagreement (Sect. 3) and I briefly discuss some familiar problems for the semantic approach (Sect. 4). Section 5 explores ways to make sense of some recent suggestions to understand logical pluralism by means of an alleged context-sensitivity of the term "valid". Although the link between pluralism and context-sensitivity has been considered in the literature (most extensively in Shapiro 2014, but also in Caret 2017 and, very briefly, in Hjortland 2013), there is no systematic investigation of how it relates to rivalry or disagreement. One aim of this paper is to fill that gap. I argue that even though the claim that "valid" is context-sensitive suggests some promising ways to capture both plurality and rivalry, the position faces rather serious problems (Sect. 6). Finally, in Sect. 7, I discuss a metasemantic conception of rivalry and argue that it is not available to indexicalist approaches. I conclude in Sect. 8 with the claim that semantic accounts of rivalry are not well suited for pluralists who aim at defending an instance of the interesting plurality thesis.

2 Supplementation, rivalry, and correctness

There is an abundance of different logics and many of them seem to qualify as a theory of validity. Does that mean that they all give *competing* accounts of validity? There is reason to suppose that this is not the case. For instance, (classical) first order predicate logic is an extension of its propositional fragment. It allows inferences that propositional logic cannot represent due to its coarse grained analysis of sentences. All inferences licensed by propositional logic, however, are also licensed by predicate logic. Predicate logic *supplements* propositional logic in so far as all additional inferences it allows involve essentially logical vocabulary not available in propositional logic. Similarly, modal propositional logic supplements non-modal propositional logic. The use of the supplementary logic is compatible with the use of the original logic (cf. Haack 1974, 2).

There are other cases, however, in which different logics use the same vocabulary—or at least seem to do so—and still give different verdicts regarding the



validity of an argument. Familiar examples are the argument from " $A \land \neg A$ " to "B" or the argument from " $\neg \neg A$ " to "A". Both are classically valid, but while the former is invalid in relevant logics and other paraconsistent logics (but valid in intuitionist logic), the latter is invalid in intuitionist logic (but typically valid in relevant logics). Assuming that the arguments in question only use vocabulary shared by all three logics, the different verdicts cannot be explained in terms of supplementation. From the intuitionists' perspective, for instance, classical logic contains theorems that are *false* or, at least, *not really* theorems $(\models A \lor \neg A)$ and deductive rules which are invalid $(\neg \neg A \models A)$. So it looks like the use of intuitionistic logic is incompatible with the use of classical logic. Intuitively, the logics give competing accounts of validity—they are rivals. There are many ways to make this intuitive sense of rivalry more precise (see, e.g. Allo 2015; Hjortland 2014; Paoli 2003; Priest 2006a). The one I focus on in this paper is formulated in terms of semantic disagreement about validity claims. In a nutshell: logics are rivals if they make jointly incompatible validity claims (see Sect. 3). This gives us the first feature of the interesting plurality thesis.

The second feature is *correctness*. Arguably, the sense of correctness relevant to the logical pluralist does not concern formal aspects of logical systems like nontriviality or soundness and completeness. It is well known that, given a suitable semantics, many logics meet those criteria. Pluralism about *pure logics* is uncontentious; "plurality is an issue of substance only if one is asking about applied logics" (Priest 2006a, 195).² Accordingly, it has been claimed that in order to get a "substantial dispute [...] off the ground, or else to show that there really is no dispute, we have to examine the relationship between a formal language in a logical system and whatever the medium of logical consequence is" (Shapiro 2014, 42). This relationship is also at the center of Susan Haack's definition of correctness:³

(1) Correctness

"[A] logical system is *correct* if the formal arguments which are valid in that system correspond to informal arguments which are valid in the extrasystematic sense [...]" (Haack 1978, 222)

I wish to remain neutral on the appropriate characterisation of *validity in the extra-systematic sense*. It involves a plethora of difficult questions about the nature of logical consequence that need not be answered here. All that (1) presupposes is that there is some kind of extra-systematic validity that is being represented in some sense or other by a formal system. It is open to different views on the *medium of logical consequence* and the question of what informal arguments may be composed of. Candidates include (fragments of) natural language (Beall and Restall 2006;



¹ The distinction between supplementation and rivalry is similar to the one made by Matti Eklund (2012), who distinguishes between the *vertical* and the *horizontal* dimension of "which logic is the right one" questions.

² Think of a *pure logic* as a "well-defined mathematical structure with a proof-theory, model theory, etc" (Priest 2006a, 195). *Applied logics* are *pure logics* interpreted as theories of specific domains.

³ A similar principle is defended by Roy Cook (2010, 495–496).

Cook 2010; Shapiro 2014), instances of vernacular reasoning (Priest 2006a), or different kinds of truth-bearers (Russell 2008).

Logical monists claim that there is exactly one logic that is correct in the sense of (1). But there are at least two reasons why one might think that there must be more than one correct logic: First, the validity of arguments might be relative to some parameter X. For instance, the argument from a set of premisses Γ to the conclusion ϕ could be valid relative to macroscopic phenomena but invalid relative to quantum frameworks (Haack 1978, 223). If there is more than one parameter X that informal arguments are relative to, then it is relative to X whether a specific account of validity is correct. This view has been called logical relativism (following Cook 2010): the correct account of validity is relative to X.⁴

Secondly, there may be more than one correct logic because there is more than one way in which an argument can be valid. Being valid according to one notion of consequence is not the same as being valid according to another. This is the central claim of *logical pluralism*: There is more than one correct account of validity (see Cook 2010, 493).

Characterized this way, relativism and pluralism are independent views. Pluralism is compatible with the relativist thesis. It could turn out that there are different ways of being valid because what it means to be valid is relative to X. However, pluralism is also compatible with non-relativist views (see, e.g. Beall and Restall 2006, 88). One may be a pluralist about different "all purpose logics" (Field 2009) because they represent different correct accounts of validity. Finally, non-pluralistic relativism would amount to a view that relativizes one *single way* of being valid to X.⁵ The distinction thus highlights an important aspect of logical pluralism, namely that different correct logics represent *different ways* of being valid. The claim is that there actually are different correct consequence relations: "[W]e take it that there are at least two distinct relations of logical consequence—and not simply two distinct relations in intension, but two distinct relations in extension" (Restall 2002, 426). I argue in Sect. 6 that the claim about distinctness in intension is problematic when it comes to opposing but equally correct validity claims.

