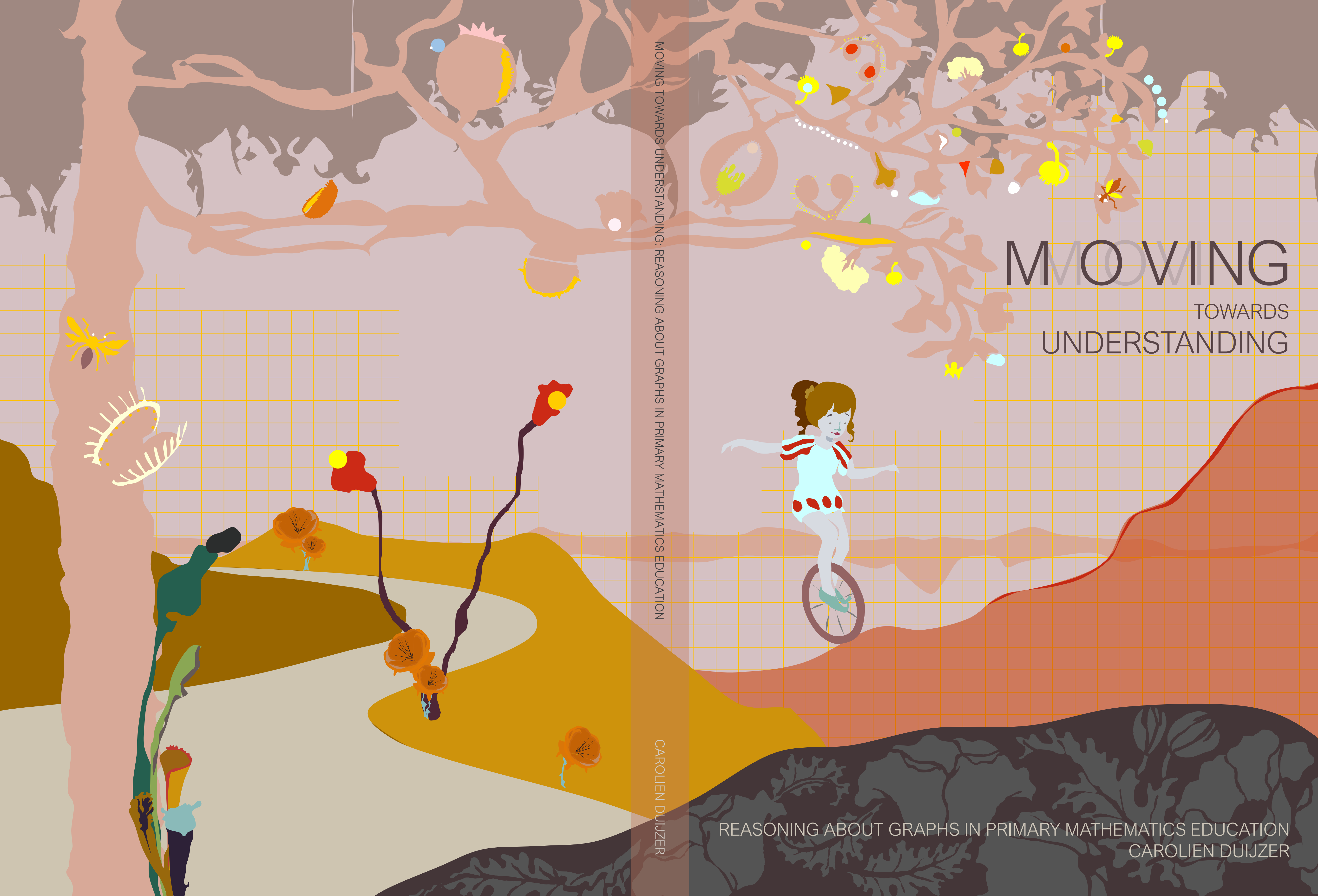
MOVING TOWARDS UNDERSTANDING: REASONING ABOUT GRAPHS IN PRIMARY MATHEMATICS EDUCATION
CAROLIEN DUIJZER
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Moving towards understanding: Reasoning about graphs in primary mathematics education

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Table of contents

CHAPTER 1

Introduction

Introduction

Figure 1 shows a drawing from the picture book *De Verrassing* (Van Ommen, 2003). The picture book tells the story of a woolly sheep who is measuring the thickness of her fur over time in order to know when she has enough wool to knit a sweater, as a present for her friend. This drawing shows the relationship between time and thickness of fur as a line in the graph. There is also a second line in the graph, presumably representing the sheep's weight over time. The intriguing story of the sheep makes it easy for children to conjecture about the graph's meaning and even recognize the relationship between the two measures and time (Van den Heuvel-Panhuizen et al., 2009). This picture book drawing is an example of how children already from a very young age are informally introduced to graphical representations of dynamic data.

Figure 1. Drawing from "The Surprise" [Dutch: De Verrassing], Van Ommen (2003, p. 3)

The ability to use graphs to produce, present, and understand complex dynamic information (e.g., quantities changing over time) by making flexible use of already given representations, is becoming increasingly important in current society. On the internet, on television, and in the newspapers graphical representations are frequently used to present data to transmit information to the viewer or reader in a presumably clear and concise manner. Yet, high levels of graphical understanding are sometimes necessary in order to interpret these graphs correctly. For example, take a look at the graph presented in Figure 2. Here we see a line graph published in an online article of the Dutch *Centraal Bureau voor de Statistiek* (Central Bureau of Statistics) (cbs.nl, 2013). The graph represents the employed versus the unemployed labor force. When taking a superficial look at this graph it almost seems as if the employed labor force is as large as the unemployed labor force. This would be a rather strange conclusion. When looking more closely at the specific values given on the y-axes of the graph (both on the left and on the right) it is shown that the left y-axis starts at the value of 0, while the right y-axis starts at the value of 6800. This difference in starting values makes the interpretation of this graph a rather complex endeavor. Although both axes have scales with steps of 100, they differ in the range they cover.

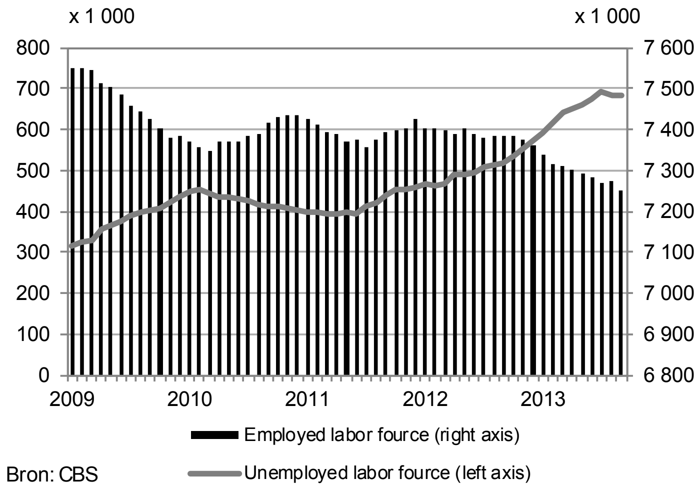

Figure 2. Graph representing the employed labor force (right axis) versus the unemployed labor force (left axis) from 15 till 65 years, given per month, cbs.nl (2013)

This example shows that high levels of graph interpretation skills, as well as the ability to critically evaluate graphs, are important when reading and interpreting complex everyday information that is presented to us. In order to interpret and recognize the deeper underlying meaning of these representations one has to develop an understanding of the formal aspects of graphical representations (e.g., the meaning of the axes, variables, the slope, and rate of change) as well as the reasoning associated with graphs (e.g., gaining a deeper understanding about the relationship between variables, drawing inferences, reasoning logically, evaluating evidence, and solving problems related to graphs) (Ainsworth, 2006; Friel et al., 2001; Shah & Hoeffner, 2002). Sophisticated skills like the ones described here, are currently referred to as 21st century or higher-order thinking (HOT) skills. The importance of problem solving and HOT is increasingly recognized both internationally (e.g., NCTM, 2000; OECD, 2019) and nationally (e.g., Thijs et al., 2014; Van den Heuvel-Panhuizen & Bodin-Baarends, 2004). Also, there is increasing awareness that the foundation of HOT in mathematics has to be laid at young age (Common Core State Standards State Initiative, CCSSI, 2010; Goldenberg et al., 2003; NCTM, 2000). However, as Kolovou et al. (2009) have revealed, the primary mathematics curriculum in the Netherlands provides very few opportunities to practice HOT, a situation which nowadays still persists (Gravemeijer et al., 2017; Van Zanten & Van den Heuvel-Panhuizen, 2018). Therefore, there is growing consensus to revise the mathematics curriculum and pay more attention to HOT (*Dutch Association for the Development of Mathematics Education*, NVORWO, 2017; Ontwikkelteam Rekenen-Wiskunde, 2019). Specifically, to meet the needs of primary school aged students, HOT should be embedded in innovative instructional settings. One promising approach would be to develop activities that include the active role of the body to build up conceptual metaphors rooted in embodied cognitions, in order to reach higher levels of mathematical understanding as embodied cognition (e.g., Gallese & Lakoff, 2005; Hall & Nemirovsky, 2012).

Against the background of providing primary school students more opportunities to develop HOT within mathematics, the *Beyond Flatland* project was initiated with a grant (405-14-303) from the Netherlands Initiative for Education Research (NRO). In this project possibilities for enriching a “flat” arithmetic-focused mathematics curriculum were explored by incorporating higher-order mathematical activities in the primary school classroom. The *Beyond Flatland* project consisted of three part-projects. This thesis is the result of one of these three part-projects and addresses the graphing of motion. The other two part-projects considered the mathematics domains of early algebra and probability. The PhD study described in this thesis focused on stimulating fifth-grade students’ reasoning with motion graphs, as an approach to incorporate HOT within mathematics activities in primary school. The graphing of motion – including both graph interpretation and graph construction activities – is rarely addressed in primary school mathematics textbooks. As a consequence, not many teachers capitalize on the opportunities this mathematical domain offers for developing students’ HOT, although there is ample evidence that students at this age can deal with representations in which motion data are visualized (e.g., diSessa et al., 1991; Van Galen et al., 2012). Students’ reasoning about motion graphs could benefit from the incorporation of bodily experiences during graph-related activities (e.g., Deniz & Dulger, 2012; Mokros & Tinker, 1987; Nemirovsky et al., 1998; Robutti, 2006). The idea that bodily experiences – including touching, gesturing, perception, and moving one's whole body – are relevant to the field of mathematics, can be positioned within contemporary work on embodied cognition (e.g., Gallagher & Lindgren, 2015; Hall & Nemirovsky, 2012; Lakoff & Núñez, 2000; Radford et al., 2017; Tran et al., 2017). In order to extend these existing lines of research, this PhD thesis aims at gaining more insight into the foundational role of bodily experiences, as concrete activities, for cognition and mathematical activity, by taking into account opportunities bodily experiences offer to support the learning of mathematical concepts, and more specifically, reasoning about motion graphs.

1. Theoretical background

1.1 HOT: Why is it important?

To be able to participate in a society characterized by vast technological innovations, skills such as collaborating, problem solving, generating and evaluating evidence, ICT literacy, critical thinking, and creativity, among others, are considered to be of increasing importance (Scott, 2015; Voogt & Pareja-Roblin, 2010). This array of skills, which are not necessarily new, but historically have not had a systematic place in education, are nowadays popularly referred to as 21st century skills (see, for example, the categorization of 21st century skills on http://www.atc21s.org). HOT is often mentioned in relation to these 21st century skills, and can be defined as: "the mental engagement with ideas, objects, and situations in an analogical, elaborative, inductive, deductive, and otherwise transformational manner that is indicative of an orientation toward knowing as a complex, effortful, generative, evidence-seeking, and reflective enterprise" (Alexander et al., 2011, p. 54). The ability to apply HOT is considered to be relevant in preparing students for a future that is currently unknown (e.g., Forster, 2014; OECD, 2016). For education, in addition to declarative (i.e., knowing "that"), and procedural knowledge (i.e., knowing "how") the question has become how to support students in developing this HOT (i.e., knowing "why") (Van Streun, 2001).

One often cited categorization of cognitive skills within an educational context is the *Taxonomy of Educational Objectives* (Anderson & Krathwohl, 2001; Bloom, 1956), namely knowledge, comprehension, application, analysis, synthesis, and evaluation. Within educational science the top three levels – analysis, synthesis, and evaluation – are often used to operationalize HOT. The underlying assumption of this classification, as well as other definitions or operationalizations of HOT presented in the educational literature (see also Resnick, 1987), is the conceptualization of HOT as domain-general. This means that there are general aspects of HOT that are shared across academic domains and that can be stimulated regardless of the academic content taught (Alexander et al., 2011). For example, many studies have focused on developing domain-general HOT such as critical thinking skills or problem solving, without specifically addressing the particularities of what one has to critically think about, or what type of problem has to be solved. According to Alexander et al. (2011) such domain-general conceptualization of HOT is not tenable. In their view, HOT "exhibit[s] distinctive qualities arising from *the nature of the domain within which the task or activity is situated*" (emphasis added, p. 51). This domain-specific view on HOT implies that thinking becomes higher order due to increasing experience within particular academic domains such as history, language, and mathematics, and as a consequence should be stimulated within these respective domains (Ericsson, 2003). Students' ability to apply HOT, including critical thinking and problem solving, is considered to be an important goal of Dutch mathematics education (NVORWO, 2017; Ontwikkelteam Rekenen-Wiskunde, 2019).

In order to study domain-specific mathematical HOT in sufficient depth, and to provide ideas about how HOT can be supported within mathematics education, the research in this thesis has taken a particular focal point: graphing and graphing motion with Grade 5 students. The domain of motion graphs offers many opportunities for HOT, such as reasoning about (graphically represented) change and relationships, reasoning about the quantities represented in the graph (e.g., distance, time), or combinations thereof (i.e., speed) as well as reasoning about the simultaneous coordination of the values (magnitudes) of the quantities in the graph (i.e., covariational reasoning, Saldanha & Thompson, 1998). The domain of motion graphs also offers ample opportunities to translate between a motion situation and its graphical representation, as well as constructing graphical representations of a motion situation. The operationalization of mathematical HOT within the domain of graphing motion offers a concrete translation of the more generally formulated educational objectives put forward for developing 21st century skills.

1.2 HOT in the context of graphing motion: Developing graph sense

In this thesis, the term *graph sense* (Friel et al., 2001; Robutti, 2006) is adopted to frame the HOT associated with graphs in primary mathematics education. This graph sense is related to *number sense* (e.g., Resnick, 1989) and *symbol sense* (Arcavi, 1994), and "develops gradually as a result of one's creating graphs and using already designed graphs in a variety of problem contexts that require making sense of data"

(Friel et al., 2001, p. 145). For example, for young children, who have little experience with graphing and the reasoning associated with graphs, answering graph-related questions implies dealing with a problem situation for which they do not yet have an appropriate, automated strategy to solve or explain them. Yet, this does not mean that they are unable to reason about such representations. When looking back at the example of the sheep in Figure 1, Van den Heuvel-Panhuizen et al. (2009) showed how this drawing stimulated five- and six-year old children to conjecture about its meaning. They saw that the sheep was measuring her weight and the thickness of her fur and when seeing this drawing they naturally made the connection between the information the sheep was gathering and the line in the graph, moving upwards. The children in their study started to reason about the meterstick the sheep is holding and the graphical representation on the wall. They even noticed that the graph has something to do with the days of the week. Presented in isolation, the graph would have been meaningless to a five- or six-year old child. Yet, the question implicitly posed in the story – *will the sheep have enough wool to knit a sweater?* – made the representation meaningful to them, enabling the children to draw some inferences from the given situation in relation to its representation in the graph. Further, in their reasoning about the graph, the children build upon their informal and intuitive understandings of certain phenomena through a few very natural everyday cognitive mechanisms. For example, in realizing that the line in the graph moving upwards was related to an increase in the sheep's fur over time, the children made use of a spatial embodied conceptual metaphor (e.g., "growth is up") (e.g., Wilson & Golonka, 2013). Conceptual metaphors like these arise naturally from correlations with experiences in everyday reality (Lakoff, 2014; Lakoff & Núñez, 2000). This example thus shows that mathematical ideas are grounded in everyday bodily experiences and intuitions, whereby the inferential structures of these experiences are mapped – through conceptual metaphors – onto abstract concepts (Núñez et al., 1999).

Whereas graph interpretation implies the reading of a graphical representation and extracting meaning from it, graph construction refers to the building of something new. Constructing a graph can, for example, be done by plotting points that are provided in a table or as a function (quantitative graph construction; Leinhardt et al., 1990) or by sketching the shape of the graph as a response to a description of a motion situation, without explicitly focusing on numerals (qualitative graph construction; Krabbendam, 1982). Graph construction is sometimes seen as more difficult than graph interpretation because interpretation "only" involves reading a representation which is already there (Leinhardt et al., 1990), whereby one can naturally build on intuitions and everyday experiences, in order to make sense of the graph. In education, graph interpretation activities are more common than graph construction activities even though there is compelling evidence that young students are very well able to construct graphical representations of motion. An example is given in the study by diSessa et al. (1991) that showed young students' ability to create graphs to describe the motion of a car that slows down, stops, and then drives away slowly (see also Sherin, 2001). When asked to come up with ways of representing this real-world motion situation using paper-and-pencil (students previously had modelled another motion situation by using a simulation program) students' drawings showed a multitude of graphical solutions. Although research reports students' difficulty with representing motion continuously (see also McDermott et al., 1987), these students showed a transition from discrete representations of the motion situation to continuous representations of that same motion situation. The setting in which graphing activities take place contributes to how well students construct, and as a consequence, interpret graphs of others. A learning environment that invites students to participate actively in developing and maintaining the practice of graphing (also as collective and shared social practice), is more likely to commit students to construct representations that are convincing and meaningful to them (Roth & McGinn, 1997).

In this PhD thesis, both graph interpretation and graph construction activities are addressed. Offering young students opportunities to interpret and construct graphs might engender high levels of reasoning. A focus on reaching higher levels of reasoning as part of generating mathematical understanding is also the objective of *Realistic Mathematics Education* (RME; e.g., Freudenthal, 1973, 1991; Treffers, 1978, 1987). RME is a domain-specific instruction theory, which has informed the development of a learning trajectory presented and evaluated in this PhD thesis. RME can be characterized by six core principles, being the *activity principle* (having students actively involved in the learning process), the *reality principle* (starting with known meaningful situations), the *level principle* (using models to bring students to a higher level of understanding), the *intertwinement principle* (integrating mathematical subdomains), the intera*ctivity principle* (creating opportunities for classroom discussions), and the *guidance principle* (having teachers in a proactive role in creating a powerful learning environment) (Van den Heuvel-Panhuizen, 2001). Although RME is one of the dominant strategies within Dutch mathematics education, as evidenced in the (more or less) RME-oriented mathematics textbooks used in most Dutch primary school mathematics classrooms, the principles of this theory are often underexposed throughout most instructional activities. In the following paragraph I will shortly introduce the value of the *reality principle* and the *level principle* as instruction strategies for this particular research project.

1.3 Learning environments supporting students' understanding of motion graphs

According to Freudenthal (1991) mathematics is first and foremost a human activity, whereas our surrounding reality can be mathematically organized, a process he called mathematization. Thus, fundamental to the process of doing mathematics is the idea that mathematics ideally emerges from real-world situations before moving on to the formal world of mathematics. Real-world here refers to situations that are experientially real to a student, and more specifically, situations that are meaningful. Asking students to solve a real-life problem that is situated in a rich meaningful context can help students attach meaning to the mathematical constructs they develop to solve the problem. Later on the context-specific model of the problem situation that can be formed in the beginning of the learning process can be generalized and can become a model that can be used to solve other problem situations and reach a higher level of mathematical reasoning (Van den Heuvel-Panhuizen, 2003). This means that models serve a so-called bridging function between these informal situation-specific solutions and formal mathematics; they shift from a *model of* to a *model for* (Streefland, 1985). A similar approach can be found in the aforementioned study of diSessa et al. (1991), in which drawings, invented by the students themselves, served as a direct model of a students' informal mathematical activity. Subsequently, these drawings were taken as a starting point on which formal approaches towards graphing were built. In some cases, models are so powerful that they can also be used for all kinds of other situations.

Over the past couple of decades technology-rich environments such as simulations (e.g., Noble et al., 2001; Roschelle et al., 2000), video modelling (e.g., Boyd & Rubin, 1996), and motion sensors (e.g., Nemirovsky et al., 1998; Robutti, 2006) have been frequently used to allow students to interactively explore and experiment with graphically represented motion. These technologies offer students opportunities to connect physical phenomena to a wide variety of representations (Hegedus et al., 2017). For example, Nemirovsky et al. (1998) described the work of two students who walked in front of a motion sensor, graphing their own movements. The reasoning of these students transformed throughout the activities: from language they used to define their own movements, to language they used to describe their own motion represented as a line in the graphical representation. This study showed how the used motion sensor became a defining element, serving as a bridge between the real-world situation and the formal representation of that situation as a mathematical graph. Research has shown that the use of motion sensor technology is relatively successful (e.g., Urban-Woldron, 2015). It comprises various elements that are helpful when coming to understand graphs: providing a real-time immediate link between situation and graph, providing students with real-time graphical representations of their own', others' or objects' motion, either in reality or via the screen of a computer, providing students the opportunity to adjust a graph simply by adjusting the motion represented in the graph (Glazer, 2011), and allowing students to start using graphs "both as objects to be talked about and as structural resources in communication" (Roth & McGinn, 1997, p. 101).

A learning environment using motion sensor technology, in which students for example are allowed to graph their own movements, capitalizes on students' perceptual-motor experiences to learn graphing conventions (e.g., Arzarello et al., 2007). This offers opportunities to connect "the mathematics of change to its historical and familiar roots in experienced motion" (Kaput & Roschelle, 2013, p. 20). For example, knowing or being told that the graph of a time-distance relationship does not go "backwards" is quite different from actually experiencing with your body that the line in the graph represents the unidirectional quantity time. This linking between a concrete physical experience and the abstraction of that experience as a mathematical graph closely aligns with theories of embodied cognition. Already from an early age we bodily experience motion as continuous change. The fundamental experience of moving through space can serve as a grounding metaphor by which the line in the graph becomes meaningful, while also providing a starting point to reason about the graph as a mathematical object (Lakoff & Núñez, 2000). The importance of embodied action and interaction for mathematical thinking and learning, as evidenced in theoretical perspectives of embodied cognition (Abrahamson & Bakker, 2016; Hall & Nemirovsky, 2012), throws yet another light on the relative success of using motion sensor technology in the classroom. There is convincing behavioral (e.g., Kelton & Ma, 2018; Ruiter et al., 2015) and neuroscientific (e.g., Gallese & Lakoff, 2005; Lakoff, 2014; Pulvermüller, 2013) evidence that bodily experiences are indeed helpful for the teaching and learning of mathematics (e.g., Abrahamson & Bakker, 2016) as well as reaching higher levels of mathematical reasoning.

1.4 Opportunities to support reasoning about motion graphs: an embodied cognition perspective

The embodiment hypothesis suggests that physical experiences are relevant not only for developing early motor skills (e.g., Piaget, 1964), but also for higher-level cognitive functioning (e.g., Koziol et al., 2012; Radford et al., 2005). An example can be found in the bodily-based experience of balance. When we were young, we probably have had countless opportunities to play on a seesaw. This fundamental experience of being in balance might serve as a grounding metaphor to aid our understanding of the equal sign (e.g., $3 + 1 = 4$) (e.g., Núñez et al., 1999). There exist different views on the exact role of the body in explaining cognitive processes (e.g., Chemero, 2011; Clark, 1999, Goldman, 2012). At the one end of the embodied continuum the role of the body is perceived as providing input for the brain, which helps generate abstract cognition. For example, providing students with activities in which they work with the bodily-based experience of being in balance, can add depth to one's understanding of the abstract concept of the equal sign. The mental processes that are activated are supposed to be similar to the cognitive processes taking place when doing mental simulation and disembodied abstract reasoning (Margolis & Laurence, 2007). At the opposite end of the embodied continuum, the role of the body is perceived as more radical, where the assumption of mental cognitive processes is regarded as unnecessary because cognition resides in the interaction of the body in and with the physical world, thus fundamentally altering the nature of cognition (Wilson & Golonka, 2013). For example, when trying to catch a fly ball, there is an ongoing real-time interaction between body and environment in order to successfully catch it (e.g., Kiverstein, 2012; Wilson & Golonka, 2013). Although differing in the ways of how cognition and abstract mental processes are defined, all these embodied views have the relevance of the body as interactional entity in common; either with oneself, others, the environment, or combinations thereof (Wilson, 2002).

2. Research directions of this thesis

The research presented in this thesis mainly focuses on the evaluation of a learning environment consisting of a six-lesson teaching sequence incorporating embodied activities related to graphing – both graph interpretation and graph construction – using motion-detecting graphing technology (e.g., Brasell, 1987; Mokros & Tinker, 1987). In this respect, we bring together – and build upon – previous work done within the field of mathematics and science education (e.g., Anderson & Wall, 2016; Brasell, 1987; Deniz & Dulger, 2012) as well as previous work done within the field of embodied cognition (e.g., Nemirovsky et al., 1998; Robutti, 2006) in order to investigate how both research strands combined might be a fruitful way to develop students' reasoning about graphs of motion. The new findings that emerge from this analysis will give us insight into students' development of graphical reasoning.

3. Structure of this thesis

This PhD thesis consists of a series of articles. The first aim of the research presented in this PhD thesis is investigating *whether* and *to what extent* mathematical activities in the domain of graphing motion are prone to elicit students' HOT. The second aim is investigating the role of bodily experiences and their potential to support students' reasoning about motion graphs. A third and final aim of this PhD thesis is whether HOT stimulated within the domain of graphing motion has the potential to foster high levels of reasoning in another slightly related mathematics domain, namely the domain of early algebra, providing further insight in the extent to which HOT can be regarded domain-specific, domain-general, or both. The empirical data gathered in this project covered students' *micro development* over the six-lesson teaching sequence and students' *macro development* over the schoolyear. An overview of the structure of this thesis, including the topics addressed in the respective chapters, is provided in Figure 3.

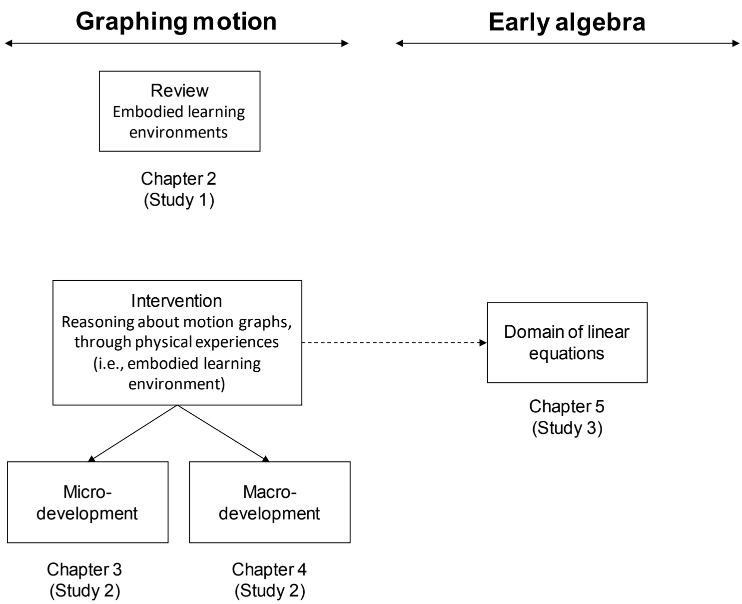

Figure 3. Overview of the structure of this thesis including the studies presented in the respective chapters

Chapter 2 reports on a systematic literature review of the research literature that made use of an embodied learning environment to support students' understanding of graphing motion. The main objective for carrying out this literature review was to obtain more insight into the characteristics of embodied learning environments and their potential for mathematics thinking and learning in general and graphing motion in particular. This chapter addresses the following main research question:

> *What does the research literature report on teaching students graphing motion using learning environments that incorporate students own bodily experiences?*

Chapter 3 reports on the study in which we investigated the potential of an embodied learning environment – consisting of a six-lesson teaching sequence – to support students' HOT, as their reasoning about graphing motion, and more specific, their reasoning about the variables represented in the graph (i.e., distance, time, and speed). In this embodied learning environment students are offered graphing activities in which their own bodily movements are visualized as a line in the graphical representation, using motion sensor technology. The analysis focused on students' micro-development over the lessons, indicated by their performance on lesson-specific graph interpretation and graph construction tasks. Moreover, we illustrated how the direct physical experiences in the embodied learning environment played a key role in students' evolving understanding about distance-time graphs, by providing an in-depth case study of one student's experiences throughout the lessons. In this case study, we focused on the interactions between the student and the motion sensor, and between the student and her peers. We answer the following research question:

> *How does students' reasoning about graphing motion develop over a six-lesson teaching sequence within an embodied learning environment?*

In *Chapter 4*, we report on the effectiveness of the six-lesson teaching sequence offering students' embodied support. Following the thesis that physical (bodily) experiences are helpful for learning and cognition we investigated whether the teaching sequence offering students direct embodied support (see *Chapter 3* of this thesis) had a differential effect on students' ability to interpret and construct graphical representations of motion than a teaching sequence offering students indirect embodied support. We made use of a cohort-sequential design in which the teaching sequence was given to the students in three successive cohorts, one class per cohort for each instructional condition. This chapter is about the longitudinal study. The analysis focused on students' macro-development over the schoolyear. We answer the following research question:

> *To what extent does embodied support in a six-lesson teaching sequence on graphing motion affect the development of students' graphical reasoning?*

And lastly, in the study that is presented in *Chapter 5* we investigated whether a teaching sequence stimulating students' domain-specific mathematical HOT has the potential to affect students' reasoning in another mathematics domain, namely linear equation solving. For this, we included both the macro-developmental data presented in the previous chapter (i.e., students' written responses to the graph interpretation and graph construction task) and additional data from the same students concerning their written responses to tasks in which they solved systems of informal linear equations. In this final study we ask:

> *To what extent does a six-lesson teaching sequence on graphing motion affect the development of students' graphical and algebraic reasoning?*

Chapter 6 brings together the findings from all studies carried out for this PhD research and consists of a summary of the main findings and conclusions. Theoretical and practical implications as well as directions for future research and practice are proposed and the limitations of this thesis are addressed.

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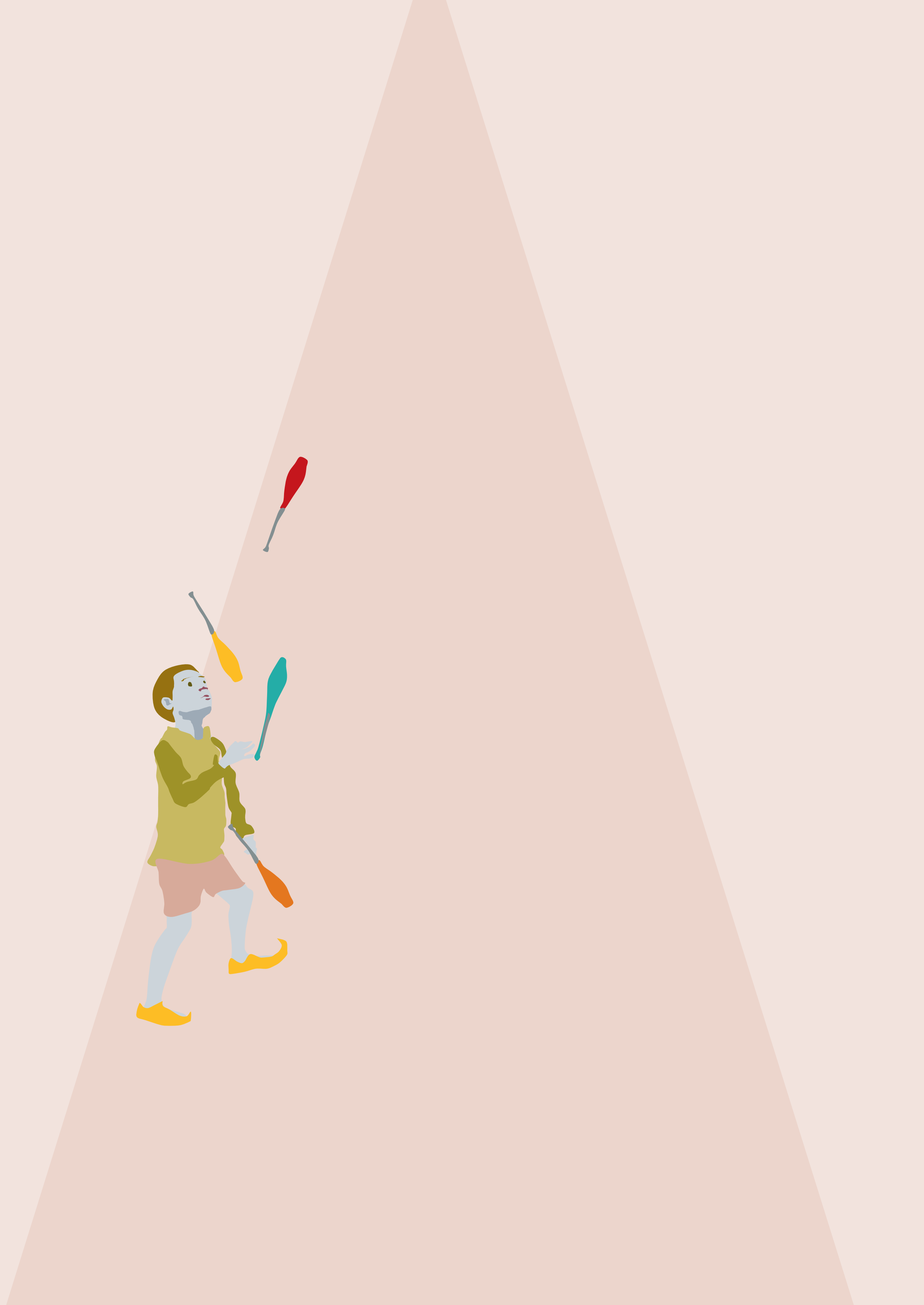

CHAPTER 2

Embodied learning environments for graphing motion: A systematic literature review

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Embedded learning environments for graphing motion: A systematic literature review

Abstract

Embodied learning environments have a substantial share in teaching interventions and research for enhancing learning in science, technology, engineering, and mathematics (STEM) education. In these learning environments, students' bodily experiences are an essential part of the learning activities and hence, of the learning. In this systematic review, we focused on embodied learning environments supporting students' understanding of graphing change in the context of modelling motion. Our goal was to deepen the theoretical understanding of what aspects of these embodied learning environments are important for teaching and learning. We specified four embodied configurations by juxtaposing embodied learning environments on the degree of *bodily involvement* (own and others/objects' motion) and *immediacy* (immediate and non-immediate) resulting in four classes of embodied learning environments. Our review included 44 articles (comprising 62 learning environments) and uncovered eight mediating factors, as described by the authors of the reviewed articles: *real-world context*, *multimodality*, *linking motion to graph*, *multiple representations*, *semiotics*, *student control*, *attention capturing*, and *cognitive conflict*. Different combinations of mediating factors were identified in each class of embodied learning environments. Additionally, we found that learning environments making use of students' own motion immediately linked to its representation were most effective in terms of learning outcomes. Implications of this review for future research and the design of embodied learning environments are discussed.

Keywords: Embodied cognition theory, Mathematics education, Graphing motion, Learning environments, Mediating factors

1. Introduction

Within the domain of STEM teaching and learning a large number of studies have been conducted incorporating *embodied mathematics activities* (e.g., Abrahamson & Lindgren, 2014; Tran et al., 2017). These are activities in which students' perceptual-motor experiences play an explicit role in the learning process (Lindgren & Johnson-Glenberg, 2013). Using perceptual-motor activities within mathematics education fits within the theoretical framework of embodied cognition (e.g., Barsalou, 2010; Gallese & Lakoff, 2005; Glenberg & Gallese, 2012; Núñez et al., 1999; Wilson, 2002). This theory emphasizes the idea that learning and cognitive processes are taking place in the interaction between one's body and its physical environment. Yet, as is described by Hayes and Kraemer (2017), little is known about how embodied processes, such as moving your body through space, contributes to STEM learning (see also DeSutter & Stieff, 2017; Han & Black, 2011; Kontra et al., 2015). Therefore, it is no surprise that recent reviews call for more research into principles of embodied (i.e., motion- and body-based) interventions for mathematics learning, as well as a systematic inventory of their presumed usefulness (Nathan et al., 2017; Nathan & Walkington, 2017). In line with these reviews, we want to shed light on the significance of embodied cognition theory for mathematics teaching and learning. Yet, we want to take a small step back and take a critical look at the extant research. We particularly focus on a mathematics domain that has a tradition of including bodily experiences for learning: graphing change in the context of modeling motion.

Reviewing the operationalization of aspects of a theory in learning environments can be a helpful strategy to elaborate a theoretical perspective (Bikner-Ahsbahs & Prediger, 2006) and can help demonstrate how theoretical considerations are useful for the teaching and learning of mathematics (Sriraman & English, 2010). Therefore, we decided to review research literature to map the existing landscape of embodied learning environments supporting students' understanding of graphing motion. In this way, we aim to elucidate the potential of these embodied learning environments for students, teachers, mathematics education researchers, and curriculum designers, and to assess their theoretical relevance in order to advance and inform the embodied cognition thesis.

1.1 Embodied cognition

Considering bodily experiences as fundamental for learning has a rather long history in the educational and developmental sciences, and has recently received an increased interest through the embodied cognition paradigm (e.g., Abrahamson & Bakker, 2016; Radford et al., 2005; Wilson, 2002). Piaget (1964) described how during the first sensorimotor developmental stage a child acquires “the practical knowledge which constitutes the substructure of later representational knowledge” (p. 177). However, according to Piaget, the significance of sensorimotor cognition would be temporary and limited to the first stages of cognitive development. In the 1980s, this interpretation changed (Núñez et al., 1999), leading to the now common proposition that “sensorimotor activity is not merely a stage of development that fades away in more advanced stages, but rather is thoroughly present in thinking and conceptualizing” (Radford et al., 2005, p. 114, see also Oudgenoeg-Paz et al., 2016). Accordingly, current embodied cognition theories emphasize that the role of perception-action structures is not limited to concrete operational thought but extends to abstract higher-order cognitive processes involved in language and mathematics as well (Barsalou, 1999). Likewise, accepting perception-action as a basic building block of cognition implies a view on cognition as, at least partly, situated (or embedded), where the interaction of the body with objects in their real spatial context is a major gateway to cognition.

