

NOTE ON COISOTROPIC FLOER HOMOLOGY AND LEAFWISE FIXED POINTS

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ABSTRACT. For an adiscal or monotone regular coisotropic submanifold N of a symplectic manifold I define its Floer homology to be the Floer homology of a certain Lagrangian embedding of N . Given a Hamiltonian isotopy $\varphi = (\varphi^t)$ and a suitable almost complex structure, the corresponding Floer chain complex is generated by the (N, φ) -contractible leafwise fixed points. I also outline the construction of a local Floer homology for an arbitrary closed coisotropic submanifold.

Results by Floer and Albers about Lagrangian Floer homology imply lower bounds on the number of leafwise fixed points. This reproduces earlier results of mine.

The first construction also gives rise to a Floer homology for a Boothby-Wang fibration, by applying it to the circle bundle inside the associated complex line bundle. This can be used to show that translated points exist.

1. Introduction. Consider a symplectic manifold (M, ω) , a coisotropic submanifold $N \subseteq M$, and a Hamiltonian diffeomorphism $\psi : M \rightarrow M$. The isotropic (or characteristic) distribution TN^ω on N gives rise to the isotropic foliation on N . A *leafwise fixed point* for ψ is a point $x \in N$ for which $\psi(x)$ lies in the leaf through x of this foliation. We denote by $\text{Fix}(\psi, N)$ the set of such points. A fundamental problem in symplectic geometry is the following:

Problem. *Find conditions under which $\text{Fix}(\psi, N)$ is non-empty and find lower bounds on its cardinality.*

This generalizes the problems of showing that a given Hamiltonian diffeomorphism has a fixed point and that a given Lagrangian submanifold intersects its image under a Hamiltonian diffeomorphism. References for solutions to the general problem are provided in [20, 22].

Example (translated points). As explained in [19, p. 97], translated points of the time-1-map of a contact isotopy starting at the identity are leafwise fixed points of the Hamiltonian lift of this map to the symplectization.

We denote

$$N_\omega := \{\text{isotropic leaves of } N\}.$$

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We call N *regular* (or “fibering”) iff there exists a smooth manifold structure on N_ω , such that the canonical projection from N to N_ω is a smooth submersion.¹ Let X be a manifold and $h \in C([0, 1] \times X, M)$. We call h a *semistrong* (N, ω) -homotopy iff for every $t \in [0, 1]$ there is a leaf F of N that contains the image of $\{t\} \times \partial X$ under h .

We denote by \mathbb{D} the closed unit disk in \mathbb{R}^2 . We call (N, ω) (or simply N) *adiscal* iff every map $u \in C(\mathbb{D}, M)$ that sends $\partial\mathbb{D} = S^1$ to a leaf of N , is semistrongly (N, ω) -homotopic to a constant map.

The main result of [20] (Theorem 1.1) implies the following. We denote by $b_i(N)$ the i -th \mathbb{Z}_2 -Betti-number of N .

Theorem 1.1 (leafwise fixed points for adiscal coisotropic). *Assume that (M, ω) is geometrically bounded, N is closed², regular, and adiscal, and that (N, ψ) is nondegenerate in the sense of [20, p. 105]. Then the following estimate holds:*

$$|\text{Fix}(\psi, N)| \geq \sum_{i=0}^{\dim N} b_i(N). \quad (1)$$

This bound is sharp if there exists a \mathbb{Z}_2 -perfect Morse function on N , see [20, Theorem 1.2]. The idea of the proof of Theorem 1.1 given in [20], is to find a suitable Lagrangian embedding ι_N of N into a geometrically bounded symplectic manifold, see (5) below. We then apply Y. Chekanov’s Main Theorem in [3], which implies the result in the Lagrangian case.³ The Lagrangian intersection points of the image of ι_N correspond to leafwise fixed points of ψ .