We now get a clearer picture of the interesting plurality thesis as it is understood in this paper. The claim is (i) that there are at least two logics such that the formal arguments valid in those logics correspond to informal arguments valid in some extra-systematic sense and (ii) that proponents of those logics endorse jointly incompatible claims about the validity of a single argument.

⁵ This allows for the possibility of non-pluralistic (in Cook's sense) implementations of the interesting plurality thesis (see Sect. 6).



⁴ A form of logical relativity which results from different choices of logical vocabulary is defended by Varzi (2002). Since I am concerned with logics that share the same logical vocabulary, I will not discuss this view here

3 Rivalry as semantic disagreement

Consider two logicians C and D, such that C accepts and D rejects the argument from Γ to ϕ . An intuitive analysis of this scenario is that C takes the proposition expressed by the validity claim " $\Gamma \models \phi$ " to be true, while D takes it to be false. This allows for a straightforward sense of rivalry in terms of semantic disagreement. The literature on disagreement is vast but given our scenario, we can narrow down the kinds of disagreement we are interested in right from the start.

First, following Cappelen and Hawthorne (2009, 60–61), there is a distinction between disagreement as an activity—as in, for instance, *disagreeing about whether* X *is the thing to do*—and disagreement as a state—as in, for instance, *disagreeing about whether to believe* p, where p is a proposition. In this paper, I am mainly concerned with the question of whether or not we should take the disagreement between C and D to be of the latter kind, namely as disagreement about whether to believe the proposition expressed by " $\Gamma \models \phi$ ".

Secondly, the disagreement between C and D concerns beliefs as opposed to desires, intentions to act, or other non-doxastic mental states. We will, therefore, be concerned only with *doxastic disagreement*, that is, with the conflict not between agents but between the content of their doxastic attitudes (cf. Marques 2014, 123). This means that, in order for C and D to disagree in the relevant sense, they need not even know of the other's beliefs.

Given these constraints, a good place to start is a simple conception of disagreement in terms of incompatibility of content as introduced by John MacFarlane:

(2) The Simple View of Disagreement (TSV)

To disagree with someone's belief that p is to have beliefs whose contents are jointly incompatible with p. (MacFarlane 2014, 121)

MacFarlane asks us to understand TSV as a special case of *noncotenability*, which gives us the following understanding of the *incompatibility* at issue: "I disagree with someone's attitude if I could not coherently adopt that same attitude (an attitude with the same content and force) without changing my mind" (MacFarlane 2014, 121). As TSV focuses on contents, this suggests that the incompatibility at play in the intended reading of rivalry as *being in a state of doxastic disagreement* is primarily semantic. It can easily be applied to validity claims: C disagrees with D's claim if C could not coherently make the same claim—a claim with the same content—without changing her mind or retracting at least one of her own claims.

TSV is only one of several notions of disagreement MacFarlane discusses and there are reasons to go into more detail in order to develop a fully-fledged account of disagreement that is faithful to the intuitions of competent speakers of natural

⁶ TSV may need some refinement when applied to dialetheist views. According to dialetheists, there are dialetheias d such that one may believe both d and it's negation, and rationally so (cf., Priest 2006b). So there are contents d, such that one can rationally believe $d \land \neg d$ without necessarily being in disagreement with anyone. Maybe dialetheias, if there are any, are indeed cases in which one can coherently believe both d and $\neg d$ without changing one's mind. I take it, however, that the disagreements at issue in this paper are not about dialetheias.



language. In what follows, however, I am interested in the "lower bounds" of disagreement, as it were. If pluralists cannot make sense of disagreement according to TSV, they will *a fortiori* not be able to get an interesting sense of disagreement according to a more demanding view.

4 How meaning variance threatens disagreement

The position that rival logicians disagree on a semantic level in a sense related to TSV is contentious. Its most influential challenge is known as the *argument from meaning variance*. It is typically attributed to Quine (1986) even though Quine himself avoids talk of meaning in his formulation of the argument. He claims that when a "deviant" logician and a classical logician are disputing a particular logical law, they are in fact talking past each other. The "deviant logician's predicament", according to Quine, is that denial of a classical logical law amounts to changing the subject. Earlier (Carnap 1937, 189) and later (Haack 1974, 8; Paoli 2003) versions of the argument are given in terms of the meaning of the logical vocabulary, which is also the reading most relevant for the purposes of this paper.

Following Haack (1978), Ole Hjortland (2013) recently provided a useful distinction of the main contemporary versions of the challenge. According to what he calls *A-variance*, the meaning of "valid" varies across logical theories; according to *B-variance*—the more traditional reading of the argument—the meaning of some logical connective varies across logical theories.

The result is allegedly that for two logical theories with the same formal language, say classical and intuitionistic logic, there is no genuine conflict between validity-attributions. In the case of (A) because *what is being attributed*, i.e. validity, is not the same in the two theories. In the case of (B) because *that to which validity is being attributed*, i.e. the argument, is not the same in the two theories. (Hjortland 2013, 359)

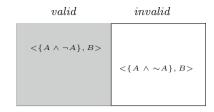
The argument from meaning variance, thus, challenges the intuitions concerning the rivalry between different logics developed above. If either of the two theses were correct, the dispute between Deidre, a 'deviant' logician who accepts the sentence " $A \land \neg A$ ", and Claire, a classical logician who rejects it, would dissolve.

According to a common interpretation of B-variance (see, Haack 1974; Hjortland 2013, 2014; Restall 2002; Shapiro 2014), the meaning of the logical constants changes with the logic in question. Thus, the argument from " $A \land \neg A$ " to "B", which Claire accepts, is not the argument that Deidre rejects. Rather, Deidre is talking about a different argument—from, say, " $A \land \sim A$ " to B, where "¬" and " \sim " do not refer to the same connective (see Fig. 1).