Embodied cognition theory refers to a variety of different but related theories varying in how the relationship between (lower-order) sensorimotor processes and (higher-order) abstract cognitive processes is conceived. The conceptualization of this relationship can be more or less radical—a distinction that relates to, but does not coincide with, the distinction between “simple” and “radical” embodiment as proposed in the research of Clark (1999). As Clark (1999) describes, simple, non-radical views of embodiment posit that bodily experiences and interactions of the body with the environment can support or influence (“on-line” and “offline”) cognitive processes like the use of finger-counting can help to build the concept of number. The bodily experiences are considered to add “color” to abstract concepts, yet without fundamentally altering the a-modal discursive nature of these concepts. This simple, non-radical view on embodiment is fully compatible with the computational (cognitivist) approach to cognition, as the embodiment of cognition is seen as an additional but not essential phenomenon (Goldinger et al., 2016; Goldman, 2012; Wilson, 2002).

A radical reading of embodiment, in contrast, holds that all human cognition emerges through, and exists in, the recurrent cycles of perception-action of the physical body in its environment (Glenberg, 1997; Kiverstein, 2012). Per this view, real knowledge resides in immediate environmental perception-action cycles (Wilson & Golonka, 2013), which make mental representations, such as abstract concepts in mathematics, “empty and misguided notions” (Goldinger et al., 2016, p. 962). Hence, the radical view has difficulty with explaining how cognition evolves in the absence of direct environmental stimuli (as in off-line cognitive activities, see also Pouw et al., 2014) or, for example, when dealing with symbolic language or mental arithmetic, which are "hungry" for mental representation (Clark, 1999; Wilson & Golonka, 2013). This view is at odds with rationalist or mentalist approaches as in computational models of cognition.

Many embodiment researchers position themselves somewhere in-between the simple and radical view in line with Goldman (2012), who claims that there is compelling behavioral and neuroscientific evidence for a moderate view of embodiment (see also Gallese & Lakoff, 2005; Lakoff, 2014; Pulvermüller, 2013). A moderate view on embodied cognition acknowledges the critical importance of bodily experiences as part of the meaning of both concrete and abstract concepts, thus as constituting the fundament of all human knowledge, but allows for two additional resources: (1) the non-immediate (off-line) grounding of cognition in bodily experiences through imagining or mentally simulating perceptions and actions by re-using the sensorimotor circuits of the brain involved in actual (on-line) perceiving or performing these actions (also referred to as mirroring, see below); and (2) the connection, based on Hebbian-associative learning, of the system of multimodal sensorimotor cognition to a system of a-modal (verbal) conceptual knowledge (Anderson, 2010; Lakoff, 2014; Pulvermüller, 2013). With these two additional resources, moderate embodiment endorses a view on human cognition as essentially situated and embodied, while allowing for grounded but abstract mental processes, such as reasoning and combining elementary embodied concepts into more complex abstract concepts. According to this view, acquired action-perception structures can be re-used through mental simulation, as perceptual symbols (Barsalou, 1999), in situations where on the basis of previous experiences and well-established skills, new (and increasingly abstract) ideas need to be constructed and understood, also in off-line contexts (Anderson, 2010; Koziol et al., 2011).

In line with embodiment theories, various studies have shown the positive effects of one's own bodily involvement on learning (e.g., Dackermann et al., 2017; Johnson-Glenberg et al., 2014; Nemirovsky et al., 2012). For example, a study by Ruiter et al. (2015) investigated the influence of task-relevant whole bodily motion on first-grade students' learning of two-digit numbers. Here, step size (small, medium, large) represented different sized number units (1, 5, 10). They found that students in the task-relevant whole bodily motion conditions outperformed students in the non-motion condition (where the movements were task-irrelevant) on students' learning of two-digit numbers. Other studies have shown the beneficial effects of part-bodily motion on learning, such as students' hand gestures (Alibali & Nathan, 2012; Goldin-Meadow et al., 2009), finger tracing (Agostinho et al., 2015), finger counting (Domahs et al., 2010), or arm movements (Lindgren & Johnson-Glenberg, 2013; Smith et al., 2014). Similarly, giving students the opportunity to observe or influence movements of other persons or of objects, instead of making these movements themselves, can lead to improved understanding as well, which suggests, in line with the moderate embodiment position, involvement of mirroring or simulation mechanisms (De Koning & Tabbers, 2011; Van Gog et al., 2009). In the study of Bokosmaty et al. (2017), fifth-grade students observed a teacher demonstrating a geometry concept. The students improved their understanding of geometry after manipulating the geometric properties of triangles as well as observing their teacher doing so. Influencing and observing the movements of others and objects entails other ways of bodily involvement than making movements of your own. A large proportion of the research on observing others or objects has been devoted to observing teachers' use of gestures (e.g., Singer & Goldin-Meadow, 2005) and observing the movements of somebody or something else through video examples or animations (e.g., De Koning & Tabbers 2011; Post et al., 2013).

Perceptual-motor experiences encompass a wide variety of bodily activities ranging from observing and influencing other (human) movements to making movements oneself. In a moderate embodiment perspective, following the mirroring systems hypothesis (e.g., Rizzolatti & Craighero, 2004), all these ways of directly and indirectly involving the body can be regarded as "embodied" (Van Gog et al., 2009). According to the mirroring systems hypothesis, the same sensorimotor areas in the brain are activated when observing actions by others as when performing these actions oneself (e.g., Anderson, 2010; Calvo-Merino et al., 2006; Gallese & Lakoff, 2005; Schwartz et al., 2012). Indeed, brain imaging studies show similar patterns of brain activation when subjects hear or read a story in which a particular action is described, when they imagine the event involving this action, or acting out the specific event (Grèzes & Decety, 2001; Pulvermüller, 2013; Pulvermüller & Fadiga, 2010), implying that understanding a concept (e.g., the verb kicking) relies on motor activation (Goldman, 2012; Pulvermüller & Fadiga, 2010).

In addition to the different levels of bodily involvement, also the immediacy of the embodiment of cognitive activities can differ between learning situations. Immediate cognitive activities are activities where immediate, or on-line, perceptual-motor interaction with the physical environment is available to the student (Borghi &

Cimatti, 2010; Wilson, 2002). For example, Smith et al. (2014) had fourth-grade students create both static and dynamic angle representations by moving their arms in front of a Kinect sensor. The angles, reflected in the position of their arms, were immediately represented on the digital blackboard. This immediate link between students' physical experiences and the abstract visual representation of angles facilitated students' improved understanding of angle measurement after completing the body-based angle task. However, many embodied learning environments present learners with non-immediate, or off-line, cognitive activities. Typically, in non-immediate learning situations students first have bodily experiences, as, for example, when they explore the shapes of particular objects, which are then followed by the learning activity where the to be learned concepts are presented (Pouw et al., 2014). In situations where an immediate task-relevant interaction with the physical environment is not available, embodied simulation mechanisms may play a crucial role. According to De Koning and Tabbers (2011), through embodied simulations, previously acquired sensorimotor experiences are made available for knowledge construction processes in the learning activity (e.g., Barsalou, 1999).

1.2 Embodied learning environments for graphing motion

1.2.1 Relevance of embodied learning environments for graphing motion

Through learning environments based on embodied cognition theory students are provided with opportunities to ground abstract formal concepts in concrete bodily experiences (Glenberg, 2010). Such embodiment-based learning environments are often used in efforts to support students' understanding of graphing motion by, for example, showing how distance changing over time is represented graphically. Like many topics within mathematics, developing an understanding of graphical representations describing dynamic situations, can be challenging for students. Among other things, students experience difficulties with distinguishing between discrete and continuous representations of change, recognizing the meaning of the represented variables and their pattern of co-variation (Leinhardt et al., 1990), and differentiating between the shape of a graph and characteristics of the situation or the construct it represents (e.g., McDermott et al., 1987; Radford, 2009a). Yet, graphical representations representing dynamic situations are foundational for the study of mathematics and science, and the absence of a solid understanding of graphical representations can make learning about rate and functions in the study of calculus and kinematics even more difficult (Glazer, 2011).

Learning environments supporting students' understanding of graphs of change and motion often incorporate students' own motion experiences. According to Lakoff and Núñez (2000), experiencing change, in the context of graphs and functions, is related to the embodied image schemes of *fictive motion* and the *source-path-goal schema*. Essentially, these embodied image schemes allow to conceptualize static representations as having dynamic components (Botzer & Yerushalmy, 2008). Metaphorical projection, by means of these image schemes, is the main embodied cognitive mechanism providing the link between the source-domain experiences (such as moving through space) and target-domain mathematical knowledge (such as developing an understanding of graphically represented motion) (e.g., Font et al., 2010; Núñez et al., 1999).

1.2.2 Operationalizing embodied learning environments for graphing motion

Over the past years, many efforts have been undertaken to categorize embodied learning. For example, taxonomies of embodied learning have been developed in the context of technology (Johnson-Glenberg et al., 2014; Melcer & Isbister, 2016), full-body interactions (Malinverni & Pares 2014), learning with manipulatives (Reed, 2018), and, more generally, for the field of learning and instruction (Skulmowski & Rey, 2018). The taxonomy of embodied learning described by Johnson-Glenberg et al. (2014) consists of four degrees of embodiment in which each degree entails a different level of bodily involvement, or *motoric engagement*. Skulmowski and Rey (2018) combined the two lowest degrees of motoric engagement found in the research of Johnson-Glenberg et al. (2014) into the category *lower levels of bodily engagement*, such as observation and finger tracing, and the two highest degrees into the category *higher levels of bodily engagement*, such as performing bodily movements and locomotion. Both taxonomies consider the conceptual link between the concrete bodily experience and the intended concept, termed *gestural congruency* (Johnson-Glenberg et al., 2014) or *task-integration* (Skulmowski & Rey, 2018). In both taxonomies, the bodily experience can be conceptually related to the learning content or not. We also see gestural congruency and task-integration as important elements on which embodied learning environments can vary. However, for embodied learning environments supporting students' understanding of graphing change, the congruency between a motion event (either experienced or observed) and the graph of that motion is already an essential element of the learning environment, which will make task integration a less informative dimension for the purpose of this review.

The aforementioned levels of bodily involvement provide us with a base to categorize embodied learning environments supporting students understanding of graphing motion. A further way to categorize embodied learning environments supporting students' understanding of graphing motion refers to the contiguity of motion and graph. The graphical representation of motion can be constructed simultaneously with the motion event or at a later moment. For this temporal aspect, we use the term *immediacy*. Because the motion and the corresponding representation are located in different representational spaces (i.e., the space in which you move/influence/observe versus the space in which the motion is represented), this distinction between immediate (or on-line) activities versus non-immediate (or off-line) activities might be especially relevant for classifying embodied learning environments supporting students' understanding of graphing motion.

In sum, to get a grip on the plethora of embodied configurations of the learning environments that one can come across in educational research literature, we propose to categorize embodied learning environments supporting students' understanding of graphing motion on two dimensions: *bodily involvement* and *immediacy* (see Figure 1). For *bodily involvement*, a distinction is made between own motion and observing others/ objects' motion. One's own motion entails a direct bodily experience, while the motion of others/objects is experienced indirectly. For the latter, *mirror neural activity* is the main embodied cognitive mechanism, as the mirror-neuron system is activated when observing movements made by others/objects. In line with this, we defined bodily involvement on a scale ranging from "motor execution," referring to one's own motion, till "motor mirroring," indicating that when observing others/objects' motion, an individual starts to rely on (neural) mirroring mechanisms (e.g., Anderson, 2010; Gallese & Lakoff 2005; Schwartz et al., 2012).

For *immediacy*, a distinction is made between immediate and non-immediate (see Figure 1), taking into account the distinction between "on-line" cognitive activities and "off-line" cognitive activities (Pouw et al., 2014; Wilson, 2002). In the first case, an immediate task-relevant interaction with the physical environment is acted out, whereas in the second case this interaction is not simultaneously available. For the latter, *embodied simulation* is the main theoretical embodied cognitive mechanism, meaning that previously acquired sensorimotor experiences are activated. Accordingly, we defined immediacy on a scale ranging from "direct enactment," referring to cognitive activity that is situated in the participant–environment interaction in the presence of direct environmental stimuli, till "reactivated enactment", indicating that within non-immediate learning environments, an individual starts to rely on embodied simulations, which are re-activations of previous sensorimotor experiences.

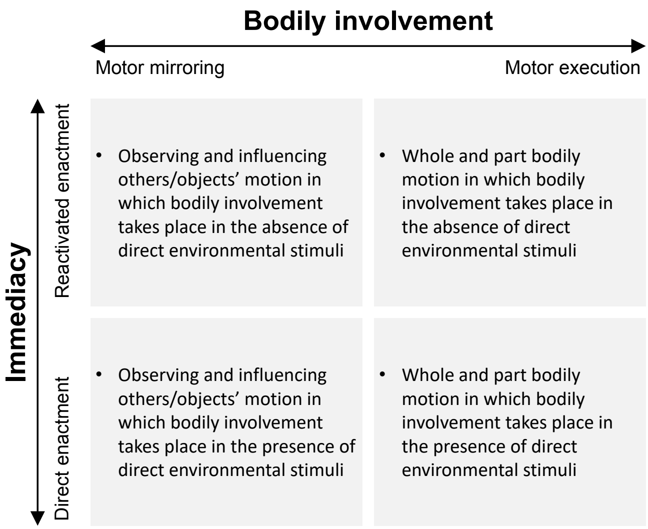


Figure 1. Taxonomy for embodied learning environments supporting students' understanding of graphing motion based on bodily involvement and immediacy

Each quadrant of the taxonomy presented in Figure 1 may give room for specific factors that are prone to mediate learning. Reviews on embodied learning have identified valuable features of embodied learning environments that impact students' learning processes. For example, in their review of embodied numerical training programs, Dackermann et al. (2017) detected three working mechanisms of embodied learning environments: mapping mechanisms between the bodily experience and the intended concept, interactions between different regions of personal space, and the integration of different spatial frames of reference. Tran et al. (2017) also found mapping mechanisms (as movements being in accordance with the mental model of the mathematical concept) to be an important factor within embodied learning environments. Additionally, they posit that the movements students make should be represented visibly to give them the opportunity to observe and reflect on these movements. Within the context of graphing motion, we expect aspects like participant–environment interactions, attentive processes, mapping mechanisms, and multimodal aspects of the learning environment to be of importance.

1.3 Research focus

In this article, we describe a review of the research literature on teaching graphing change and, more specifically, graphing motion (e.g., graphical representations of *distance* changing over *time*). We focused on learning environments in which students' bodily experiences are an essential part of the learning activities and the learning. We were especially interested in articles in which these embodied learning environments are used, described, and empirically evaluated, for example, by means of an experiment. Based on these articles, we aimed to specify the embodied configurations that constitute these learning environments; identify the presumed factors that mediate learning within these learning environments, as described by the authors; and evaluate the efficacy of these learning environments by considering the learning outcomes. Since graphing motion is a key topic within both mathematics and science and already present within the early grades, we decided to include studies from primary education to higher education. To guide our review, we formulated the following four research questions: *What does the research literature on teaching students graphing motion using learning environments that incorporate students own bodily experiences report on…*

1. *...the embodied configuration (in terms of bodily involvement and immediacy) of these learning environments?*

2. *...the presumed factors mediating learning within these learning environments?*

3. *...the relationship between the learning environments' embodied configuration and the factors that mediate learning?*

4. *...the efficacy of embodied learning environments for graphing motion?*

2. Method

2.1 Literature search

The literature search was carried out in four databases: Web of Science, ERIC, PsycINFO, and Scopus. As a first quality criterion, we searched for empirical research articles published in peer-reviewed journals and written in the English language. We did not set a publication date restriction to the articles because we are also interested in articles not (yet) mentioning embodied cognition as the main or related theory, but still applying its core features, for example, in the field of kinesthetic learning. There were no further methodological restrictions, so we included articles with qualitative studies, quantitative studies, and mixed-method studies. In a stepwise process, we defined a query consisting of Education × Learning facilitator × Domain × Graph × Graph variables (for the full query, see Appendix 2.1). Our initial search, conducted on April 6, 2017, generated 1953 journal articles (see Figure 2). After deduplication, 1651 unique publications remained.

2.2 Selection of articles

The selection process was facilitated by organizing all publications and coding information in a database, using Excel. Selection decisions were frequently discussed with all authors. We first performed a quick scan of the full text of the 1651 articles to identify the articles on graphing motion. Articles not written in English (153), not about education and learning (979), not in the STEM domain (307), not including graphing activities (80), not containing motion data (94), or not having a full-text available (2) were excluded (see Figure 2). This resulted in 36 relevant articles for the purpose of the review. By snowballing the reference lists of these articles, 13 additional articles of interest were found. Then we inspected the full texts of these 49 articles' methodology and results, only including articles in which the embodied learning environments were sufficiently described (i.e., containing a clear description of tools and tasks) and the bodily experiences could be considered task relevant. This resulted in the exclusion of five articles and the final selection of 44 articles for our analysis.

2.3 Data extraction and analysis

The 44 articles were first coded in terms of the contextual information regarding the studies carried out, comprising school level, sample, subject matter domain, research design, tools, learning activities, intervention length, dependent measures, and reported learning outcomes. Then we zoomed in on the learning environments, our units of analysis. A learning environment is a setting (e.g., a classroom) in which a set of activities is provided to the participants (e.g., a teaching sequence given to a group of Grade 5 students). In many articles, the learning environments differed between conditions. The 44 articles contained a total of 62 different embodied learning environments. Some of these learning environments were used as a control condition and some as experimental conditions. Hereafter, we coded the learning environments on their *bodily involvement* and *immediacy* as an indication of their embodied configuration. Finally, we extracted the presumed mediating factors for students' understanding of graphing motion from each article and looked at the four classes of embodied learning environments in which they were mentioned.

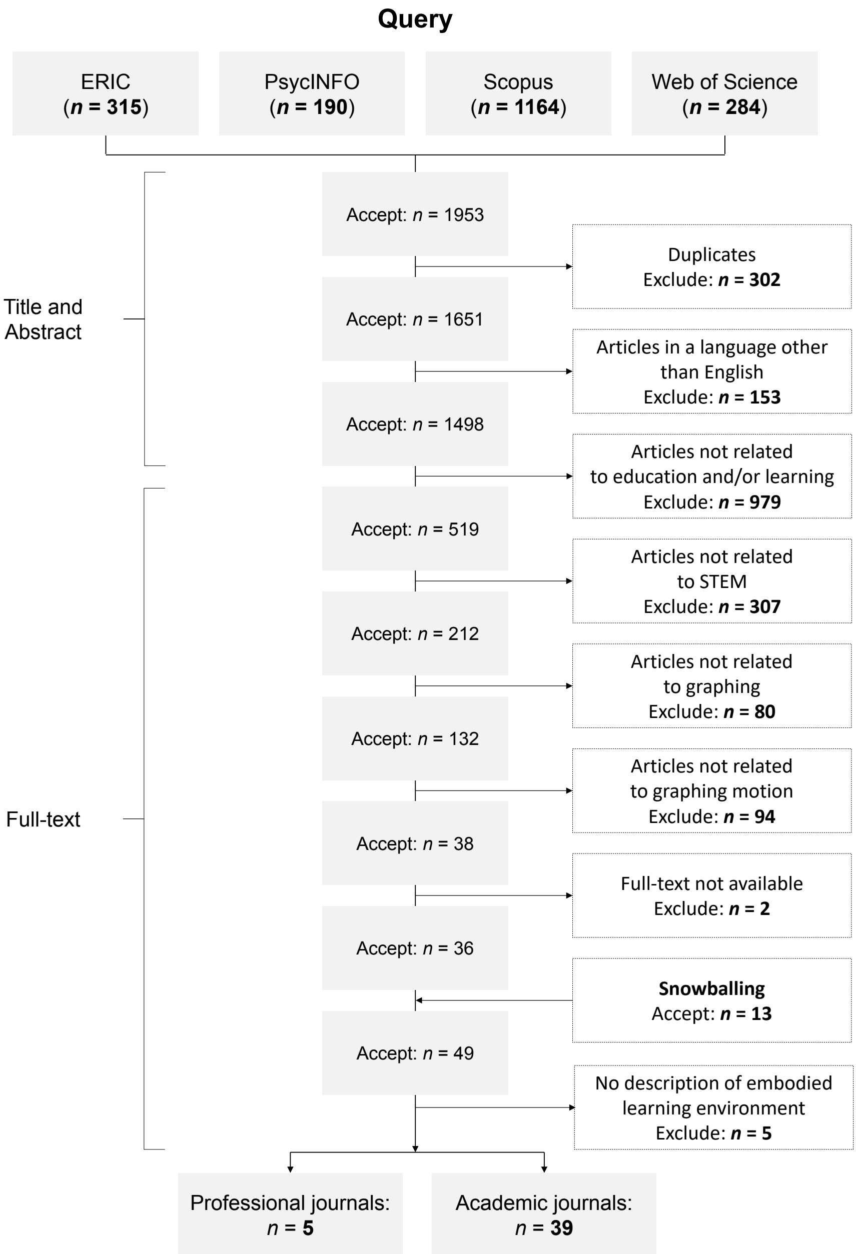


Figure 2. Flowchart of search strategy showing the numbers of included and excluded articles

2

Bodily involvement gives an indication of students' engagement with a movement, ranging from an action of the whole body to observing the movement of others. For example, a learning environment in which a student has to move a small toy car over the table by moving part of her/his body was qualified as part bodily motion. However, due to lacking information in most of the articles, the number of bodily actions and their duration was not coded. *Immediacy* gives an indication of the temporal alignment of motion and graph. This temporal alignment relates to whether or not there is an immediate task-relevant interaction with the physical environment. For example, a learning environment in which a student has to move in front of a motion sensor and later constructs a graph using this data was qualified as non-immediate, whereas a learning environment where the graphical representation is constructed in parallel with the movement of that student was qualified as immediate. These latter learning environments were often technology enriched since technology eases the immediate representation of a graphical representation alongside a motion event. See Table 1 for a description of the degrees of bodily involvement and immediacy.

Table 1

Coding categories of bodily involvement and immediacy of embodied learning environments

Category	Description
Bodily involvement	
Own motion	
Whole bodily motion	Students move their body from one point to another and exert control over the graphical representation of the movement.
Part bodily motion	Students move part(s) of their body (e.g., an arm or a hand) and exert control over the graphical representation. The students' body is stationary (i.e., it does not move through space).
Others/objects' motion	
Influencing and observing others' or objects' motion	Students influence others' or objects' motions, represented in the graphical representation. This can happen in reality (setting a real pendulum in motion) or in a computer environment (putting in values that influence a motion).
Looking at or observing others' or objects motion	Students observe the motion of other persons' or objects' motion. The students do not affect the motion or representation in any way.
Immediacy	
Immediate	The graphical representation of the motion is constructed in parallel to the motion. There is no delay.
Non-immediate	The motion is not directly translated into a graphical representation. The construction of the graph based on the data happens at a different (later) stage.

Learning environments containing more than one degree of bodily involvement and immediacy were assigned to the highest degree. For example, when a learning environment included both whole-bodily motion and influencing and observing others' or objects' motion, the learning environment was assigned to the category whole bodily motion. The same holds for immediacy. Learning environments containing both immediate and nonimmediate bodily experiences were assigned to the category immediate. An independent second rater coded a subsample of 12 articles containing 20 learning environments (> 25%). Inter-rater reliability was very good for the bodily involvement dimension (Cohen's Kappa = 1.00) and good for the immediacy dimension (Cohen's Kappa = 0.74). We clustered the learning environments into four main classes in which the degrees of bodily involvement and immediacy are combined: Class I – Immediate Own Motion, Class II – Immediate Others/Objects' Motion, Class III – Non-immediate Own Motion, and Class IV – Non-immediate Others/Objects' Motion.

In order to extract the mediating factors from the described studies, the articles were carefully read and indications of mediating factors, presumed by the authors, were recorded. First, these mediating factors were recorded based on the terminology used by the authors. Later, these factors were clustered in categories. Finding a new mediating factor sometimes led to changing the categories or combining and splitting mediator categories. For example, an article mentioning gesturing as supporting students' understanding of graphing motion first fell in a category labeled "gestures." Later, we decided to create a category "semiotics" in which we grouped all mediating factors related to meaning supported signs systems. In this respect, throughout several iterations of reading and data extraction, we came to eight overarching mediator categories: *real-world context*, *multimodality*, *linking motion to graph*, *multiple representations*, *semiotics*, *student control*, *attention capturing*, and *cognitive conflict*.

2.4 The subject matter domains addressed in the articles

As a result of our search query, all articles either addressed topics from the domains of mathematics and physics or integrated topics from both domains. The mathematics-oriented articles used motion to address the teaching and learning of graphs as visual representations of dynamic data (e.g., Boyd & Rubin, 1996; Robutti, 2006). Some of these articles also included more advanced topics like functions and the mathematics of change (calculus) (e.g., Ferrara, 2014; Salinas, et al., 2016). Most of the articles in physics addressed the relation between distance traveled, velocity, and acceleration (kinematics) (e.g., Anderson & Wall, 2016; Mitnik et al., 2009).

Articles that used an integrated approach addressed both aspects from physics, such as distance traveled, velocity, and acceleration and from mathematics, like slope and rate of change (e.g., Nemirovsky et al., 1998; Noble et al., 2001).

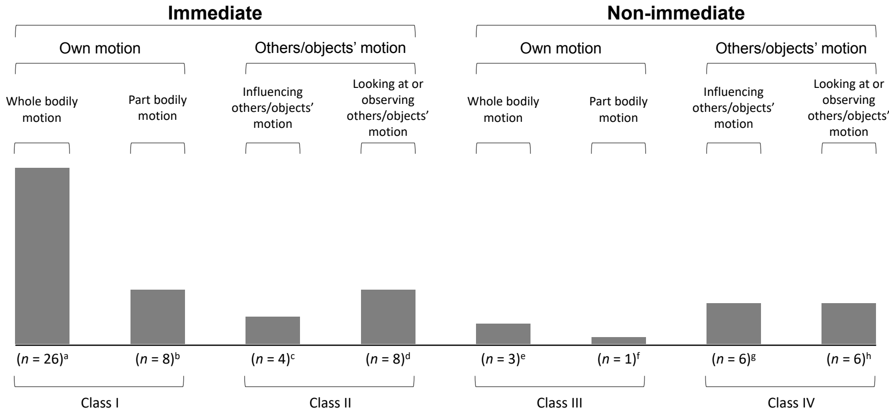
All articles, in both mathematics and physics, included learning environments in which data are represented by means of graphs. These data can be first-order data such as distance and time measures, which can be represented in distance–time graphs (e.g., Deniz & Dulger, 2012; Kurz & Serrano, 2015) or derived data resulting in velocity–time graphs or acceleration–time graphs (e.g., Anderson & Wall, 2016; Nemirovsky et al., 1998; Struck & Yerrick, 2010). Also, in some mathematical learning environments, graphs were drawn of functions (e.g., linear and quadratic functions) (e.g., Noble et al., 2004; Salinas et al., 2016; Stylianou et al., 2005; Wilhelm & Confrey, 2003) or, related to physics, of uniform and oscillatory motion (e.g., Kelly & Crawford, 1996; Metcalf & Tinker, 2004).

2.5 Efficacy of embodied learning environments for graphing motion

Of all included articles (n = 44), 26 articles gave information about the efficacy of the embodied learning environments for graphing motion. In these articles, the learning outcomes of multiple groups or pre- and post-tests were compared. To ensure the robustness of our evaluation of the reported learning outcomes, we carried out a quality check of the research design of the articles and the reported learning outcomes per learning environment (Appendix 2.2). The study design of these 26 articles was either (quasi)experimental (n = 15) or descriptive (n = 11). The mean quality rating (range, 5–20) for this subset of articles was 11.77, with a standard deviation of 2.93. From this quality rating, we infer that the methodological quality of this subset of articles is sufficient.

3. Classes of embodied learning environments

The 62 learning environments were classified on bodily involvement and immediacy (see Figure 3). Class I – Immediate Own Motion was the largest (34 learning environments) Class II – Non-immediate Own Motion was the smallest (4 learning environments). The other two classes contained the same amount of learning environments (12 learning environments each).



a Anderson & Wall, 2016**; Brasell, 1987; Deniz & Dulger, 2012; Espinoza, 2015; Ferrara, 2014; Kelly & Crawford, 1996; Kurz & Serrano, 2015; Metcalf & Tinker, 2004; Mokros & Tinker, 1987; Nemirovsky et al., 1998; Radford, 2009; Robutti, 2006; Solomon et al., 1991; Struck &Yerrick, 2008; Stylianou et al., 2005; Svec et al., 1995; Svec, 1999; Taylor et al., 1995; Thornton & Sokoloff, 1990; Wilhelm, & Confrey, 2015; Wilson & Brown, 1998; Zucker et al., 2014****

b Anastopolou et al., 2011; Botzer & Yerushalmy, 2006; Botzer & Yerushalmy, 2008; Holbert & Wilensky, 2014; Kuech & Lunetta, 2002; Nemirovsky, 1994; Noble et al., 2001; Russell et al., 2003

c Altiparmak, 2014; Espinoza, 2015; Kozhevnikov & Thornton, 2006; Salinas et al., 2016

d Anastopolou et al., 2011; Brungardt & Zollman, 1995; Ferrara, 2014; Kozhevnikov & Thornton, 2006**; Noble et al., 2004; Skordoulis et al., 2006; Zucker et al., 2014

e Anderson & Wall, 2016; Brasell, 1987; Deniz & Dulger, 2012

f Heck & Uylings, 2006

g Anderson & Wall, 2016; Carrejo & Marshall, 2007; Roschelle et al., 2010**; Simpson et al., 2006; Woolnough, 2014

h Boyd & Rubin, 1996; Brungardt & Zollman, 1995; Mitnik et al., 2009**; Zajkov & Mitrevski, 2012**

Note. *(***) = The number of asterisks indicates the number of similar embodied learning environments within an article.

Figure 3. Four classes of learning environments based on bodily involvement and immediacy

3.1 Class I – Immediate own motion

In 30 out of the 34 learning environments that belonged to Class I – Immediate Own Motion, motion sensor technology was used (e.g., Anderson & Wall, 2016; Ferrara, 2014; Nemirovsky et al., 1998), allowing for the immediate representation of a student's motion as a graph. For example, in the study of Robutti (2006), students started by interpreting a description of a motion situation, which was followed by sketching a graph of this situation. Finally, students acted out the motion event by walking in front of the motion sensor. The translation of their movements into a graphical representation happened immediately and was represented on the screen of a graphing calculator. An example where students used parts of their body can be found in Anastopoulou et al. (2011). They asked students to replicate distance–time and velocity–time graphs by moving their hands in front of a motion sensor. Again, an immediate translation of the motion into a graphical representation was provided. In other studies, it was not the motion of students' hands that was represented, but the motion of an object that students moved with their hands, for example, a motion sensor attached to a wheel which was rolled over a table (Russell et al., 2003). In another study, students were asked to replicate given distance–time, speed–time, or acceleration–time graphs by rotating a disc-shaped handle on top of a rotational motion sensor (Kuech & Lunetta, 2002). In the remaining studies of this class, no motion sensor technology was used. Instead, students were for example asked to move a computer mouse over a mousepad, while at the same time this motion was represented on the screen of the computer (Botzer & Yerushalmy, 2006, 2008).

3.2 Class II – Immediate others/objects' motion

A total of 12 learning environments fell within the category of activities in which students influenced or observed the motion of another person or object without moving (parts of) their own body while getting an immediate representation of that motion. Most studies dealing with moving physical objects were situated in kinematics laboratory settings within physics classes. The used objects varied widely. In one learning environment (Espinoza, 2015), a pendulum system was used, allowing students to exert control over its movement, while a graph of the pendulum's movement was immediately presented to the students by means of motion sensor technology.

Other learning environments in this class dealt with simulated motion using computer software, such as *SimCalc Mathworlds*. In Salinas et al. (2016), students controlled the movements of an animated avatar by building and editing mathematical functions. The students pressed play to see the corresponding animation, while both the animation and graph were presented simultaneously to the students. Another example of using software can be found in Noble et al. (2004). They provided students with a simulation of an elevator moving up and down and a two-dimensional graph with unlabeled axes, representing the velocity in floors per second on the y-axis and the time in seconds on the x-axis.

Finally, in some learning environments within Class II—Immediate Others/objects' Motion, another person demonstrated motion events. For example, in Anastopoulou et al. (2011), a teacher demonstrated hand movements that were captured by a motion sensor and transferred to distance–time and velocity–time graphs, thus allowing the students to see the teacher's hand motion and the corresponding graphs in real time (see also Kozhevnikov & Thornton, 2006; Zucker et al., 2014).

3.3 Class III – Non-immediate own motion

In three out of the four learning environments belonging to Class III – Non-immediate Own Motion, the data collection occurred manually, which caused a slight delay between the motion event and its graphical representation (e.g., Anderson & Wall, 2016; Heck & Uylings, 2006). For example, in Deniz and Dulger (2012), students walked at varying speeds while carrying a bottle of water with a hole in the bottom. Every second, one drop of water fell through this hole. Thus, by measuring the time of traveling and the distance between the drops of water, the students could construct position–time graphs. In the fourth learning environment within this class, the construction of the graphical representation was intentionally delayed. Brasell (1987) tested whether different time delays between the whole-bodily motion and the graphical representation could facilitate an equivalent linking in memory.

3.4 Class IV – Non-immediate others/objects' motion

In 6 of the 12 learning environments within Class IV – Non-immediate Others/objects' Motion, students had to construct a graph after they had observed the movements of physical objects (e.g., Anderson & Wall, 2016; Carrejo & Marshall, 2007; Mitnik et al., 2009) or the movements within a video or a simulation environment (e.g., Boyd & Rubin, 1996; Zajkov & Mitrevski, 2012). For example, in Carrejo and Marshall (2007), students had to record time and distance measures of a ball, using a spark timer, and then construct several graphs of the ball's motion. Here, graph construction happened some time after the motion was finished. Similarly, in another learning environment (Anderson & Wall, 2016), students built ramps and had to choose three objects to roll off the ramp while collecting time and distance measures with timers and measuring tapes. In the article of Mitnik et al. (2009), students observed the movements of a robot moving through space. After all data were collected (i.e., the robot had completed the movement), the students combined distance and time measures of the robot's movements and used this for constructing distance–time and velocity–time graphs. In Boyd and Rubin (1996), students watched videotaped motion events and analyzed these videotaped motion events at a later stage.

In the learning environment described by Brungardt and Zollman (1995), the delay between motion and graph was deliberately used. Students were shown graphs of object motion, several minutes after they had seen the real videotaped motion event, to assess whether the real-time nature of simultaneously presenting graph and motion had an effect on students' understanding of graphs. Finally, some of the simulation environments within this class asked students to first program the movements of an animated object, either in algebraic or graphical form, after which they could see the movements of the objects (e.g., Roschelle et al., 2010). Also, a simulation environment (*ToonTalk*) was used in the article of Simpson et al. (2006). Using this software, students were asked to define the properties of a spacecraft in such a way that it could successfully land on the moon. Here, the graphical representation was not immediately presented after the movement. First, students saw the movements of the *ToonTalk* object, and second, position–time and velocity–time graphs were plotted from the data.

4. Mediating factors within embodied learning environments

Our analysis uncovered eight mediating factors: *real-world context*, *multimodality*, *linking motion to graph*, *multiple representations*, *semiotics*, *student control*, *attention capturing*, and *cognitive conflict*. These mediating factors are to a different extent theoretically aligned with the embodiment framework. Authors sometimes attributed more than one mediating factor to a learning environment. For the 62 embodied learning environments, we found 127 instances in which authors mentioned a mediating factor. In two articles, with two learning environments each, the authors did not mention mediating factors at all (Brungardt & Zollman, 1995; Deniz & Dulger, 2012). In Table 2, the eight mediating factors and the articles in which they were mentioned are presented.