Similarly to Theorem 1.1, in [21] for a regular N , we defined monotonicity and the minimal Maslov number $m(N)$, and we proved the following result ([21, Theorem 3]):

Theorem 1.2 (leafwise fixed points for monotone coisotropic). *Assume that (M, ω) is geometrically bounded or convex at infinity⁴, $N \subseteq M$ is closed, monotone, and regular, and (N, ψ) is non-degenerate. Then the following estimate holds:*

$$|\text{Fix}(\psi, N)| \geq \sum_{i=\dim N - m(N) + 2}^{m(N) - 2} b_i(N). \quad (2)$$

The idea of the proof of this theorem given in [21], is to use the same Lagrangian embedding as in the proof of Theorem 1.1. We then apply P. Albers’ Main Theorem in [2], which states Theorem 1.2 in the Lagrangian case.

Finally, the main result of [22] (Theorem 1) implies that leafwise fixed points exist for an *arbitrary* closed coisotropic submanifold if the Hamiltonian flow is suitably C^0 -small. More precisely, it implies the following:

Theorem 1.3 (leafwise fixed points for C^0 -close coisotropic). *Let (M, ω) be a symplectic manifold and $N \subseteq M$ be a closed coisotropic submanifold. Then there exists*

¹Such a structure is unique if it exists. In this case the symplectic quotient of N is well-defined.

²This means compact and without boundary.

³[20, Theorem 1.1] is formulated in a more general setting than Theorem 1.1. Chekanov’s result is needed to deal with that setting, whereas in the setting of Theorem 1.1 Floer’s original article [5] suffices.

⁴[20, Theorem 1.1] is stated for the geometrically bounded case, but the proof goes through in the convex at infinity case.

a C^0 -neighbourhood $\mathcal{U} \subseteq C(N, M)$ of the inclusion $N \rightarrow M$ with the following property. Let $(\varphi^t)_{t \in [0,1]}$ be a Hamiltonian flow on M satisfying $\varphi^t|_N \in \mathcal{U}$, for every $t \in [0, 1]$. If (N, φ^1) is nondegenerate then the following estimate holds:

$$|\text{Fix}(\varphi^1, N)| \geq \sum_{i=0}^{\dim N} b_i(N). \quad (3)$$

This result is optimal in the sense that the C^0 -condition cannot be replaced by Hofer smallness of φ^1 . The idea of the proof of Theorem 1.3 given in [22], is to find a suitable embedding of N as a Lagrangian submanifold \tilde{N} of some symplectic manifold \tilde{M} that is a Weinstein neighbourhood of \tilde{N} , see (7) below. We then use Weinstein's neighbourhood theorem and the existence of Lagrangian intersection points for the zero-section in the cotangent bundle. These points correspond to leafwise fixed points for φ^1 . Since N is not assumed to be regular in Theorem 1.3, in contrast with Theorems 1.1 and 1.2, we can construct \tilde{M} only locally around \tilde{N} .

The point of this note is to reinterpret the proofs of Theorems 1.1 and 1.2 in terms of a version of Floer homology for an adiscial or monotone regular coisotropic submanifold. I also outline a definition of a local version of Floer homology for an arbitrary closed coisotropic submanifold and use it to reinterpret the proof of Theorem 1.3. Details of the construction of this homology will be carried out elsewhere. For the extreme cases $N = M$ and N Lagrangian, local versions of Floer homology were developed in [7, 15, 16, 4, 18, 9]; see also the book [17, Section 17.2].⁵

Potentially a (more) global version of coisotropic Floer homology may be defined under a suitable condition on N that is weaker than regularity, so that the C^0 -condition on (φ^t) in Theorem 1.3 can be relaxed.⁶ This may also yield a lower bound on $|\text{Fix}(\psi, N)|$ that is higher than the sum of the Betti numbers of N , for a suitably generic pair (ψ, N) .

Based on the ideas outlined below, one can define a Floer homology for certain regular contact manifolds and use it to show that a given time-1-map of a contact isotopy has translated points. Namely, consider a closed manifold N and a contact form α on N . Assume that α is regular, i.e., that its symplectic quotient is well-defined. Then N is naturally a smooth principal S^1 -bundle, which is called a Boothby-Wang fibration. The associated real two-dimensional vector bundle E is equipped with a natural symplectic form, see e.g. [8, proof of Lemma 3, p. 200]. The idea is now to define the Floer homology of (N, α) to be the Floer homology of the circle bundle in E of some radius r . If the symplectic quotient of N is monotone then for a suitable choice of r this circle bundle is a monotone coisotropic submanifold with minimal Maslov number at least 2. Hence its Floer homology is well-defined. Since part of the symplectization of (N, α) symplectically embeds into E , it follows that translated points exist under suitable hypotheses.