 $^{^{7}}$ I am grateful to two anonymous referees of this journal for substantial and very helpful feedback on this section.



Fig. 1 The dispute according to the argument from "meaning variance"



According to this analysis, it no longer follows that the Claire and Deidre disagree. Since they are talking about different connectives, their beliefs are compatible and the apparent dispute turns out to be merely verbal.⁸

Some pluralists (e.g. Kouri Kissel 2018; Shapiro 2014; Terrés Villalonga 2019) endorse this result to a certain extend. After all, it may still be argued that both Claire's negation and Deidre's negation correspond to different extra-systematic negations. Thus, both their logics may be correct in so far as arguments involving negation in the formalisms correspond to different extra-systematic arguments, respectively. But note that in interpreting Claire and Deidre as talking about different arguments, the resulting pluralism does not amount to an instance of the interesting plurality thesis. On the current picture, Claire and Deidre do not disagree about the validity of a single argument.

There are further options, however. The argument from B-variance as construed above is contentious and there are a number of suggestions in both proof-theoretic and model-theoretic terms on how to delineate the meaning of connectives in such a way that one can plausibly maintain that they have a common meaning in different logics. ¹¹ It is beyond the scope of this paper to discuss those proposals in detail, but the basic ideas are as follows.

Proof-theorists usually distinguish between the inference rules for the connectives, on the one hand, and a deducibility relation specifying structural properties of a system, on the other hand. On the assumption that the inference rules of a connective are its meaning constitutive laws, it may then be argued that systems with identical rules for the connectives give a common meaning to those connectives. Examples include Gentzen's sequent calculi LK and LJ (see, Haack

¹¹ For proof-theoretic approaches, see Dicher (2016), Haack (1974), Hjortland (2013), Paoli (2003), Paoli (2014), Restall (2014); model-theoretic approaches can be found in Hjortland (2013), Priest (2008), Restall (2002).



⁸ Quine's original threat is more subtle than this (see, Restall 2002; Paoli 2003, 2014). The current analysis of B-variance may also fail to do justice to Quine's holism about meaning. It does seem adequate, however, on the accounts of meaning endorsed in the current discussion of logical pluralism.

⁹ In response to the challenge from B-variance, Kouri Kissel (2018) sketches a *pragmatic* account of the disagreement of two interlocutors debating negation. Her approach is interesting and it merits a proper discussion elsewhere (I think it is vulnerable to arguments similar to the ones raised in Sect. 7, but I will not develop these arguments, here). What is important for the purposes of this paper is that her endorsement of B-variance precludes a semantic analysis of logical disagreement in terms of TSV.

Note also that care must be taken when combining connectives of different logics in one language. In particular, intuitionist and classical negation are provably equivalent when unrestrictedly combined in the same language (see Harris 1982).

1974, 10) or the sequent calculi of classical logic and subexponential linear logic without additive constants (see, Paoli 2003, 541).

On the model-theoretic approach, the meaning of the connectives is typically given in terms truth-conditions. Thus, the analogous thought is that identical truth conditions are enough to give a common meaning to the connectives. Take, for instance, the truth tables for the connectives in both Kleene's three-valued paracomplete K_3 and Priest's paraconsistent LP (Priest 2008) or Restall's (2002, 438) (minimalist) truth conditions for negation (" $\neg A$ is true iff A is not true") in both classical and relevant logics.

The overarching thought of these counterarguments against B-variance is that if there are formalizations of two logics, such that their inferential rules (or truth conditions) for the connectives are the same, then those logics assign the same meaning to the connectives.

It is still debatable whether this is enough to support the intuition that the logics in question are genuine rivals. ¹² One may think that the problem raised by the argument from meaning variance does not target the connectives exclusively but also the notion of validity or logical consequence. According to this line of reasoning, the challenge from A-variance remains even if the challenge from B-variance can be met. ¹³ Take Haack's example: the deducibility relations of LK and LJ are different. While LJ allows at most one formula in the succedent of the sequents, LK imposes no such restrictions. Accordingly, one may argue, the notions of validity in those logics differ. Similarly for K_3 and LP: both logics have the truth values $V = \{0, i, 1\}$, but while in K_3 the only designated value D is $1, D = \{1\}$, both i and 1 are designated in LP, $D = \{1, i\}$. Again, one may argue that the notions of validity in those logics differ.

In terms of Claire and Deidre's example, the arguments from common meaning of the connectives may allow us to maintain that the two logicians are indeed discussing one and the same argument from " $A \land \neg A$ " to "B". However, supposing that the logic endorsed by Claire invokes a notion of validity different from the one endorsed by Deidre, the dispute will have to be understood along the lines of (3).

(3) a.
$$A \land \neg A \vDash_1 B$$

b. $A \land \neg A \nvDash_2 B$

Again, some pluralists (e.g., Beall and Restall 2006; Caret 2017; Dicher 2016; Hjortland 2013) endorse this result to a certain extent. It is not clear, however, that it leaves room for an *interesting* form of pluralism. Taking into account that one can believe both (3-a) and (3-b) without changing one's mind, there is certainly no disagreement in terms of TSV. Again, the apparent dispute turns out to be merely verbal, so it is not obvious that this is much of an improvement.

¹³ Hjortland 2014 calls this the "Meta-Quinean challenge".



Francesco Paoli, in his 2003 thinks it is, but see Hjortland (2013, 2014) and Paoli (2014).