Table 2

Mediating factors for students' understanding of graphing motion in the learning environments and the articles in which they are mentioned

Mediating factors	*n*	Article
1. Real-world context *Referring to experiences of students with or in the real world*	15	Altiparmak, 2014; Boyd & Rubin, 1996; Carrejo & Marshall, 2007; Holbert & Wilensky, 2014; Mitnik et al., 2009**; Mokros & Tinker, 1987; Noble et al., 2004; Solomon et al., 1991; Struck & Yerrick, 2010; Taylor et al., 1995; Thornton & Sokoloff, 1990; Heck & Uylings, 2006; Wilhelm & Confrey, 2003; Woolnough, 2000
2. Multimodality *Referring to intertwining modalities*	14	Anastopolou et al., 2011; Anderson & Wall, 2016***; Botzer & Yerushalmy, 2006; Botzer & Yerushalmy, 2008; Ferrara, 2014**; Mokros & Tinker, 1987; Nemirovsky et al., 1998; Noble et al., 2004; Radford, 2009; Robutti, 2006; Russell et al., 2003
3. Linking motion to graph *Linking motion to a graphical representation*	31	Anastopolou et al., 2011**; Anderson & Wall, 2016**; Botzer & Yerushalmy, 2006; Botzer & Yerushalmy, 2008; Boyd & Rubin, 1996; Brasell, 1987; Brungardt & Zollman, 1995; Deniz & Dulger, 2012; Espinoza, 2015; Ferrara, 2014; Holbert & Wilensky, 2014; Kozhevnikov & Thornton, 2006***; Kurz & Serrano, 2015; Metcalf & Tinker, 2004; Mitnik et al., 2009**; Mokros & Tinker, 1987; Nemirovsky et al., 1998; Robutti, 2006; Russell et al., 2003; Simpson et al., 2006; Skordoulis et al., 2014; Struck & Yerrick, 2010; Stylianou et al., 2005; Svec, 1999; Thornton & Sokoloff, 1990; Heck & Uylings, 2006
4. Multiple representations *Referring to multiple representations of a particular motion*	29	Altiparmak, 2014; Anastopolou et al., 2011**; Botzer & Yerushalmy, 2006; Botzer & Yerushalmy, 2008; Brasell, 1987**; Espinoza, 2015**; Kelly & Crawford, 1996; Kozhevnikov & Thornton, 2006***; Kuech & Lunetta, 2002; Nemirovsky, 1994; Noble et al., 2001; Roschelle et al., 2010**; Salinas et al., 2016; Simpson et al., 2006; Skordoulis et al., 2014; Svec, 1999; Wilhelm & Confrey, 2003; Wilson & Brown, 1998; Zucker et al, 2014*****
5. Semiotics *Referring to the use of meaning-supported sign systems*	9	Anastopolou et al, 2011; Botzer & Yerushalmy, 2006; Botzer & Yerushalmy, 2008; Ferrara, 2014; Nemirovsky, 1994; Nemirovsky et al., 1998; Noble et al., 2004; Radford, 2009; Robutti, 2006
6. Student control *Referring to students being in control of the learning environment*	9	Anastopolou et al., 2011; Anderson & Wall, 2016; Botzer & Yerushalmy, 2008; Brasell, 1987; Mokros & Tinker, 1987; Nemirovsky et al., 1998; Russell et al., 2003; Salinas et al., 2016; Struck & Yerrick, 2010
7. Attention capturing *Referring to aspects in the learning environment that capture students' attention*	12	Botzer & Yerushalmy, 2008; Boyd & Rubin, 1996; Brasell, 1987**; Deniz & Dulger, 2012; Holbert & Wilensky, 2014; Kozhevnikov & Thornton, 2006***; Nemirovsky et al., 1998; Noble et al., 2004; Russell et al., 2003
8. Cognitive conflict *Referring to conflicting conceptions*	8	Carrejo & Marshall, 2007; Kuech & Lunetta, 2002; Nemirovsky, 1994; Simpson et al., 2006; Svec et al., 1995; Woolnough, 2000; Zajkov & Mitrevski, 2012**
Total	127	

Note. *(****) = The number of asterisks added to an article indicates the number of embodied learning environments described in this article for which the mediating factor applies.

2

4.1 Real-world context

When authors mention the *real-world context* as a mediating factor, they refer to experiences of the students with the real world (e.g., Boyd & Rubin, 1996; Carrejo & Marshall, 2007; Heck & Uylings, 2006; Struck & Yerrick, 2010; Wilhelm & Confrey, 2003). Mitnik et al. (2009) gave students the opportunity to study the motion of a robot in the real world, by making the environment more explorative and immersive. In another example, specific parts of the learning environment were related to both the real world and formal contexts, by having authentic player-created graphs that looked like typical velocity–time graphs (Holbert & Wilensky, 2014). Also, other authors claim their learning environments to be almost identical to the real world (e.g., Mitnik et al., 2009; Thornton & Sokoloff 1990). Solomon et al. (1991) use the term "micro world" to indicate that the used learning environment consisted of a world less complex than the real world. According to Thornton and Sokoloff (1990) through a learning environment containing real-world elements, links can be made between students' personal experiences, physical actions, and formal mathematics or physics concepts.

Another finding was that embodied learning environments using a *real-world context* are often presented as a natural venue for scientific exploration (Holbert & Wilensky, 2014; Thornton & Sokoloff, 1990; Woolnough, 2000). For example, Mokros and Tinker (1987) emphasize how the use of microcomputer-based laboratories provided students with genuine scientific experiences. Using elements from the real world also has the advantage of being prone to draw on students' prior knowledge and experiences (Altiparmak, 2014; Taylor et al., 1995). For example, in a simulation environment used in Noble et al. (2004), students, over the course of the activities, started recognizing the movement of an elevator in the graph.

4.2 Multimodality

The articles describing learning facilitators related to the multimodality aspect of the learning environment are all referring to the role of intertwining modalities. This means that by the nature of the tool or the instruction, at least two of the modalities of seeing, hearing, touching, imagining, or motor actions are simultaneously activated. In most of the learning environments, seeing and motor action are involved (Anderson & Wall, 2016; Botzer & Yerushalmy, 2006; Nemirovsky et al., 1998; Noble et al., 2004; Radford, 2009b; Russell et al., 2003). Additionally, Anastopoulou et al. (2011) mention how the interactive technology in their learning environment activated communicative modalities together with these two sensory modalities. In the same line, Mokros and Tinker (1987) emphasize how their use of microcomputer-based laboratories gave students valuable kinesthetic experiences, sometimes using their own bodily motion as data, thus activating the learning modalities perception and motor action (see also Robutti, 2006). Furthermore, Botzer and Yerushalmy (2008) mention the modality touching. In their learning environment, students' hand motion with a computer mouse was captured and shown in graphs. When students retraced the graphs with their fingers on the visual display of the computer, a blend of seeing, touching, and motor action manifested itself. A similar intertwining of multiple modalities is discussed by Ferrara (2014) focusing on the multimodal nature of mathematical thinking. The motion of a student walking in front of a motion sensor was represented on a larger screen in front of the classroom. When the student tried to make sense of the graphical representation of his own motion, this resulted in perceptual, perceptual-motor, and imaginary experiences, manifested by the student's verbal expression of thinking.

4.3 Linking motion to graph

Linking motion to graph as a mediating factor can either refer to the motion of the student (e.g., Anderson & Wall, 2016; Espinoza, 2015), to the motion of somebody else (e.g., Anastopoulou et al., 2011; Skordoulis et al., 2014), or to the motion of objects (e.g., Brungardt & Zollman, 1995; Simpson et al., 2006). In these learning environments, students experienced or observed a link between motion and the corresponding graphical representation. In some instances, authors primarily focus on how the learning environment provided this linkage between motion and graph (e.g., Kurz & Serrano, 2015; Metcalf & Tinker, 2004; Stylianou et al., 2005; Svec, 1999), while other authors focus more on how students were engaged in connecting the graph to the motion (e.g., Anastopoulou et al., 2011; Deniz & Dulger, 2012; Nemirovsky et al., 1998; Heck & Uylings, 2006). A few authors emphasize how this linkage might facilitate a corresponding linking in memory, whereas the information in the graph is a direct result of students' own motion (e.g., Brasell, 1987; Brungardt & Zollman, 1995; Kozhevnikov & Thornton, 2006; Mokros & Tinker, 1987; Struck & Yerrick, 2010). While in almost all learning environments the linkage between an actual (or simulated) motion and the corresponding graph is explicit, some authors also refer to *linking motion to graph* at a more abstract level (e.g., Botzer & Yerushalmy, 2006, 2008; Boyd & Rubin, 1996; Espinoza, 2015; Ferrara, 2014; Holbert & Wilensky, 2014; Robutti, 2006; Russell et al., 2003; Thornton & Sokoloff, 1990). This means that the actual motion helped to conceptualize what lies behind the graphical representation, such as the sensory aspects of the motion experience (Mokros & Tinker, 1987) or mathematical abstractions (Mitnik et al., 2009; Svec, 1999).

4.4. Multiple representations

All learning environments mentioning the mediating factor *multiple representations* refer to multiple representations of a particular motion. Sometimes one and the same motion is represented in multiple graphs (e.g., Anastopoulou et al., 2011; Brasell, 1987; Kelly & Crawford, 1996; Kozhevnikov & Thornton, 2006; Nemirovsky, 1994; Skordoulis et al., 2014; Wilson & Brown, 1998; Svec, 1999). For example, in Kuech and Lunetta (2002), the same motion was represented as a position–time, velocity–time, or acceleration–time graph. Also in the article of Botzer and Yerushalmy (2006, 2008), the students' own motion was visualized in multiple graphical formats. Here, the two dimensions of the motion of the students' hand over the mousepad were represented in two graphs.

Within other learning environments, the motion was represented to the students in multiple formats (e.g., Altiparmak, 2014; Espinoza, 2015; Nemirovsky, 1994; Wilhelm & Confrey, 2003; Wilson & Brown, 1998). In these learning environments, a motion was represented by, for example, a graph, table, or formula (e.g., Kuech & Lunetta, 2002). Furthermore, some articles mention acting out of the motion itself as a representation. In this respect, Anastopoulou et al. (2011) refer to kinesthetic, in addition to graphical and linguistic, representations of motion. Similarly, Zucker et al. (2014) mention how the representations in their learning environment included the "physical motion of an object in front of the sensor" (p. 443) in addition to words, graphs, tables, and animated icons (see also Simpson et al., 2006).

4.5 Semiotics

The mediating factor *semiotics* entails the use of meaning-supporting sign systems. This means that in the learning environment, symbols, signs, gestures, and language, including metaphors, are explicitly used to signify meaning. Botzer and Yerushalmy (2006) describe how gestures served as "an intermediate stage between the sensory experience and the use of formal language" (p. 8) (see also Anastopoulou et al., 2011; Ferrara, 2014). Representing the graphs' mathematical features through gesturing enabled students to elaborate on the meaning of graphs (Botzer & Yerushalmy, 2008). Another important component of *semiotics* is the role of (conceptual) metaphor and metaphorical projection. For example, Botzer and Yerushalmy (2008) mention the possible activation of the fictive motion mechanism, when students actively explored graphical representations, enabling them to conceptualize static graphs as representing motion (see also Ferrara, 2014; Nemirovsky et al., 1998). Nemirovsky (1994) and Noble et al. (2004) describe how the learning environment and its tools offered the student a so-called field of possibilities with graphically represented symbols which had to be interpreted. In this respect, Nemirovsky (1994) refers to symbol-use, in which symbol-use not only depends on the configuration of the learning environment but also on personal intentions and specific histories, conceptualized as extra symbolic components. Other authors concentrate on students' knowledge objectification (i.e., the meaning making process) from an explicit semiotic perspective. Robutti (2006) uses semiotic mediation to refer to the objectification of knowledge, consisting of several steps marked by different semiotic means, including gestures, words, metaphors, and cultural elements to explain the graphical representation (see also Botzer & Yerushalmy, 2008). Similarly, Radford (2009b) provides a semiotic analysis of the way students used their semiotic means in the process of knowledge objectification. Throughout this analysis, the interplay of action, gesture, and language is emphasized.

4.6 Student control

The mediating factor *student control* explicitly refers to students being in control in the learning environment allowing them to manipulate either the motion event or its graphical representation. Most of these articles refer to *student control* as being in control of the (physical) motion (e.g., Anderson & Wall, 2016; Nemirovsky et al., 1998; Russell et al., 2003; Struck & Yerrick, 2010). In this respect, students are able to directly manipulate the visual display (Anastopoulou et al., 2011). Brasell (1987) adds how this direct manipulation of the graphical representation made the graphs "more responsive […] and more concrete" (p. 394). Moreover, when students are able to control the movement represented in the graphical representation, they might feel more engaged (Anastopoulou et al., 2011), making the learning activities more meaningful (Mokros & Tinker, 1987). Other articles refer to *student control* as being in control of the graphical representations already present in the learning environment. In this respect, Botzer and Yerushalmy (2008) mention how student control over the graphical tools was stimulated by actions as dragging, stretching, and shrinking the graphs, and that these actions strongly contributed to students' understanding of graphical signs. Similarly, in the learning environment of Salinas et al. (2016), students performed their own actions on a graphical representation, which resulted in a change of the graph. For example, an action on a position–time graph led to a corresponding change in a velocity–time graph. These actions in the learning environment produced by the students are an essential component of doing mathematics (e.g., formulating and testing mathematical conjectures).

4.7 Attention capturing

Learning environments mentioning the mediating factor *attention capturing* as a learning facilitator refer to affordances in the learning environment that direct students' attention. In most learning environments, *attention capturing* implies directing students' attention toward important visual features of the graphical representation (e.g., Botzer & Yerushalmy, 2008; Deniz & Dulger, 2012; Nemirovsky et al., 1998; Russell et al., 2003). These visual features are especially prominent when the graph is displayed alongside the motion event, making specific changes in the motion event (e.g., changes in speed or changes in direction) directly observable to the student (e.g., Brasell, 1987). Moreover, because changes in motion are highlighted in the graphical representation, it becomes clearer to the student what the relevant aspects of the graph are that they have to attend to (Kozhevnikov & Thornton, 2006). In the learning environment of Holbert and Wilensky (2014), students explored the relationship between a car's velocity and acceleration using several game mechanics, which ultimately allowed students to relate the car's graphically represented speed with visual environmental cues. Other learning environments intend to capture the students' attention by making changes in the representations. Boyd and Rubin (1996) mention how the changes between video frames in their video environment drew students' attention to the differences between the frames. The learning environment of Noble et al. (2004) involves activities related to velocity, using different but related representations. These authors talk about the active nature of perception and how, in a familiar display, students are prone to recognize, and focus on, what is new.

4.8 Cognitive conflict

The final mediating factor we identified in the articles is *cognitive conflict*, which refers to students' conflicting conceptions. In general, this means that students, by means of a tool, are confronted with new information that conflicts with their existing knowledge or ideas (e.g., Simpson et al., 2006; Zajkov & Mitrevski, 2012). The student taking part in the learning environment of Nemirovsky (1994) had already some ideas about the concept of velocity and the meaning of velocity graphs. While progressing through the activities, she continuously had to deal with symbolic representations of her movement that did not make any sense to her. This conflict made her rethink the meaning of the graphs. Something similar is described in the article of Svec et al. (1995) who use the term disequilibrium to denote the conflict between the students' own beliefs and the gathered data. As opposed to cognitive conflicts within a person, also the cognitive conflict between students, initiated through (small) group discussions, is mentioned (Kuech & Lunetta, 2002). In a matching activity, students disagreed about the specific motion that would best match a particular graph, which resulted in cognitive conflict among the students. Other articles use the mediating factor *cognitive conflict* to indicate not only students' personal conflicting beliefs about a certain concept or phenomenon but also conflicting beliefs generated within different educational domains (i.e., mathematics and physics). Both Carrejo and Marshall (2007) and Woolnough (2000) focus on students' personal experiences with a concept and the concept taught within the domain of mathematics and the domain of physics. It appeared to be difficult for students to integrate similar concepts within these different areas, causing cognitive conflict (Woolnough, 2000).

5. Mediating factors within the four classes of mathematical learning environments

In this section, we elaborate on the relationship between the eight mediating factors and each class of embodied learning environments. The bar chart given in Figure 4 shows the occurrence of the perceived mediating factors per learning environment for each class.

In Class I – Immediate Own Motion, all eight perceived mediating factors were present, as opposed to seven mediating factors for Class II – Immediate Others/Objects' Motion, five mediating factors for Class II – Non-immediate own motion, and six mediating factors in Class IV – Non-immediate Others/Objects' Motion. Class I – Immediate Own Motion was the largest class, containing most learning environments (n = 34). Moreover, Class III – Non-immediate Own Motion, only containing four learning environments in total, mentioned five (different) mediating factors in total. Therefore, each mediating factor has a share of 20%.

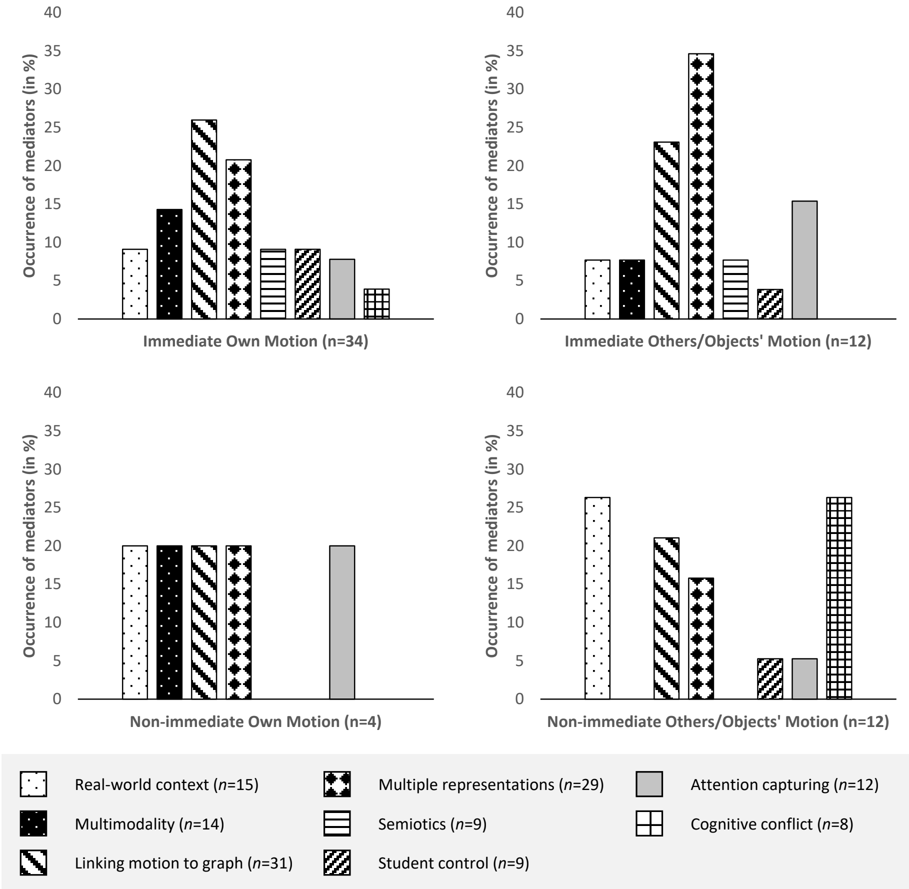


Figure 4. The occurrence of mediating factors per learning environment for each class of embodied learning environments

The mediating factors *linking motion to graph* and *multiple representations* are present in each of the four classes. Moreover, these two mediating factors have a substantial share within each class (between 16% and 35%). The mediating factors *real-world context* and *attention capturing* are present in each of the four classes as well (between 5% and 20%). *Multimodality* is mentioned as a mediating factor in learning environments present in Class I – Immediate Own Motion (14%), Class II – Immediate Others/Objects' Motion (8%), and Class III – Nonimmediate Own Motion (20%), but not in Class IV – Non-immediate Others/Objects' Motion.

The mediating factor *cognitive conflict* is rather present within Class IV – Non-immediate Others/Objects' Motion. *Cognitive conflict* has a substantial share within this class (26%), especially when compared with the other three classes, where *cognitive conflict* either is not perceived as a mediating factor (Class II – Immediate Others/Objects' Motion and Class III – Non-immediate Own Motion) or holds a minor share (Class I – Immediate Own Motion, 4%). *Student control* is mentioned relatively little as a mediating factor when compared with the other mediating factors (less than 9%). Something similar holds for the mediating factor *semiotics*. *Semiotics* is only present in the first two classes, representing learning environments containing an immediate translation of the embodied experiences (less than 10%).

6. Impact of embodied learning environments

For 26 articles, we conducted a more fine-grained analysis of the reported effects on students' learning in order to give an indication of the efficacy of each class of embodied learning environments. A summary of these 26 articles, including study design, tools, intervention length, description of activities, outcome measures, effect sizes, reported results, and quality appraisal score, are given in Appendix 2.3.

6.1 Effect sizes

We calculated effect sizes using the common standardized mean difference statistic Hedges *g* for all learning environments for which adequate statistical information regarding their effectiveness was provided ($n = 11$). A positive *g* value indicates that the experimental group has a higher outcome score than the control group, or that a posttest outcome score was higher than a pretest outcome score (e.g., in the case of pre–post comparisons, see also Borenstein et al., 2009). When articles made a comparison between groups and included a pretest, we calculated an adjusted effect size by subtracting the pretest effect size from the post-test effect size (Durlak, 2009). Additionally, we corrected for upwards bias for samples smaller than $N = 50$ (Durlak, 2009). Information regarding the statistical significance of the mean differences as provided by the authors was documented as well. Since there was variability in the experimental design, the data used, and outcome measures between the reviewed studies, we could not directly compare effect sizes or compute an overall effect size.

6.2 Learning outcomes and reported effects

All 20 learning environments within Class I – Immediate Own Motion reported positive learning outcomes. In most of these learning environments, comparisons were made with other embodied learning environments and/or a control condition

(n = 13), while for the other learning environments, pre–post comparisons were made (n = 7). In some articles, a statistical analysis was performed on the data (n = 6), all of which resulted in statistically significant differences. Among these articles, three reported at least one significant medium to large effect (g > 0.50; Ellis, 2010) on a measure associated with graphing motion (Brasell, 1987; Mokros & Tinker, 1987; Svec, 1999). For example, in Brasell (1987), viewing an immediate representation of one's own motion seems to be advantageous for pre-university students' understanding of distance–time graphs (g = 1.22 and g = 1.01), although students in the nonimmediate motion representation environment also outperformed the controls on distance– time graphs, but not on velocity–time graphs.

For five of the ten learning environments within Class II – Immediate Others/objects' Motion, comparisons were made with other learning environments (n = 3), or a control condition (n = 2). When comparisons were made with learning environments belonging to the first class (n = 2), students in the second class performed less well (e.g., Anastopoulou et al., 2011; Zucker et al., 2014). When a comparison was made with a learning environment belonging to the fourth class (n = 1), the reported learning outcomes were positive, but nonsignificant with a small effect (g = 0.29, Brungardt & Zollman, 1995). When comparisons were made with a control condition (n = 2), the results were either positive with one statistically significant medium to large effect (g = 0.81, Altiparmak, 2014), or a non-significant small effect (g = 0.20, Espinoza 2015). For the other five learning environments, pre–post comparisons were made. Three of these belonged to the same article (Kozhevnikov & Thornton, 2006) in which the reported learning outcomes were all positive, of which one showed a significant large effect (g = 2.09) of the intervention on students' understanding of force and motion and a moderate effect on spatial visualization ability (g = 0.62).

For both learning environments within Class III – N on-immediate Own Motion, a comparison was made with a learning environment belonging to the first class and/or a control condition, for which no strong results in favor of this class were reported (Brasell, 1987; Deniz & Dulger, 2012). In the article of Deniz and Dulger (2012), students seemed to benefit less from the intervention, which consisted of walking with a bottle of water with a hole in it, than the students who received an immediate graphical representation of their own movement on the screen of the computer, with a relatively small negative effect size (g = − 0.39).

Finally, in Class IV – Non-immediate Others/objects' Motion (n = 10), six learning environments were compared with another learning environment, belonging to Class I (n = 1) or Class II (n = 1), Class IV (n = 2) or being part of a control condition (n = 2). In the article making a comparison with a learning environment in the second class, a non-significant small negative effect was found (g = − 0.29; Brungardt & Zollman, 1995). In Mitnik et al. (2009), two learning environments belonging to Class IV were compared. In one condition, students made use of a real robot whereas in the other condition students watched a simulation of a robot. Results were in favor of the first condition with a statistically significant large effect (g = 0.76). The two learning environments in Roschelle et al. (2010) involved Smartgraphs software, one for Grade 7 and one for Grade 8 students. Students using this software seemed to score higher on the outcome measures than students in the control conditions, especially on outcome measures associated with reasoning about and representing change over time. The four learning environments in which pre–post comparisons were made all reported positive learning outcomes regarding outcome measures associated with graphing change, for example, students' ability to interpret and calculate slope (Woolnough, 2000).

7. Discussion and conclusion

7.1 Summary of the results

In this study, we evaluated 62 embodied learning environments supporting students' understanding of graphing motion, derived from 44 research articles. In order to know more about the embodied configurations of these learning environments, we developed a taxonomy in which embodied learning environments were juxtaposed on their degree of *bodily involvement* (own and others/objects' motion) and *immediacy* (immediate and non-immediate). This resulted in four classes of embodied learning environments: Class I – Immediate Own Motion, Class II – Immediate Others/objects' Motion, Class III – Non-Immediate Own Motion, and Class IV – Non-Immediate Others/objects' Motion. Our analysis showed that immediate own motion experiences were most common in the embodied learning environments; 34 out of the 62 learning environments belonged to this class.

According to the authors of the reviewed articles, a large variety of situations or characteristics of the embodied learning environments mediated the learning of students. After clustering these situations or characteristics, we recognized eight mediating factors, namely, *real-world context*, *multimodality*, *linking motion to graph*, *semiotics*, *attention capturing*, *multiple representations*, *student control*, and *cognitive conflict*. All these factors have their own specific role in how and why they mediate learning within such a learning environment. Each class of embodied learning environments has a particular embodied configuration and entails different combinations of mediating factors. Within some classes, particular mediating factors are more common than within other classes. For example, within Class I – Immediate Own Motion, the factors *multimodality*, *linking motion to graph*, and *multiple representations* were most common. This implies that the embodied configuration in this learning environment gives students the opportunity to deploy *multiple modalities* in order to link their own motion to the graphically represented motion and interacts with *multiple representations* throughout this process. Out of the eight mediating factors, two were mentioned in all four classes: *real-world context* and *multiple representations*. These mediating factors seem to be relevant for learning environments supporting students' understanding of graphing motion, regardless of their embodied configuration.

Our analysis of the 26 studies in which a comparison was made with another learning environment, with a control condition, or between pre- and post-measures to demonstrate the possible impact of embodied learning environments on students' learning revealed how embodied learning environments with only lower levels of bodily involvement, irrespective of the immediacy of the translation of the embodied experiences, seemed to be less effective. These findings imply that students' own motion experiences might be most beneficial for learning graphs of change and that learning by observing others/objects' motion is not as effective within this domain. Consequently, these findings give more weight to the mediating factors (e.g., *multimodality*, *linking motion to graph*, and *multiple representations*) present in Class I – Immediate Own Motion, than to the mediating factors in the other learning environments with different embodied configurations.

7.2 Limitations

For quality purposes, we only reviewed research articles published in peer-reviewed journals. As a result of this we might have missed studies on embodied learning environments that have been published elsewhere. Another limitation of our study is related to the challenges we met when classifying the embodied learning environments. Due to the fact that the learning environments often included multiple activities with different levels of *bodily involvement* and different levels of *immediacy*, for each learning environment we assigned the highest level of bodily involvement and immediacy that was found in the activities. Consequently, the learning environments with multiple activities were more often classified as whole bodily motion and immediate than as one of the lower levels of bodily involvement and immediacy. Moreover, regarding the levels of bodily involvement, to avoid further complexity of our review, we did not distinguish between observing the movement of a human model and observing the movement of an object, even though some studies have suggested that this is a relevant aspect to consider (Höffler & Leutner, 2007; Van Gog et al., 2009). These studies posit how non-human movements are less likely to trigger the mirror-neuron system as opposed to human movements. We, however, consider our topic of interest (graphs of objects/humans moving through space) to have such a clear and direct link to the human action repertoire of moving through space, that if it is an object such as a cart or a car moving that is observed, the same brain activation will take place as with the observation of a human (see also Martin, 2007). Another shortcoming of our review is that we did not limit our investigation of mediating factors to those factors that were empirically tested. To have a broad scope of possible mediating factors, we included each mediating aspect mentioned in the articles in our review and clustered them. Therefore, since these mediators are based on the self-reported information provided by the authors of the articles, the evidence for the found mediating factors is not equally strong. Interestingly though, regarding the mediating factors that were found, not many authors seem to link their study to *semiotics* while this factor can be seen as fundamental to these kinds of learning environments. Similarly, even though all learning environments seem to contain some aspects of *multimodality* and *multiple representations*, again not all authors see those aspects as essential or helpful elements of their learning environments, which is yet another interesting finding of our study. Therefore, more research is needed into the effects of these mediating factors, to determine whether, and to what extent, they are helpful in learning environments supporting students' understanding of graphing motion. A further limitation we would like to address is the wide variety of articles included in our review. This variety led to considerable variation in, for example, the level of education of the participants (ranging from primary education to higher education) and intervention length (ranging from two tasks to 20 class sessions). Even though including this wide variety of articles gives insight in the breadth and depth of the research conducted in this mathematics domain, it also makes it difficult to generalize the results found in the various articles. This especially pertains to the presumed mediating factors per class of embodied learning environments and to the respective efficacy of each class of embodied learning environments. To be more precise regarding the efficacy of embodied learning environments, for example regarding the influence of age, topic, or intervention length, more targeted research is necessary. Our review is also limited by the fact that the reviewed articles include few comparison studies and, as a result, our evaluation of the efficacy of embodied learning environments on student learning was rather constrained. A final limitation of our study relates to our choice to focus on the embodied approach to teaching graphing motion. This means that we did not address the full spectrum of a mathematical topic such as graphing change. Moreover, in addition to offering students an embodied learning environment other approaches can also be used for teaching students graphing motion. The embodiment approach is just one perspective, yet in our opinion, a valuable one.

7.3 Future directions

In this review, we presented a new taxonomy to classify embodied learning environments (see Figure 1). The taxonomy was based on two important embodied cognitive mechanisms: mirror neural activity and embodied simulation, which were operationalized by looking into the levels of *bodily involvement* and *immediacy*. This combination was not encountered in any of the already existing embodied learning taxonomies, although some taxonomies also considered levels of bodily involvement (e.g., Johnson-Glenberg et al., 2014; Skulmowski & Rey, 2018). By including *immediacy* as a further way to classify embodied learning environments, we developed a method to consider learning environments that deal with both immediate, or on-line, cognitive activities and non-immediate, or off-line, cognitive activities. To be precise, observing others/ objects can be theoretically aligned with off-line cognitive activity because the mirroring systems hypothesis and embodied simulation share some common characteristics (i.e., the sensorimotor circuits of the brain are re-used through embodied simulation when perceiving someone else performing a particular action). For that reason, we think our taxonomy to be especially relevant for embodied learning environments supporting students' understanding of graphing motion because when graphing motion, the bodily experience (e.g., the space in which you move) is often separated from the visualization of the motion (e.g., the graphical representation) in both space and time (e.g., Gallese & Lakoff, 2005; Rizzolatti et al., 1997), which is conveniently captured as immediacy in our scheme.

Although we think that combining *bodily involvement* and *immediacy* in our taxonomy provides a more precise insight in the understanding of embodied learning environments than previous taxonomies did, further research is necessary into the distinctions between the different quadrants of our taxonomy. Therefore, comparisons are needed of the different configurations of embodied learning environments to find out whether and how the embodied configurations of learning environments affect learning. Moreover, the specificity of this taxonomy, including the two embodied cognitive mechanisms, *mirror neural activity* and *embodied* *simulation*, presented along two dimensions, is crucial for categorizing learning environments concerned with dynamic representations. This expresses the need for different subfields of embodied learning research to take into account tailored taxonomies when systematically reviewing the embodiment literature.

We think of at least two ways to use our newly developed taxonomy. First, the classification of embodied learning environments on *bodily involvement* and *immediacy* may be used to inform and design new learning environments. The levels of bodily involvement and immediacy can be used as guidance for researchers and curriculum designers, providing them with concrete support on how to incorporate important theoretical distinctions (i.e., mirror neural activity and embodied simulation) for the design of embodied learning environments supporting students' understanding of graphing motion. Second, the taxonomy may also serve as a framework to categorize embodied learning environments not specifically related to the graphical representation of motion, but to other changing quantities as well, for example temperature. Temperature as well as motion can be directly experienced through the senses. Reed and Evans (1987) found how students' experiences with the mixing of water at two different temperatures helped them to perform a similar task in an unfamiliar domain (mixing acid solutions) and helped them to understand functional relations. Additionally, we suggest our taxonomy to be valuable for embodied activities outside the domain of graphing change, within, for example, the domain of number learning and number representation. As with graphs of change in the context of modeling motion, embodied activities within this domain often involve (whole) bodily activities. In the article of Fischer et al. (2011), students were given tasks that aim to train their basic numerical competencies by making whole bodily movements. In this respect, kindergartners saw a number on a digital blackboard after which they had to jump left or right depending on whether this number was smaller or larger than a standard value. Here, the experience is immediate since students are immediately confronted with the position of the numbers on the number line.

In our review, we included a wide variety of learning environments, originating from different traditions of views on cognition. Namely, besides research taking an explicit embodied cognition perspective (e.g., Botzer & Yerushalmy, 2008; Robutti, 2006), we also incorporated research investigating, for example, real-time versus delayed aspects of video learning within the area of kinematics graphing (e.g., Brasell, 1987). This has given us the opportunity to re-evaluate existing research from an embodied cognition perspective. These various theoretical perspectives have led to eight mediating factors. We propose that the mediating factors identified in our review can be seen as situated between the theoretically grounded embodied configurations of learning environments, that is, the allocation of learning environments on *bodily involvement* and *immediacy*, and the learning that takes place. For example, the mediating factor *real-world context* stems from viewing cognitive activity as grounded in the real world (e.g., Morse & Ziemke, 2007; Wilson, 2002), by which a connection is established between real-world activity and the intended concept. In the case of graphing motion, there should be a link between the real-world activity of motion and the graphical representation of this motion as the intended concept. The mediating factor *linking motion to graph* is related to the mapping mechanisms that structure the abstract mathematical concept by means of bodily experiences (Font et al., 2010) and through (perceptual-grounding) processes like embodied simulation (Barsalou, 2010). *Multimodality* as a mediating factor is an essential aspect of embodied cognition as well. A multimodal view on cognition encompasses the idea that conceptual knowledge depends upon a rich interrelated coordination of modality-specific systems (Barsalouet al., 2003). This is also where the mediating factor *semiotics* plays a role, as "mathematical ideas are conveyed using rich, multimodal forms of communication, including gestures and tangible objects in the world" (Nathan et al., 2017, p. 1499). Also, the mediating factor of *attention capturing*, as the processing of perceptual information linked to sensory motor experiences, is necessary to trigger (intentionally or unintentionally) a response (e.g., Gibson, 1979; Grafton, 2009). Next, in many learning environments, students are given *multiple representations* of their bodily experiences, letting them perceive multiple variations of the same concept in relation to their movements. This can help them in developing a solid understanding of graphically represented motion. Moreover, *student control* of the bodily experience implies subsequent agency over the resulting graphical representation. According to Johnson-Glenberg (2018), this is a feature of (virtual reality) embodied learning environments and a function of students' ability to manipulate content. And finally, *cognitive conflict* might arise when the graphical representation contradicts expectations (e.g., misalignment between motion and students' ideas about the shape of the graph). Even though the eight mediating factors can be fitted within embodied cognition theory, it is important to note that not every mediator will be readily associated with this theory. For example, the mediator *cognitive conflict* will more likely evoke cognitivist ideas such as conflicting conceptions when interacting with other students. We think that for this mediator—as well as for the other mediators—to have a clear link with embodied cognition theory, the workable element has to be located at the sensorimotor level, for example, as was primarily the case in the included articles, when a student walks in front of a motion sensor and is then confronted with a graphical shape contradictory to what they expected.

The wide variety of the found mediating factors can be a function of the focus we had in our study on the domain of graphing motion. Therefore, more research is needed to establish the genuine role of each mediating factor within the learning environments and their link with embodied learning. One concrete suggestion for doing so is investigating the mediating factors present in embodied learning environment within different mathematical domains. This would provide more specific information about mediating factors that are related to the embodied configuration of learning environments (i.e., domain-general mediating factors) and mediating factors that are related to a particular domain (i.e., domain-specific mediating factors). When we know more about the concrete working mechanisms of each mediating factor, this might ultimately lead to a better understanding of embodied learning within this (and other) mathematics domain.

7.4 Concluding remarks

The ubiquity of embodied activities in mathematics and science learning environments raises important questions regarding the embodied configurations of these learning environments and in what ways and to what extent these bodily experiences are helpful for the learning process (e.g., Nathan et al., 2017). We hope that the insights regarding our classification of these learning environments and the eight factors mediating learning within the four classes of learning environments will provide researchers and curriculum designers with an integrative approach for designing embodied learning environments. Furthermore, we hope that this will lead to new insight in how mathematics and science activities including bodily experiences can be used for mathematics teaching and learning.

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CHAPTER 3

Supporting primary school students' reasoning about motion graphs through physical experiences

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Abstract

Reasoning about graphical representations representing dynamic data (e.g., distance changing over time), including interpreting, creating, changing, combining, and comparing graphs can be considered a domain-specific operationalization of the general 21st century skills of creative, critical thinking and solving problems. This paper addresses the issue of how these 21st century skills of interpreting and creating graphs can be supported in a six-lesson teaching sequence about graphing motion. In this teaching sequence, we focused on the potential of an embodied learning environment to facilitate the development of primary school students' reasoning about motion graphs by having primary school students (9-11 years) "walk" graphs in front of a motion sensor to generate distance-time graphs. We asked: *How does students' reasoning about graphing motion develop over a six-lesson teaching sequence within an embodied learning environment?* Based on the collected data, we examined changes in students' level of reasoning on graph interpretation and graph construction tasks using a repeated measurement design. Additionally, we present two teaching episodes showing instances of how perceptual-motor experiences during the lessons aided students' reasoning about graphical representations of motion. Results show that students went from iconic understanding towards understanding in which they reasoned based on one or two variables when interpreting and constructing graphical representations of motion events. At these higher levels of reasoning these students showed understanding of modelling motion in line with the intended 21st century skills of generating, refining, and evaluating graphs.

Keywords: Distance-time graphs, Embodied cognition, Graphing motion, Motion sensor technology

1. Introduction

Twenty-first century competences include the need for equipping students with an integrated set of knowledge, skills, and attitudes, for example being creative, innovative, and communicative (see the categorization of 21st century skills on http://www.atc21s.org). Such competences are typical of STEM learning in general and mathematics learning in particular (English, 2016; Honey et al., 2014). The capacity to deal with large amounts of new information through media and technology is becoming increasingly important in today's society. This includes the ability to use graphs to produce, present, and understand complex dynamic information (Binkley et al., 2012), as well as making flexible and creative use of representations. Reasoning about graphical representations, making connections between the variables on the horizontal and vertical axes, such as time and distance, creating graphical representations, critically evaluating data represented in graphs, using graphs to communicate findings to others, and also, making comparisons within and between graphs, are important components of higher-order thinking skills within science and mathematics (e.g., Boote, 2014). Promoting students' fluency with graphs, as well as stimulating related higher-order reasoning can therefore be a fruitful way to incorporate 21st century skills within mathematics classrooms. This is also in line with the framework for robust learning, which suggests that domain-specific learning environments are needed to support students in "becoming knowledgeable, flexible, and resourceful disciplinary thinkers" (Schoenfeld, 2016, p. 3). There is general consensus (see also NCTM, 2000) that laying a strong foundation for these higher-order thinking skills should start in primary school and that this also applies to the introduction of graphs (e.g., Friel et al., 2001).