Various versions of coisotropic Floer homology may play a role in mirror symmetry, as physicists have realized that the Fukaya category should be enlarged by coisotropic submanifolds, in order to make homological mirror symmetry work, see e.g. [11].

⁵In [1] a Lagrangian Floer homology was constructed that is "local" in a different sense.

⁶This can only work under suitable conditions on N . The reason is that by [10, Theorem 1.1], there exists a closed hypersurface N in \mathbb{R}^{2n} and a Hamiltonian diffeomorphism on \mathbb{R}^{2n} that is arbitrarily Hofer-close to id and has no leafwise fixed points w.r.t. N .

2. Floer homology for an adiscal or monotone regular coisotropic submanifold. To explain the coisotropic Floer homology in the regular case, consider a geometrically bounded symplectic manifold (M, ω) , and a closed, regular coisotropic submanifold $N \subseteq M$.

Suppose first also that N is adiscal. We define the *Floer homology of N* as follows. Let $\varphi = (\varphi^t)_{t \in [0,1]}$ be a Hamiltonian isotopy starting at id, such that (N, φ^1) is nondegenerate. We call a point $x_0 \in M$ a (N, φ) -contractible leafwise fixed point iff the path $x : [0, 1] \rightarrow M, x(t) := \varphi^t(x_0)$, is semistrongly (N, ω) -homotopic to a constant path.⁷ We define

$$\begin{aligned} \text{Fix}_c(N, \varphi) &:= \{(N, \varphi)\text{-contractible leafwise fixed points}\}, \\ \text{CF}(N, \varphi) &:= \bigoplus_{\text{Fix}_c(N, \varphi)} \mathbb{Z}_2. \end{aligned} \tag{4}$$

Remark. By definition this direct sum contains one copy of \mathbb{Z}_2 for each point in $\text{Fix}_c(N, \varphi)$.

We now define a collection of boundary operators on $\text{CF}(N, \varphi)$, one for each (N, φ^1) -regular time-dependent almost complex structure. To explain this, observe that, since N is regular, N_ω carries canonical smooth and symplectic structures $\mathcal{A}_{N,\omega}$ and ω_N . We define

$$\begin{aligned} \widehat{M} &:= M \times N_\omega, \quad \widehat{\omega} := \omega \oplus (-\omega_N), \\ \iota_N : N &\rightarrow \widehat{M}, \iota_N(x) := (x, \text{isotropic leaf through } x), \widehat{N} := \iota_N(N), \\ \widehat{\varphi}^t &:= \varphi^t \times \text{id}_{N_\omega}. \end{aligned} \tag{5}$$

The map ι_N is a $\widehat{\omega}$ -Lagrangian embedding of N into \widehat{M} , see [20, Lemma 3.2]. It follows from [21, Proposition 61, p. 43] that the restriction of the map ι_N , given by

$$\begin{aligned} \iota_N : \text{Fix}_c(N, \varphi) &\rightarrow \text{Fix}_c(\widehat{N}, \widehat{\varphi}) = \\ \left\{ \widehat{x} \in \widehat{N} \cap (\widehat{\varphi}^1)^{-1}(\widehat{N}) \mid t \mapsto \widehat{\varphi}^t(\widehat{x}) \text{ contractible with endpoints in } \widehat{N} \right\} \end{aligned} \tag{6}$$

is well-defined and injective. A straightforward argument shows that it is surjective.