There is more to be said about A-variance, however. In the next section, I discuss the resources available to contextualist approaches to "valid" when it comes to the arguments from meaning variance. I argue that if the type of pluralism in question is a claim about the semantics of "valid" in natural language, then it is hard to console the view with the most widely accepted account of context-sensitivity in natural language, namely David Kaplan's LD. ¹⁴

5 Context-sensitivity and levels of meaning

A number of authors have suggested that the *character-content* distinction of Kaplanian formal semantics (Kaplan 1989b) offers a fruitful way of understanding logical pluralism (e.g., Hjortland 2013; Caret 2017, or Shapiro 2014), but a detailed analysis of these suggestions is still lacking in the literature. ¹⁵

The Kaplanian framework not only allows for elegant semantic implementations of the thesis that there may be more than one correct account of some phenomenon X, the discussions of those implementations also offer rich resources for approaching the question of how to best think of the relation between different accounts of X. Given the prominence of the notion of disagreement in the debates about the context-sensitivity of natural language expressions (like, for instance, predicates of personal taste, epistemic modals, future contingents, or knowledge ascriptions), ¹⁶ this looks like a promising framework for formulating an interesting version of logical pluralism.

One advantage of the Kaplanian picture is that the distinction between the *character* and the *content* of an expression offers a more fine grained picture of meaning than the one we have considered so far. Roughly, the *character* of an expression *x* is set by linguistic conventions. We can think of it as meaning in the sense of what is known by the competent language user. Secondly, the *content* of an expression is its *intensional meaning*. Finally, there are the usual extensions, the *referential meaning* of an expression (cf. Kaplan 1989b, 501–506).

Contexts of utterance, on the Kaplanian picture, can be thought of as *n*-tuples containing at least an agent, a time, a location, and a possible world. This way, indexical expressions like "I" or "here" can be assigned the values delivered by the context in which they are uttered. "I" refers to the agent of the context, "here" refers to its location. In order to accommodate the idea that "valid" is an indexical expression, we have to extend the Kaplanian picture. Contexts will have to include some value that determines a particular notion of validity at issue at a context of

¹⁶ See MacFarlane 2014 for a useful overview as well as for details on the semantics.



¹⁴ There are also interesting proof-theoretic approaches to the challenge from A-variance (Paoli 2014; Dicher and Paoli 2018) that have similarities to some of the model-theoretic Kaplanian strategies (see Sect. 5).

¹⁵ Shapiro's treatment of context-sensitivity is extensive, but Shapiro endorses a sort of metacontextualist view in which the question of whether to endorse an indexical or a nonindexical contextualism for different kinds of logical vocabulary is itself a context-sensitive matter (see, e.g., Shapiro 2014, 154–155). This point is orthogonal to the issues discussed here.

utterance. This is not quite as straightforward as the other contextual notions. It is reasonably clear how to think of the agent or the location of a context, but how do we determine the notion of validity at a context? Shapiro proposes that "each 'context' includes a specific mathematical theory or structure. It would be a mathematical theory being advanced at any given time by a mathematician or a group of mathematicians" (Shapiro 2014, 89). Caret suggests that contexts include a parameter for a deductive standard, where "a deductive standard is an admissible class of cases that function as logically salient alternatives" (Caret 2017, 753). Theses are quite demanding conceptions of context, but the worries I develop in what follows are independent of these particular proposals.

Characters can be represented by functions from contexts to contents, which means that they determine the *content* of an expression *x* in every context. Contents, in turn, can be represented by functions from circumstances of evaluations to extensions. Think of circumstances of evaluation as "both actual and counterfactual situations with respect to which it is appropriate to ask for the extensions of a given well-formed expression" (Kaplan 1989b, 502). According to Kaplanian orthodoxy, they contain at least a parameter for a possible world, a time, and a location, but I will get back to this below. Given this picture, we arrive at *x*'s extensions by evaluating the content of *x* at a circumstance. Validity claims are sentences, so the character of a given validity claim is a function from contexts to propositions. These propositions, in turn, are functions from circumstances of evaluations to truth-values.

Recall the interesting plurality thesis: there are at least two opposing, but equally correct, answers to the question of whether a single argument is valid. We saw in Sect. 4 that this is incompatible with meaning-variance of the connectives. What about a view allowing for meaning variance of "valid"? The Kaplanian levels of meaning offers three options.

The first is that the linguistic meaning of "valid", its character, changes with a change of logic. This amounts to claiming that the expressions in question are only superficially similar but have, in fact, different lexical entries. Polemically put, this means that the paraconsistent logician's complaint against classical logic that the argument from " $A \land \neg A$ " to "B" is *invalid* would be similar to insisting that one cannot sit on a bank, on the grounds that one cannot sit on a financial institution. This is a change of subject, if anything is, and it clearly reduces Claire and Deidre's dispute to a verbal dispute.

In fact, there is a plausible candidate for a common character of "valid": Beall and Restall's (2006, 29) *Generalized Tarski Thesis* (GTT).

GTT An argument is valid_x if and only if, in every $case_x$ in which the premises are true, so is the conclusion.

This takes us to the second option: the content of "valid" changes with a change of logic. Given that characters are functions from contexts to contents, a suitable accompanying thesis would be that the context of utterance provides the value x for

^{17 &}quot;Class of cases" in Caret's formulation refers to the notion of case in Beall and Restall's Generalized Tarski Thesis (see below).



the type of cases and, therefore, for the notion of "validity" in question. Modifying GTT slightly, we can also make sense of Shapiro's (2014, 89) claim that contexts include a mathematical theory or structure:

GTT* An argument is valid_x if and only if, in every context_x in which the premises are true, so is the conclusion.

Accordingly, contexts including a classical theory yield classical validity, contexts including an intuitionist theory yield intuitionist validity, and so on. Abstracting from the details, these proposals amount to something along the lines of (4), where "valid" refers to the character of "valid" and " c_x " indicates the logic x of the context c_x .¹⁸

(4) The indexicality claim:

"valid" has a *context-sensitive character*, s.t. $valid(c_{\mathcal{L}_1}) \neq valid(c_{\mathcal{L}_2})$ if \mathcal{L}_1 is relevantly different from \mathcal{L}_2 .

The character of an expression x applied to a context c is, of course, identical to x's content in c so that "valid $(c_{\mathcal{L}_1})$ " can also be written as valid \mathcal{L}_1 . Thus, (5) can express different contents when uttered in relevantly different contexts:

(5) The argument from " $A \land \neg A$ " to "B" is valid.

In a context including Claire's classical logic, the content expressed would be (6-a), in a context including Deidre's paraconsistent logic it would be (6-b).