2. Background of the study

2.1 Reasoning about graphical representations

Similarly to *number sense* (e.g., Resnick, 1989) and *symbol sense* (Arcavi, 1994), students have to acquire a *graph sense*, which "develops gradually as a result of one's creating graphs and using already designed graphs in a variety of problem contexts that require making sense of data" (Friel et al., 2001, p. 145). Graph sense can be considered as representing a way of thinking, rather than as a specific set of rules and skills that can be transmitted to others (Friel et al., 2001). Such graph sense includes the interpretation or construction of graphs and the ability to distinguish between discrete and continuous representations. It also includes the ability to recognize the meaning and significance of the represented variables, the slope, and the more general visual characteristics of the graph (e.g., Friel et al., 2001; Robutti, 2006). In this article, we focus on graphs representing the bivariate relationship of *distance* changing over *time*. In such graphs, varying the scale of the graph changes the shape of the graphically represented motion, which offers opportunities for students to reason about the relationship between the represented variables and the (qualitative) understanding of slope (e.g., Nemirovsky et al., 2013; Zaslavsky et al., 2002). Even when students are not focusing on numerals and symbols, they can develop graph sense (see also Krabbendam, 1982). This graph sense reflects the ability to look at the represented information at a qualitative global level, becoming sensitive to and focusing on a general trend in the graph itself (Leinhardt et al., 1990). More specifically, graph interpretation on a global scale implies "looking at the entire graph (or parts of it) and gaining meaning about the relationship between the two variables and, in particular, their pattern of co-variation" (Leinhardt et al., 1990, p. 11), whereas graph construction implies the visualization of a certain relationship as representing shapes of trends on the graphs' axes (Matuk et al., 2019).

In summary, graph sense equals the development of a robust understanding enabling a student to overcome most of the difficulties often associated with making sense of graphs. One such difficulty is iconic interpretation of a graph, which occurs when students connect the overall shape of the graph with visual characteristics of the situation represented in the graph. A common example of iconic interpretation would be to interpret a rising line in a distance-time graph as an actual representation of a physical situation such as a car driving up a hill (see also Clement, 1985). Resisting the temptation to interpret a graph by its superficial characteristics might also equate to aspects of critical thinking.

2.2 Developing graph sense: A 21st century skill

The development of graph sense is an important component of 21st century learning. It includes the skills of interpreting and creating graphs, but also more generally, learning to use flexibly and creatively and evaluate critically graphical representations not earlier encountered, and the ability to apply this understanding in different problem situations. Developing graph sense can be challenging even for university students (e.g., Brasell & Rowe, 1993). Nevertheless, younger students already possess the ability to reason with, and construct (graphical) representations of dynamic situations. For example, a study by DiSessa et al., (1991), investigated the ability of students aged 11 to 12 to generate, critique, and refine representational forms. These authors showed how these students developed understanding of different kinds of representations, by drawing graphs of a given motion situation. Here, students started with discrete representations of a motion event before moving on to continuous representations of that motion event. The instructional approach used in this study can be considered as emergent modelling, which implies that students make a specific "model of" a situation which at a later stage can be used as a "model for" formal mathematical reasoning (Streefland, 1985). For example, students are provided with a task about a particular motion situation for which they have to develop a graphical solution which is situation-specific. The produced graph can be seen as a model of the original motion situation. During this so-called reinvention process, graphs emerge from the students' own activities. When looking at a graph representing different motion situations (or with different represented variables), students can apply their acquired understanding of the graph as a mathematical model of a particular motion situation as a model for reasoning about the represented variables in the graph (Doorman & Gravemeijer, 2009). Therefore, students' own inventions (or close approximations), based on experiences with a real-world phenomenon are a powerful starting point on which to build conventional graphing (DiSessa et al., 1991).

Researchers have often designed tasks involving software environments linking animations and graphs (Roschelle et al., 2000) or motion sensor technology, where a link is forged between a student's own motion and the corresponding graphical representation. Following Mishra and Henriksen (2018), taking advantage of such technology is an important aspect of learning in the 21st century, since "technology can powerfully change how and what we teach" (p. 15). For example, students can manipulate specific elements of a graphical representation by means of graphing software. In a fairly easy way, technology can show how zooming in on the graphs' axes might affect the graphical representation of a situation but does not change the situation itself (Godwin & Sutherland, 2004). This offers students more opportunities to not only generate, but also refine and critically evaluate graphical representations. When using motion sensor technology students not only learn to use technological tools but are also stimulated to test their hypotheses about the graphs produced by these tools. Nemirovsky et al. (1998) showed how two students (aged 9 and 10) became fluent tool users when using a computer-based motion detector for creating distance-time graphs of their own movements. Throughout the activities the students developed ways of seeing the graphical representation as a representation of—and as a response to—their bodily actions. Initially, the students experienced how distance-time graphs have some specific idiosyncratic traits (e.g., the line in the graph cannot go backwards), while eventually, the graphical representation became an object they understood and were able to reason with. The use of motion sensor technology has proven to be powerful in offering students a direct experience of their own bodily movement (e.g., Deniz & Dulger, 2012; Mokros & Tinker, 1987). Through the support of motion sensor technology, students' perceptual-motor experiences are employed to learn graphing conventions (e.g., Arzarello et al., 2007) and thus offer opportunities to connect "the mathematics of change to its historical and familiar roots in experienced motion" (Kaput & Roschelle, 2013, p. 20). This linking between a physical experience and the abstraction of that experience as a graph closely aligns with an embodied cognition approach.

2.3 Embodied cognition

Embodied cognition theory posits that both concrete and abstract higher-order thinking and reasoning, like language and mathematics, are embedded in sensorimotor schemes that one can acquire through physical interactions of one's body with the environment (see also Pouw et al., 2014). Hence, learning takes place by enacting knowledge or concepts through bodily activities. This entails that gestural and other bodily activity are fundamental constituents of cognition (e.g., Radford, 2009b). When adopting a moderate position towards embodied cognition (Goldman, 2012), it is assumed that even when concrete actions and perceptions are not readily available, previously acquired action-perception structures can be simulated, in terms of re-use or re-activation, and may serve the formation of new (abstract) ideas and thoughts (e.g., Barsalou, 2010).

2.3.1 Embodied learning environments

Following this idea of embodied cognition, developing graphical reasoning has often been investigated in learning environments enriched with direct physical experiences. In these embodied learning environments, bodily experiences are an essential part of the learning activities (e.g., Johnson-Glenberg et al., 2014; Skulmowski & Rey, 2018). In the context of graphing motion these bodily experiences can be manifold and range from making whole-, or part-bodily movements, to observing someone, or something else, moving (Duijzer et al., 2019, see *Chapter 2* of this thesis). In some studies, the focus has (inter alia) been on students' use of gestures and their supportive role in expressing ideas and supporting learning graphical reasoning with motion sensor technology (e.g., Radford, 2009a, 2009b; Robutti, 2006). Radford (2009a, 2009b) focused on the semiotic process in which signs, words, and gestures all work in unison to develop students' graph sense. In particular, the work of a small group of Grade 8 students showed that throughout the graphing activities, including a motion sensor, the students slowly abandoned their iconic interpretation of the graph and reformulated their interpretation in terms of the movements present in the graph (Radford, 2009a). Throughout this process students pointed towards characteristics of the graph, data points, lines, and axes. They also made gestures expressing the shape of graphs and indicating motion represented in the graphs (see also Robutti, 2006). Other studies more specifically addressed the role of whole bodily motion in learning activities, for example by looking into how perceptual and motor activities merge when students are engaged in a mathematics activity (Nemirovsky et al., 2013). Similarly, Ferrara (2014) presented two teaching episodes, focusing on a 7-year-old student's perceptual, bodily, and imaginary experiences when walking in front of a motion sensor. This student became able to connect his movements with the graph(s) representing his movements and, a year later, was also able to communicate his understanding of the graph to others.

3. The current study

Although the aforementioned studies illustrate the importance of perceptual-motor experiences when reasoning about graphs of motion, they all have a rather laboratory character. These studies presented in-depth analyses of the development of a few students (see also Nemirovsky et al., 1998; Robutti, 2006). It is unclear whether the experiences of those few students could also be elicited in a whole classroom setting, and if so, to what extent this engenders the development of higher-order reasoning, and the potential of these activities to stimulate 21st century skills. In the context of graphing motion, we operationalize the 21st century skills of being creative and thinking critically as students' ability to generate, refine, and evaluate graphical representations as well as making flexible and creative use of representations. Following embodied cognition theory, we assume that students – when interacting with motion sensors and collaborating with their peers – start to reason about the connection between their bodily movements and the representation of those movements as a graph. Thus, developing graph sense might be a fruitful way to integrate 21st century skills in the mathematics classroom.

Most studies conducted in the primary grades using motion sensor technology focused on instructional activities concerning graph interpretation, for example, interpreting a graph as a response to one's own movements in front of the motion sensor (e.g., Nemirovsky et al., 1998). Moreover, in studies in which students' developing understanding about motion graphs was actually measured, tests often include graph interpretation items, using a multiple-choice format (see also Deniz & Dulger, 2012). Graph sense however, does not contain only graph interpretation, but also graph construction (Friel et al., 2001). Research has shown that students of all ages experience difficulties with both graph interpretation and graph construction. Moreover, the skill to present data and communicate this data to others is vital for many future professions (e.g., Leinhardt et al., 1990). Therefore, it is important that both interpretation and construction skills are addressed in lesson activities, as well as afterwards on lesson-specific tasks, and that students are given the opportunity to show their reasoning when interpreting and constructing graphs.

In the current study, we investigate the development of primary school students' understanding of – and reasoning about – motion graphs in a whole class teaching and learning setting. Our focus is on graphing change in the context of modelling motion. To elicit students' reasoning about motion graphs, we developed an embodied learning environment consisting of a six-lesson teaching sequence. In this embodied learning environment, students made distance-time graphs of their own movements by moving in front of a motion sensor. As such, we expected that students would no longer consider the resulting graphical representation as a standalone, isolated entity, but as a reference to their own bodily experiences. Students were given ample opportunities to reason about the resulting graphical representations—throughout the lessons and afterwards on lesson-specific graph interpretation and graph construction tasks—enabling them to communicate their understanding about the graphs. More specifically, we answer the following research question:

> *How does students' reasoning about graphing motion develop over a six-lesson teaching sequence within an embodied learning environment?*

In answering this research question, we first investigate the development of students' reasoning over the six-lesson teaching sequence by looking at their performance on graph interpretation and graph construction tasks. We then provide an in-depth analysis of how the embodied learning environment might have supported the students in their ability to generate, refine, and reason about graphical representations of motion. For this, we particularly focus on one student and her interactions with the motion sensor technology and her peers.

4. Method

4.1 Participants

To answer the research question, the teaching sequence was taught in three primary school classes (Grade 5; 9–11 years) in the area of Utrecht, The Netherlands, between October 2016 and June 2017. Only schools sharing similar demographics were contacted. Classes were chosen based on teachers' willingness to participate. Participation was voluntarily. A total of seventy students participated in this study; 28 girls (40%) and 42 boys (60%) ($M_{\text{age}} = 10.4$, $SD = 0.45$). For seven students (out of 77) we did not get parental consent to use the data. The research was conducted in accordance with the ethical guidelines of the Institutional Review Board of the faculty of Social and Behavioural Sciences at Utrecht University. All students took part in the teaching sequence as part of their regular classroom instruction.

4.2 Procedure

Each lesson took about 50 minutes and was taught by the first author supported by a teaching assistant. The lessons were given weekly in 6 consecutive weeks. In Lessons 3–5, in the first part of the lessons, the class was split into four small groups to allow students to work individually with the motion sensors. These small groups were supervised by research team members who followed a lesson script in order to ensure implementation fidelity. The small-group activities took 30 minutes. The remainder of the lessons was given to the entire class. After each lesson, students responded to two lesson-specific tasks: one task related to graph interpretation and the other to graph construction. A repeated measurement design was used: every second task was also provided to the students after the subsequent lesson (i.e., the second task provided after Lesson 1 was the same as the first task after Lesson 2).

4.3 Data collection

We collected data from various sources: videotaped lessons, student material, and student responses to lesson-specific tasks after each lesson. Within one class we videotaped a small group of students (n = 7) from whom video permission was obtained. Video sections in which students were engaged with the motion sensor technology were identified and transcribed. We chose to mainly focus on one student, Celine. She showed a progression over the lessons, which was representative for many of the other students. Throughout the lessons Celine showed high motivation, creativity, and attention towards the activities. She was also able to explain her thinking fluently. Based on her score in the mathematics test of the Dutch student monitoring system and the population data of this test (Cito LOVS; Janssen et al., 2010), she can be considered a slightly above average student. We present two teaching episodes, consisting of interactions between Celine and the students with whom she worked.

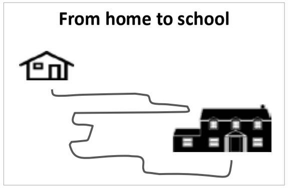
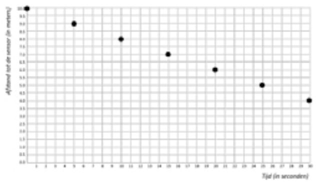
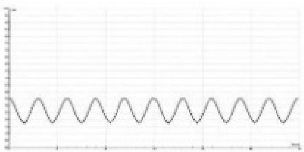
4.4 Materials

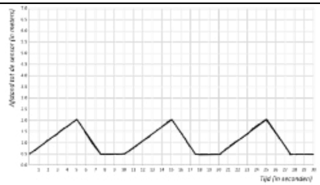
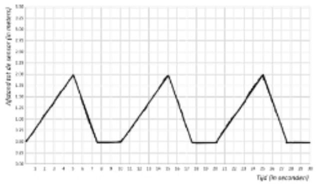
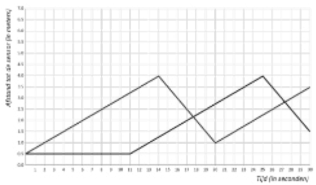
4.4.1 Teaching sequence on graphing motion

In the first lesson, students developed their own representations of a motion situation. In the following lessons, situations in which distances were measured at particular moments were proposed. Finally, students solved problems by modelling dynamic data and reconstructing events from continuous graphs. The problems or tasks did not explicitly ask them to perform calculations, the required reasoning was mainly global and qualitative (see also Leinhardt et al., 1990). In particular, the teaching sequence started from informal graphs to working with discrete graphs and finally continuous graphs. During this trajectory the concepts of scaling on the graphs' axes, qualitative understanding of slope, and qualitative methods of graph interpretation and construction were addressed. See Table 1 for an overview of the teaching sequence, examples of the types of graphs presented in each lesson, and a description of key activities.

Table 1

Overview of the six-lesson teaching sequence on graphing motion

Lesson		Main topic	Key activities
1. Motion: reflecting and representing		Informal graphical representations	*Reason with variables and construct representations of a real-world situation*
2. From discrete to continuous graphs		Measuring distance	*Measure distance in discrete intervals and continuously, and reason about differences between discrete and continuous graphs*
3. Continuous graphs of “distance to” (1)		Generate, refine, and reason with continuous graphs	*Coupling specific movements to their representation as a line in the graph* *Coupling a concrete situation to a graphical representation*

4. Continuous graphs of "distance to" (2)		Generate, refine, and reason with continuous graphs	*Coupling specific movements to their representation as a line in the graph* *Investigating how speed is represented in the steepness of slope*
5. Scaling on the graphs' axes		Reason about the relationship between two variables through scaling	*Construct graphs with different scales on the axes*
6. Multiple movements and their graphical representations		Generate, refine, and reason about simultaneous movements and their representation as a graph	*Critically evaluate points of intersection and their meaning*

In addition to the activities per lesson, students also received a problem-solving task that spanned the entire teaching sequence. A description of this problem-solving task and how it is related to the respective lessons of the teaching sequence can be found in Appendix 3.1.

In order to provide an immediate link between the dynamic situation of moving in space and its graphical representation, we used a motion sensor which directly represented the dynamic situation as a distance-time graph. As such, during the lessons, students' embodied experiences of moving in front of the motion sensor played a central role. The motion sensor was used in all lessons except Lesson 1. In Lesson 2 and Lesson 6 most students observed other students who were walking in front of the motion sensor. From Lesson 3 to Lesson 5 all students walked in front of the motion sensor individually. The craggy graphs created by the motion sensor offered students opportunities to reason about and critically reflect on these graphs, to separate essential elements from noise (e.g., neglecting the vertical strokes caused by someone being out of the sensor), and to discuss how these elements relate to the movement in front of the sensor (see also Figure 4, right panel). Throughout the lessons the teacher coordinated the small group and classroom discussions by asking open-ended questions such as "What do you think would happen if…?", "Why do you think so?", "Can you think of more ways to achieve a similar result?", "Do you see a pattern?", thus stimulating students' thinking and argumentation but leaving them free to come up with their own ideas.

4.4.2 Motion sensor technology

We made use of two ultrasonic €Motion sensors, developed by CMA, in conjunction with Coach6 Software (Heck et al., 2009). The tool was programmed to provide a single graph in which the distance between the sensor and the nearest object was displayed over a period of 30 seconds. The graph was projected either on the digital classroom board (Lesson 2 and 6) or on the screen of laptop computers (Lesson 3-5). When a student moved backwards, the distance between the sensor and the student increased, when a student moved forwards, this distance decreased.

4.4.3 Lesson-specific tasks: graph interpretation and graph construction

In the lesson-specific tasks all provided graphs were distance-time graphs (with and without measurement units) describing the motion of a person or object. The graph interpretation tasks consisted of a graph for which the students had to decide whether three different situations or graphs could fit the given graph, see Figure 1. The graph construction tasks consisted of a description of a motion situation including multiple variables on the basis of which students had to draw a graph representing that situation, see Figure 2. For each graph interpretation task students also had to answer an open-ended question, which probed them to make their reasoning explicit.

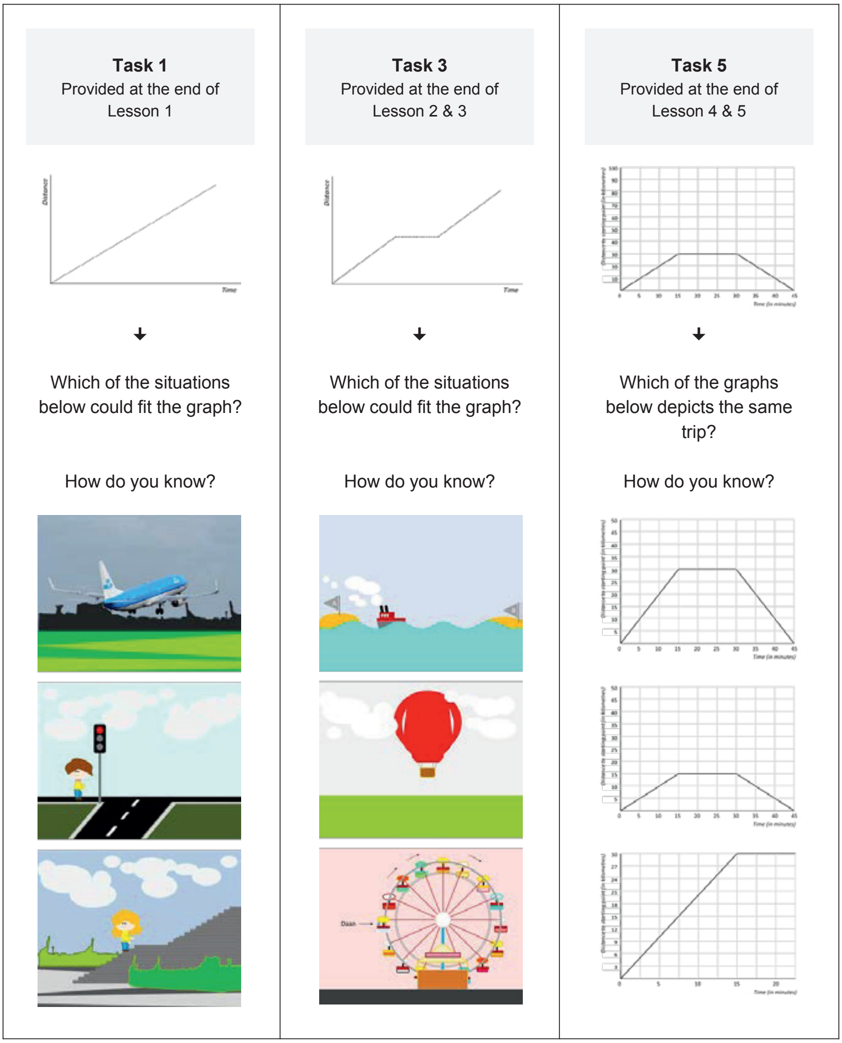


Figure 1. Lesson-specific graph interpretation tasks

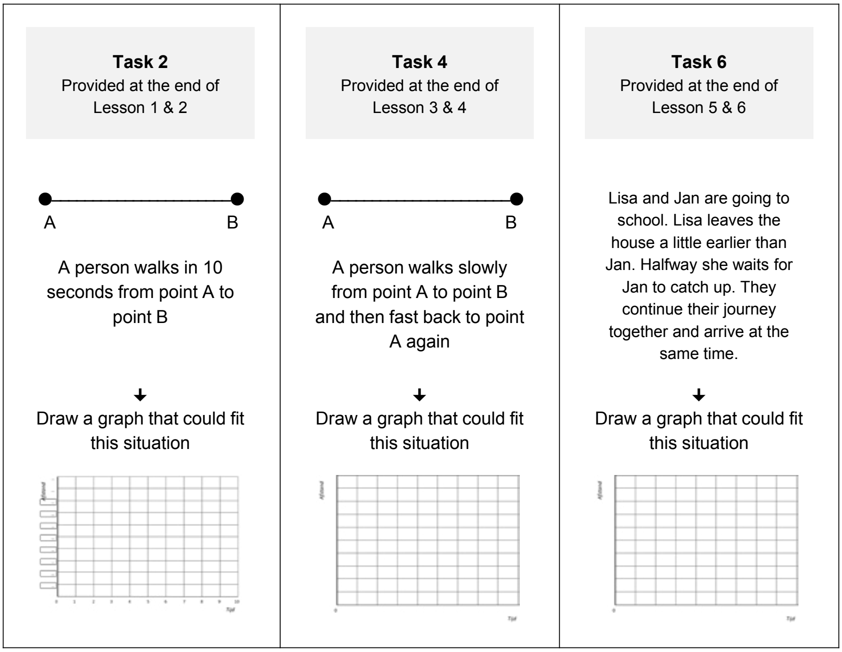


Figure 2. Lesson-specific graph construction tasks

4.5 Data analysis

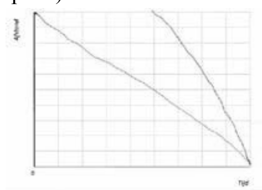
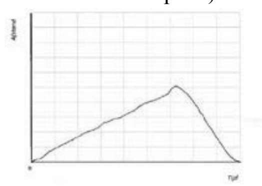
4.5.1 Coding scheme for students' level of reasoning about graphs

Students' responses to the lesson-specific tasks were analyzed to investigate the development of their level of reasoning over the teaching sequence. These responses and explanations were categorized by means of a coding scheme. The development of this coding scheme occurred in conjunction with the analysis of the qualitative data. First, research team members individually classified the student responses and later these classifications were compared and if necessary revised. After several discussion and revision rounds, we agreed upon having four main codes that enabled us to categorize student responses to the lesson-specific tasks of graph interpretation and graph construction, from less to more sophisticated: unrelated reasoning, iconic reasoning, single variable reasoning, and multiple variable reasoning (for details and examples of these codes, see Table 2). In this categorization, the 21st century skills of generating, refining, and evaluating graphs are captured by focusing on the extent to which students included the information about the different variables in their reasoning. To validate our categorization into iconic reasoning, single, and multiple variable reasoning, we compared this process to the categories used in several other studies on graphical reasoning. For example, Lingefjärd and Farahani (2017) identified three categories: (1) intuitive and iconic interpretations, (2) scientifically grounded interpretations and (3) a combination with influences from both 1 and 2. Similarly, Johnson et al. (2019) included an iconic category and three other categories: motion of objects, individual quantities, and relationship between quantities. Our categories resemble these earlier categorizations.

The three different graph interpretation tasks were all of similar difficulty: the same type of reasoning led to the correct answer and explanation. In the three graph construction tasks, students had to represent a given situation graphically. The difficulty of these graph construction tasks gradually increased in the sense that the situations students had to model became more complex. We chose this approach because during graph construction, a student has to generate something that is not there yet (Leinhardt et al., 1990). When interpreting a graph, a student has to evaluate and recognize elements that are already apparent. Therefore, our graph interpretation tasks could, from the start, be more complex.

Table 2

Coding scheme used for students' level of reasoning on the graph interpretation and graph construction tasks

		Description of students' reasoning	
		Graph interpretation	Graph construction
Level of reasoning	Code	*Example*	*Example*
		Student reasons...	**Student constructs graph...**
Unrelated reasoning	R0	...without referring to the graphical representation or the motion event *"It is a guess."*	...without taking the description of the motion event into account 
Iconic reasoning	R1	...on the basis of the shape of the graphical representation or superficial characteristics of the motion event *"The staircase has almost the same shape as the graph."* [Task 1C]	...on the basis of superficial characteristics of the description of the motion event 
Single variable reasoning	R2	...on the basis of a single variable (distance or time or speed) *"The boat moves forwards and the graph as well, if the graph goes up it means you go forward."* [Task 3A]	...taking into consideration a single variable (distance or time or speed) 
Multiple variable reasoning	R3	...on the basis of multiple variables (distance and/or time and/or speed) *"No, because the graph represents time and distance, it would if it represents the distance upwards!"* [Task 1C]	...taking into consideration multiple variables (distance and/or time and/or speed) 

Note. The complete coding scheme, including examples of student responses per task, can be found in Appendix 3.2 (graph interpretation) and Appendix 3.3 (graph construction).

4.5.2 Teaching episodes

To investigate how students' perceptual-motor experiences in front of the motion sensor aided their reasoning about the graphical representations, we analyzed two teaching episodes in more detail. We focus particularly on Lesson 3 (Episode 1) and Lesson 5 (Episode 2). In these lessons crucial moments in which the relation between students' experiences and their reasoning in terms of generating, refining, and evaluating graphs were apparent could be distinguished.

Lesson 3 marks the beginning of the critical evaluation of how motion is represented in a continuous graph. Students encounter situations in which iconic interpretations lead to incorrect conclusions. They experience that particular aspects of movements relate to features of the graph (e.g., relating the direction of a motion to an increasing distance from a given point), which offers opportunities for the students to extend, refine, and develop reasoning by challenging their pre-existing conceptions of motion graphs. In this lesson, students have their first individual experiences in front of the motion sensor.

In Lesson 5 the element of scale is explicitly introduced. Students interpret the shape of the graphs in relation to the graphs' features and connect points in the graph to distance and time values on the axes and to locations in space. Additionally, making the students sensitive for how changes in scale on the graph's axes relate to what the graph will look like challenges students' critical thinking skills. Again, in Lesson 5, students enact motion in front of the sensor individually.

The two teaching episodes took place in the same class. We zoom in on one particular student, named Celine. At the end of the first and second lesson Celine had only shown instances of iconic reasoning. In our description, we took a micro-analytic approach (see also Nemirovsky et al., 2013), focusing on Celine's reasoning when interacting with the motion sensor, including her gestures and movements, to get a good grasp of her developing understanding. We also describe the actions of her peers, thus showing how she, in interaction with her classmates, comes to correctly interpret the concepts of time, distance, and speed as represented in the graphs.

5. Results

5.1 Students' level of reasoning over the teaching sequence

Students' answers on the graph interpretation and construction tasks improved over the teaching sequence, as shown by more frequent occurrences of high levels of reasoning (Level R2 and R3) towards the end of the teaching sequence. Figure 3 shows the proportion of students with a particular level of reasoning for both graph interpretation and construction tasks. For Task 1, the lesson-specific task administered after the first lesson, students' reasoning could be qualified as iconic reasoning (Level R1: 55%). That is to say, reasoning in which the graphical representation was interpreted as an analogous depiction of the represented situation, for example on Task 1A "In the graph it goes up and here the airplane also goes up". A smaller proportion of the students reasoned while taking into account a single variable (Level R2: 40%), for example on Task 1A "He travels a distance and then he continues", whilst only 3% of the students showed reasoning in which they referred to the graph as representing a bivariate relationship (Level R3), for example on Task 1A "Because he travelled a certain amount of distance within a certain amount of time".

Over the course of the teaching sequence the frequency of students' iconic reasoning on graph interpretation gradually decreased (from about 50% in Lesson 2 to 9% in Lesson 5), whereas the frequency of students' single variable reasoning increased towards the third lesson (from about 46% in Lesson 2, to 78% in Lesson 3) and then decreased in Lesson 4 and again slightly increased towards Lesson 5 (62% in Lesson 4 and 66% in Lesson 5). Examples of students' single variable reasoning were responses such as "The distance increases just as with the hot air balloon and this is shown in the graph" (Task 3B) and "The one in the top is stretched more but there it is 30 kilometer and here as well" (Task 5A). The decline in the frequency of single variable reasoning co-occurred with an increase of the frequency of students' multiple variable reasoning (from less than 7% in Lesson 1-3 to 25% in Lesson 5). An example of such reasoning was on "He moves till 30 kilometers in 15 minutes, then waits, and after 30 minutes back again" (Task 5A). Students' reasoning on the graph construction tasks showed a comparable pattern. From the first lesson onwards most students were able to incorporate at least two variables correctly when creating a graph of a given motion situation. This skill continued to increase towards Lesson 6 (from about 54% in Lesson 1 to 78% in Lesson 6). This increase co-occurred with a gradual decrease on iconic reasoning from Lesson 3 onwards (Level R1: from about 38% in Lesson 3 to 0% in Lesson 6).

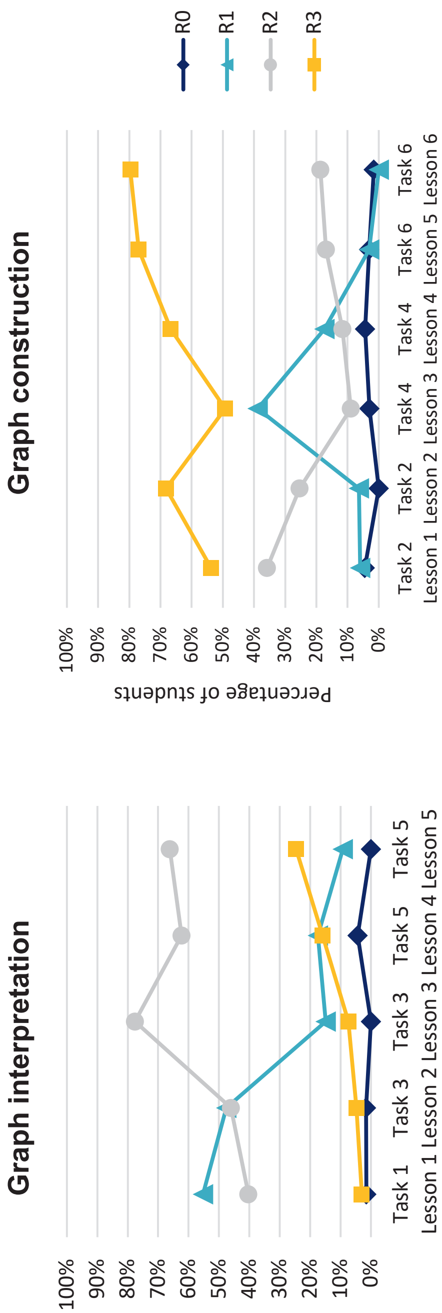


Figure 3. Students' level of reasoning ($N_{\text{Lesson 1}}$=67, $N_{\text{Lesson 2}}$=63, $N_{\text{Lesson 3}}$=67, $N_{\text{Lesson 4}}$=69, $N_{\text{Lesson 5}}$=65, $N_{\text{Lesson 6}}$=64)

5.2 Perceptual-motor experiences and developing graph skills

When looking at the changes in students' level of reasoning on the lesson-specific tasks, Lesson 3 and 5 appear to be benchmark lessons. First, for graph interpretation, there were more students who reasoned based on a single variable on Task 3 after Lesson 3 than after Lesson 2 (+32%). In the first teaching episode, we show students' interactions with the motion sensor and related reasoning in the third lesson. Second, from Lesson 4 till 5 (Task 5), there were fewer students who reasoned based on single variable (–5 %). Additionally, there were more students who reasoned in a covariational manner, taking into account multiple variables when interpreting the graph(s) (+18%). During the fifth lesson, students moved in front of the motion sensor individually. Furthermore, during this lesson, more emphasis was placed on the graphs' axes (e.g., focusing on scale and intervals on the x-axis and y-axis), prompting students to be critical of how a particular scaling on the graph's axes would change its appearance, which Task 5 assessed more explicitly. In Episode 2 we show a short excerpt of an interaction between a few students to show the kind of reasoning during the fifth lesson.

5.2.1 Episode 1: *Walking a given graph*

In Lesson 3, after some exploration of the motion sensor, students had to replicate the distance-time graph depicted in Figure 4 (left panel), to do this they had to interpret the graph in terms of distance from the sensor, where and when to start and stop, and about time, because each wave takes a certain amount of time.

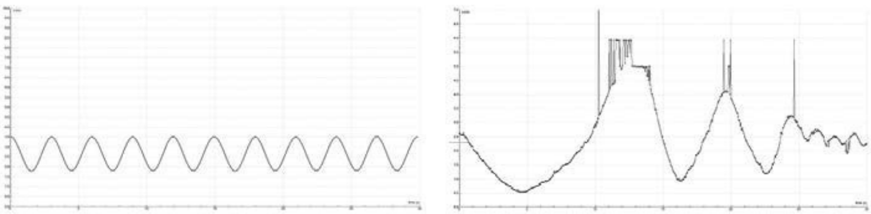


Figure 4. Given graph of back-and-forth movement in front of the motion sensor (left panel) and a graph produced by a student (right panel).

First, the graph was discussed with the students, then Mark was chosen to walk the graph. Initially, Mark walked faster than the graph required. Amir and Celine discussed Mark's movements and what they thought he should change. The graph walked by Mark showed fewer curves than the given graph in the same time-interval, while covering more distance.

1. **Celine**: He is making them bigger. [*Gestures the shape of the*
2. *curve Mark is walking*]
3. **Celine**: They have to be closer together
4. **Teacher**: How could we make the graphs more similar?
5. **Celine**: A little faster... and a slightly shorter distance?

While Mark was walking the graph, it became clear that his graph was not entirely similar to the given graph. Celine noticed that Mark's waves were larger than the ones in the given graph. He covered more distance, while it also took him longer to make each curve. In sharing her ideas, she used gestures to describe the shape of the graph (Lines 1-2). Initially, Celine did not describe the wave shape of the graph verbally, but her gesture clarified what she meant. She corroborated this gesture when she said "they have to be closer together" (Line 3). Celine shared specific ideas about the movements associated with a certain shape in the graph. She further mentioned that someone should walk "a little faster" covering a "slightly shorter distance" (Line 5). Celine's interpretation includes the variables time and distance in a covariational manner. Her reasoning, considering both time and distance, and combining them with speed, was prompted by her observation of Mark's movements in front of the sensor.

This is an example of how technology can be used to our advantage in strengthening the domain-specific 21st century skills of generating, refining, and evaluating graphical representations. The technology and the activity allowed the students to critically look at what went on with the graphical representation, when desired results are absent. Words and gestures helped the students to connect the shape of the graph (e.g., concavity of the curve) to the movements needed to reproduce the graphs (e.g., walking faster). Moreover, the students became increasingly able to communicate the relational aspect of the variables distance and time, when interpreting and describing the graphs. Celine's reasoning shows how the embodied learning environment prompted students to move beyond iconic interpretations of the graph, illustrated by some students who initially started to jump in front of the sensor. The graph that Mark walked, prompted Celine to pose a specific hypothesis, showing that Celine was very well able to describe in words what should be changed in the situation in order to change the appearance of the graph. This was rather exemplary, over the course of the lesson students became increasingly able to distinguish between relevant and irrelevant movements, and between relevant and irrelevant parts of the graph. The students became aware of the graphs' shape at a global level (a curve, possible and impossible shapes).

5.2.2 Episode 2: *Experiencing that speed matters*

In Lesson 5, students' interpretation of the scale on the graph's vertical axis was explicitly addressed. At the start of this episode the teacher asked what motion was represented by the graph on the left in Figure 5. After a short discussion, the students reached the conclusion that someone should walk slowly away from the sensor, then fast towards the sensor, and finally stand still, while performing this sequence three times in total.

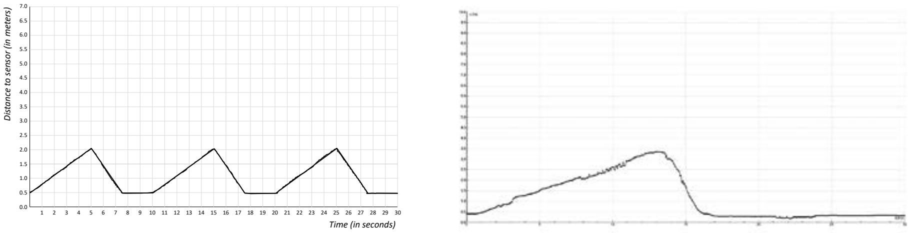


Figure 5. Given graph (left) and a graph (right) produced by a student by moving in front of the motion sensor.