Let $p \in (2, \infty)$ and ψ be a Hamiltonian diffeomorphism on M . We call a t -dependent $\widehat{\omega}$ -compatible smooth almost complex structure \widehat{J} on \widehat{M} (N, ψ) -regular, iff the vertical differential of its Cauchy-Riemann operator is surjective for all finite-energy \widehat{J} -holomorphic strips with compact image and boundary on \widehat{N} and $\widehat{\psi}(\widehat{N})$. (For definitions see [5, Proposition 2.1].⁸) We define $\mathcal{J}_{\text{reg}}(N, \psi) := \mathcal{J}_{\text{reg}}(M, \omega, N, \psi, p)$ to be the set of all (N, ψ) -regular \widehat{J} .⁹ For every $\widehat{J} \in \mathcal{J}_{\text{reg}}(N, \varphi^1)$ we define the *Floer boundary operator*

$$\partial_{N,\varphi,\widehat{J}} : \text{CF}(N, \varphi) \rightarrow \text{CF}(N, \varphi)$$

to be the (Lagrangian) Floer boundary operator of $(\widehat{M}, \widehat{\omega}, \widehat{N}, \widehat{\varphi}, \widehat{J})$, where $\text{CF}(N, \varphi)$ is as in (4). (See [5, Definition 3.1].)

To see that this operator is well-defined, recall that it is defined on the direct sum of \mathbb{Z}_2 's indexed by the set occurring on the right hand side of (6).¹⁰ Using the

⁷By definition, for every such point x_0 , the point $x(1) = \varphi^1(x_0)$ lies in the isotropic leaf of x_0 . Hence x_0 is a leafwise fixed point of φ^1 .

⁸The exponent p enters the Banach bundle setup of that proposition.

⁹It follows from the proof of [5, Proposition 2.1] that this set is dense in the set of all t -dependent $\widehat{\omega}$ -compatible almost complex structures, and therefore nonempty. See [6, Theorem 5].

¹⁰Sometimes this is called the “ \mathbb{Z}_2 -vector space spanned by this index set”. Here we identify the intersection $\widehat{N} \cap (\widehat{\varphi}^1)^{-1}(\widehat{N})$ with $\widehat{\varphi}^1(\widehat{N}) \cap \widehat{N}$ via the map $\widehat{\varphi}^1$.

bijection (6), we identify this direct sum with $\text{CF}(N, \varphi)$. Hence $\partial_{N, \varphi, \widehat{\mathcal{J}}}$ is defined between the right spaces.

We check the conditions of [5, Definition 3.1]. Since N is closed, the same holds for N_ω . Since (M, ω) is geometrically bounded, it follows that $(\widehat{M}, \widehat{\omega})$ is geometrically bounded. The Lagrangian \widehat{N} is diffeomorphic to N via ι_N , and therefore also closed. Since N is adiscial, it follows from [21, Proposition 61, p. 43] that the same holds for \widehat{N} . Since (N, φ^1) is nondegenerate, by [20, Lemma 3.2(c)] the intersection of \widehat{N} and $\widehat{\varphi}^{-1}(\widehat{N})$ is transverse. Hence the conditions of [5, Definition 3.1] are satisfied, and therefore the boundary operator $\partial_{N, \varphi, \widehat{\mathcal{J}}}$ is well-defined.¹¹ By [5, Lemma 3.2] it squares to 0. Hence we may define the *Floer homology of $(N, \varphi, \widehat{\mathcal{J}})$* to be the homology

$$\text{HF}(N, \varphi, \widehat{\mathcal{J}}) := H(\text{CF}(N, \varphi), \partial_{N, \varphi, \widehat{\mathcal{J}}}).$$

Let G be a \mathbb{Z} -grading of the \mathbb{Z}_2 -vector space $\text{CF}(N, \varphi)$, which via the identification (6) is compatible with the Viterbo-Maslov index for strips satisfying Lagrangian boundary conditions.¹² Such a G induces a grading on $\text{HF}(N, \varphi, \widehat{\mathcal{J}})$. For every pair $\widehat{\mathcal{J}}_0, \widehat{\mathcal{J}}_1 \in \mathcal{J}_{\text{reg}}(N, \varphi^1)$ we denote by

$$\Phi_{\widehat{\mathcal{J}}_0, \widehat{\mathcal{J}}_1} : \text{HF}(N, \varphi, \widehat{\mathcal{J}}_0) \rightarrow \text{HF}(N, \varphi, \widehat{\mathcal{J}}_1)$$

the canonical isomorphism provided by the proof of [5, Proposition 3.1, p. 522]. This isomorphism respects the grading G . It does not depend on the choice of G .