- (6) a. That the argument from " $A \land \neg A$ " to "B" is valid.
 - b. That the argument from " $A \land \neg A$ " to "B" is valid.

Given that different contents can be represented by different functions, it is no surprise that (6-a) and (6-b) can have different extensions, that is, different truth values. Note that they could even have different truth values when evaluated at the same circumstances of evaluation—they are different propositions after all. On the assumption that the falsity of (6-b) is equivalent to the claim that the argument from " $A \land \neg A$ " to "B" is invalid_D, it seems that we get different verdicts on one and the same argument. As pointed out in the previous section, however, it is not clear that those verdicts are really *opposing* verdicts. (6-a) and (6-b) are perfectly compatible, which means that there is no disagreement in the sense of TSV.

To see this more clearly, imagine that Deidre accepts the claim " $A \land \neg A$ ". Claire protests on the grounds that anything follows from a contradiction, uttering (7-a). Deidre, in turn, complains that (7-a) is false/not true, as contradictions do not entail everything, uttering (7-b) and (7-c).

- (7) a. [C:] The argument from $A \land \neg A$ to B is valid.
 - b. [D:] That's false/not true!
 - c. [D:] The argument from $A \land \neg A$ to B is not valid.

 $^{^{18}}$ Notation: characters are printed in typewriter font, contents in small caps.



Now, while (7-a) and (7-c) come out true, (7-b) is false according to the indexicalist option. ¹⁹ Assuming that the indexical *that* in (7-b) refers to what Claire said—that is, to the proposition expressed by Claire's utterance—(7-b) would amount to claiming that it is false that the argument from $A \land \neg A$ to B is classically valid. This seems to give us the wrong result as it commits us to interpret Deidre in such a way that she is mistaken either about classical validity or else about what is expressed by her utterance. In any case, the dispute would turn out to be verbal. So far, this is no improvement on the arguments sketched in Sect. 4.

There is a third option, however: The thought is that although the character *and* the content of "valid" are semantically invariant across different contexts in the sense that they express the same functions, its extension can vary nonetheless. This amounts to a variant of *nonindexicalism* as introduced by John MacFarlane (2009, 2014). A natural candidate for the invariant character of "valid" is the unparameterized Tarski Thesis (TT):

TT An argument is valid if and only if, in every case in which the premises are true, so is the conclusion.

Abstracting from the details again, the proposal amounts to something along the lines of (8), where "valid" refers to the character of "valid", "valid" refers to its content, " c_x " indicates the logic x of the context c, and " e_x " indicates the logic x of the circumstance of evaluation e.

- (8) Nonindexical context-sensitivity:
 - a. "valid" has a *stable character*, s.t. for all contexts c_x and c_y valid(c_x) = valid(c_y).
 - b. "valid" has a *context-sensitive content*, s.t. $VALID(e_{\mathcal{L}_1}) \neq VALID(e_{\mathcal{L}_2})$ if \mathcal{L}_1 is relevantly different from \mathcal{L}_2 .

For this view, and in particular for the clause (8-b) to work, we need to assume that the circumstances of evaluation contain a parameter for the salient logic. This move is somewhat non-standard. I pointed out above that, according to semantic orthodoxy, the circumstances of evaluation contain a parameter each for the world, the time, and the location of the context (cf. Kaplan 1989b; Lewis 1981), although Kaplan suggests that, in the end, this boils down to a question of *language engineering*:

 $^{^{20}}$ A note on terminology: *Nonindexical Contextualism* and *Relativism* (as defined in MacFarlane 2014) give different accounts of which logic is relevant. Contextualists claim that the logic governing the context of utterance c_U is the one which fixes the extension, relativists claim that it is the logic governing the context of assessment c_A . The distinction is important for some stronger notions of disagreement and the phenomenon of *retraction* i.e. the speech act one performs in saying "I take that back" or "I retract that." These differences, although important in general, can be neglected for the purposes of this paper as I am only concerned with the simple view of disagreement. In what follows I use *nonindexicalism* as an umbrella term for nonindexical contextualism and relativism in MacFarlane's sense.



¹⁹ Suppose that Claire and Deidre are in different contexts here, maybe because the context switches midway during their conversation. If you insist that they share the same context, then the indexicalist account predicts that one of them is wrong. Change the scenario, then, so that they are not in involved in a direct conversational exchange but that they evaluate each others' claims from different contexts.

It is a question of which features of what we intuitively think of as possible circumstances can be sufficiently well defined and isolated. If we wish to isolate location and regard it as a feature of possible circumstances we can introduce locational operators: "Two miles north it is the case that", etc. [...] However, to make such operators interesting we must have contents which are locationally neutral. (Kaplan 1989b, 504)

Nonindexicalists wish to isolate *logic* and regard it as a feature of possible circumstances. What is needed, thus, are logical operators such as "In classical logic it is the case that" or, rather, "It is classically valid that" or "It is paraconsistently valid that". Further, the contents in question need to be logically neutral, i.e. they cannot be assigned a truth-value without indicating a logic in the circumstances of evaluation. This line of reasoning is contentious²¹ and defenders of the view would ultimately have to go into more detail, but I do not want to defend the view here. In fact, I argue that this option is problematic for independent reasons.

Let us assume for the moment, however, that we can make sense of a nonindexicalist position about "valid". In contrast to the indexicalist, the nonindexicalist holds that validity claims like (5) express the same content in every context:

- (5) The argument from " $A \land \neg A$ " to "B" is valid. Thus, in both Claire's and Deidre's contexts the content of (5) would be (9).
- (9) That the argument from " $A \land \neg A$ " to "B" is valid

So far, nonindexicalism about valid is compatible with logical monism. It differs from monism in claiming that (9) cannot be assigned a truth-value without indicating a logic at the circumstances of evaluation. It is the latter claim that enables the nonindexicalist to hold that the content expressed by (5) may be true at one circumstance of evaluation $e_{\mathcal{L}_1}$ containing logic \mathcal{L}_1 and not true at another that contains logic \mathcal{L}_2 .