Vanessa was chosen to walk the graph. First, the students discussed from where to start walking. This required them to connect a specific point in the graph to a specific position in the classroom, considering that the graph describes the distance from the sensor, represented on the vertical axis. Then, the teacher activated the sensor. Vanessa started walking the graph, first walking backwards, then forwards, standing still, and walking backwards again. All students paid close attention to her movements and the graph on the screen of the computer. Initially, Vanessa walked away and towards the sensor at a constant speed.

Celine: Now, walk faster forwards again. [*Raises her arm, gesturing Vanessa to walk faster towards the sensor, indicating in her gesture an increase in speed and repeating her gestures several times very quickly*]

[*Vanessa walks towards the sensor, stands still, and walks backwards*]

Celine: Slowly backwards and then fast forwards. [*Repeats the gestures she just made, urging Vanessa to walk faster towards the sensor*]

3

Immediately hereafter, Vanessa walked towards the sensor a little faster than she did before. After this, the resulting graph on the screen was compared with the given graph. A short discussion unfolded about the aspects that differed between the two graphs. The highest point in the graph just walked by Vanessa did not match the given graph.

15. **Teacher**: How high is the highest point here? [*Points*
16. *towards the graph*]
17. **Celine**: Two meters *so… [Takes one large step backwards,*
18. *and another large step backwards] …over here*

Even though the teacher discussed the graph and the different paces of walking with the students beforehand, Vanessa did not yet incorporate this in her walking. Celine, however, seemed to understand that in order to make the graph similar to the given graph, Vanessa should walk faster towards the sensor than when walking away from it. In order to communicate her ideas, Celine resorted to the use of metaphoric-representational gestures, enacting Vanessa's walk. With her gestures, she explicated how Vanessa should walk faster, as if she were conducting the movement herself, corroborating it by saying "walk faster forwards" (Lines 6-9). Furthermore, as shown in the second half of the interaction, for Celine it was self-evident that, when walking towards a specific point, the vertical axis conveys positional information of the variable distance (Line 17-18). This understanding is indicated by Celine's walking while saying "two meters, so…" (Line 17) and "...over here" (Line 18), which can be interpreted as deictic signs, showing where someone should be according to the highest point in the graph. This seems to be an important step in the development of Celine's reasoning. By using "here" (Line 18), to denote this point, Cecile is explicitly linking the position in the graph to a position in walking space, even without having the direct feedback of the motion sensor. Throughout the activity she shows her ability to deploy the 21st century skills of flexibly and creatively using the graphical representation and relate the representation to the real world situation it represents. Moreover, the link made by Celine is quantitative, making two large steps, indicating the first and the second meter.

6. Discussion

By offering students opportunities to interpret and create graphical representations of motion, this study proposed a domain-specific operationalization of the 21st century skills of using graphs to produce, present, and understand complex dynamic information. Students participated in activities that were situated in an embodied learning environment in which they were asked to interpret, create, change, combine, and compare graphical representations of their own and other's motion.

6.1 Students' development in levels of reasoning on graph interpretation and graph construction

Based on our analyses of students' level of reasoning, at the beginning of the lessons, students' graph interpretation skills were relatively weak. Students were inclined to interpret a graph as a literal depiction of the situation, which was also found in previous research investigating students' graph interpretation skills (e.g., Clement, 1985; Mokros & Tinker, 1987). These iconic interpretations were quite persistent in students. Even after students were introduced to graphs describing distance-time relationships (from Lesson 2 onwards) iconic reasoning was still the most prominent level of reasoning on graph interpretation tasks. Only after Lesson 3, which was the first lesson in which students enacted motion in front of the motion sensor individually, which provided them with opportunities to generate, refine, and critically evaluate motion graphs, the iconic level of reasoning became less common. For the graph construction tasks students also improved over the lessons, showing an increase in students' answers including higher levels of reasoning towards the end of the six-lesson teaching sequence, despite the fact that the motion situations students had to model became more complex. From the third lesson onwards students rarely constructed iconic graphs, whereas they often drew graphs in which more than one variable was correctly taken into account.

Overall, we found that multiple variable reasoning was more often present in students' answers on graph construction than on the graph interpretation tasks, which more often included single variable reasoning. According to Leinhardt et al. (1990) graph construction is more complex than graph interpretation because "interpretation relies on and requires reaction to a given piece of data (e.g., a graph, an equation, or a data set) [whereas] construction requires generating new parts that are not given" (Leinhardt et al., 1990, p. 12). In this same line, Berg and Smith (1994) conjectured how graph construction tasks might force students to consider both local and global aspects of graph construction which leads to higher levels of cognitive engagement. They contrast this with graph interpretation tasks in which students do not have to consider local aspects of the graph and more often choose a graph that fits the picture of the situation, in an iconic way. This is consistent with our results. In Lesson 3, when comparing students' reasoning on Task 2 and Task 3, both with the same graph, they more often showed iconic interpretations on the graph interpretation task and more multiple variable reasoning on the graph construction task. In our study, students reached high levels of reasoning when constructing graphs of motion, taking into account various aspects of distance-time relations present in the motion situations. This indicates that the graph construction tasks challenged the students to deploy high-levels of cognitive engagement, illustrating the usefulness of such tasks for higher-order thinking activities, in line with the intended 21st century skills of interpreting and creating graphical representations.

6.2 The role of perceptual-motor experiences in developing graphing skills

Over the two episodes Celine's reasoning went from iconic interpretations towards covariational interpretations (i.e., distance changing over time) (see also Radford, 2009a). The first episode focused on students' modelling of motion represented in a given graph by moving in front of the motion sensor individually. Celine incorporated signs, words, and gestures to come to a deeper understanding of graphically represented motion (see also Radford, 2009a), by coordinating the (observed) motion with the graphical representation on the screen. For example, Celine made use of iconic representational gestures (Roth, 2001). Botzer and Yerushalmy (2008) argue how such gestures imply that Celine mentally stretched the graph in order to compare it with the original one, as such revealing her perceptual-motor and analytical thinking (see also Robutti, 2006). The second episode introduced speed more explicitly, noticeable in the steepness of slope as a result of walking at varying speeds in front of the sensor. In both Vanessa's and Celine's reasoning, moving, and gesturing the concept of speed was apparent (see also Radford, 2014).

6.3 Limitations of the study

There are some limitations to the current study. First, this study is only based on students in three classes. Including more classes would enhance the robustness of our findings. Second, in order to show students' development over time we primarily focused on students' writing on the lesson-specific tasks and we illustrated how this reasoning was elicited during the lessons in the teaching episodes. According to Radford et al. (2004) "a direct translation of actions into symbols require[s] the students to undergo a dynamic process of imagining, interpreting and reinterpreting" (p. 73). More research is necessary to establish how students' physical experiences in the lessons relate to their answers on the lesson-specific tasks students performed on paper. For example, to what extent do students use their experiences of moving in front of the motion sensor? A research methodology with think-aloud protocols when solving the lesson-specific tasks might be suitable. A third, related limitation is that students' reasoning on the lesson-specific interpretation tasks might not be a precise reflection of their understanding. It could be that they did not write down their entire reasoning. We observed students' covariational reasoning throughout the lessons but did not see this level of reasoning in their answers to the lesson-specific graph interpretation tasks.[1] In that sense, the results for the graph interpretation tasks might underestimate their full understanding which provides another explanation for the limited occurrence of multiple variable reasoning on these tasks.

6.4 Concluding remarks

This study contributes to theories of mathematical thinking and learning by showing how embodied activities engenders high levels of mathematical reasoning. As such, our study was an extension of previous research that showed the capability of students this age to model dynamic data and reason about the relationship between multiple variables, when engaging in immediate own motion learning activities. Experiences in primary grades do not usually provide children the opportunity to engage in mathematics and science activities that involve modelling motion. We found that embodied activities using technology can be applied in an authentic and realizable classroom setting (see also Deniz & Dulger, 2012). As opposed to previous studies incorporating graphing activities, we asked students to also create graphs instead of only interpret given graphs. The lesson-specific tasks used in our study were fit to capture the intended domain-specific 21st century skills of generating, refining, and evaluating (motion) graphs. We saw a gradual decrease in the occurrence of iconic reasoning over the lessons while higher levels of reasoning (i.e., reasoning with a single variable or multiple variables) were more noticeable towards the end of the lessons. Students' thinking about these graphs went beyond merely replicating factual information and can be considered, for students at this age, as higher-order thinking. Students' perceptual-motor experiences in front of the motion sensor seemed to have been crucial in achieving this. The activities allowed them to reason about and critically evaluate graphical representations while using their creative thinking skills in adjusting their movements in order to replicate graphs more closely. This illustrates the potential of a sequence of embodied, constructive and reflective activities using technology.

[1]In a later carried out analysis we found that the students sometimes did show, on the lesson-specific graph interpretation tasks, instances of covariational reasoning similar to the reasoning students showed on the pre- and post tasks used to measure their development over the lessons; see *Chapter 4* of this thesis.

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CHAPTER 4

Moving towards understanding: Students interpret and construct motion graphs

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Moving towards understanding: Students interpret and construct motion graphs

Abstract

Bodily experiences are associated with powerful forms of understanding, yet not much research has investigated to what extent bodily experiences benefit the development of graphical reasoning. We examined the effectiveness of providing embodied support in a teaching sequence of six lessons on motion graphs, including both graph interpretation and graph construction activities, on fifth-grade students' reasoning about graphically represented motion. Divided over nine classes 218 students took part in our study. Students in three classes received lessons on graphing motion with direct embodied support, three classes received lessons on graphing motion with indirect embodied support, and three classes served as a baseline condition and received lessons on a different mathematics topic. Development of students' graphical reasoning was measured on four tasks. All students were given these same tasks four times with two months intervals. The teaching sequence on graphing motion took place either after the first, second, or third measurement. We used a cohort-sequential design to assess the intervention effect, the condition effect and the fading effect. Results showed that students improved their graphical reasoning at post-intervention-measurements when compared to their performance before the intervention. Moreover, students in the teaching sequence with direct embodied support showed a slightly larger gain in their graphical reasoning than students in the teaching sequence with indirect embodied support. These results suggest that embodied support as a learning facilitator can improve reasoning about graphing motion in primary school classrooms.

Keywords: Distance-time graphs, Embodied cognition, Graphing motion, Motion sensor technology

1. Introduction

The ability to understand and reason about graphical representations is a core part of science and mathematics proficiency and, therefore, an important topic in education (OECD, 2000; Roth & Bowen, 2003). Reasoning about graphical representations involves a broad range of skills ranging from encoding basic visual and spatial information in the graph, such as the scaling of the axes, the slope or the intercept, to relating these features to the conceptual or scientific phenomenon they represent, such as a sloped straight line in a distance-time graph reflecting constant speed (Shah & Hoeffner, 2002). Since graphing is often addressed within mathematics lessons, when graphing linear functions, students are mostly confronted with idealized examples, whereas graphs representing real-world phenomena often contain ambiguous elements such as noise or non-linearity (Lai et al., 2016). This might be one of the reasons that students are unable to apply their apparent understanding of graphs within mathematics lessons to graphs they encounter outside the mathematics classroom (McDermott et al., 1987). In the Dutch primary school mathematics curriculum, graphing is only briefly treated. Since graph comprehension – and reasoning about graphs – can be challenging, even for otherwise capable learners and expert users (e.g., McDermott et al., 1987; Roth & Bowen, 2003), it is generally agreed upon that students should be offered ample opportunities to acquire the skills associated with graph interpretation and construction, and to reason about these graphs (e.g., NCTM, 2000; Wang et al., 2012; Wavering, 1989).

In this study, we aimed to foster students' graphical reasoning in primary school. To this end we developed a teaching sequence on motion graphs representing the real-world phenomenon of distance changing over time. In such graphs, students are prompted to connect elements of the graphical representation to the physical event that is represented and to reason about the relationship between the variables on the horizontal and vertical axis as well as their pattern of covariation (Leinhardt et al., 1990). We investigated both short-term and middle-long-term effects of this teaching sequence on students' reasoning about graphs. Following recent proposals to include bodily experiences in teaching graphing (e.g., Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman & Leseman, 2019, see *Chapter 2* of this thesis), stemming from the wider embodied cognition approach to learning and development (see below), we investigated in particular whether a teaching sequence on motion graphs incorporating direct physical experiences has a stronger effect on students' graphical reasoning than a teaching sequence without such direct physical experiences.

2. Theoretical background

2.1 Graphical reasoning

Recognizing visual features of a graph, such as data points and values on the axes, interpreting relationships represented by these features, and connecting these relationships to what the graph actually represents, are three essential processes for comprehending graphs (Shah & Hoeffner, 2002). Graph comprehension is related to developing *graph sense* (Friel et al., 2001; Robutti, 2006). Graph sense, like *number sense* (Resnick, 1989) and *symbol sense* (Arcavi, 1994), is a holistic construct. It is a way of thinking, of becoming sensitive for what various graphs might represent and for how various (non-standard) phenomena might be graphed, both locally and globally. It also includes the ability to distinguish between discrete and continuous representations, to recognize the meaning and significance of the slope, and the more general visual characteristics of the graph (e.g., Robutti, 2006). A student should become flexible in recognizing and using these components, and should also be able to explain their thinking and communicate it to others using graph related language (Friel et al., 2001). When, for example, reasoning about representing the dynamic situation of distance changing over time in graphs, students should be given the opportunity to connect the represented physical situation (i.e., motion) with visual elements of the graphical representation (e.g., the slope, rate of change), and vice versa (e.g., McDermott, 1987). Graph sense encompasses both graph interpretation and graph construction (Friel et al., 2001), although the latter has only rarely been addressed in research on lesson activities (e.g., Leinhardt et al., 1990; Mevarech & Kramarski, 1997).

The extent to which students are able to comprehend and reason about graphical representations depends upon many factors such as prior personal experiences, basic everyday intuitions, and familiarity with the graph's conceptual content (Friel et al., 2001; Janvier, 1981; Shah & Hoeffner, 2002; Vitale et al., 2015). When graphs represent changes over time (e.g., increase of distance or length), which are particularly difficult for students to understand (Arzarello & Robutti, 2004), several misconceptions about interpreting and constructing graphs can arise (Glazer, 2011). For example, a student can interpret a graph as an iconic representation of a real event (Bell & Janvier, 1981; Leinhardt et al., 1990). This might happen when a student interprets the intersection of two lines in a speed-time graph as the moment when two persons or objects meet. Such reasoning about the graph is not necessarily illogical, because the student simply builds upon informal and intuitive understandings encountered in everyday reality, and applies this knowledge to the graph (e.g., Elby, 2000; Lakoff & Núñez, 2000). Similarly, when asked to construct a graph, a student might draw a line representing the actual path of motion like a map (e.g., McDermott, 1987; Mevarech & Kramarski, 1997). Various studies have shown that such an iconic or pictorial way of reasoning about graphs representing change over time can be quite persistent in students (Clement, 1985; Mokros & Tinker, 1987). These superficial interpretations might hamper the deeper conceptual understanding of graphs as representing a specific meaningful relationship between more than one variable (Lai et al., 2016; Leinhardt et al., 1990). Being able to resist superficial interpretations and instead draw correct inferences about what a graph actually represents is an important part of graphical reasoning.

2.2 Fostering graphical reasoning

In order for students to develop their graphical reasoning, teachers should preferably build on a students' informal and natural intuitions, and as a consequence circumvent aforementioned misconceptions. It is thus important that students should be offered ample opportunities to discover the deeper relationship between the variables on the axes and reason about their pattern of covariation (e.g., Friel et al., 2001; Lai et al. 2016; Leinhardt et al., 1990; Mokros & Tinker, 1987). Covariational reasoning, for young students, entails the mental coordination of the values of two quantities, while keeping in mind that at every moment the other quantity also has a value (Carlson et al., 2002; Saldanha & Thomspon, 1998). This covariational reasoning is important when interpreting and constructing graphical representations, because it enables students to make a connection between the two variables represented on the graph's axes (Saldanha & Thompson, 1998).

Instructional approaches targeting students' graphical understanding can be divided in two main categories; on the one hand, approaches in which the focus is more on quantitative or local aspects of graphing, on the other hand approaches in which the focus is more on qualitative or global aspects (e.g., Leinhardt et al., 1990). Choosing scales, fitting the paper, reading points in the graph, and letting students plot points from data given in tables, are instructional activities that lead to a focus on graphs' local aspects when interpreting the meaning of a graph or when drawing a graph (e.g., Berg & Smith, 1994; Hattikudur et al., 2012; Lai et al., 2016). When following these more or less fixed routines, a deeper conceptual understanding of the relational aspect of the represented variables might not be sufficiently supported (Yerushalmy & Schwartz, 1993). For example, when a student plots points in a graph and produces a correct slope, this does not necessarily imply understanding of what the slope represents (Vitale et al., 2016). Additionally, Thompson and Carlson (2017) argue how the plotting of points in the graph and "connecting points" without a deeper discussion of the values between successive points, often hampers a deeper understanding of the line in the graph as representing a relationship between two continuously changing quantities.

In contrast, without instructional emphasis on numerals and procedures, students have been found to look at the represented information at a more qualitative and global level (e.g., Krabbendam, 1982). An advantage of a more qualitative, global approach is that it resembles how one might judge a graph in real-life, which often excludes performing calculations on the graph's represented values (Cleveland & McGill, 1984). Another advantage is that when *interpreting* a graph, students can focus on the graph's general shape (Leinhardt et al., 1990), and when *constructing* a graph students can visualize a relationship between two variables as shapes of trends mapped onto the graphs' axes (Matuk et al., 2019). As described by Castillo-Garsow et al. (2013), thinking about the relationship between two variables as continuously changing necessarily involves thinking about motion. This thinking about motion might act as an embodied conceptual metaphor (Lakoff & Núñez, 2000), which maps early everyday experiences with motion to the abstract concept of (graphically represented) continuous change (see also Lakoff, 2014).

In addition to a focus on local or global aspects of graphing, particular learning facilitators that are included in the design of learning environments have been found to foster students' graphical reasoning. For example, in a study by diSessa et al. (1991) students (11-12 years) invented representations of a motion story about a car travelling through the desert by first drawing discrete representations and then moving on to continuous representations of this motion event. This meaningful motion situation and the emphasis on students' own inventions turned out to be powerful learning facilitators for the development of students' qualitative reasoning about these motion representations. Another example can be found in the work of Noble et al. (2004). Sixth-grade students were asked to make block representations of a moving elevator, using physical cubes. The block representations were then transferred into a simulation environment. The elevator in the environment moved in accordance with the motion represented by the blocks. Over the course of the activities, the students were reasoning about the "fastness" of the elevator, without explicitly referring to more quantitative ratio-based descriptions of the movement. Students' reasoning about this particular motion situation was presumed to support more formal reasoning about multiplicative relationships. In both of these examples the real-world context, thus the context of the travelling car and the moving elevator, supported students' (qualitative) reasoning about the (graphical) representations, which allowed them to further develop their formal mathematical reasoning as well as to partake in more conventional graphing practices.

Another often used learning facilitator, already shortly mentioned, known to facilitate students' qualitative reasoning about graphs is the use of real-time motion and simulation environments (Stroup, 2002). For example, in a study of Nemirovsky et al. (1998) students familiarized themselves with the graphical representation of their own movements in front of a motion sensor that was connected to a desktop computer. This approach allowed the students to reason about the relationship between changes in their own movements and the resulting changes in the graphical representation. In learning environments making use of motion sensor technology, physical experiences are an explicit part of students' learning activities. Moreover, through the use of motion sensor technology, the line in the graphical representation becomes meaningful to the students since the line in the graph is connected to their own bodily movements and thus in experienced motion (Kaput & Roschelle, 2013). Using motion sensor technology by which a graphical representation appears in real-time also provides a valuable entry-point into reasoning about continuous change represented in graphs (e.g., distance changing over time), because motion experienced with your own body, or observed, must have a value at every point in time. The explicit introduction of bodily experiences in learning activities is in accordance with an embodied cognition approach.

2.3 Enriching graph instruction: An embodied perspective

Learning environments, in which students' own bodily experiences are an explicit part of the learning activities, are also termed embodied learning environments (e.g., Johnson-Glenberg et al., 2014; Skulmowski & Rey, 2018). The ways in which students are provided with opportunities for bodily engagement in learning environments supporting students understanding of graphing motion can vary widely, ranging from whole- or part-bodily movements to observing someone or something else moving (Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman & Leseman, 2019). Including bodily experiences in learning environments is based on the premise that all cognitive processes originate from the perceptions and actions of our body in interaction with our immediate environment (e.g., Pouw et al., 2014; Wilson, 2002). The resulting action-perception schemes are considered to be the fundament of our cognitive architecture. Also, observing movement of others or mentally simulating actions by activating previously acquired action-perception structures are considered to be part of the embodied cognition continuum. Our brain enables us to simulate particular action-perception structures (and invent new ones) (Van Gog et al., 2014), by re-using the sensorimotor circuits of the brain that were involved in previous experiences of perceiving and acting (e.g., Anderson, 2010; Pulvermüller, 2013). More specific, through the (simulated) enactment of mathematical structures with our body, content-specific action-perception structures evolve which constitute a source domain that can be metaphorically projected to target concepts (Abrahamson & Bakker, 2016; Lakoff & Núñez, 2000).

In a recent review of research into embodied learning environments (Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman & Leseman, 2019) it was shown that, although physical experiences are often utilized in learning environments supporting students' understanding of graphing motion, not much comparative research into the development of primary school students' understanding of motion graphs has been conducted to date. Of the six studies that did investigate this age group, only one study (Deniz & Dulger, 2012) took a quasi-experimental approach in a classroom setting, the other studies reported (short-term) case studies, involving one or two students (e.g., Ferrara, 2014; Nemirovsky et al., 1998), or observational research (e.g., Anderson & Wall, 2016). Deniz and Dulger (2012) compared two inquiry-based lesson sequences on motion and temperature of which one was enriched with real-time graphing technology and the other with traditional non-digital laboratory equipment. Both lesson sequences incorporated physical experiences, yet only the technology group received immediate feedback provided by the tool. These technology lessons inter alia consisted of specific movements students had to perform in front of a motion sensor (three lessons on motion, three lessons on temperature, six hours in total), which were displayed in real-time on a computer screen. Afterwards the graphs were discussed with the students. Results showed that using the real-time graphing technology significantly improved students' ability to interpret motion and temperature graphs. Based on their systematic review, Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman and Leseman (2019) concluded that embodied learning environments making use of students' own motion immediately linked to its representation, which was often done through the use of motion sensor technology, were most effective. Thus, embodied learning environments providing students with direct physical experiences have been found to be helpful in supporting students' understanding of motion graphs.

3. The present study

In the present study, we investigated the middle-long-term learning outcomes of a six-lesson teaching sequence, supporting students' reasoning about motion graphs, featuring a particular sequencing of mathematical graphing tasks. Embodied learning environments supporting students' understanding of graphing motion have been found to be effective in small-scale one-to-one settings, however, to date, in the primary grades their effects have rarely been studied in whole-classroom settings (Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman & Leseman, 2019). To this end, we developed a teaching sequence on graphing motion for primary school students. Following the proposal that higher levels of (mathematical) understanding are grounded in physical experiences regarded as embodied cognitions, we developed two parallel versions of this teaching sequence differing in their degree of directness. The teaching sequence in which students were offered direct embodied support, involved graphing activities in which students' own bodily movements were visualized as a line in the graph, using motion sensor technology. The teaching sequence in which students were offered indirect embodied support involved graphing activities that were mostly paper-and-pencil based or projected on the digital blackboard. Students did work with an image of the motion sensor context, but without the presence of the physical tool. A third group of students served as a baseline condition and received lessons on a different mathematics topic.

The study was carried out in primary school classrooms. As a truly randomized design was not feasible, we used a cohort-sequential design with three cohorts which received the lesson sequence in the first, second and third trimester of the school year, respectively. Each cohort comprised of two classes who received either the direct or the indirect embodied support instruction in the trimester where the lesson sequence was provided. A fourth cohort was included as baseline condition. This cohort received a series of lessons on another mathematical topic. We wanted to investigate the potential effects of the embodied learning activities on students' graphical reasoning ability in the context of modelling motion. We formulated the following research question:

> *To what extent does embodied support in a six-lesson teaching sequence on graphing motion affect the development of students' graphical reasoning?*

To assess students' learning progress as a result of the teaching sequence, tests were administered before and after the teaching sequence. The tests consisted of a number of graphical reasoning tasks and required students to explain in writing their reasoning when solving the tasks. Students' written responses were subsequently evaluated with regard to the level of graphical reasoning displayed. We will analyze changes in students' graphical reasoning by performing a longitudinal analysis on the task level following Item Response Theory (IRT), allowing us to model intra- and inter-individual changes in growth. This approach enables us to increase this study's power, to disentangle faulty reasoning from simple mistakes, and to get better insight in changes in levels of reasoning over time. We hypothesize that students taking part in a teaching sequence on graphing motion will, on average, change in their graphical reasoning from lower to higher levels of reasoning more than can be expected based on mere maturation or multiple testing. Additionally, in line with existing research on embodied learning environments, we hypothesize that students receiving a teaching sequence with direct embodied support will outperform students taking part in a teaching sequence with indirect embodied support.

4. Method

4.1 Participants and study design

Schools and classes were chosen based on the willingness of the teachers to participate, resulting in a convenience sample. A total of 237 fifth-grade students from seven elementary schools, divided over nine classes participated in our study. From 19 students we did not obtain written parental consent to collect data. The final sample consisted of 218 students (Grade 5; $M = 10.47$, $SD = 0.47$; 94 female, 43%) divided over two instruction conditions (indirect support condition, $n = 68$; direct support condition, $n = 70$) and a baseline condition ($n = 80$). All schools were located in the area of Utrecht, the Netherlands. The study was conducted between October 2016 and June 2017. The research was approved by the Ethical Review Board of the faculty of Social and Behavioral Sciences at Utrecht University.

All students participated in a teaching sequence of six lessons on graphing motion (with direct and indirect embodied support) or a non-related topic (probability) in the baseline condition as part of their regular classroom instruction. The study adopted a cohort-sequential design, meaning that for each research condition, one cohort of students participated in the teaching sequence in the first trimester of the school year, the second cohort of students in the second trimester, and the third cohort of students in the third trimester. To compose the cohorts, the six classes that would receive the teaching sequence on graphing motion were first clustered in three pairs on matching general school characteristics. Next, in consultation with the teachers, each pair was assigned to one of the three cohorts. Finally, per cohort, the two classes were randomly assigned to one of both instruction conditions. This design allowed us to (1) have the same researcher teaching all the lessons on graphing motion, and (2) to compare the learning curve during the six-lesson teaching with the baseline condition and post intervention conditions (when not yet having had the teaching sequence). Table 1 gives an overview of the study research design..

Table 1

The cohort-sequential design of the study

Condition	Cohort		Phase						
				Oct. – Nov. 2016		Jan. – Feb. 2017		Apr. – May 2017	
Baseline	0	($n = 80$)	M1		M2		M3		M4
With indirect embodied support	1	($n = 24$)	M1	*Teaching sequence Graphical reasoning*	M2		M3		M4
	2	($n = 23$)	M1		M2	*Teaching sequence Graphical reasoning*	M3		M4
	3	($n = 21$)	M1		M2		M3	*Teaching sequence Graphical reasoning*	M4
With direct embodied support	1	($n = 21$)	M1	*Teaching sequence Graphical reasoning*	M2		M3		M4
	2	($n = 22$)	M1		M2	*Teaching sequence Graphical reasoning*	M3		M4
	3	($n = 27$)	M1		M2		M3	*Teaching sequence Graphical reasoning*	M4

4

4.2 Teaching sequence and procedure

The main goal of the teaching sequence was to help students become acquainted with graphs representing the bivariate relationship of distance changing over time, and foster students' reasoning about these graphs. The instruction sequence started with informal graphing activities (Lesson 1), followed by a transition from discrete to continuous graphs (Lesson 2), and to continuous graphs (Lesson 3 onwards). Table 2 gives an overview of the teaching sequence, including the main topic per lesson and its key activities.

The teaching sequences in the conditions with indirect and direct embodied support were taught by the first author of this paper, and in the case of direct embodied support with the help of a teaching assistant. Each teaching sequence consisted of six lessons, about 50 minutes each, one lesson per week, divided over 6 weeks. Two weeks before the start of the intervention a general reasoning test was administered. One week before a cohort started with the teaching sequence all students completed the graphical reasoning assessment; this was done for the three cohorts (M1-M3). Finally, after all cohorts had completed the teaching sequence there was a final assessment (M4; see also Table 1).

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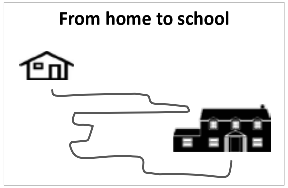
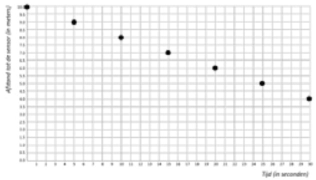
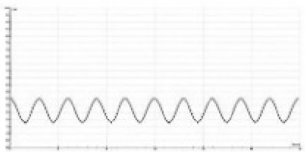
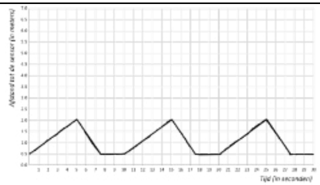
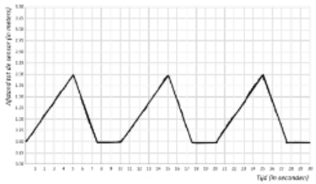
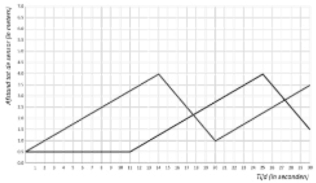
Table 2

Overview of the six-lesson teaching sequence on graphing motion

Lesson		Main topic	Key activities
1. Motion: reflecting and representing		Informal graphical representations	*Reason with variables and construct representations of a real-world situation*
2. From discrete to continuous graphs		Measuring distance	*Measure distance in discrete intervals and continuously, and reason about differences between discrete and continuous graphs*
3. Continuous graphs of "distance to" (1)		Generate, refine, and reason with continuous graphs	*Coupling specific movements to their representation as a line in the graph* *Coupling a concrete situation to a graphical representation*

4. Continuous graphs of "distance to" (2)		Generate, refine, and reason with continuous graphs	*Coupling specific movements to their representation as a line in the graph* *Investigating how speed is represented in the steepness of slope*
5. Scaling on the graphs' axes		Reason about the relationship between two variables through scaling	*Construct graphs with different scales on the axes*
6. Multiple movements and their graphical representations		Generate, refine, and reason about simultaneous movements and their representation as a graph	*Critically evaluate points of intersection and their meaning*

4.3.1 Instruction conditions

In the instruction condition with indirect embodied support (hereafter: indirect support condition) the students were provided with graphing activities that were paper-and-pencil based (including spoken narratives as well as illustrations of a motion sensor), presented on work sheets or on the digital blackboard. The activities on the digital blackboard were sometimes visualized dynamically, but mostly consisted of non-dynamic illustrations of motion situations of non-human moving objects, such as a toy car travelling a particular distance within a particular period of time. Although the motion situations referred to source-domain embodied experiences (e.g., moving your body through space), the graphing activities in the indirect support condition did not involve students enacting the movements in the classroom. Therefore, the degree of embodied support in this instruction condition was low. Similar motion situations and graphing activities were also provided to students in the instruction condition with direct embodied support (hereafter: direct support condition), but instead of only providing the context as an illustrated narrative, students were explicitly prompted to physically enact the situations, using a motion sensor technology. The motion sensor registered enaction and provided students with a direct linkage between their movements and the representation of their movements as a line in the distance-time graph presented on the screen of a computer or the digital blackboard. Therefore, the degree of embodied support in this instruction condition was high. In Figure 1, the difference between both instruction conditions is further explained by giving an example of the lessons' setup. Shown is an activity part of Lesson 2, in which distance is measured at discrete time-intervals (5 seconds).

With indirect embodied support	With direct embodied support
• Motion sensor illustrated on digital blackboard (see white circle) • Sensor measures every 5 seconds • Toy car simulates movement in front of motion sensor • A graph appears in real-time	• Motion sensor physically present (see white circle) • Sensor measures every 5 seconds • Student walks in front of motion sensor • A graph appears in real-time

Figure 1. Difference in set-up between the conditions with indirect and with direct embodied support

4.3.1.1 Motion sensor technology

In the direct support condition, we made use of two ultrasonic €Motion sensors, together with Coach6 Software (CMA, Heck et al., 2009). The motion sensor was set to measure the distance between the sensor and the nearest object or person over a 30-second trial, providing a single distance-time graph. The graph was presented on the digital blackboard (Lesson 2 and 6) or on the screen of laptop computers (Lesson 3-5). When moving toward the sensor, the distance between the sensor and the student decreased. When moving from the sensor this distance increased.

4.4 Measures

4.4.1 General mathematics performance

In order to obtain an indication of students' overall mathematics performance, data from the Dutch student monitoring system (CITO LOVS: Janssen et al., 2010), provided by the schools, were used. In this system, schools record their students' results on the biannual standardized mathematics tests. We used the scores of the students on the end-term Grade 4 tests as an indication of their overall mathematics performance (norm population end-term Grade 4: $M = 91.9$, $SD = 10.6$, CITO, 2015).

4.4.2 General reasoning

As a measure of students' general reasoning ability, an abbreviated version of the Raven Standard Progressive Matrices (Raven SPM: Raven et al., 2000), consisting of two sets of 9-items, was used (Bilker et al., 2012). Raven's SPM is a test of general reasoning ability and fluid intelligence. Each item consists of a set of pictorial geometric design elements, in black and white. Students are asked to identify the missing element which completes the specific pattern represented by the set. The test was administered to all students in their classrooms during class time, following the instructions in the test's manual.

4.4.3 Graphical reasoning

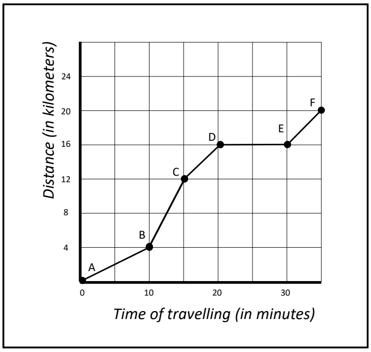
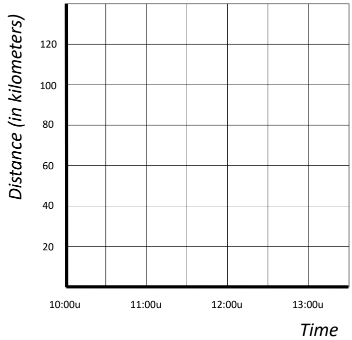
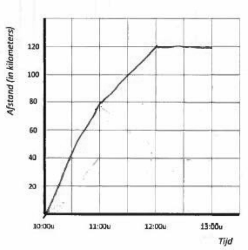
Students' graphical reasoning about distance-time graphs was assessed four times by a paper-and-pencil test consisting of exactly the same four tasks at each measurement moment: three graph interpretation tasks and one graph construction task. The four tasks were part of a larger test that also included nine other problems related to two other mathematical domains, namely algebra (four tasks) and probability (five tasks). In this study we only include students' performance on the tasks related to graphing motion. Students' received a correctness score on their answer to each task (correct = 1, incorrect = 0; minimum score = 0, maximum score = 4). On Task 2 students could receive partial credit (i.e., resulting in three possible scores for this task "0", "0.5", 1."). In addition, in order to assess students' reasoning, all tasks included an open-ended question, which probed students to make their thinking explicit, by asking them "how do you know?" Students were requested to explain their reasoning in writing and the written responses were coded afterwards for the level of reasoning displayed (see below).

Table 3 shows two tasks as examples. The tasks were developed in such a way that students with different levels of understanding, could show different levels of reasoning in solving them. For example, Task 1 shows a distance-time graph representing the movement of a car. The speed of the car – the hidden quantity – can be visually deduced by inspecting the steepness of slope. Discovering this hidden quantity can be corroborated with reasoning in which a student explicates that the car in this particular segment travels the largest distance (e.g., when compared to the other segments within the graph), or with reasoning in which a student explicates how the steepness of slope qualitatively represents "distance changing over time" or quantitatively, by taking into account the numerals on the axes. At these higher levels of reasoning a student also reasons about the given quantities on the axes in an (informal) covariational manner.

Task 3 represents the graph construction task, including an empty graph and a description of a motion situation. The motion situation consists of three separate parts, in which the train travels at different speeds. Each part of the motion situation implies different rates of change ("twice as fast between 11 and 12 o'clock"). These differences should be made visible by the students in the empty graph. In order to construct a correct graph a student should take into account the relative differences in speed between the three different segments, by quantifying them. In this task, applying the principle "steeper slope means faster movement" does not necessarily result in the correct graph.

Table 3

Example tasks graph interpretation (left panel) and graph construction (right panel)

<table>
<tr><th>Task 1</th><th>Task 3</th></tr>
<tr><td>A car drives through town</td><td>A train ride.
A train travels twice as fast between 10:00 and 11:00 o'clock than between 11:00 and 12:00 o'clock. The train stands still from 12:00 to 13:00 o'clock.</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>1a. Between which points does the car goes fastest?
1b. How do you know?</td><td>2a. Draw a graph that fits the description above.
2b. How do you know?</td></tr>
<tr><td>Correct answer for this task:
B-C</td><td>Correct answer for this task:

</td></tr>
<tr><td>Score: correct (1), incorrect (0)</td><td>Score: correct (1), incorrect (0)</td></tr>
</table>

4.4.3.1 Coding scheme for students' level of reasoning

To evaluate students' explanations of how they arrived at a particular solution of the three graph interpretation tasks and the graph construction task, a coding scheme was developed based on an open exploratory analysis of students' explanations. At first, the work of a few students was examined. All research team members first individually categorized these students' responses. Later these classifications were compared, discussed, and revised until agreement was obtained. Finally, this resulted

4

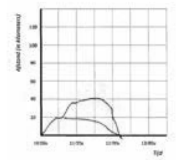
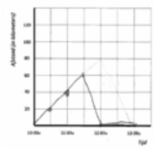
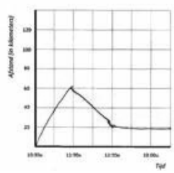
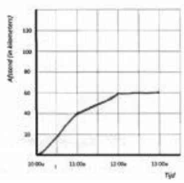
in one coding scheme, applicable to reasoning on both graph interpretation and graph construction tasks, consisting of four categories with increasing sophistication in level of reasoning: *unrelated reasoning* (R0), *iconic reasoning* (R1), *single variable reasoning* (R2) and *multiple variable reasoning* (R3).