Definition 2.1 (Floer homology for adiscal coisotropic). We define the *Floer homology of (N, φ)* to be

$$\text{HF}(N, \varphi) := \left(\left(\text{HF}(N, \varphi, \widehat{\mathcal{J}}) \right)_{\widehat{\mathcal{J}} \in \mathcal{J}_{\text{reg}}(N, \varphi^1)}, \left(\Phi_{\widehat{\mathcal{J}}_0, \widehat{\mathcal{J}}_1} \right)_{\widehat{\mathcal{J}}_0, \widehat{\mathcal{J}}_1 \in \mathcal{J}_{\text{reg}}(N, \varphi^1)} \right).$$

Remarks. • This is a collection of graded \mathbb{Z}_2 -vector spaces together with grading-preserving isomorphisms.

- Philosophically, the Floer homology of (N, φ) is defined to be $\text{HF}(N, \varphi, \widehat{\mathcal{J}})$, for some choice of $\widehat{\mathcal{J}}$. The collection of isomorphisms $\left(\Phi_{\widehat{\mathcal{J}}_0, \widehat{\mathcal{J}}_1} \right)$ encodes the sense in which this does “not depend” on this choice.

By the proof of [5, Theorem 1] $\text{HF}(N, \varphi)$ is isomorphic to the singular homology of \widehat{N} (hence of N) with \mathbb{Z}_2 -coefficients. Since $\text{HF}(N, \varphi)$ is generated by the (N, φ) -contractible leafwise fixed points of φ^1 , this reproves Theorem 1.1.

Suppose now that N is monotone in the sense of [21] and of minimal Maslov number $m(N) \geq 2$.¹³

Definition 2.2 (Floer homology for monotone coisotropic). We define the *Floer homology of (N, φ)* to be the Floer homology of $(\widehat{N}, \widehat{\varphi})$, as defined in [13, 14].

¹¹In [5] Floer assumes that the symplectic manifold is closed. However, the same construction of Floer homology works for geometrically bounded symplectic manifolds. Here we use that we only consider Floer strips with compact image.

¹²By [5, Proposition 2.4] such a grading exists and each two gradings differ by an additive constant.

¹³We continue to assume that (M, ω) is geometrically bounded, N is closed and regular, and that (N, φ^1) is nondegenerate.

Since N is monotone, the same holds for \widehat{N} . The minimal Maslov numbers of N and \widehat{N} agree, see the proof of [21, Theorem 3, p. 17]. It follows that the Floer homology of (N, φ) is well-defined. By [2, Corollary 2.1] it is isomorphic to singular homology in degrees $i = \dim N - m(N) + 2, \dots, m(N) - 2$. It follows that (2) holds. This reproves Theorem 1.2.

3. Local Floer homology for an arbitrary coisotropic submanifold. Consider now the situation in which (M, ω) is any symplectic manifold, N an arbitrary closed coisotropic submanifold of M , and $\varphi = (\varphi^t)$ a Hamiltonian flow on M whose restriction to N stays “ C^0 -close” to the inclusion $N \rightarrow M$, such that (N, φ^1) is nondegenerate. We also fix an ω -compatible almost complex structure J on M . Heuristically, we define the local Floer homology $\text{HF}(N, \varphi, J)$ as follows. Its chain complex is generated by the points $x \in \text{Fix}(N, \varphi^1)$, for which there is a “short” path from x to $\varphi^1(x)$ within the isotropic leaf through x .

To explain the boundary operator $\partial = \partial_{N, \varphi, J}$, we denote by $i_N : N \rightarrow M$ the inclusion map. We equip the product $M \times N$ with the presymplectic form $\omega \oplus (-i_N^* \omega)$. By [22, Lemma 4] there exists a symplectic submanifold \widetilde{M} of $M \times N$ that contains the diagonal

$$\widetilde{N} := \{(x, x) \mid x \in N\} \quad (7)$$

as a Lagrangian submanifold. We shrink \widetilde{M} , so that it is a Weinstein neighbourhood of \widetilde{N} . The flow φ induces a Hamiltonian flow $\widetilde{\varphi}$ that is defined on an open neighbourhood \widetilde{N} of \widetilde{M} . The structure J induces an almost complex structure \widetilde{J} on \widetilde{M} that is $\widetilde{\omega}$ -compatible.