- (10) a. That the argument from $A \wedge \neg A$ to B is valid is true at $e_{\mathcal{L}_1}$.
 - b. That the argument from $A \wedge \neg A$ to B is valid is not true at $e_{\mathcal{L}_2}$.

²² Dicher and Paoli (2018) recently formulated a position that may provide a *proof-theoretic* variant of just this view. They defend the claim that the consequence relation relevant for proof-theoretic semantics is given by the sequent-to-sequent derivability in Gentzen systems. According to their proposal, this relation is also the only candidate discriminating enough to fix the meaning of the connectives for the logic in question. As I understand them, different logics may still emerge if one allows the internal structural rules—which are considered not to be meaning constitutive for either the connectives or the consequence relation—to vary. A pluralist may then argue that there is more than one *correct/adequate/right* set of internal structural rules. The resulting position seems to avoid A-variance as well as B-variance while still allowing opposing, but equally correct, answers to the question of whether a single argument is valid.



 $^{^{21}}$ Actually, even the inclusion of time and location is contested (cf., e.g., King 2003; Cappelen and Hawthorne 2009).

The interesting aspect of the view is that it does better than indexicalism with respect to disagreement. Given that (9) is the content expressed by (5), Claire cannot coherently adopt Deidre's belief without changing her mind, because this would mean endorsing what Deidre said, namely that the argument from " $A \land \neg A$ " to "B" is invalid, while at the same time believing that it is valid. The nonindexicalist is in a position to claim that all the utterances in (7) are true.

- (7) a. [C:] The argument from $A \land \neg A$ to B is valid.
 - b. [D:] That's false/not true!
 - c. [D:] The argument from $A \land \neg A$ to B is not valid.

The interpretation of (7-a) and (7-c) follows the analysis above. Supposing again that the indexical *that* in (7-b) refers to what Claire said—that is, to the proposition expressed by Claire's utterance—Deidre's utterance of the sentence in (7-b) would amount to claiming that that proposition is false in her context. This gives us the right result as it allows us to interpret Deidre in such a way that she is rejecting what Claire said. Further, the proposition expressed by Claire's utterance is indeed false/ not true in Deidre's context, which is governed by a logic relevantly different from the one in Claire's context.

This is a significant step toward an implementation of the interesting plurality thesis: We now have a proposal according to which there are two opposing, but equally correct, answers to the question of whether a single argument is valid. In effect, "opposing" and "correct" can even be interpreted in a quite natural way, namely that there are two validity claims which are *incompatible* in the sense that they cannot be adopted within one and the same context, but which can both be *true* when evaluated at appropriate circumstances of evaluation.

6 Worries about the semantic options

Despite all this, the analyses offered here are partly incomplete and partly downright problematic for a number of reasons. Generally, the options sketched in Sect. 5 simply posit the context-dependency of the expression "valid". If this is to be taken seriously, the view needs to be supported by linguistic data.²³ Context-sensitivity of natural language expressions is not something that can be postulated at will (see

²³ An anonymous referee suggested that Glanzberg's (2015) arguments against the view that natural language determines a logical consequence relation could support the indexicality claim. The idea would be that while there is no (general) notion of logical consequence in natural language, there may be different context-sensitive ways of reconstructing it. Glanzberg argues that "we only get to logic *proper by a significant process of* identification, abstraction, *and* idealization" (Glanzberg 2015, 71). Maybe there is a way to link this process to contexts of utterance such that it supports the indexicality claim. My point here is merely that we still need a story about how this may work. Note also that Glanzberg himself briefly considers logical pluralism in connection with his *permissive* views of logics. As I understand him, he thinks that permissive views are compatible with pluralism, but he also argues that "we still have good reason to think the logic in natural language thesis is false, even if we adopt a permissive view." (Glanzberg 2015, 81).



also, Williamson 2014, 227). After all, the view to be defended is not just that logicians freely talk of, say classical validity and intuitionistic validity, but that the expression "valid" may express those different notions of validity depending on the context of utterance. I do not want to claim that no such data could be provided, I only want to point out that it still needs to be provided.²⁴

On a more theoretical note, both the indexicalist and the nonindexicalist about "validity" treat "valid" like any other context-sensitive object language expression. But, in fact, there is also a general definition of validity in the Kaplanian framework—call that *Validity*. According to that definition, a formula is Valid in Kaplan's LD just in case it is true in *every* context (Kaplan 1989b, 547). This raises the question of what (non-)indexicalists about "valid" want to say about arguments satisfying Validity. After all, these are arguments that are valid according to every context-dependent object language notion of validity.

This points to a more general problem for logical pluralism raised by Priest (2006a, 202-203): Why not simply take the intersection of all admissible logics and be a monist about that? In the case at hand this would amount to logical monism about Validity. It is the latter notion, the monist could claim, that gives us the core of universally correct inferences—inferences which are valid in every context. She may even allow those core inferences to be augmented if we are reasoning in certain kinds of contexts, for instance, in classical contexts. The contextualist about "valid", in contrast, claims that each kind of context requires different notions of validity, but she could also accept that there are certain inferences that are valid in every context. Priest calls this duck/rabbit pluralism: "The facts are agreed upon. What is at issue is only how to describe them." (Priest 2006a, 203)

So the contextualist analysis not only turns the dispute between partisans of competing logics into a verbal dispute, it also turns the dispute between monists and pluralists into a verbal dispute. If the facts are all agreed upon in the debate between the relevant kind of pluralism and monism, then all that is at issue is the label. This also raises further questions about the stability of the pluralist position: If there are contexts in which classical inferences fail, logical pluralism may just be another non-classical logic. Alternatively, if the normativity objection against logical pluralism succeeds, then all sub-classical logics may be normatively overridden by classical logic (see, Caret 2017; Keefe 2014; Read 2006; Stei 2017, 2019).