For the graph interpretation tasks, students' written explanations were coded. For the graph construction tasks we took another approach. The students in our sample showed a richness of graphical solutions, yet the majority of the students explained these solutions by simply restating the description of the motion situation as their answer. We assumed students' graphical solutions to be a direct indication of their levels of reasoning outlined above. Therefore, for the graph construction task, we coded students' reasoning as a function of students' ability to correctly take into account the variables on the graph's axes. We distinguished between students who constructed: an illogical graph without taking into account the description of the motion situation (Level R0), a graph based on superficial characteristics of the motion event (Level R1), a graph taking into account a single variable correctly (Level R2), and a graph taking into account multiple variables correctly (Level R3). This highest level of reasoning included, yet was not restricted to, responses that showed a student's informal covariational reasoning.

The coding of the graph interpretation and graph construction tasks resulted in four reasoning scores per measurement moment. In Table 4, the four codes can be found including a description and examples of student's reasoning per category.

Table 4

Coding scheme used for students' level of reasoning on the graph interpretation and graph construction tasks

		Description of students' reasoning	
		Graph interpretation *Example*	Graph construction *Example*
Level of reasoning	Code	**Student reasons...**	**Student constructs graph...**
Unrelated reasoning	R0	...without referring to the graphical representation or the motion event *"You can see"* *"I guessed"*	...without taking the description of the motion event into account 
Iconic reasoning	R1	...on the basis of the shape of the graphical representation or superficial characteristics of the motion event *"Because those two points are the highest"* *"Over there the line is the longest"*	...on the basis of superficial characteristics of the description of the motion event 
Single variable reasoning	R2	...on the basis of a single variable (distance or time or speed) *"Between B and C, the line goes upwards from 4 till 12, so he gives a lot of gas"* *"There he drives 8 kilometers and everywhere else this is 4 or less"*	...taking into consideration a single variable (distance or time or speed) 
Multiple variable reasoning	R3	...on the basis of multiple variables (distance and/or time and/or speed) *"The car drives 8 kilometers in 5 minutes. So, in the shortest period of time, the most kilometers."*	...taking into consideration multiple variables (distance and/or time and/or speed) 

Note. The complete coding scheme, including examples of student responses per task, can be found in Appendix 4.1 (graph interpretation) and Appendix 4.2 (graph construction).

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An independent second rater coded the four tasks on the four measurements of a subsample of 21 students (336 responses, approximately 10% of all responses). Inter-rater reliability was high with an overall inter-rater reliability of Cohen's Kappa = .92.

4.5 Data analysis

4.5.1 General mathematics performance

We provide sample means and standard deviations for students' general mathematics performance and general reasoning. One-way analyses of variance (ANOVAs) were conducted in order to compare the baseline and the two instruction conditions for differences on general mathematics performance and general reasoning prior to the intervention. A Pearson chi-square test was conducted to test for unintended differences in students' level of reasoning on M1, so before any lessons. Further, we used frequencies of students' level of reasoning (R0, R1, R2, R3) on the graphical reasoning test to calculate the proportion of students using a particular level of reasoning for the baseline condition, and both instruction conditions.

4.5.2 Modelling change in underlying ability

To model students' development in graphical reasoning we adopted an approach in which we combined multi-group Latent variable Growth curve Modelling (LGM), suitable to study longitudinal trends, with assumptions from Item Response Theory (IRT), suitable for categorical data. LGM is a versatile approach for modelling systematic intra- and interindividual differences in change over time and offers many advantages for the modelling of longitudinal data compared to more traditional statistical methods (Willet & Bub, 2005). In our study, we assumed that a student's graphical reasoning would change over the four measurement occasions. We expected a slight increase in reasoning level due to growing familiarization with the tasks and maturation, and a larger increase due to the teaching sequence on graphing motion. The IRT assumption is that graphical reasoning ability itself cannot be directly observed: it is a hypothetical latent ability that underlies the observed reasoning levels in the students' written answers; scored as unrelated reasoning (R0), iconic reasoning (R1), single variable reasoning (R2) and multiple variable reasoning (R3). Thus, the four reasoning levels can be mapped onto the underlying latent graphical reasoning ability. According to IRT, the reasoning levels shown by students on particular tasks are a function of students' unobserved (latent) reasoning abilities and the difficulty of the different levels of reasoning on these tasks. Students' abilities and the tasks' difficulties are placed on the same scale, allowing to express students' reasoning abilities as the probability of showing particular levels of reasoning on these tasks and to express the difficulties of the tasks as the proportions of students showing particular levels of reasoning on these tasks. LGM with IRT yields estimates of students' growth in reasoning ability expressed as the increased probability of showing higher levels of reasoning on a particular set of tasks. We estimated students' individual growth trajectories based on four partial individual effects. Students may show individual differences in their reasoning on the pre-measurement (*intercept effect*) and in the rate of change over time (*slope effect*) for the subsequent three measurements. In addition to the intercept and the slope effect, we included an *intervention effect* and a *weakening effect*. With the intervention effect we model students' change in ability after partaking in the teaching sequence. For example, an intervention between M1 and M2, might lead to a change in students' graphical reasoning ability between the measurements on M1 and M2, and may extend to a change between M3 and M4. The weakening effect takes into account the possibility that the intervention effect might fade-out over time. Two control variables (general mathematics performance and general reasoning) were included as predictors in the LGM analyses to control for individual differences in general mathematical ability and general reasoning ability. Finally, to answer the main question of the current study, condition was added as a predictor into the model since we assumed that the intervention effect might depend on the specific condition students are in (indirect or direct embodied support). Hence, by adding condition as a predictor we could investigate whether the instruction condition impacted changes in students' reasoning ability over time, thus answering the question whether students in different instruction conditions differ in growth trajectories. In a stepwise procedure we first estimated an unconditional model that served as our baseline model only including the intercept effect and the slope effect. In the next step we added the intervention effect and the weakening effect. We then added the two general measures (general mathematics performance and general reasoning) as predictors of the intercept and the slope effect. Both predictor variables were grand mean centered. In the final step, we added condition as a predictor of the intervention effect. The multi-group latent growth curve model, with time varying effects added, was estimated using Mplus (Version 8; Muthén & Muthén, 2012-2017). A logit link was used to map the likelihood of using a certain level of reasoning (Level R0, R1, R2, or R3) onto students' latent graphical reasoning ability. The logit link implies that we had to use robust Maximum Likelihood Estimation (MLR). As a consequence, because MLR provides no chi-square goodness of fit index, we used the Aikake Information Criterion (AIC) and the Bayesian Information Criterion (BIC) as relative overall fit measures. We report the change in AIC (ΔAIC) and BIC (ΔBIC) for each comparison between models. Both fit indices take into account sample size and the number of parameters. We followed the commonly applied rule that lowest AIC and BIC represent the best model fit. Further, we provide parameter estimates and significance values of the separate effects and the predictors.

4.5.3 Missing data

Of the 218 students in this study, 213 had complete data on general mathematics performance, and 217 had complete data on general reasoning. For the students with missing data on these measures, values were imputed based on class averages. Four students in the conditions with direct or indirect embodied support missed either M2 or M3, while the subsequent measure was present. To avoid having missing post-measurements, we decided to substitute the missing measurement point with the subsequent one. For example, a student in Cohort 1, receiving the intervention between M1 and M2, missed M2. For this student we treated M3 as if it were M2 and M4 as if it were M3.

5. Results

5.1 Preliminary analyses and descriptive statistics

There were no significant differences on students' general mathematics performance ($F(2, 210) = 0.77$, $p = .465$, partial $\eta^2 = .007$), general reasoning ($F(2, 214) = 0.29$, $p = .752$, partial $\eta^2 = .003$), and level of graphical reasoning on M1 ($\chi^2(6) = 10.88$, $p = .092$) between the baseline condition and the two instruction conditions. Table 5 presents per condition, for each cohort, the means and standard deviations of general mathematics performance and general reasoning, as well as the correctness scores on the graphical reasoning test for all four measurement moments. Although they did not have an intervention on graphing motion, students in the baseline condition did seem to improve in their correctness scores over the school year (+ 0.70), as did students in the indirect (+1.03) and direct support condition (+ 1.08).

Table 5

Means and standard deviations for general mathematics performance, general reasoning (abbreviated raven's spm) and graphical reasoning $M_{measure\ 1} – M_{measure\ 4}$ (correctness scores)

	Cohort	General measures: Mathematics performance CITO E6	General measures: general reasoning Raven's SPM	Macro measures: M1	Macro measures: M2	Macro measures: M3	Macro measures: M4
		M(*SD*)	*M*(*SD*)	*M*(*SD*)	*M*(*SD*)	*M*(*SD*)	*M*(*SD*)
Indirect embodied support [Instruction Condition 1]	1	96.75(9.07)	10.50(2.47)	**1.25(0.97)**	**2.46(0.96)**	2.38(0.86)	2.57(1.08)
	2	100.38(8.16)	11.65(2.79)	2.27(1.13)	**2.93(1.02)**	**2.83(1.02)**	3.20(0.75)
	3	96.67(12.83)	10.24(2.43)	1.98(1.27)	2.36(1.37)	**2.50(1.21)**	**2.75(0.85)**
Total		97.88(10.16)	10.81(2.61)	1.81(1.19)	2.59(1.13)	2.56(1.03)	2.84(0.93)
Direct embodied support [Instruction Condition 2]	1	89.75(13.68)	9.30(2.74)	**0.93(0.78)**	**2.35(0.85)**	2.05(1.01)	2.31(1.30)
	2	101.73(12.16)	12.23(2.81)	1.34(0.96)	**2.16(1.10)**	**2.86(0.98)**	2.50(1.12)
	3	94.48(11.63)	10.04(2.72)	2.14(1.31)	2.60(1.42)	**2.26(1.40)**	**2.90(1.23)**
Total		95.42(13.13)	10.52(2.97)	1.52(1.17)	2.38(1.17)	2.39(1.22)	2.60(1.23)
Baseline Condition	4	97.31(12.77)	10.49(2.74)	1.45(1.06)	1.91(1.18)	1.97(1.23)	2.15(1.20)

Note. E6=End-term Grade 4. Boldened scores indicate scores on measurements in between which the intervention took place.

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The development of students' level of reasoning on the graphical reasoning test for all four tasks together is shown in Figure 2 for the baseline condition. The proportions of level of reasoning are shown for each measurement occasion. There was some decline of R1 reasoning over time, but a slight increase of R3 reasoning. Overall, the proportions of level of reasoning (R0-R3) in the baseline condition stayed rather stable over time.

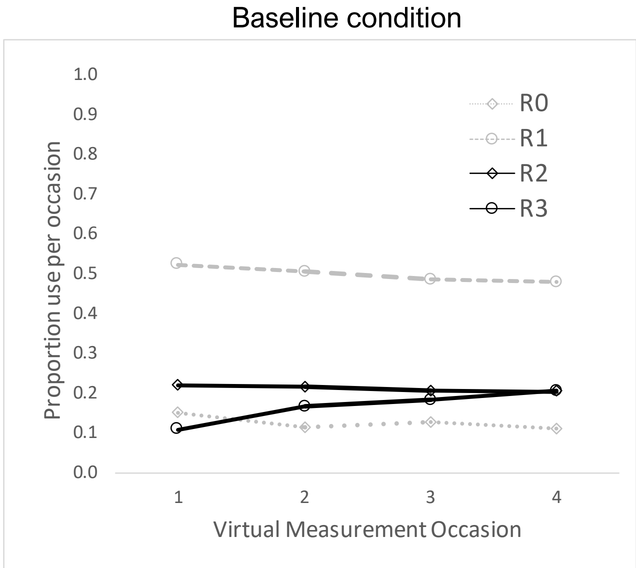

Figure 2. Proportions of level of reasoning (R0, R1, R2, R3) for each measurement occasion for the Baseline condition

In Figure 3, the development of students' reasoning is shown for the indirect support condition (left panel) and the direct support condition (right panel). In this figure measurement occasions are aligned between cohorts, such that the intervention is set to start and end at the same virtual time points for all cohorts. This alignment was necessary in order to be able to visually compare the development of students in the different cohorts, since students in the different cohorts participated in the teaching sequence in different time periods. Students in Cohort 1 participated in the teaching sequence during the first time-period (October – November), directly after the first measurement occasion; students in Cohort 2 received the teaching sequence during the second time-period (January – February), performing two measurements before the teaching sequence; and students in Cohort 3 participated in the third time-period (April – May), performing three measurements before the teaching sequence. When aligned in Figure 3, students in Cohort 1 are shown as having participated in virtual measurements 3 to 6, students in Cohort 2 in virtual measurements 2 to 5, and students in Cohort 3 in measurements 1 to 4. This allows for a direct comparison of the improvement of students in all cohorts following their participation in the teaching sequence by inspecting the change between virtual measurement occasions 3 and 4.

After partaking in the teaching sequence more students in both the direct and indirect support condition showed reasoning on the basis of a single variable (R2) as well as reasoning on the basis of multiple variables (R3). Additionally, students in the direct support condition exhibited a larger gain in the frequency of R2 and R3 reasoning (R2: + 21% points and R3: + 15% points) than students in the indirect support condition (R2: + 7% points and R3: + 6% points).

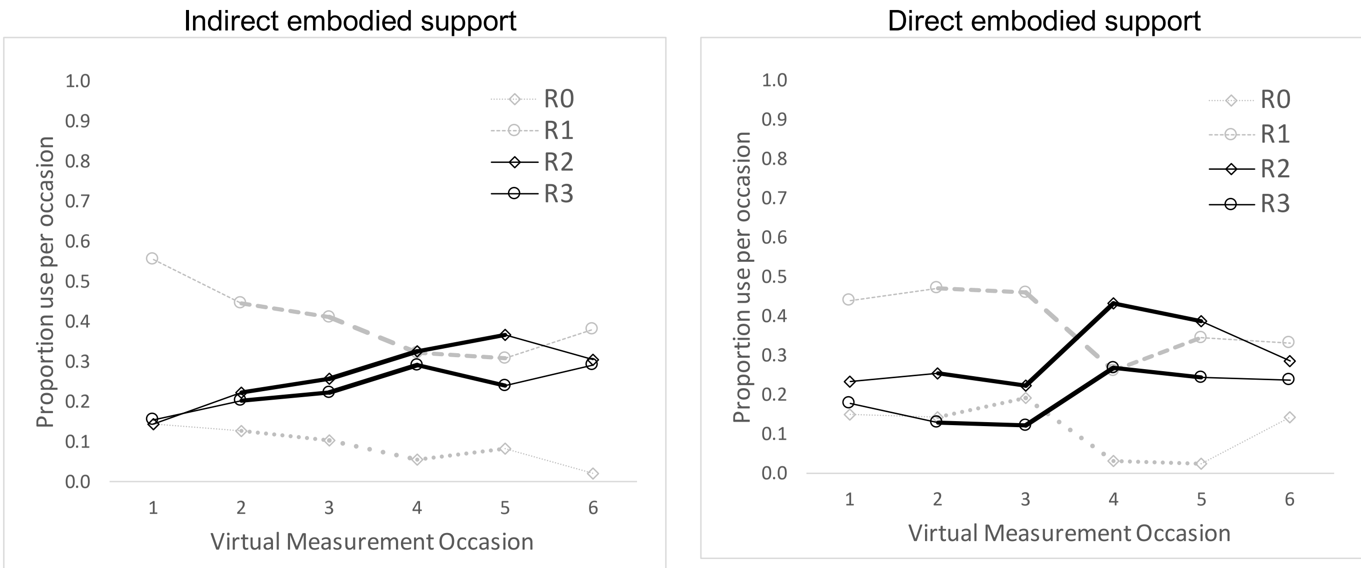

Figure 3. Proportions of level of reasoning (R0-R3) for each virtual measurement occasion. Between virtual measurement occasions 3 and 4 the teaching sequence took place for the indirect embodied support (IES) condition and the direct embodied support (DES) condition. Thin line-segments are based on one cohort (VM1, IES n = 83, DES n = 107; VM6, IES n = 92, DES n = 84) thicker line-segments on two cohorts (VM2, IES n = 171, DES n = 195; VM5, IES n = 188, DES n = 168), and thickest line-segments are based on all three cohorts (VM3, IES n = 271, DES n = 280; VM4, IES n = 263, DES n = 274)

5.2 Effects of embodied support on students' graphical reasoning ability

To investigate the general effectiveness of both instruction conditions in terms of immediate (post-test) and middle-long-term (follow-up) effects, latent growth curve analysis was used to model intra-individual change in graphical reasoning over the four measurement points, corrected for general mathematics ability and general reasoning. First, an unconditional growth model, including the intercept effect and the slope, but no other effects was estimated. The fit of this model (AIC = 7970.16; BIC = 8031.08) serves as our baseline. Adding the intervention effect and the weakening effect to the model resulted in an improvement in the overall relative model fit ($\Delta AIC = 83.69$; $\Delta BIC = 73.54$). In addition to the overall fit measures also structural parameters of the model are of interest (Wald tests). The effect of the intervention on students' reasoning was significant (1.10, $p < .001$). There was also a significant weakening effect on the delayed measures after the intervention (-0.47, $p < .001$). The addition of general mathematics performance and general reasoning as predictors of the intercept further improved our model ($\Delta AIC = 86.21$; $\Delta BIC = 79.44$). Both predictors are significant predictors of the intercept effect (general mathematics performance: 0.52, $p < .001$, general reasoning: 0.23, $p = .001$).To investigate the effect of embodied support on students' reasoning about motion graphs on the immediate and delayed post-test, instruction condition was added as a predictor of the intervention effect. In this way we modelled the relationship between students' changes in graphical reasoning over the four measurement points and the specific condition they are in. After adding the condition effect to our model, we found an improvement in model fit ($\Delta AIC = 7.64$; $\Delta BIC = 4.25$). Condition turned out to be a significant predictor of the intervention effect ($p = .001$), explaining 25% of the variance of the intervention effect. Thus, students receiving direct embodied support during the teaching sequence displayed higher levels of reasoning after the intervention than students that received indirect embodied support.

In order to gauge the effect of instruction condition, it is helpful to visualize the results. Figure 4 shows these effects for the baseline (left) and the three cohorts separately. The lines in the graphs show the visualization of the additive relationship between the intercept effect, the slope effect, the intervention effect, and the weakening effect, for students in the direct support condition (top line) and students in the indirect support condition (bottom line).

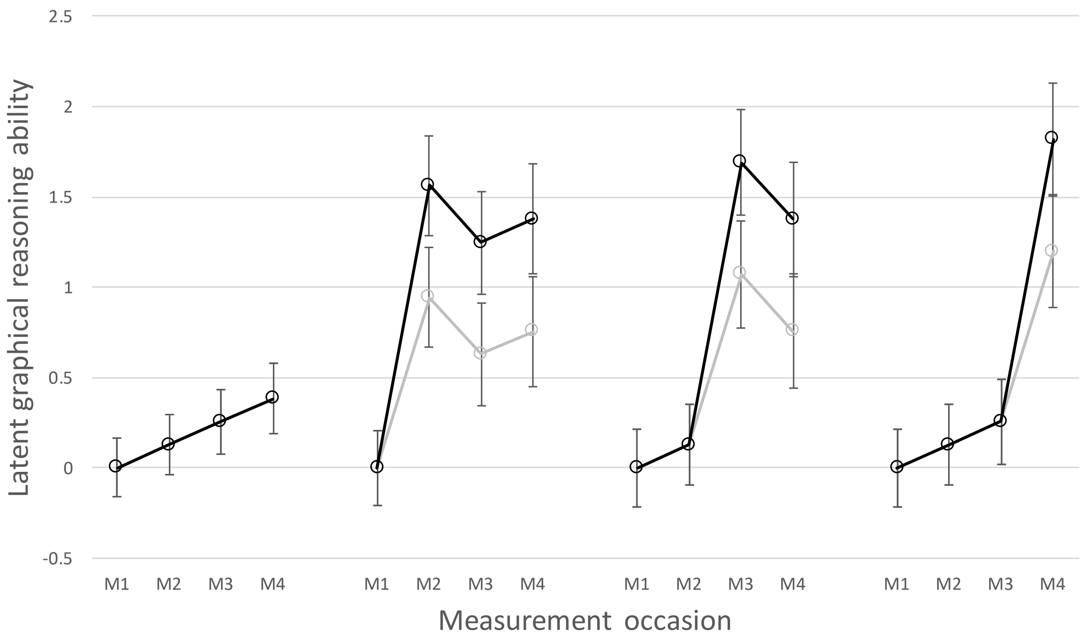

Figure 4. Additive relationship between the effects of the intercept, the slope, the intervention, and the weakening separately for the baseline (left), Cohort 1, Cohort 2, and Cohort 3, for the direct support condition (black line) and the indirect support condition (grey line), for students' latent graphical reasoning ability: Error bars indicate 95% confidence intervals

Table 6 presents the fit indices and parameter estimates of our final model including all four partial effects (i.e., intercept, slope, intervention, weakening), as well as the three predictors (general mathematics performance, general reasoning, condition).

Table 6

Fit indices and parameter estimates of the final LGM model including all partial effects, control measures, and the effect of condition

Model	AIC/BIC	df	Model parameter	Estimate	*p*-value (two-tailed)
Intercept, Slope, Intervention effect, Weakening effect + Condition as predictor of the intervention effect + General mathematics performance + Non-verbal reasoning	7792.624/ 7873.852	193	Intercept (mean)	0.0	fixed
			Slope (mean)	0.128	.003
			Intervention (mean)	1.125	< .001
			Weaken (mean)	-0.443	< .001
			General mathematics performance (mean)	0.044	< .001
			General reasoning (mean)	0.088	.001
			Condition (regression β)	0.309[1]	.001

Note. [1]Condition was coded as 1 Direct support condition and -1 Indirect support instruction condition

5.3 Reaching higher levels of reasoning: Examples of two growth trajectories

In order to explicate what the above quantitative analysis implies in relation to the activities that were conducted in the classroom, and the reasoning of the students on the tasks used to assess their levels of reasoning, in this final section we provide the growth trajectories of two students over the schoolyear (see Table 7). We focus on Task 1. The trajectories given below are not representative for the entire sample of students, they serve as an illustration. Both trajectories show growth in reasoning ability as a result of the intervention and some post-intervention fading of this effect. Following the findings of the quantitative analysis, indicating that the direct support condition was more effective on students' growth in graphical reasoning, we restrict ourselves to the instruction condition offering direct embodied support.

Table 7
Growth trajectories of Elliot and Levi showing their reasoning on the four measurement moments

Name	M1	M2	M3	M4
Elliot	[CD-EF] *"I think so, because these are small pieces"*	[B and C] *"because in 5 minutes they travel 12 kilometers"*	[BC] *"Because between these points you have the most kilometers in a short time period"*	[BC] *"I looked and then I have written down the answer"*
Levi	[B-C] *"It is the longest"*	[b and c] *"I looked at which one was the longest and the time"*	[b to c] *"I looked at where the lines in the graph were going up the highest"*	[BC] *"Nowhere it goes as fast in the graph. He travels in 5 minutes, 8 kilometers, he never does this at another moment in the graph"*

5.3.1 Trajectory 1 – Cohort 1: Elliot

On the measurement before the intervention (M1), Elliot based his answer on some superficial characteristics of the graph, resulting in Level R1. The answer of Elliot is "C-D and E-F", which is an incorrect answer. Elliot corroborates his answer with: "Because these are the shortest pieces". With shortest pieces this student refers to the line segments in the graph. On the measurement directly after the intervention (M2) the reasoning of Elliot has changed. He now uses the variables distance and time in an informal covariational manner: "because in 5 minutes they travel 12 kilometers", using both quantities represented on the axes of the graph in his reasoning. On the third measurement moment (M3), Elliot still reasons according to the highest level (R3), still showing reasoning in an informal covariational manner, yet without explicitly mentioning the numerals. Instead he qualitatively refers to the given quantities "most kilometers" and "little time". On the final measurement (M4), Elliot does not show reasoning that is related to the graphical representation anymore. Instead his reasoning is merely procedural, resulting in Level R0. The growth trajectory of Elliot illustrates how a student can show an increase in level of reasoning from pre- to post intervention and a weakening effect on one of the delayed measures, as was found in the quantitative analysis described above.

5.3.2 Trajectory 2 – Cohort 3: Levi

On the first measurement moment, Levi shows reasoning according to Level R1, see Table 7. He, like Elliot, focuses on a particular line segment being "the longest". Although his answer is correct: "BC", the reasoning associated with his answer can be considered superficial. On the second and third measurement moment, without having had an intervention, Levi shows reasoning according to Level R2. For example, on measurement moment 2 he states: "I looked at which one was the longest and the time". Although the first part of this answer is similar to his answer given on measurement moment 1, this time he corroborates his answer with explicitly mentioning the variable time, indicating that he incorporated the quantity time given on the *y*-axis of the graph. Finally, on the fourth measurement moment, directly after having had the intervention, he shows reasoning according to Level R3: "Nowhere it goes as fast in the graph, he travels in 5 minutes, 8 kilometers, he never does this at another moment in the graph." The growth trajectory of Levi shows how Levi throughout the schoolyear shows growth, regardless of having had an intervention. Yet, his reasoning after the intervention clearly is more elaborate.

6. Discussion

In this study, we examined whether a six-lesson teaching sequence on motion graphs raised students graphical reasoning. We defined graphical reasoning as a mixture of qualitative and quantitative reasoning about a single variable or about multiple variables, as opposed to reasoning in an iconic or pictorial way. We took students' written responses to the open-ended graph interpretation and graph construction tasks as reflecting their reasoning and coded this reasoning on four levels of increasing complexity and appropriateness. In line with previous research, the present study investigated the added benefit of direct bodily experiences, compared to indirect bodily experiences in the teaching sequence. We thus asked: *To what extent does embodied support in a six-lesson teaching sequence on graphing motion affect the development of students' graphical reasoning?* The teaching sequence focused on problem situations involving motion, situated in a real-world context that was presented on worksheets and modelled on the digital blackboard in the instruction condition offering indirect embodied support and was presented on paper and physically enacted in the instruction condition offering direct embodied support. In our method and analyses, we took into account both short-term and middle-long-term effects of the intervention.

We modelled individual changes in graphical reasoning ability using latent growth modelling. We found that students' graphical reasoning improved after taking part in the teaching sequence on motion graphs. Students more often used reasoning taking into account a single variable (Level R2) or taking into account multiple variables (Level R3). We also found that students taking part in the direct embodied support condition benefited more from the intervention than students in the indirect embodied support condition. Students receiving direct embodied support showed more often higher levels of graphical reasoning (Level R2 and Level R3) after partaking in the teaching sequence than students receiving indirect embodied support. This shows that an embodied learning environment incorporating immediate whole-bodily motion activities is more helpful in stimulating students' reasoning about graphs than when students do not perform immediate whole-bodily motion activities, and instead receive an illustrated model of this motion sensor context on worksheets and the digital blackboard. This finding underscores previous research within this specific mathematics domain (e.g., Deniz & Dulger, 2012), and other mathematics domains (e.g., Fisher et al., 2011). For a review on this topic, see Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman and Leseman (2019). The difference in terms of estimated abilities, between the two conditions, was about one standard deviation. The proportion explained variance, however, was small (r^2 = .25). This can be explained by the fact that students in the indirect support condition, were also confronted with activities that capitalize on bodily-based experiences. For example, the object of the toy car used in the indirect support instruction condition, to some extent, might have caused neural activity in the human brain similar to the neural activity induced when viewing another person's action or performing an action (see also Beauchamp & Martin, 2007; Chao & Martin, 2000; Chouinard & Goodale, 2010). Additionally, the graphing of motion itself capitalizes on experienced motion, whereby these experiences with real motion can act as metaphorical mappings between source-domain experiences (such as real movements through space) and the graphical representation, even in the absence of direct physical experiences (e.g., Barsalou, 1999; see also Castillo-Garsow et al., 2013).

In previous research it has been established that when students partake in graphing activities, using for example a motion sensor and desktop laptop, several graph reading errors, such as iconic and pictorial interpretations of graphs can be overcome (e.g., Brasell, 1987; Deniz & Dulger, 2012; Duijzer, Van den Heuvel-Panhuizen, Veldhuis & Doorman, 2019, see *Chapter 3* of this thesis; Mokros & Tinker, 1987). These findings were mostly based on tests consisting of multiple-choice questions. In our study, we added complexity and depth to the analyses by taking into account students' written explanations as indications of their level of reasoning and changes therein over a prolonged period of time. We illustrated these changes by incorporating two qualitative examples presenting the growth trajectories of two students. At the highest level of reasoning (Level R3) these students reasoned about the variables distance and time in an informal covariational manner. Additionally, these qualitative examples showed the added value of including students' written explanations in the statistical analysis. For example, Levi gave the correct answer on each of the four measurement moments, yet his written explanations show a clear increase in the level of understanding over time. At the first measurement, he incorporates a superficial characteristic of the graph in his reasoning, while at the final measurement (M4) his reasoning changed to reasoning in which he took into account both variables. Thus, including students' written explanations gave us more information regarding their understanding than when we would have only looked at students' correctness scores. This approach is in line with Lai et al. (2016), who show the importance of incorporating a direct measure of reasoning by giving students the opportunity to elaborate on their answers in achievement tests. In this sense, we demonstrated that students' reasoning taking into account iconic or pictorial aspects of the graphs (Level R1), was often replaced by reasoning in which they took into account one or more of the relevant variables (Level R2 and Level R3), regardless of the correctness of their answer.

6.1 The value of direct versus indirect embodied support

The motion sensor context used in our study is just one example of digital technology that has been utilized over the past couple of decades to support learning in mathematics and science classrooms. The digital element of the motion sensor entails the real-time translation of movement into a digitalized graphical representation of that movement. The context of the motion sensor was used extensively in the teaching sequence offering direct embodied support. In the instruction condition offering indirect embodied support, the students did not have the opportunity to benefit from a motion sensor in the physical way. They were offered this context on paper and on the digital blackboard. Thus, on the basis of our comparison between instruction conditions, we cannot determine exactly which specific elements of the teaching sequence were most helpful in facilitating students graphical reasoning. Both instruction conditions involved sense making activities that were perceptually experienced (Barsalou, 1999; see also Goldman, 2012).

Further, we operationalized direct embodied support as making whole bodily movements in front of the motion sensor. Yet, due to the nature of the motion sensor context, the whole bodily motion activities in front of the sensor to some extent has more advantages than the physical experience of motion alone. It includes physical movement as well as immediate feedback provided by the tool. Even though this immediate feedback was sometimes also provided to the students in the teaching sequence with indirect embodied support, the combination of physical experiences with real-time feedback in one instruction condition makes it difficult to disentangle their respective unique effects. Future research could address this by creating a condition in which students for example do not receive immediate real-time feedback, but delayed feedback (see also Brasell, 1987), to isolate the effects of the real-time feedback provided by the tool. Another possibility is to isolate the unique contribution of own bodily motion experiences. For example, by letting students work with a dynamic model of the activities' set-up. An example of such a learning environment is presented in the study of Salinas et al. (2016), who gave students the opportunity to control an animated avatar in a computer software program. The movement of the avatar is presented alongside the corresponding graph. The students could influence or control the motion of the avatar, but could not move their selves, eliminating the possibility of direct physical experiences.

6.2 Limitations, strengths, and future research

This study has some limitations that we have to mention here. First, even though students' reasoning on the test items provided us with a window into their thinking processes, we cannot be sure that we captured the full breadth of students' understanding, when only looking at their written responses to the tasks. It might be worthwhile to include more extended measures such as think-aloud protocols when solving the tasks. A second, related limitation, is that we included only four tasks to measure students' development in reasoning about motion graphs. Even though using few tasks is a considerable advantage when thinking about the mental effort imposed on the students, future research might consider using more tasks, specifically more graph construction tasks. A third, and final, limitation worth mentioning is that even though we have investigated the teaching sequence in a realistic classroom setting, which enhanced the ecological validity of our study and the applicability of the approach in education, a drawback of this approach is that some of the teaching time was consumed by the procedural aspects of setting up the equipment. Also, the use of motion sensor technology in the classroom might have had a distracting effect as well. Since not all students are walking at the same time in front of the sensor some students sometimes were disengaged, either by the other small group working with the sensor, or by talking with their peers (see also Anderson & Wall, 2016). A suggestion for future research is to let students work in even smaller groups (e.g., three or four students) on the tasks.

This study also has several strengths. First, an important difference between previous research on graph understanding in the primary grades and the current study is that we looked at the development of students' graphical reasoning over a year. We included multiple measurements to look at students' longitudinal development and to take into account fade-out effects of the intervention. We indeed found a fade out effect for the intervention. Second, from a statistical point of view, this study is innovative in the sense that the used latent growth curve model incorporated categorical responses to the tasks, which allowed us to model gradual changes in levels of reasoning (Boom & Ter Laak, 2007). Third, our cohort-sequential research design enabled us to "re-use" student groups per instruction condition, whereby the groups served as their own control group, depending on the specific cohort. This resulted in the need of fewer participants overall, which is an advantage from both a practical and ethical point of view. Fourth, we incorporated a baseline condition that helped us to more accurately estimate the intercept effect and the slope effect, thus increasing this study's statistical power. As a fifth strength we would like to mention the contribution of our study to the existing literature, by presenting a way of incorporating whole bodily movements in whole-classroom lesson activities.

6.3 Conclusions and implications for education

The aim of this study was to incorporate (physical) experiences during graphing activities as embodied support in mathematics lessons in order to positively contribute to fifth-grade students' understanding of distance-time graphs. This study showed that the used activities resulted in higher levels of graphical reasoning, thus demonstrating the usefulness of incorporating graphing activities in the primary school mathematics classroom. Additionally, this study showed the added value of physical activities, as whole bodily movements in front of the motion sensor, on students' graphical reasoning. The current study adds to a growing body of evidence that physical experiences are indeed helpful for mathematics learning in general and graphical understanding in particular. Yet, what exactly caused this growth is something further research could explore.

Even though on the basis of this study we cannot make strong statements, we do think our study has some implications for graphing motion in primary school mathematics classrooms. First, through carefully designed lesson activities involving problem situations situated in a real-world context, capitalizing on students' intuitive understandings of representing motion, students' graphical reasoning can be improved. Second, our study shows that it is possible to implement embodied activities, that are activities enriched with immediate whole-bodily motion experiences, in an authentic classroom setting (see also Deniz & Dulger, 2012), which adds to research investigating practical applications of embodied cognition approaches for education and learning. In this respect, our study confirms findings from previous research into embodied mathematics learning showing the feasibility of incorporating these type of physical bodily-based activities in whole classrooms.

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CHAPTER 5

Fifth grade students' reasoning on graphs of motion and linear equations

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This chapter will be combined into a journal article with a parallel chapter having the focus on linear equation solving which is published in the PhD thesis of Mara Otten.

Fifth grade students' reasoning on graphs of motion and linear equations

Abstract

Although the domains of graphing and algebra involve similar aspects of domain-specific mathematical higher-order thinking (HOT), including reasoning about covariation, research investigating the conceptual overlap between elementary understanding of graphing and algebra is scarce. In this study, we investigated the effects of a six-lesson teaching sequence about graphing motion on students' graphical reasoning and students' algebraic reasoning. We assessed 138 fifth-grade students' development in reasoning on four graphing motion tasks (graph interpretation and construction) and four linear equation solving tasks, four times over one school year. Both mathematical domains draw on the application of covariational thinking, which in both task-groups was operationalized as the HOT skills *extracting*, *using and combining sources of information about mathematical relationships.* Results from our analyses using latent growth curve modelling showed that the lessons on graphing led to a significant improvement in students' graphical reasoning, as well as an – albeit smaller – significant improvement on students' algebraic reasoning. There was also a strong correlation between initial level of reasoning on both domains, yet no correlation between development on both domains was found. This implies that the intervention on graphing motion did affect students' algebraic reasoning, but that this relationship was not related to individual improvements in both mathematics domains.

Keywords: Motion graphs, Linear equations, Domain-specific mathematical higher-order thinking

1. Introduction

Ollie and Eve are going to school. Eve leaves home a little earlier than Ollie. Halfway she waits for Ollie to catch up. They continue their journey together and arrive at the same time.[1]

In order to draw an accurate distance-time graph of the situation above, a 10-year old student should understand the relationship between time and distance and how this relationship can be represented graphically to represent the movements of Eve and Ollie. This task is far from easy. When constructing a line-graph, students should be able to visualize a relationship between two changing variables with the graph's axes as a reference. This implies a deep understanding of how combining these two variables can be represented as a line in the graphical representation. Constructing graphical representations, reasoning about graphically represented change (e.g., motion), making connections between variables on the two axes, and being able to critically reflect on the information presented in these (or similar) graphs, among others, can be regarded for 10-year old students as domain-specific mathematical higher-order thinking (HOT) (e.g., Boote, 2014; Kramarski & Mevarech, 2004). We consider these activities as requiring HOT for these students, due to their non-algorithmic nature and their deviation from routine procedures (e.g., Murray, 2014).

HOT in interpreting and constructing graphical representations involves reasoning about complex mathematical concepts. An important example of such a concept is covariation (Fitzallen, 2012; Leinhardt et al, 1990). Within a graph, covariation is depicted as the relationship between two sets of measurements that vary along numerical scales, with each data point referring to a particular value of two variables at the same time (Fitzallen, 2012; Hattikudur et al., 2012). According to Saldanha and Thompson (1998), for young children, covariational reasoning entails the mental activity of coordinating the values of two quantities, while thinking about each quantity in turn (e.g., first time, then distance, then time, and so on). This covariational reasoning is important for students in order to make a connection between the two variables represented on the axes of a graph. The notion of covarying quantities is also important in other mathematical domains, like functions and algebra, when students have to think about how changes in values of one variable are related to changes in another variable. This is considered a prerequisite for the development of functional thinking (Panorkou & Maloney, 2016).