The boundary operator ∂ is now defined to be the boundary operator of the “local Lagrangian Floer homology” of $(\widetilde{M}, \widetilde{\omega}, \widetilde{N}, \widetilde{\varphi}, \widetilde{J})$. This map counts finite energy \widetilde{J} -holomorphic strips in \widetilde{M} that stay “close” to \widetilde{N} , have Viterbo-Maslov index 1, map the lower and upper boundaries of the strip to \widetilde{N} and $(\widetilde{\varphi}^1)^{-1}(\widetilde{N})$, and connect two intersection points of \widetilde{N} and $(\widetilde{\varphi}^1)^{-1}(\widetilde{N})$. Such points correspond to points $x \in \text{Fix}(N, \varphi^1)$, for which there exists a short path from x to $\varphi^1(x)$ within a leaf. (See [22, Lemma 6].)

To understand why heuristically, the boundary operator ∂ is well-defined and squares to zero, observe that \widetilde{N} intersects $(\widetilde{\varphi}^1)^{-1}(\widetilde{N})$ transversely, since (N, φ^1) is nondegenerate. (See [22, Lemma 7].) Therefore, for generic \widetilde{J}^{14} , the moduli space of \widetilde{J} -strips is a 0-dimensional manifold in a natural sense. (Here we divided by the translation action.) It is compact for the following reasons:

- Holomorphic strips with boundary on \widetilde{N} and $(\widetilde{\varphi}^1)^{-1}(\widetilde{N})$ stay inside some fixed compact neighbourhood of \widetilde{N} , provided that $\widetilde{\varphi}^1$ is close enough to the identity. This follows from the fact that there is a neighbourhood \widetilde{U} of \widetilde{N} and an exhausting \widetilde{J} -plurisubharmonic function on \widetilde{U} .
- Disks or spheres cannot bubble off. This follows from our assumption that \widetilde{M} is a Weinstein neighbourhood of \widetilde{N} , which implies that \widetilde{N} is an exact Lagrangian in \widetilde{M} .
- Index-1-strips generically do not break.

It follows that heuristically, ∂ is well-defined. For similar reasons we have $\partial^2 = 0$.

¹⁴Here one needs to work with a family of almost complex structures depending on the time t .

Given two choices of symplectic submanifolds $\widetilde{M}, \widetilde{M}' \subseteq M \times N$ containing \widetilde{N} , one obtains a symplectomorphism between open neighbourhoods of \widetilde{N} in \widetilde{M} and \widetilde{M}' , by sliding \widetilde{M} to \widetilde{M}' along the isotropic leaves of N . This symplectomorphism intertwines the corresponding $\widetilde{\varphi}$'s and \widetilde{J} 's. It follows that the boundary operator does not depend on the choice of \widetilde{M} , and therefore, heuristically, is well-defined.

To make the outlined Floer homology rigorous, the words “close” and “short” used above, need to be made precise. To obtain an object that does not depend on the choice of “closeness”, the *local Floer homology of (N, J)* should really be defined to be the *germ* of the map

$$\varphi \mapsto \text{HF}(N, \varphi, J)$$

around $\text{id} : M \rightarrow M$.

By showing that $\text{HF}(N, \varphi, J)$ is isomorphic to the singular homology of \widetilde{N} , it should be possible to reproduce the lower bound (3) of Theorem 1.3.

Remark (local presymplectic Floer homology). A presymplectic form on a manifold is a closed two-form with constant rank. By [12, Proposition 3.2] every presymplectic manifold can be coisotropically embedded into some symplectic manifold. By [12, 4.5. Théorème on p. 79] each two coisotropic embeddings are equivalent. Hence heuristically, we may define the local Floer homology of a presymplectic manifold to be the local Floer homology of any of its coisotropic embeddings.

Remark (relation between the constructions). Assume that N is regular. Then the constructions of its “global” and local Floer homologies are related as follows. Namely, the symplectic submanifold $\widetilde{M} \subseteq M \times N$ can be viewed as a local version of \widehat{M} . More precisely, shrinking \widetilde{M} if necessary (so that it still contains \widetilde{N}), the manifold \widetilde{M} symplectically embeds into \widehat{M} via the map

$$(x, y) \mapsto (x, \text{isotropic leaf through } y).$$

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