There are two further problems for the nonindexicalist option in particular. The first concerns the generality and necessity of validity statements. ²⁵ According to the nonindexicalist, the truth of those statements depends on the circumstances of evaluation indexed by logics. We have already seen that a pluralist who endorses some paraconsistent logic will have to claim that there are circumstances in which



²⁴ An anonymous referee suggested that the vagueness of "valid" may be enough to support the claim that "valid" is context-sensitive, but this clearly depends on the account of vagueness at issue. While contextualists about vagueness (e.g. Graff Fara 2000; Shapiro 2006) obviously endorse this idea (Sorensen (2018) explicitly links contextualism about vagueness to the Kaplanian *character/content* distinction), epistemicists like Williamson (1994) do not. I take it, therefore, that only specific *theories* of vagueness support the indexicality claim—the vagueness of "valid" in itself does not.

²⁵ Thanks to an anonymous referee for pressing me on this point.

the content of "The argument from $A \land \neg A$ to B is valid" is not true. According to Kaplan, necessity is a property of the content of a sentence. The content that a sentence expresses in a given context is necessary if it would be true no matter what the circumstances were (Kaplan 1989a, 597). Thus, on the nonindexicalist picture, "THAT THE ARGUMENT FROM $A \land \neg A$ TO B is VALID" and similar classical or intuitionistic laws will not be necessary. This need not be a problem as some pluralists explicitly reject at least the generality of logic (e.g. Shapiro 2014), but in this case this is an immediate result of the semantics. Note, that the indexicalist can avoid this outcome as, on her view, the sentence "The argument from $A \land \neg A$ to B is valid" in a, say, classical context will express the content "THAT THE ARGUMENT FROM $A \land \neg A$ TO B Is VALID C" and there is no semantic reason why this should not be true at every circumstance of evaluation even on a pluralist picture.

The second worry about nonindexicalism is that the view may lack the main characteristic of pluralism as introduced in Sect. 2. The reason is that nonindexicalism amounts to a relativisation of the *truth* of validity claims to a parameter at the circumstances of evaluation not to a pluralisation of *validity*. The idea is that a validity claim ϕ (expressing the *same content* in every context of utterance) is true in the context c iff ϕ is true *relative to* the logic \mathcal{L}_c of that context. That means that the (unqualified) expression "valid" does not give rise to a plurality of ways of being valid. Rather, it expresses one and the same notion of validity. Of course, the extension of "valid"—the set of arguments denoted by the expression—may vary with the logic-parameter at the circumstances of evaluation. However, that does not seem to be enough to deliver genuinely different notions of validity—it is the same function from contents to extensions in all contexts, after all.

Assume, by analogy, that nonindexicalism about epistemic modals is correct. Then the extension of the phrase *is epistemically possible* relative to Claire may be different from its extension relative to Deidre. That does not mean, however, that there are different ways of being epistemically possible. In fact, the whole point of nonindexicalism is that this is not the case.

Similarly, there is only one nonindexicalist way of being valid, and this particular way of being valid is relative to a parameter X, namely the logic-parameter in the circumstances of evaluation. Note that according to Cook's characterisation of pluralism and relativism (Sect. 2), this clearly amounts to a form of relativism. This is not particularly worrying in itself. Many pluralist views also have relativistic elements (e.g., Shapiro 2014). My worry is, rather, that it is a *non-pluralistic* form of relativism as the nonindexicalist account does not have the resources to semantically represent different ways of being valid. So even if nonindexicalism manages to give a successful implementation of the interesting plurality thesis, it does so at the cost of giving up a decisive pluralistic feature at the level of semantics.

The results seem to be that indexicalism, while being plainly on the pluralist side, is not an instance of the interesting plurality thesis as long as the notion of opposition is understood semantically. Nonindexicalism, in contrast, is interesting in the sense that it accommodates the opposition between the correct logics, but it lacks the resources to semantically model different ways of being valid.

This may or may not to be enough to rule out nonindexicalism as a serious pluralist option. Pluralism is not a particularly well-defined position, so this will



most certainly be a matter of debate. Given that it provides an implementation of an intuitive reading of the interesting plurality thesis, it may be argued that nonindexicalism is a viable option for a defender of pluralism who is unimpressed by giving up the generality of logic and the necessity of logical laws. However, some pluralists, most notably Beall and Restall, reject this approach:

We are pluralists about logical consequence because we take there to be a number of different consequence relations, each reflecting different precisifications of the pre-theoretic notion of deductive logical consequence. This is a pluralism, not a relativism. (Beall and Restall 2006, 88)

The nonindexicalist strategy does not represent logical consequence as a pretheoretic notion which may be made precise in a number of different ways. Rather, it offers one unique precisification of the pretheoretic notion and relativizes this to a further parameter. At least as far as the semantics is concerned, there is no plurality left. I take this to be particularly worrisome on a framework that is supposed to capture plurality on the level of (natural language) semantics—like the contextualist proposal discussed here. ²⁶

Whatever one thinks about nonindexicalism, there may be hope left for indexicalism. In what remains, I discuss one further, broadly semantic argument that aims at establishing a sense of rivalry for indexical pluralism—the argument from disagreement about concepts. It is not available to the nonindexicalist as, on her view, "valid" expresses the same concept in every context.

7 Disagreement about concepts

Maybe the rivalry we should be looking for is not about whether or not a given argument is valid, but about something else. Here is a suggestion:

There is real competition, genuine rivalry, here, not over which arguments are valid in an agreed sense of "valid", but over what conception of validity is most proper and adequate. [...] Due regard for the significance of this disagreement seems to require that one combine a kind of pluralism about logical systems with a recognition that there may be real competition at the level of metaconcepts. (Haack 1978, 229)

This may not result in an instance of the interesting plurality thesis, but maybe the resulting position would be interesting enough. It may give us a dispute on par with the one between, say, a defender of *de re* modality and a Quinean about modality. Being a pluralist about this type of dispute would, of course, involve the claim that both parties are right in some sense or other.

I do not think this kind of rivalry is available to the indexicalist. To see why, consider an example from the philosophy of language based on the following scenario which Mark Richard uses to argue against the *criteria view* about "rich":



²⁶ Thanks to an anonymous referee for pressing me on this issue.