[1] Adapted example from Lesson 6 of the teaching sequence used in the current study.

Covariational reasoning, on the one hand can be seen as domain-specific: used within graphs (e.g., using shapes of graphs to describe change) or within algebra (e.g., using the structure of equations to identify the similarity of $a + 2b = 3$ and $2a + 4b = 6$). On the other hand, it calls upon processes that are similar across mathematics domains, such as covariational reasoning in terms of extracting, using, and combining sources of information about mathematical relationships (e.g., simultaneously coordinating the values of two quantities) within both graphs and algebra. HOT is considered to be of increasing importance in our knowledge intensive society (Forster, 2014; OECD, 2019). There is general consensus that laying a strong foundation for these HOT skills should start in primary school (e.g., NCTM, 2000) and that this also applies to the introduction of graphs (Friel, et al., 2001) and early algebra (Kaput, 2008). Yet, opportunities within primary school mathematics education to raise the level of students' mathematical thinking, have been found to be rather scarce. For example, in Dutch mathematics textbooks, opportunities for students to show and develop HOT are virtually absent (Kolovou et al., 2009; Van Zanten & Van den Heuvel-Panhuizen, 2018).

In this study, we investigated opportunities the domain of motion graphs offers for promoting transfer of HOT to the domain of linear equations. We analyzed the effect of a teaching sequence on graphing motion, including activities targeting students' domain-specific HOT (e.g., *making connections between a motion event and its representation in the graph, reasoning about changing quantities*), while also touching upon more general components of mathematical HOT (e.g., *extracting, using, and combining sources of information about mathematical relationships*), on students' reasoning about graphing motion and solving systems of informal linear equations (hereafter: linear equation solving) (informal: without formal notation). The findings of this study will provide further insight into the extent to which mathematical HOT can be stimulated within and across mathematics domains.

2. Theoretical background

2.1 The nature of higher-order thinking

Within educational science a distinction is often made between higher- and lower-order cognitive abilities or thinking skills (e.g., Lewis & Smith, 1993). Higher-order cognitive abilities, such as creativity, reasoning, and concept formation, are based on – and influenced by – lower-order cognitive abilities, such as attention, perception, and motor development (Shuxian, 2009). A similar division can be found in Bloom's taxonomy (1956), in which the three bottom levels; knowledge, comprehension, and application, are assumed to serve the transition towards the three upper levels;

analysis, synthesis, and evaluation. Within educational science the top three levels are often used to operationalize HOT. According to Levine (1999), HOT "enables students to grapple with intellectually sophisticated challenges, integrate multiple ideas and facts, undertake difficult problems, and find effective and creative solutions to dilemmas whose answers are not immediately obvious" (p. 217). This definition stresses how HOT is quite different from memorization, factual recall, and the following of routine or fixed solution procedures. Rather, HOT implies a deep command of these basic and more advanced skills, while also knowing how to apply them within new contexts (e.g., Murray, 2014). What can be considered as HOT for one individual, might be a routine thinking procedure for someone else. Therefore, the application of HOT in the classroom (and what makes it different from lower-order thinking activities) also depends on the nature of a task and a person's intellectual experience (Alexander et al., 2011; Lewis & Smith, 1993).

Most conceptualizations of HOT reflect the longstanding belief that thinking skills largely consist of generic components, such as the three top levels of Bloom's taxonomy, which can be applied to any academic domain, regardless of disciplinary knowledge (e.g., Greeno, 1987; Leighton, 2004; Resnick, 1987). Yet, others have taken a different position. Alexander et al. (2011, p. 54) conceptualize HOT as "the mental engagement with ideas, objects, and situations in an analogical, elaborative, inductive, deductive, and otherwise transformational manner that is indicative of an orientation toward knowing as a complex, effortful, generative, evidence-seeking, and reflective enterprise" while also "exhibit[ing] distinctive qualities arising from *the nature of the domain within which the task or activity is situated*" (emphasis added, p. 51-53). In order to make specific use of the resources within a domain, one will always need the incorporation of disciplinary knowledge (Tricot & Sweller, 2014). Per this view, HOT originates from – and is intricately linked to – specific topics within academic domains (e.g., Alexander et al., 2011; Ericsson, 2003). For example, within the domain of motion graphs, critically evaluating a graph can be seen as domain-specific HOT. This is also in line with the framework *Teaching for Robust Understanding* (TRU), which suggests that domain-specific learning environments are needed to support students in "becoming knowledgeable, flexible, and resourceful disciplinary thinkers" (Schoenfeld, 2016, p. 3).

2.2 Reasoning about motion graphs in the primary school mathematics classroom

A graph is a visible object yet entails invisible mathematical concepts or relationships that are to be constructed by the student. When interpreting a motion graph, students should be able to extract the relevant pieces or segments from the graph and give an interpretation of this information in relation to the physical situation the graph represents (e.g., Friel et al., 2001; Janvier, 1981; Shah & Hoeffner, 2002; Vitale et al., 2015). For example, the slope in a distance-time graph represents the relationship between two variables, distance and time, which simultaneously represents another physical quantity, namely speed. Students can derive speed from the distance-time relationship as represented in the graph, by qualitatively or quantitatively inspecting the slope. Moreover, speed is visually present in the steepness of slope: a steeper slope means faster movement, as more distance is covered (on the vertical axis) in the related time interval (on the horizontal axis). Slope is an important concept within graphs in both mathematics and physics (Planinic et al., 2012). Another important concept is scale. The axes of a graphical representation have a certain scale that can be adjusted. Through the adjustment of the scale of the axes, the shape of the represented relationship changes, which offers opportunities to reason about this relationship as well as about the (qualitative aspects of) slope (Nemirovsky et al., 2013; Zaslavsky et al., 2002). When reasoning about representing the dynamic situation of distance changing over time, students are prompted to connect the represented physical situation (i.e., motion) with visual elements of the graphical representation (i.e., the slope, rate of change, scaling on the axes). For example, understanding that adapting the scale of a graph changes the appearance of the graph but does not alter the information represented in the graph is an important step when coming to understand and work with graphical representations. It involves flexibility and sensitivity regarding the visualization of change and relationships as well as the ability to reason about the relationship between the two variables and their pattern of covariation (Leinhardt et al., 1990).

2.3 Domain-specific HOT in graphing motion and linear equation solving

Graphing and linear algebra, including graphing motion and linear equation solving, are often addressed together in mathematics education. This connection can be explicit, for example writing an equation to represent the relationship between distance and time in a problem involving motion at constant speed (e.g., Thompson & Carlson, 2017) or implicit, as in the research of Nemirovsky and Rasmussen (2005). Nemirovsky and Rasmussen describe a learning arrangement incorporating kinesthetic activity with a physical tool, called the water wheel, which was supposed to support students in their understanding of motion graphs. Interestingly, this specific activity also led to the construction and interpretation of formal algebraic expressions, while the construction and interpretation of these formal algebraic expressions was not explicitly taught. They also describe that to date, few studies have been conducted investigating the interplay between kinesthetic activities and equations, or other symbolic expressions. This idea that using kinesthetic activities as direct perceptual-motor experiences in mathematics learning activities can be helpful for learning within and across mathematics domains, is informed by theories of embodied cognition. Theories of embodied cognition posit that all thinking and learning (including formal abstract mathematics) is grounded in concrete physical interactions of our body with the surrounding world (Lakoff & Núñez, 2000).

Within the domain of motion graphs various mathematics concepts are addressed. A graphical representation is a formal symbol system, representing a relationship between two variables, showing a pattern of covariation. Within a distance-time graph, speed is a hidden quantity which can be deduced by synthesizing the information represented on the *x*- and *y*-axis. When constructing a distance-time graph, speed can be qualitatively visualized in the steepness of slope, or quantitatively, by taking into account the values of the variables on the *x*- and *y*-axis. In order to solve a graphing question for which there is no fixed solution procedure (e.g., questions involving trends or relationships that cannot be directly answered by extracting information regarding specific points), HOT is required, because information found in the graph has to be combined and visual comparisons have to be made within and between graphs. This requires flexibility of students to switch between representations, descriptions of situations, or between other ways of representing data, such as tables or equations.

Within the domain of linear equations, equality is an important concept, meaning that the expressions on both sides of the equal sign represent the same value. During the process of solving for the unknown this equality of the equation should be maintained. This makes a correct understanding of equality crucial for solving linear equations (e.g., Bush & Karp, 2013; Kieran et al., 2016). When solving a system of linear equations, the information from multiple equations needs to be combined in order to find the values of the unknowns. For this, students need to reason about the relationships between these unknowns and their pattern of covariation (i.e., how changes in the one result in changes in the other). Consider the following example: *Lotte buys one pizza and one soda for €10. The next week, she buys three pizzas and two sodas for €27. What is the price of one pizza and what is the price of one soda?*[2] To solve this problem, a student needs to reason about the unknown price of a pizza in relation to the unknown price of a soda. In addition, when combining the

[2] Adapted example from one of the algebra tasks used in the current study

information from both equations, a student has to reason about the relationship between the value of unknowns in one equation in relation to the relationship between the value of the unknowns in the other equation. In the example above, the first equation (i.e., pizza + soda = 10) fits two times in the second equation (i.e., 3 pizzas + 2 sodas = 27). In order to isolate the price of one pizza, a student for example might reason about changes in the total price when subtracting one pizza and one soda from the second equation, or when they replace the pizza and the soda by the price of 10 (on the basis of the first equation).

Within the domains of motion graphs and linear equations, covariation, as the simultaneous coordination of two quantities' values, is a core concept. We cannot automatically assume that this concept is similar across domains, yet we can describe this essentially domain-specific concept as also involving more general HOT skills occurring within both mathematical domains. In particular, reasoning about covariation involves extracting, using, and combining sources of information about mathematical relationships. For example, students can extract the information found on the graphs' axes, take into account their interrelatedness, and combine the given quantities into something new. Similarly, students can extract the information provided in equations in a system of equations, take into account their interrelatedness, and combine this information to find unknown values or relationships. Given that the concept of covariation is important to both domains, it would be worthwhile to investigate whether stimulating reasoning about such HOT within one domain, might potentially result in the development of HOT within the other domain. Due to similarity in general elements of covariational reasoning across both domains (i.e., extracting, using, and combining sources of information about mathematical relationships), achieving application of HOT in the other mathematical domain, even when this reasoning is not targeted explicitly, seems promising.

Challenging domain-specific mathematics activities could offer a fruitful starting point to elicit HOT in the domain of motion graphs. To this end, we developed two parallel versions of a six-lesson teaching sequence on graphing motion, resulting in two instruction conditions, in which fifth-grade students explored graphs representing the bivariate relationship of distance changing over time (see also Duijzer et al., 2020, see *Chapter 4* of this thesis). The two instruction conditions offered either *direct* or *indirect* embodied support to the students. In the instruction condition offering direct embodied support, students were allowed to "walk graphs" in front of a motion sensor. Students experienced directly, with their own body, how changes in movement resulted in changes in the graphical representation. In a previous study (Duijzer et al., 2019, see *Chapter 3* of this thesis), it was shown how these direct physical experiences during the lessons engendered high levels of graphical reasoning. In the instruction condition offering indirect embodied support, students received this motion sensor context on worksheets and the digital blackboard. Students partaking in the instruction condition offering direct embodied support improved slightly more than students partaking in the instruction condition offering indirect embodied support (Duijzer et al., 2020). In the present study, both instruction conditions are included.

3. The present study

In the present study we investigated transfer of receiving lessons within the domain of motion graphs towards linear equation solving. The domain-specific mathematical HOT that was stimulated throughout these lessons might have the potential to transfer to the domain of linear equations. In parallel, another study was carried out to investigate the effect of receiving lessons within the domain of linear equations, towards students' ability to reason about graphing motion (Otten, Duijzer et al., 2020). We assume that transfer might take place on the basis of students' reasoning about covariation, which in both mathematical domains plays an important role. More specifically, reasoning needed within both mathematical domains requires the HOT skills of extracting, using, and combining sources of information about mathematical relationships. We formulated the following research question: *To what extent does a six-lesson teaching sequence on graphing motion affect students' graphical and algebraic reasoning?*

We used two series of four tasks to assess primary school students' improvement of graphical and algebraic reasoning. On the graphing tasks, students were asked to reason about problems with two changing variables presented on the horizontal and the vertical axes of a graph or constructing the relationship between two changing variables as a graph. On the algebra tasks, students were asked to reason about problems involving a system of informal linear equations. Solving these mathematical problems requires handling the underlying covarying relationship between variables. We hypothesized that after partaking in the teaching sequence on graphing motion students would show an improvement in their ability to reason about linear equation solving. The presence or absence of an intervention effect on students' algebraic reasoning, which was not intentionally taught, would give us more insight regarding the extent to which domain-specific mathematical HOT can also stimulate more general components of mathematical HOT as indicated by the presence of HOT in the other mathematics domain.

4. Method

4.1 Participants

Participants were 150 fifth-grade students from six classes from six different elementary schools. From 12 students we did not receive permission to use the collected data. Our final sample consisted of 138 students (53 female, 38%). The average age of the students was 10.5 years (*SD* = 0.4). Schools, teachers, and students participated on a voluntary basis. All schools were located in the area of the city of Utrecht, the Netherlands. Data were collected between October 2016 and June 2017. The Ethical Review Board of the faculty of Social and Behavioural Sciences at Utrecht University approved of this study.

4.2 Study design and procedure

Participants six-lesson teaching sequence on graphing motion was provided to the students as part of their regular classroom instruction at different moments during the school year. We adopted a cohort-sequential design, meaning that Cohort 1 received the teaching sequence in the first trimester of the school year. Cohort 2 received the teaching sequence in the second trimester of the school year. And Cohort 3 received the teaching sequence in the third trimester of the school year. Table 1 gives an overview of the study design.

Table 1
The cohort-sequential design of the study

Cohort		Phase						
			Oct. – Nov. 2016		Jan. – Feb. 2017		Apr. – May 2017.	
1	(*n* = 45)	M1	*Teaching sequence Graphical reasoning*	M2		M3		M4
2	(*n* = 45)	M1		M2	*Teaching sequence Graphical reasoning*	M3		M4
3	(*n* = 48)	M1		M2		M3	*Teaching sequence Graphical reasoning*	M4

The six-lesson teaching sequence was taught to the students by the first author of this paper, with the help of a teaching assistant. Each lesson took approximately 50 minutes. The lessons were divided over 6 weeks, one lesson per week. Two weeks before participating in the teaching sequence, all students completed an abbreviated version of Raven's Progressive Matrices (Bilker et al., 2012). One week before the first cohort of classes participated in the teaching sequence, all students in all cohorts completed a mathematical HOT test, consisting of four tasks related to graphing motion and four tasks related to linear equation solving. After each cohort completed the teaching sequence there was another assessment, so four times in total (M1-M4). Students were tested in their own classroom. All students completed the test at the same time, which took approximately 45 minutes.

4.3 Teaching sequence

The aim of the teaching sequence was to teach students about graphs representing the bivariate relationship of distance changing over time (i.e., distance-time graphs), and elicit students' reasoning about these graphs. An overview of the teaching sequence, including the topic and key activities per lesson, is given in Table 2 (see also Duijzer et al., 2019; Duijzer et al., 2020). Students' reasoning about graphical representations was stimulated by asking them to explain, hypothesize, evaluate, compare, and discuss their ideas with other students in small groups.

4.4 Instruction condition

Two parallel versions of the teaching sequence were developed, resulting in two instruction conditions. In the instruction condition offering direct embodied support, physical experiences with graphing motion, using motion sensor technology, were a major part of the lesson activities. In the instruction condition offering indirect embodied support, the students were given the motion sensor context with graphing activities that were paper-and-pencil based or presented on the digital blackboard. The graphing motion activities on the digital blackboard were mostly non-dynamic and contained motion situations dealing with non-human moving objects, such as a toy car travelling a particular distance within a particular period of time. Similar situations were also provided to students in the instruction condition offering direct embodied support, but instead of only providing students illustrated versions of these situations, students were explicitly prompted to physically enact the situations, in front of the motion sensor. The movements performed by the students in front of the motion sensor directly corresponded with the real-time representation of those movements as a line in the graph. Both instruction conditions draw on the source-domain bodily experiences of moving through space. Previous research showed that both conditions were effective in stimulating students' graphical reasoning (Duijzer et al., 2020).

Tabel 2
Overview of the six-lesson teaching sequence on graphing motion

Lesson title	Main topic *Activities*
1. Motion: reflecting and representing	Informal graphical representations *Reason with variables and construct representations of a real-world situation*
2. From discrete to continuous graphs	Measuring distance *Measure distance in discrete intervals and continuously, and reason about differences between discrete and continuous graphs*
3. Continuous graphs of "distance to" (1)	Reason with continuous graphs *Coupling specific movements to their representation as a line in the graph* *Coupling a concrete situation to a graphical representation*
4. Continuous graphs of "distance to" (2)	Reason with continuous graphs *Coupling specific movements to their representation as a line in the graph* *Investigating how speed is represented in the steepness of slope*
5. Scaling on the graphs' axes	Reason about the relationship between two variables through scaling *Construct graphs with different scales on the axes*
6. Multiple movements and their graphical representations	Generate, refine, and reason about simultaneous movements and their representation as a graph *Critically evaluate points of intersection and their meaning*

4.5 Measures

4.5.1 General mathematics performance

To obtain a measure of students' general mathematics performance we used test performance data from the Dutch student monitoring system (CITO LOVS: Janssen et al., 2010). This information was provided by the schools. Schools use this (or a similar) system to monitor students' performance on the biannual standardized mathematics tests. We collected students' results from the end-term Grade 4 test (norm population end-term Grade 4: $M = 91.9$, $SD = 10.6$, CITO, 2015).

4.5.2 General reasoning

In order to obtain a measure of students' general reasoning we administered an abbreviated version of the Raven Standard Progressive Matrices (Raven SPM: Raven et al., 2000), consisting of two sets of 9-items (Bilker et al., 2012). Raven's SPM is a test of non-verbal reasoning ability and fluid intelligence. We administered the test to all students in one session in their classroom, following the test manual's instructions.

4.5.3 Mathematical HOT

Mathematical HOT was measured by collecting students' responses on four graphing tasks and four algebra tasks. For all tasks, students were invited to elaborate on their answer by answering the question "How do you know?" These written responses of the students were coded as an indication of their graphical reasoning.

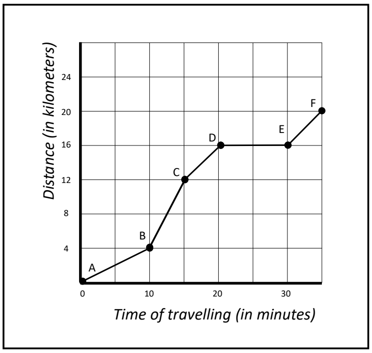
4.5.3.1 Graphing tasks

Table 3 shows two tasks. Task 1 (left) shows a distance-time graph. The graph represents the movement of a car, as indicated by the graph's heading. The speed of the car – the hidden quantity – can be deduced by a global visual inspection of the slope of the line or by looking at the specific numerals on the x-axis and y-axis and calculating the distance travelled within a period of time (and compare this with the other segments present within the graph), thus comparing rate of change. In order to respond to the interpretation task, the students have to grasp the meaning of the variables on the x-axis and the y-axis and compare segments within the graph. Task 2 (right) shows an empty graph and a description of a motion situation. The motion situation consists of three separate parts, in which the train travels at different speeds, which implies different rates of change in the graph ("twice as fast between 11 and 12 o'clock"). These differences should be quantified and visualized in the graph. Simply applying the principle "steeper slope means faster movement" does not necessarily lead to a correct graph, because the position versus time curves corresponding to the movements described in the motion situation and the relative differences in speed between the three segments have to be taken into account.

Table 3

Example tasks graphing motion including exemplary solutions of two students

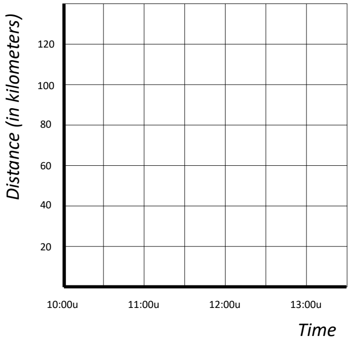
A car drives through town



1a. Between which points does the car goes fastest?
1b. How do you know?

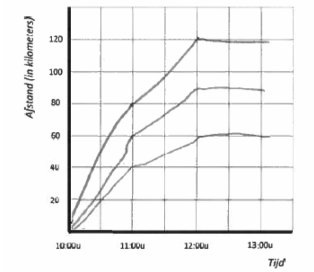
Solution:
"The car travels in 10 minutes 12 kilometres, that is the fastest of what has been shown in the graph."

A train ride.
A train travels **twice as fast** between **10:00 and 11:00** o'clock than between **11:00 and 12:00** o'clock. The train stands still from **12:00 to 13:00** o'clock.



2a. Draw a graph that fits the description above.
2b. How do you know?

Solution:



"because of the text"

Note. The complete coding scheme, including examples of student responses for each task, can be found in Duijzer et al., 2020 (See *Chapter 4* of this thesis).

4.5.3.2 Graphing tasks: Combining sources of information

In Table 3, Task 1 (bottom left panel), an example of a student's reasoning is given. In this answer, the student explicitly refers to the car driving "fastest" between point B and D and compares this with the other segments in the graph "of what has been visualized in the graph". Furthermore, by providing the answer "B and D", and corroborating this answer by stating that "the car travels 12 kilometers within 10 minutes", a reference is made to the correct quantities for time and distance. In Table 3, Task 2 (bottom right panel), a student draws three possible lines in the empty graph. These three solutions are all correct translations of the accompanying text. In that sense, this student seems to understand the relative differences in speed between the three different segments in the story, as well as differences between the distances the train travels when the first segment has different speeds. In both examples, students showed their HOT by combining the information found on the *x*- and *y*-axis of the graph.

4.5.3.3 Algebra tasks

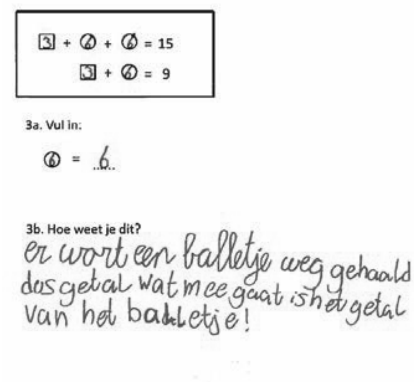
The algebra set of tasks consisted of four tasks in which students were asked to solve a system of informal linear equations. Two tasks required students to find the value of unknown variables. See Table 4 (left) for an example of this type of task. Two other tasks required students to find relationships between variables. See Table 4 (right) for an example of this type of task. In order to find the correct solution to the problems students have to combine information from both the two given equations.

4.5.3.4 Algebra tasks: combining sources of information

In Table 4 (bottom left panel), a student's solution on the algebra task is given. It shows how this student coordinates the value of the little ball in the two equations and combines the information from both equations to come to her final answer. This shows the simultaneous coordination of the values of two quantities. In Table 4 (right panel), another student's solution on another algebra task is shown. This particular student includes information ("4□:2□" and "□:□") taken from the equation on the right, while also showing that □ equals ••. In both these examples students show their HOT by combining information from both given equations.

Table 4

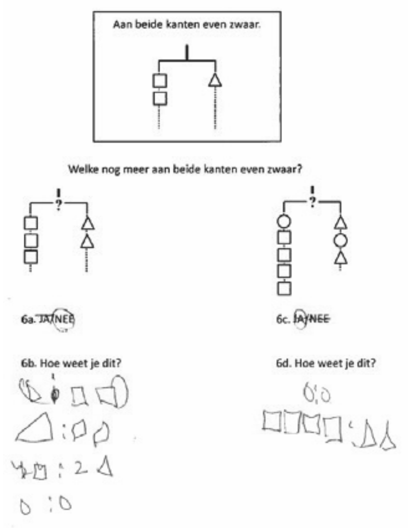
Example tasks algebra including exemplary solutions of two students



1a. Fill in:
1b. How do you know?

Solution:

"A little ball is taken away, so the adjoining number is the number of the little ball!"



2a. Fill in:
2b. How do you know?

Note. The complete coding scheme, including examples of student responses for each task, can be found in Otten, Van den Heuvel-Panhuizen et al., 2020.

4.6 Coding scheme for students' reasoning showing HOT

For both the graphing tasks and the algebra tasks separate (domain-specific) coding schemes were developed to indicate students' level of reasoning, that shared a common structure to qualify reasoning in terms of three levels of complexity (Duijzer et al., 2019, 2020; Otten et al., 2019, Otten, Van den Heuvel-Panhuizen et al., 2020). For the graphing tasks, reasoning about the variables in the graph was taken as point of departure. The highest level of reasoning, Level R2, indicates reasoning on the basis of multiple variables (distance, time, and/or speed), which can be considered equivalent to the HOT of extracting, using, and combining sources of information about mathematical relationships is present. The intermediate level of reasoning, Level R1, indicates reasoning on the basis of one variable (distance, time, or speed).

When students reason according to Level $R0_1$, they do not take into account any of the variables but rather reason on the basis of iconic or superficial characteristics of the graph. The level of reasoning, Level $R0_2$, indicates no apparent reasoning (e.g., “I do not know”). For the algebra tasks a similar distinction between levels of reasoning was made. The highest level of reasoning, Level R2, indicates reasoning on the basis of two given equations, indicating the HOT of extracting, using, and combining sources of information about mathematical relationships. The intermediate level of reasoning, Level R1, indicates reasoning on the basis of one of the two given equations. The lowest level of reasoning, Level R0, indicates reasoning without taking into account any of the given equations. Table 5 shows the alignment between both coding schemes and the overarching HOT.

Table 5

Alignment between the coding schemes of graphing and algebra in relation to HOT in terms of extracting, using, and combining sources of information about mathematical relationships

Code	Graphing	Algebra	HOT
$R0_1$	No reasoning	No reasoning	No reasoning
$R0_2$	Iconic/superficial reasoning		
R1	Reasoning with a single variable	Reasoning on the basis of one equation	Reasoning taking into account one source of information
R2	Reasoning with multiple variables	Reasoning on the basis of two equations	Reasoning taking into account more than one source of information

4.7 Data analysis

4.7.1 Preliminary analysis and descriptive statistics

Sample means and standard deviations are given for students’ general mathematics performance and general reasoning. Proportions of students using a particular level of reasoning per measurement moment (M1 – M4) were also calculated. To calculate these proportions we summed the occurrences of a particular level of reasoning (graphical reasoning: $R0_1$, $R0_2$, R1, R2; algebraic reasoning: R0, R1, R2), per measurement moment, for all four tasks together, and divided this by the total occurrence of all levels of reasoning, for all four tasks together, for that same measurement moment. Finally, students’ changes in graphical and algebraic reasoning from pre- to post-intervention were mapped as either positive change (+), no change (=), or negative change (–). The nine resulting possible combinations between graphing and algebra (e.g., + on graphing and + on algebra; + on graphing and – on algebra) were reported in percentages of students showing this particular combination.

4.7.2 Modelling change in graphical and algebraic reasoning

Modelling shifting frequencies in (or proportions of) students' levels of graphical reasoning and students' levels of algebraic reasoning over the four measurements can be realized if we assume two underlying continuous latent abilities for each participant, one for each domain. The latent ability for graphical reasoning represents the probability of achieving a level of reasoning on graphical tasks, and, likewise, the latent ability for algebraic reasoning represents the probability of achieving a level of reasoning on algebraic tasks. Item Response Theory (IRT) modeling was used to map the reasoning levels (i.e., Level $R0_1$, Level $R0_2$, Level R1, or Level R2 for graphical reasoning, and Level R0, Level R1, or Level R2 for algebraic reasoning) to a student's latent ability, in each domain. The probability of using a particular level of reasoning in the two domains is determined by both the task difficulty and students' latent ability in each domain. Latent variable Growth curve Modeling (LGM) was used to model changes in latent ability for each domain over measurements.

LGM offers many advantages for the modelling of longitudinal data when compared to more traditional statistical methods (Willet & Bub, 2005). LGM assumes an underlying latent ability that varies between individuals and can change over time (e.g., due to repetition or experience, or due to participating in an intervention). LGM also permits participants to have different values for the individual growth parameters allowing us to model intra- and inter-individual differences in change over time. We specified one integrated LGM model incorporating a growth trajectory for graphical reasoning and a separate growth trajectory for algebraic reasoning. In order to model changes in both graphical and algebraic reasoning, these individual growth trajectories were estimated based on four partial individual effects: the *intercept effect* (representing individual differences in reasoning on the pre-measurement), the *slope effect* (representing rate of change over time for the subsequent three measurements), the *intervention effect* (representing students' change in ability after partaking in the teaching sequence), and the *weakening effect* (accounting for the possibility that the intervention effect might fade-out over time).

Extending the LGM to a cohort sequential multi-group LGM by including the three cohorts of the intervention as groups, allowed us to evaluate the unique effect of the intervention on students' reasoning in addition to possible spontaneous development that can be attributed to a baseline growth trajectory not related to the intervention, thus allowing a stronger causal interpretation of the effects (Duncan & Duncan, 2009). Because for each cohort the intervention took place at a different moment between measurements, the loadings for the intervention and weakening effect differed between cohorts. For example, for Cohort 1, for which the intervention took place between measurement moment M1 and M2, the intervention could only have an effect on measurements moments M2 to M4. Similarly, for this cohort weakening could only have an effect on the delayed measurement moments M3 and M4. Apart from this, no differences between cohorts were allowed in the model.

We included general mathematics performance and general reasoning ability as time-invariant predictors of the graphing motion intercept and the linear equation solving intercept, in a stepwise process. Both predictor variables were grand mean centered. Including both predictors turned out to be a severe complication in the process of model estimation. Because general mathematics ability and general reasoning ability were correlated, we decided to only use general mathematics ability in the LGM model.

4.7.3 Assessment of model fit

We used Mplus 8, with the Weighted Least Squares Means and Variances adjusted estimator (WLSMV), a PROBIT link, and Delta parameterization (Muthén & Muthén, 1998- 2017). As an evaluation of model fit, we report the root mean square of approximation (RMSEA; Browne & Cudeck, 1993), the comparative fit index (CFI), and the Tucker-Lewis-index (TLI) (Little, 2013). Conventional recommendations are that the RMSEA should be lower than .08, and the CFI and the TLI should be higher than 0.90 (Little, 2013).

4.7.4 Missing data

Of the 138 students in this study, 135 students had complete data on general mathematics performance and 137 on general reasoning. For the missing data of these students, values were imputed based on class averages. Our model could only be estimated if all levels of reasoning, on all tasks, for each of the three cohorts, and for each measurement moment appear at least once. This means that when none of the students showed a certain level of reasoning on any of the tasks on any of the four measurement moments, the model could not be estimated. For the second item, on the graphing tasks, Level R2 showed three empty cells, therefore we decided to join Level R1 with Level R2 for this item.

5. Results

5.1 Preliminary analyses and descriptive statistics

Students' scores on general reasoning ability differed between Cohort 1 ($M = 9.97$, $SD = 2.60$), Cohort 2 ($M = 11.93$, $SD = 2.78$.), and Cohort 3 ($M = 10.25$, $SD = 2.57$), $F(2, 135) = 7{,}64$, $p < .001$, partial $\eta^2 = .102$. Tukey's post-hoc test revealed a significant mean difference ($p = .002$) between Cohort 1 and Cohort 2.

Figure 1 shows the development of students' level of reasoning on the graphical reasoning tasks (left panel) and the algebraic reasoning tasks (right panel). In this figure the measurement occasions (M1 – M4) are aligned between cohorts. Each cohort of students participated in the teaching sequence at different time periods. Students participating in Cohort 1 received the intervention between measurement moments 1 and 2. This cohort is shown in virtual measurement moments 3 to 6. Students participating in Cohort 2 received the intervention between measurement moment 2 and 3. This cohort is shown in virtual measurement moments 2 to 5. Students participating in Cohort 3 received the intervention between measurement moment 3 and 4. This cohort is shown in virtual measurement moment 1 to 4. Aligning the three cohorts in this way allows for a direct visual comparison of the development of students' graphical and algebraic reasoning.

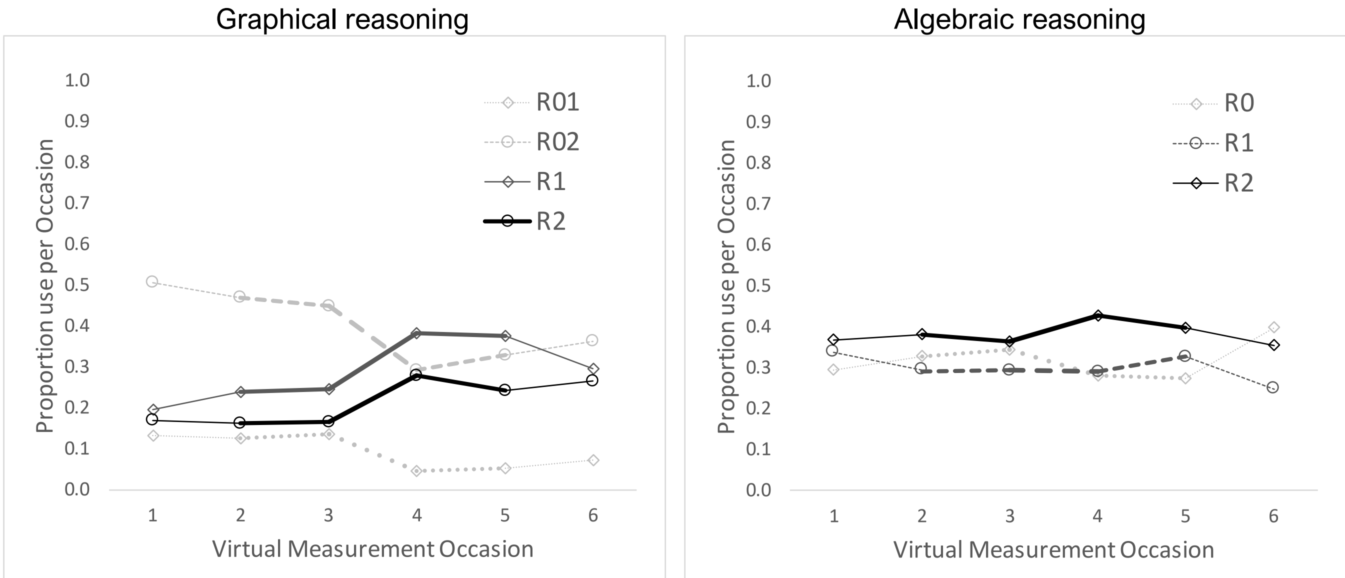


Figure 1. Proportions of level of reasoning (R0-R3) for students' graphical reasoning (left panel) and students' algebraic reasoning (right panel) for each virtual measurement occasion. Between virtual measurement occasions 3 and 4 the teaching sequence took place in all cohorts. Thin line-segments are based on one cohort (VM1, GR n = 190, AR n = 187; VM6, GR n = 176, AR n = 173) thicker line-segments on two cohorts (VM2, GR n = 366, AR n = 358; VM5, GR n = 356, AR n = 354), and thickest line-segments are based on all three cohorts (VM3, GR n = 551, AR n = 545; VM4, GR n = 538, AR n = 527)

5

With regard to students' levels of graphical reasoning, taking part in a six-lesson teaching sequence resulted in an overall increase in the frequency of reasoning on the basis of a single variable (R1: +14%) as well as an overall increase in the frequency of reasoning on the basis of multiple variables (R2: +11%), here shown as group averages per cohort of students. Also, we saw an overall decrease of answers in which students did not show any reasoning ($R0_1$: –11%) and answers in which students reasoned on the basis of iconic or superficial characteristics of the graph ($R0_2$: –15%). Likewise, with regard to students' levels of algebraic reasoning, after partaking in the teaching sequence we saw a decrease of answers in which students did not show any reasoning (R0: –6%), while reasoning on the basis of one equation remained the same. Also, reasoning on the basis of both equations occurred more often (R2: +7%). This effect was short-term as it faded out over time. The frequency of students' reasoning on the basis of both equations decreased again from measurement moment 4 to measurement moment 5 (R2: –3%) and measurement moment 6 (R2: –5%). Regarding students' initial levels of graphical reasoning and algebraic reasoning a difference is noticeable between students' reasoning on the graphing tasks and students' reasoning on the algebraic tasks. The frequency of higher levels of reasoning was higher at the start of the intervention on the algebraic reasoning tasks than on the graphical reasoning tasks, giving students more room for improvement on the graphical reasoning tasks than on the algebraic reasoning tasks.

5.2 Frequencies of students' combined levels of graphical and algebraic reasoning on the pre- and post-intervention measures

We subsequently looked at students' development on either their graphical or algebraic reasoning from pre- to post intervention. Table 6 shows students' development on graphical and algebraic reasoning combined.

Although there were students who showed a negative change from pre- to post-intervention or stayed the same on both measures (29/138), the majority of the students seemed to improve on both their graphical reasoning as well as their algebraic reasoning (51/138) or stayed the same on both measures (36/138). Yet, these descriptive statistics do not take changes in students' levels of reasoning before and/or after the pre- and post-intervention measures into account. The LGM analysis, which we will turn to now, does precisely that.