Suppose [...] that Mary wins a million dollar lottery. Didi is impressed, and remarks to a friend "Mary's rich." Naomi, for whom a million dollars is not really all that much, remarks in a conversation disjoint from Didi's, "Mary is not rich at all." Suppose the salient comparison class is the same in both cases. (Richard 2004, 218)

The analogy to the debate about the proper conception of validity becomes apparent once we consider the idea that Didi and Naomi's disagreement is over the criteria which should be used with the word "rich". The rivalry that Haack has in mind seems to be of just this kind. Applied to the dispute between Claire and Deidre we considered earlier, it would amount to the claim that what Claire and Deidre disagree over are which criteria should be used with the word "valid": Claire's classical or Deidre's paraconsistent critera.

Richard claims that, in the case of "rich", there is indeed some kind of conflict over the correct criteria for "rich", but that this conflict is due to a more basic disagreement over what it is for someone to be rich (cf. Richard 2004, 221). The claim is supported by the following test aiming at the *source* of the disagreement:

(11) Richard's Test

"[D]o [Didi and Naomi] disagree as to whether 'rich' applies to Mary because they disagree about what it is to be rich, or do they disagree about what it is to be rich because they disagree as to whether 'rich' applies to Mary?"

For Richard, the answer is clear:

Surely the answer to the first question is affirmative, the answer to the second negative. But if they disagree over criteria because they disagree over the nature of wealth, then it is a difference over what it is to be rich which is at the root of their differences. And if so, then the right answer to the question, "What is their substantive disagreement?", is surely one in which the word "rich" is used, and not (just) mentioned. (Richard 2004, 221)

The issue here is not about whether or not there is disagreement about the correct criteria—there is. Rather it is about the *source* of that disagreement. In the case of "rich", Richard argues, it is about the nature of wealth, about what it means to be rich. I propose an analogous test for validity:

(12) Richard's Test applied to "valid" Suppose C and D disagree over whether the argument from Γ to ϕ is "valid". Do they disagree as to whether "valid" applies to the argument

"valid". Do they disagree as to whether "valid" applies to the argument because they disagree about what it is to be valid, or do they disagree about what it is to be valid because they disagree as to whether "valid" applies to the argument?

It think the answer ought to be the same as in the case of "rich". Plausibly, nonclassical logicians oppose classical logic (at least partly), because they are unhappy with inferences that come out valid according to classical logic, inferences which they take to be counterexamples to classical validity. Arguments, that is, which are



not *really* valid. If this is correct, then non-classical logicians oppose the classical notion of validity because they disagree with classical logicians about what it is to be valid. Admittedly, the alternative, non-classical criteria they propose may be designed such that the contested inferences do not come out as valid, but it seems that the basic disagreement is about whether or not a particular argument is valid. Take, for instance, Anderson and Belnap's comment about the paradoxes of material implication:

We agree with the Official view that there are no paradoxes of implication, but for reasons which are quite different from those ordinarily given. To be sure, there is a misunderstanding involved, but it does not consist in the fact that the strict and material "implication" connectives are "odd kinds" of implication, but rather in the claim that material and strict "implication" are "kinds" of implication at all. (Anderson and Belnap 1975, 4)

The passage is primarily concerned with implication connectives and their relation to implication, but it is not too much of a stretch to think that the reason why Anderson and Belnap think that the material conditional is not an *implication* connective is that they think the inferences it licenses are not really implications. They are not really valid. Similarly, Ackermann motivates the introduction of his calculus by pointing out that, "one would reject the validity of the formula $A \rightarrow (B \rightarrow A)$ " (Ackermann 1956, 113). The reason to develop an alternative calculus and with it an alternative account of validity, then, is that a specific argument is taken to be invalid.

Analogously, it is natural to suppose, for instance, that proponents of nonclassical accounts of vagueness reject classical validity *because* of specific borderline cases, that dialetheists reject classical validity *because* of specific paradoxical sentences, and maybe even that intuitionists reject classical validity *because* of specific instances of " $\phi \lor \neg \phi$ " (e.g., where ϕ is the twin prime conjecture).²⁷

If this is correct, then the "criteria view" gives the wrong analysis of the nature of the disagreement between advocates of competing logics. The fundamental disagreement is about the validity of specific arguments, not about the correct criteria for the expression "valid". This means that indexical pluralism gives an uncharitable analysis of the disputes between logicians. More importantly, it also means that the case for an indexical pluralism combined with rivalry about concepts is significantly weakened. It is hard to see how the move to the debate about criteria is even available to indexicalists. They declare, after all, that what seems to be the fundamental disagreement between logicians is merely a verbal dispute. But if the fundamental disagreement is dissolved in such a way that, in the end, all parties involved are right, why disagree about the most proper and adequate conception of validity in the first place? Either the pluralist is right, which means that all admissible conceptions of validity are adequate and proper, or else there is only one

²⁷ I'm not overly confident about the intuitionists. Maybe the reason they oppose classical logic is just anti-realism about mathematics. In case it is related to questions of meaning (as in Dummett 1983), however, the Richard point may apply.



adequate and proper conception of validity, in which case it is hard to make sense of pluralism. The case for rivalry about concepts undermines the case for pluralism.

8 Conclusion

Logical pluralism is a controversial view only if it can make sense of the potential rivalry between the logics it is pluralistic about. In this paper, I presented two ways of implementing logical pluralism within a standard framework for context-sensitive expressions. I showed that indexicalism does not constitute an instance of the interesting plurality thesis. Nonindexicalism does, but I argued that it comes short on the kind of plurality relevant for the Kaplanian framework. The overall features of the semantic analysis of rivalry—relying on the incompatibility of content—suggest that the prospects of capturing the aspect of *rivalry* between logics in terms of semantic disagreement without giving up core features of logical pluralism are rather dim. If my conjectures about the source of the disagreement between partisans of different logics are correct, then turning to competition at the level of metaconcepts is of no help for the pluralist. This does not, by itself, refute logical pluralism. It does suggest, however, that the approach from context-sensitivity, although popular among pluralists, is not a promising way to go.

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