Table 6

Number of students' showing either positive change (+), no change (=), or negative change (–) on their graphical and algebraic reasoning from pre- to post-intervention

		Graphs							
		+		=		-		*Total*	
Algebra	+	51	(37%)	8	(6%)	5	(4%)	64	(46%)
	=	22	(16%)	6	(4%)	5	(4%)	33	(24%)
	-	23	(17%)	4	(3%)	14	(10%)	41	(30%)
	Total	97	(70%)	18	(13%)	24	(17%)	138	(100%)

5.3 Effect of the intervention modelled with a multi-group LGM

The LGM model including all four measurement moments had an overall fit in terms of RMSEA that was acceptable (.062, 90% CI [.050 - .072]). However, fit in terms of CLI and TFI was insufficient, with fit indices below the critical cut-off values (CFI = .789, TLI = .811). Extensive exploration of analysis and model options, including suggestions given by modifications indices provided by Mplus, did not lead to clear improvements of CLI- and TFI-fit, however they did reveal robustness of the relevant parameter estimates. We suspect that the strict assumptions of our model (needed to test our hypotheses) combined with the small sample size per cohort (<49) made it difficult to obtain a better overall model fit, but that the main results are nevertheless informative and trustworthy.

Due to the use of a Probit model, the effects, shown in Table 7, are scaled such that they represent standard deviations for the latent ability. Therefore, the given values can be interpreted as standard effect sizes. There was a clear positive effect of the intervention on students' graphical reasoning (0.59 *SD*, $p < .001$), which was also found in a previous study (Duijzer et al., 2020). There was also a small positive effect of the intervention on students' algebraic reasoning (0.30 *SD*, $p = .003$).

Table 7

Parameter estimates of the final multi-group LGM model

	Graphing			Algebra		
Model parameter	*M*	*p*-value	*var*	*M*	*p*-value	*var*
Intercept[a]	@0	xx	0.22	@0	xx	0.45
Slope	0.05	.305	0.02	-0.06	.227	0.02
Intervention	0.59	.000	0.31	0.30	.003	0.31
Weakening	-0.16	.055	@0	-0.03	.812	@0
Predictor regression *β*						
General reasoning ability on Intercept	0.45	.000		0.40	.000	
Covariances *β*						
Intercept G with Intercept A	0.96	.000				
Intervention G with Intervention A	0.12	.917				

Note. [a] Although non-significant we allowed the intercepts in cohort 2 and cohort 3 to deviate from 0.

In order to support the interpretation of the effect size of the intervention on either latent ability, it is helpful to visualize the results, see Figure 2a and Figure 2b. Due to the scaling of the Probit model with the delta parametrization in Mplus, both figures show a standard normal distribution representing the latent abilities of all participants, for the graphical reasoning Task 1 and the algebraic reasoning Task 1. Scales are anchored at zero for the average ability on the measurement directly before the intervention. The shift of the curve, therefore, represents the increase in average ability due to the intervention. In both Figure 2a and Figure 2b, a clear shift of the curve to the right is present, representing the positive intervention effects that were found. The area under the curve limited by the vertical borders shows the probability of a student reasoning according to a particular level. This holds that the larger the area under the curve for a particular level of reasoning, the larger the probability that a particular student reasons according to that level. Because the thresholds did not change, the intervention effects become salient. When comparing both Figure 2a and Figure 2b, for students' graphical reasoning (Figure 2a) a larger shift to the right is shown than for students' algebraic reasoning (Figure 2b), which corresponds with the aforementioned effects.

The correlation between the hypothetical abilities of graphical reasoning and algebraic reasoning, for the four tasks, on the first measurement moment, was moderate and significant (r = .346–.349, p < .001). This indicates that students' reasoning within one domain did covary with reasoning in the other domain. However, we were more interested in correlations between improvements possibly due to the six-lesson teaching sequence. The unstandardized covariance between the intervention effect on graphical reasoning and the intervention effect on algebraic reasoning was small and non-significant (r = .004, p = .922). Because the variance of the intervention effects of graphical reasoning and algebraic reasoning was constrained to be equal (to avoid negative variance on the latent ability for algebraic reasoning) the standardized covariance (also low and non-significant r = .116, p = .917), is not trustworthy. Nevertheless, it is clear that improvement in one domain was not related to improvement in the other domain.

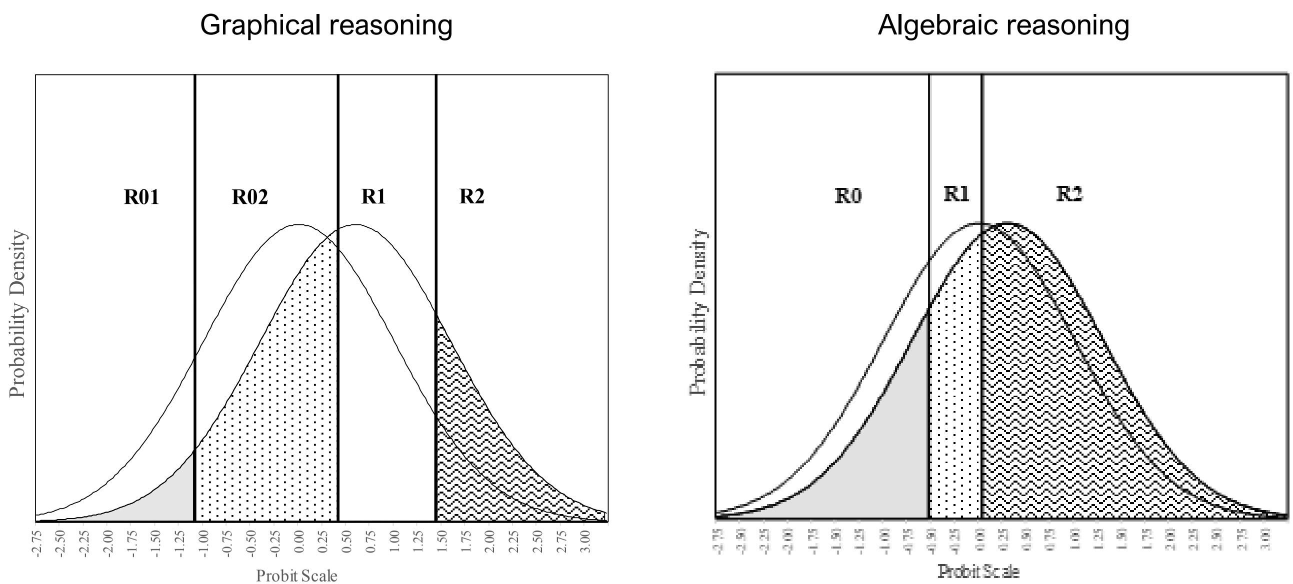


Figure 2a and 2b. Intervention effects (shifts to the right) on students' graphical reasoning ability (left panel) and on students' algebraic reasoning ability (right panel)

6. Discussion

The present study focused on the development of students' mathematical HOT across two distinct, but related domains (graphing motion and linear equation solving). This HOT was formulated in terms of extracting, using, and combining sources of information, for which students draw on their ability to reason about covariation. We investigated whether an intervention, consisting of six lessons targeting graphing motion would affect not only students' graphical reasoning but also affected their ability to reason about problems in which they were asked to solve a system of informal linear equations. In a cohort-sequential design with overlapping measurement moments between cohorts, students received the intervention either at the beginning, halfway, or at the end of the school year. LGM analysis allowed us to investigate students' development in these domains. The dependent variables were students' graphical reasoning and algebraic reasoning measured with four graphing tasks and four algebraic tasks which were coded with regard to students' level of reasoning.

6.1 Summary of the results

Initial exploration of changes in students' levels of graphical and algebraic reasoning (i.e., positive, negative, or no change) revealed that a large number of students showed positive change from pre- to post-intervention, on both their graphical and their algebraic reasoning. These descriptive statistics demonstrated a possible effect of the intervention, as well as an indication of a relationship between growth in both domains. A more precise, sensitive, and trustworthy analysis was looked after with LGM. Results of the LGM analysis showed that students indeed significantly improved their graphical reasoning as was visible in a positive linear growth of their graphical reasoning ability (see also Duijzer et al., 2020). Further inspection showed that students also significantly improved their algebraic reasoning. This effect (0.30 *SD*) was not as strong as the effect on students' graphical reasoning (0.59 *SD*). Students at the start of the intervention already showed relatively high levels of algebraic reasoning, which could be an explanation for this smaller effect. We found a strong relationship between students' initial levels of graphical and algebraic reasoning. However, when evaluating the relationship between students' growth in graphical reasoning and students' growth in algebraic reasoning, when taking into account this initial correlation, no correlation between improvement between the domains was found. This means that students who improved their graphical reasoning after the intervention on graphing motion did not systematically improve their algebraic reasoning, and vice versa. Even though students as a group improved, this improvement was not related to individual students' improvement in each of these mathematics domains. No transfer of HOT to the domain of solving systems of informal linear equations appeared to take place.

6.2 The nature of mathematical HOT in this study

The domains of graphing motion and linear equation solving require domain-specific HOT, in terms of reasoning about covarying quantities. Nonetheless, the HOT elicited in the domain of graphing motion did not result in the application of HOT in the domain of linear equation solving. This finding, that some students' graphical reasoning and others' algebraic reasoning improved without the presence of a relationship between both, implies that the intervention on graphing motion did not structurally elicited general elements of mathematical HOT relevant to both mathematical domains. Somehow, for some students the domain-specific teaching sequence only affected reasoning within the domain of graphing motion, whereas for other students this only affected reasoning within the domain of linear equation solving. This underscores a view on HOT as essentially situated within, and emerging from, the domain in which the teaching and learning activities were carried out. We thus may conclude that transfer of HOT to another – slightly related – mathematics domain cannot be taken for granted. HOT is domain-specific even within a particular academic discipline like mathematics, which is in line with the domain-specific view on HOT as advocated by Alexander et al. (2001).

We did see students improving on their algebraic reasoning. Yet, what precisely caused this development in algebraic reasoning, stimulated or not through the intervention, and measured by the algebraic tasks is currently unknown. We cannot explain this development conceptually on the basis of their development in HOT. Yet, we can think of a few alternative explanations, in terms of learner characteristics and contextual factors. For example, with regard to learner characteristics, the observed development in graphical reasoning could to a certain extent be motivational. The students partaking in our study could have become more interested in the tasks they recognized as a result of the intervention, or vice versa, students could have become more interested in tasks they did not recognize as a result of the intervention. This motivational factor could potentially affect students' performance on the tasks within the domain of graphing motion or the tasks within the domain of linear equation solving, which in turn could have affected the relationship between graphical reasoning and algebraic reasoning. Also, contextual factors, such as extra-curricular activities, may contribute to growth in either one of the domains. A comparison of these relations was not possible in the current study due to our use of a limited set of measurements. Further research that includes child characteristics and additional contextual factors may enhance our understanding of the relationship between development in either one, or both of these mathematical domains.

6.3 Limitations and future directions

This study has some limitations we would like to point out. First, although covariation plays a role in both the domain of graphing motion and the domain of linear equation solving, there was limited conceptual overlap between the graphing motion intervention and the tasks used to measure students' development in algebraic reasoning. In contrast, Nemirovsky and Rasmussen (2005) designed activities with undergraduate students in which the chosen graphical and algebraic tasks were informationally equivalent, sharing the same underlying quantitative structure. Our primary interest was not in creating such overlap between intervention and tasks, but rather in stimulating students' thinking, including the underlying covariational thinking that was deemed relevant to both mathematical domains. Future research could investigate more precisely whether conceptual overlap between tasks could result in the improvement of HOT in the domain of graphing motion parallel to the domain of solving linear equations. Also, in order to elicit HOT within these two separate yet related mathematics domains, the chosen activities could also explicitly incorporate the learning strands of both graphing motion and linear algebra within lesson activities. For example, within secondary mathematics education, graphs and functions are often addressed together. In Dutch primary mathematics education both mathematics topics are not an explicit part of the main curriculum. Yet, difficulties students have with the function concept are often related to difficulties they experience with graphs, especially with the understanding of time-dependent graphs (Arzarello & Robutti, 2004). Strengthening graph sense while also exploring, for example, linear functions in either graphical or algebraic form, could be a more explicit aim of learning activities in primary education. This exploration of the function concept can be taught through activities involving distance-time graphs (e.g., Robutti, 2006; Gjøvik & Sikko, 2019) and can be supported through the use of technology-rich environments, including motion sensors (Robutti, 2006) or simulation software (Roschelle et al., 2010; Sinclair & Armstrong, 2011). When following such approach, the learning of graphs of motion and the learning of the function concept becomes strongly connected to their origin within particular mathematical domains, touching upon the whole array of knowledge and concepts that are deemed relevant to that domain. According to Schoenfeld (2016) in order to coming to grips with the thinking and learning within any discipline, it is important to submerge oneself in the specific practices, habits, and knowledge of a particular discipline. We can assume that such approach paves the way to HOT since both domain-specific HOT (as reasoning within each domain) as general elements of HOT (as reasoning across both domains) are stimulated simultaneously.

Second, the rather sophisticated LGM model, with categorical outcome measures, used in our study posed some serious difficulties throughout the process of model estimation. Although we acquired sufficient model fit in terms of RMSEA, which in the context of using categorical data is the most reliable fit measure to attend to (Little, 2013), fit measures in terms of CFI and TLI were below the critical cut-off values. For that reason, one should be careful in interpreting the obtained parameter estimates as a summary of the relationship between the variables (West et al., 2012). Therefore, the extent to which these significant results are meaningful, should not be overestimated. Yet, extensive exploration of analyses and model options, including suggestions given by modifications indices provided by Mplus revealed robustness of the relevant parameter estimates. We assume that the strict assumptions of our model combined with the rather small number of students per cohort in relation to the complex data analyses that were conducted, made it difficult to obtain a better overall model fit. Further research is necessary including a larger sample of students. Third, we analyzed the students' reasoning with a coding scheme for their written explanations. These writings not always reflect their full understanding of the task. For example, when a student writes down "I do not know" (coded Level R01) this does not necessarily mean that the student is unaware of the answer to the question. It could also be that the student is unable to write down this understanding. One solution to circumvent this problem would be to let students think-aloud during solving these tasks or interview them afterwards.

6.4 Concluding remarks

The present study provides some preliminary insights regarding the development of mathematical HOT across two distinct but related mathematical domains. We have found that after partaking in an intervention students' graphical reasoning improved, yet without structurally improving their algebraic reasoning, and vice versa. This begs the question as to whether there are general elements within HOT that can be stimulated regardless of the mathematical domain from which the HOT originated. This finding is in line with contemporary views on the development of HOT, advocating that thinking becomes higher order due to increasing experience within a particular academic domain (e.g., Alexander et al., 2011; Ericsson, 2003), as opposed to long-held and persistent beliefs that HOT can be supported regardless of academic content (e.g., Bloom, 1956; Resnick, 1987). Our study shows that even within an academic domain such as mathematics transfer of HOT from one mathematical domain to another mathematical domain (in this study: on the basis of graphical reasoning or as a result of similarity between both domains) cannot be taken for granted. As such, this study contributes to a further conceptualization of the domain-specific view on HOT and proposes that if one aims to promote students' reasoning in another mathematical domain than explicitly taught, providing students with an explicit conceptual link between the targeted mathematical domains might be essential.

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5

CHAPTER 6

“Look! The line in the graph... cannot go backwards!” A summary and general discussion

"Look! The line in the graph... cannot go backwards!"
A summary and general discussion of students' reasoning about graphs in primary mathematics education.

The teaching of motion graphs is rarely included in the regular Dutch primary school mathematics curriculum. This is a missed opportunity because more advanced mathematical topics, such as dynamic graphs showing a change in distance over time, have the potential to foster high levels of mathematical thinking and reasoning (e.g., reasoning about variables, slope, or covarying quantities). Against the background of providing primary school students more opportunities to develop higher-order thinking (HOT) within mathematics, as described in *Chapter 1* of this thesis, the first aim of this PhD thesis was to investigate whether and to what extent mathematical activities in the domain of graphing motion could elicit fifth-grade students' reasoning about motion graphs. A second aim of this PhD thesis was to investigate the role of bodily experiences for mathematical cognition. We did this by taking into account the opportunities bodily experiences offer to support the development of students' reasoning about motion graphs. To this aim, we conducted a systematic literature review and developed, implemented, and evaluated an intervention consisting of a six-lesson teaching sequence to teach the graphing of motion to fifth-grade students. The literature review was conducted to shed light on the significance of embodied learning environments supporting students' understanding of graphing motion. Here, we took a critical look at the extant research and included research conducted in science, technology, engineering, and mathematics (STEM) education. The teaching sequence we developed, implemented, and evaluated in parallel, incorporated embodied mathematical activities, following the assumption that reaching higher levels of mathematical thinking and reasoning depends to a large extent upon opportunities for physical movement and embodied interactions (e.g., Gallese & Lakoff, 2005; Hall & Nemirovsky, 2012; Núñez et al., 1999).

Mathematical HOT in the context of graphing motion can be considered domain-specific. Higher levels of mathematical reasoning arise as a consequence of experiences within this particular domain (Alexander et al., 2011). This domain-specific mathematical HOT inter alia draws on a student's covariational reasoning capacity (see also Radford, 2009). Covariational reasoning is the simultaneous coordination of the magnitudes of quantities in the graph, while keeping in mind that at every moment the other quantity also has a value (e.g., Saldanha & Thompson, 1998). This covariational reasoning, as the simultaneous coordination of the values of quantities, is also present within other mathematical domains. One domain for which this particularly holds is the domain of early algebra (e.g., Kieran et al., 2016). A third and final aim of this PhD thesis, therefore, was to investigate the potential of the teaching sequence on graphing motion to engender mathematical HOT in the domain of early algebra, as an indication of the extent to which HOT stimulated within a particular mathematics domain can be regarded domain-specific, domain-general, or both.

In this final chapter, I first summarize the findings of the studies reported in this thesis. Thereafter, the implications of these findings for theory and practice are discussed and suggestions for further research are given. The limitations of this thesis are addressed. This chapter ends with the main conclusions of our research project.

1. Summary of the results

1.1 Embodied learning environments supporting students' understanding of motion graphs

The research literature reports on a wide variety of embodied learning environments, originating from different traditions of views on cognition. In *Chapter 2*, we reported on a systematic literature review of research that incorporated embodied learning environments to support students' understanding of graphing motion. We did so in order to gain more insight in the breadth and depth of these embodied learning environments and their educational potential. To get a grip on the defining characteristics of these embodied learning environments, we categorized them on two dimensions: bodily involvement and immediacy. For bodily involvement we distinguished between own motion (direct bodily experience) and observing others/objects' motion (indirect bodily experience). For immediacy we distinguished between *immediate* ("on-line" cognitive activities) and *non-immediate* ("off-line" cognitive activities). Combining both dimensions resulted in a taxonomy of embodied learning environments with four classes, each representing a specific embodied configuration: Class I – Immediate own motion, Class II – Immediate others/objects' motion, Class III – Non-immediate own motion, Class IV – Non-immediate others/objects' motion. Embodied learning environments that made use of students' own motion immediately linked to its representation (Class I), were most common across the sample of reviewed articles.

6

The review then uncovered eight characteristics specific to embodied learning environments supporting students' understanding of graphing motion as described by the authors of the reviewed articles, which we referred to as mediating factors: *real-world context*, *multimodality*, *linking motion to graph*, *multiple representations*, *semiotics*, *student control*, *attention capturing*, and *cognitive conflict*. These eight mediating factors have their own role in how they support learning within these embodied learning environments. Some of these factors potentially bridge the gap between source domain embodied experiences and the learning taking place. Two examples are: *linking motion to graph*, by enabling students to observe a direct link between motion and the corresponding graphical representation, and *multimodality*, as through the nature of the tool or the instruction, at least two of the modalities of seeing, hearing, touching, imagining, or motor actions are simultaneously activated. Other factors have a more facilitating role in the learning process, for example *multiple representations* (i.e., receiving multiple representations of a particular motion event) and *student control* (allowing students to control the learning environment by letting them manipulate the motion or the graphical representation).

The four classes that we specified, together with the eight mediating factors illustrate the variety and, often, complexity of embodied learning environments as occurring in education and research. Each class of embodied learning environments contained different sets of mediating factors, increasing the number of qualitative different learning environments substantially. Embodied learning environments that made use of students' own motion immediately linked to its representation (Class I) were found to be most effective in terms of learning outcomes. In this particular class, the three mediating factors, *multimodality*, *linking motion to graph*, and *multiple representations* were most common. The two-dimensional framework and the identified mediating factors, together with the synthesis of the evidence so far regarding the efficacy of each class, can inform the future design – and evaluation – of embodied learning environments.

One limitation of the review was that we based the mediating factors on the reported information provided by the authors of these articles, which made the evidence for the mediating factors not equally strong. A second limitation of this study pertained to the wide variety of articles included in the review, which not only led to considerable variation in various participant characteristics, it also made it difficult to integrate the findings of the various articles regarding the effectiveness of the studied learning environments. Still little (comparative) research had been done on whether and to what extent embodied activities (that vary in their degree of embodied support as direct or indirect physical experiences in learning activities) are helpful in stimulating primary school students' reasoning about motion graphs. More comparative research is needed to determine which embodied configurations are most effective.

1.2 The effect of an embodied learning environments on students' reasoning about motion graphs

Chapter 3 and *Chapter 4* of this thesis described the design, implementation and evaluation of a six-lesson teaching sequence on graphing motion that was developed in our research project. This teaching sequence had the explicit aim of stimulating students' reasoning about graphs representing the bivariate relationship of *distance* changing over *time*. The results of the review described in *Chapter 2* suggested that learning environments in which students' own motion becomes immediately linked to its representation as a graph (e.g., distance-time graph, speed-time graph) are most promising to support students in their understanding of these graphs. Following the idea, based on embodied cognition theories, that perceptual-motor experiences are an important entry-point into reaching higher levels of mathematical reasoning, we developed two parallel versions of the teaching sequence. In one version, students were offered direct embodied support, involving graphing activities in which students' own bodily movements were visualized as a line in the graph, using motion sensor technology. In the other version, students were offered indirect embodied support, involving graphing activities that were mostly paper-and-pencil based or projected on the digital blackboard. Students did work with an image of the motion sensor context and the related activities, but without the presence of the real tool.

Chapter 3 reported on the teaching sequence offering students direct embodied support. The focus of the study presented in this chapter was on students' *micro-development* in reasoning about motion graphs over the six-lesson teaching sequence, and the pivotal role of bodily experiences therein. We captured students' micro-development over the lessons by assessing, after each lesson, students' graphical reasoning using a series of graph interpretation and graph construction tasks. The interpretation and construction tasks were alternated over the lessons. We analyzed students' written responses to these tasks. We found that from lesson 1 to 6 students went from iconic understanding towards understanding in which they reasoned on the basis of multiple variables when interpreting and constructing graphical representations of motion events. At this higher level of reasoning, students' often showed instances of reasoning in an informal covariational manner (i.e., "covering more distance in less time") (see also Radford, 2009), in which they took into account the two variables represented on the axes of the graph. In the analysis of two teaching episodes of one student's interaction with the teacher and other students, it was shown in which ways this student started to reason about graphs at higher levels in relation to her perceptual-motor experiences in front of the motion sensor. We found that the student made sense of the problem of walking a specific graph by coordinating various modality-specific systems, including seeing, hearing, gesturing, and moving, which she linked to the graphical representation that unfolded on the screen of the computer. These movements, as well as anticipated and unanticipated features of the graphical representation which emerged in real-time, were reflected in this student's reasoning about the graph. The technology used throughout the lessons was an important facilitator in this process. For this and the other students, we found that the bodily activities in front of the motion sensor engendered high levels of mathematical reasoning about the variables distance, time, and speed, as well as combinations thereof in an informal covariational manner (e.g., "Covering more distance in less time results in a steeper slope").

Chapter 4 reported about the evaluation study in which a cohort sequential longitudinal design was used to investigate students' changes in graphical reasoning over the school year. We assessed students' development in graphical reasoning over the six-lesson teaching sequence by analyzing their written responses on four graph interpretation and graph construction tasks. We compared the instruction condition offering students direct embodied support (see *Chapter 3*) with an instruction condition offering students indirect embodied support. The lessons in the indirect support condition contained activities that were similar to the ones in the direct support condition, yet without giving students the opportunity to physically enact the given motion situations in front of a motion sensor. In this condition the movement of an object (i.e., a toy car) was taken as point of departure. A third group of students served as a baseline condition and received lessons on another mathematics topic. We found a strong effect of the intervention on students' reasoning about graphs of motion over the school year, indicating that both versions of the teaching sequence were highly effective. When comparing both instruction conditions, we found an effect in favor of the instruction condition offering direct embodied support, indicating that these students improved more in their graphical reasoning than students' in the indirect support condition.

As reported in *Chapter 2* of this PhD thesis, and supported by empirical observations in *Chapter 3*, the advantage of having embodied (perceptual-motor) experiences when graphing motion, for example through the use of motion sensor technology, is presumed to be mediated by the rich interrelated coordination of modality-specific systems, the immediate link between motion and graph, and the various representations of motion as a result of movements in front of the sensor. Each of these three mediating factors has a specific role as to how and why they enable learning within the embodied learning environment developed for this thesis.

Multimodality as a mediating factor is an often mentioned and essential aspect of embodiment. A multimodal view on cognition encompasses the idea that conceptual knowledge depends upon a rich interrelated coordination of modality-specific systems (Barsalou et al., 2003). The mediating factor *linking motion to graph* is related to the mapping mechanisms that structure the abstract mathematical concept by means of bodily experiences (Font et al., 2010), such as graphically represented motion present in the physical world. *Multiple representations* as a mediating factor refers to multiple representations of a particular motion event. Experiencing such variation in motion and representation, and distinguishing between what changes and what remains invariant, can be considered a necessary condition for learning (Runesson, 2006).

1.3. Transfer of HOT to a related mathematics domain: linear equations

Mathematical HOT in the domain of graphing motion inter alia draws on a student's covariational reasoning capacity. This covariational reasoning is also relevant in other mathematical domains. It seems plausible that elements of HOT relevant to multiple mathematical domains can be strengthened within one mathematical domain and transfer to the other mathematical domain, based on the assumption of domain-general mathematical reasoning. In the final study of this thesis, reported in *Chapter 5*, it was investigated whether the domain-specific teaching sequence on graphing motion also resulted in HOT, on the basis of reasoning about covarying quantities, in another mathematics domain, i.e., solving linear equations. We used a similar approach as with graphical reasoning to assess students' algebraic reasoning. The findings of this study indicated that the domain-specific teaching sequence on graphing motion apparently also resulted in a significantly higher mean level of algebraic reasoning at the group level. This effect was not as strong as the effect on students' graphical reasoning. Yet, at the individual level, no correlation between students' growth in graphical reasoning and students' growth in algebraic reasoning was found. Thus, students who improved in graphical reasoning did not systematically improved in algebraic reasoning, and vice versa. Based on these findings we drew the tentative conclusion that the HOT that was targeted in our research did not transfer to the domain of linear equation solving. Rather, the HOT that students developed in the motion graph lessons was primarily domain-specific. The finding that the students did show improvement in their algebraic reasoning after partaking in the intervention warrants further research.

2. Implications of the main findings and ways to move forward

Taken together, the findings of the studies reported in this thesis have several implications for theory and practice. In the following paragraphs, I will first elucidate some theoretical implications of the findings regarding embodied activities in mathematics lessons and the nature of mathematical HOT, followed by some recommendations for how to move forward. Subsequently, I will describe some practical implications of the research presented in this thesis and how to implement embodied graphing activities to support students' mathematical HOT within primary school mathematics education.

2.1 Theoretical implication: The value of direct and indirect embodied support in mathematics activities

To frame the first theoretical implication of this thesis I would like to start with a short example of a classroom interaction. During the first part of Lesson 3 given in Cohort 1 (direct embodied support condition), students were prompted to think about movements in front of the motion sensor and about possible and impossible graphs, by asking them: "Can you make a letter of the alphabet?" This rather simple question resulted in a wealth of ideas. Students came up with many letters, most of which would not be reproducible as a distance-time graph (e.g., the letter "A" or "O"). After some time one of the students, who had not yet walked in front of the sensor, wanted to make an O-shaped graph. After this 30-second trial the students as a group reached the conclusion that making an O is impossible. This insight in the unidirectionality of time was explicitly mentioned by one of the students who stated that: "It [the line in the graph], cannot go backwards". Then two other students added: "It [the motion sensor] keeps measuring the distance." The students' conclusion – that making an O is impossible – is a critical moment when students comes to understand the particularities of motion graphs as graphically representing elapsed distance over a certain period of time (e.g., Arzarello & Robutti, 2004), where time can only increase.

The importance of directly experiencing movements in front of the motion sensor seemed twofold: (1) for students to see with their own eyes how a specific movement can or cannot be represented as a time-distance graph, and (2) to prompt rich interactions between students in which they collectively formulated hypotheses (i.e., making the letter O) and drawing conclusions (i.e., making an O is impossible: "The line cannot go backwards"). These relatively simple activities, "walking" letters as presented in the example above, among other activities, performed in front of the motion sensor caused students to make a connection between their own movements in front of the sensor and the line in the graphical representation. Through walking understanding emerged. This process was further facilitated through interaction and reflection between the students and between the students and the teacher.

As just one example, the activity described above shows how whole-bodily movements in front of the motion sensor resulted in the development of new metaphorical thought, which was grounded in – or became connected to – their already existing intuitive ideas about (the representation of) certain phenomena. To be more precise, from their experiences in everyday reality students presumably were aware of the unidirectionality of time (e.g., Friedman, 2000; McCormack & Hoerl, 2017). This understanding of time is an embodied understanding, which receives its meaning through our everyday use of various conceptual metaphors such as: "the summer is ahead of us", "time passes by", or "it takes a long time" (Lakoff & Núñez, 2000). These embodied understandings of the passing of time were implicitly and explicitly present in the reasoning of the students (e.g., "it keeps measuring the distance"), and became connected to the abstract formal representation of motion as a line in the distance-time graph (e.g., "the line in the graph cannot go backwards"). Students' reasoning about the passage of time represented in the graph, as a consequence of their bodily movements in front of the motion sensor, might seem trivial at first. But, as discussed by Thompson and Carlson (2017), thinking about the passage of (measured) time as continuous change when graphically representing dynamic phenomena such as changes in height or changes in distance is a rather complex endeavor (a learning progression from "chunky" images of change, to "smooth" images of change), and essential for developing a robust understanding of the concept of function. According to these authors, developing this understanding could potentially benefit from metaphoric ideas such as fictive motion, which involves thinking about a subject as if it is moving, while in reality nothing moves (e.g., "The A2 goes from 's-Hertogenbosch to Amsterdam"). In a nutshell, this short excerpt shows how the formation of new relevant metaphorical mappings between source-domain experiences and target-domain knowledge took place, and as a consequence, resulted in higher levels of reasoning about the graph as grounded in bodily experiences (see also Lakoff & Núñez, 2000).

Based on the studies presented in *Chapters 2 to 4*, and general literature on the role of embodiment in human cognition, we infer that students who had only been asked to draw graphs on paper and saw graphs on the digital blackboard acquired their understanding about these graphs differently. The opportunity to directly experience the line in the graph through physical activity was not present in the instruction condition offering students only indirect embodied support. In this condition, a direct physical link with relevant source-domain embodied experiences was absent. Yet, this does not mean that students could not make sense of the distance-time graphs and the activities as grounded in bodily experiences. For example, in making sense of the motion graphs these students were also provided the opportunity to build on their intuitive understandings of motion phenomena in order to represent motion as a line in the graph. Yet, the source-domain bodily experience of moving through space was activated in a qualitatively different way, namely through observation and off-line cognitive processing (see also *Chapter 2*), presumably on the basis of embodied simulation (Barsalou, 1999). Embodied simulation theory explains how source-domain experiences are activated when a direct (physical) link with that source-domain is absent. Through embodied simulations, previously acquired sensorimotor experiences are re-activated or re-used for knowledge construction processes in the learning activity (e.g., Barsalou, 1999, 2010; De Koning & Tabbers, 2011).

Although both instruction conditions drew on universal source-domain bodily experiences, resulting in conceptual metaphors that might be similar across conditions, in the one condition these metaphors were implicit and internal, in the other condition they were explicit and active (e.g., Gallagher & Lindgren, 2015), and this likely explains why a stronger learning effect was found in the latter condition. However, more fine-grained research with appropriate measurement tools (e.g., eye-tracking, Lai et al., 2013; Worsley & Blikstein, 2014) is needed to obtain a deeper understanding of how perception-action processes in embodied learning environments activate, change, combine and blend elementary embodied cognitions to ground abstract mathematical concepts, while extending the scope of this research to other complex (and pivotal) concepts in mathematics as well (e.g., Abrahamson & Sánchez-García, 2016; Duijzer et al., 2017).

2.2 Theoretical implication: The nature of mathematical HOT

Based on *Chapter 3* to *Chapter 5* we can draw the tentative conclusion that both direct and indirect embodied support in the teaching sequence stimulated higher levels of graphical reasoning among primary school students, and more so in the instruction condition offering direct embodied support (see *Chapter 4*). Transfer of the effect of the learning environment for graphical reasoning to the domain of algebra, however, could not be unambiguously established (see *Chapter 5*).

Potential transfer of the domain-specific concept of covariation and the related HOT to another mathematics domain, such as linear equation solving, could possibly benefit from letting students explicitly see and experience the interrelatedness and parallel forms of this concept across domains (Dreyfus & Eisenberg, 1982). I would like to propose two aspects that could be considered relevant in doing so. The first aspect pertains the stimulation of mathematical HOT by explicitly incorporating the learning strands of both graphing motion and linear algebra within lesson activities. For example, in secondary mathematics education, graphs and functions are often addressed together. In Dutch primary mathematics education both mathematics topics are not often an explicit part of the main curriculum. Yet, difficulties students have with the function concept are often related to difficulties they experience with graphs, especially with the understanding of time-dependent graphs (Arzarello & Robutti, 2004). Strengthening graph sense while also exploring, for example, linear functions in either graphical or algebraic form, could be more explicitly incorporated in learning activities in primary mathematics education. For example, the exploration of the function concept can be taught through mathematical activities involving distance-time graphs (e.g., Gjøvik & Sikko, 2019; Robutti, 2006) and can be supported through the use of technology-rich environments, including motion sensors (Robutti, 2006) or simulation software (Roschelle et al., 2010; Sinclair & Armstrong, 2011).

The second aspect pertains the activation of relevant source-domain bodily experiences. As described in this thesis, the bodily experience of moving through space can be considered one of the relevant source-domain bodily experiences for graphically represented motion. Metaphorical projection, by means of image schemes such as *fictive motion* or the *source-path-goal* schema, is the main embodied cognitive mechanism providing the link between the source-domain experiences (such as moving through space) and target-domain mathematical knowledge (such as developing an understanding of graphically represented motion) (e.g., Font et al., 2010; Núñez et al., 1999). The source-domain bodily experience of moving through space served as a grounding mechanism for students' graphical reasoning, through elicited or instructed metaphorical projections (e.g., linking movement to projected graphs, eliciting discussion about for example irreversible time), but not for students' algebraic reasoning. Although this specific source-domain experience is also relevant for students' informal covariational reasoning or functional thinking (cf., Nunez et al., 1999), pivotal to the domain of early algebra, this was not enough to achieve spontaneous transfer to the domain of linear equation solving. Further research could make an effort to build on the source-domain bodily experience of moving through space to build up metaphorical projections that are relevant to both the domain of graphing motion and the domain of linear equations (see also Nemirovsky & Rasmussen, 2005). Also, a recent study (Otten et al., 2020) showed how a balance model, either physically or presented on worksheets was particularly helpful for stimulating students' algebraic reasoning. The balance model used in their research drew on the source-domain bodily experience of being in balance (e.g., Núñez et al., 1999). Combining the source-domain experiences relevant to both the domain of graphing (moving through space) and the domain of algebra (being in balance), to build shared metaphorical projections might be a fruitful approach to advance the research in this area. These metaphorical projections could be more explicitly activated through, for example, interaction and reflection on the resulting domain-specific reasoning (e.g., reasoning about covarying quantities), in order to increase the potential for transfer across domains (see also Gallagher & Lindgren, 2015).

2.3 Practical implications

There is a pressing need to incorporate sophisticated skills such as higher-order thinking skills or 21st century skills in education, as is recognized at the international (e.g., OECD, 2019) and national level (e.g., Ontwikkelteam Rekenen-Wiskunde, 2019; Thijs et al., 2014). In the Netherlands, the NVORWO (2017) has emphasized that within mathematics education both basic mathematical skills (e.g., *declarative*, *procedural*, *factual knowledge*) and mathematical HOT skills (e.g., *mathematical reasoning*, *modelling*, *visualizing*, *problem solving*, *developing a mathematical attitude*) should be supported. Furthermore, according to the NVORWO, these HOT skills should be formulated in terms of longitudinal learning strands which can make the transition from primary mathematics education into secondary mathematics education more fluent. A recent analysis of the Inspectie van het Onderwijs (*Dutch school inspectorate*, Onderwijsinspectie, 2019) showed that currently little attention is paid to HOT activities in both primary and secondary mathematics education. Also, primary school teachers appear to have little knowledge of the longitudinal learning strands beyond primary school, into secondary mathematics education, which is especially detrimental for high-performing students.

In line with the educational agenda outlined above, the question now is how the results of this PhD thesis can contribute to incorporate HOT at the primary school level. More specifically, what are the practical implications of the findings of this PhD thesis? First, we think the learning environment that was developed and implemented as part of this PhD thesis, and the activities therein, can serve as a domain-specific operationalization of mathematical HOT at the primary school level. Second, following recent proposals around embodied cognition, the learning environment offering direct embodied support, using motion sensor technology, can be seen as a worthwhile approach to stimulate students' domain-specific HOT, and more specific their reasoning about these motion graphs.

Three elements are relevant to consider when implementing the graphing activities as developed, implemented, and evaluated in this PhD thesis in primary mathematics education. First, when students enter the classroom, they already have some intuitive and informal notions of representations and motion phenomena, either from previous education but more likely from the world outside school (e.g., an intuitive understanding of the passage of time as fictive motion). It is important that education provides students with opportunities to build on these intuitive understandings of motion related phenomena in lesson activities. One way of doing so is to take these intuitive understandings as a starting point, and on them build new understandings. For example, by letting students invent their own representations of motion situations, before moving on to formal mathematical representations. Having students actively involved in the learning process, starting with known meaningful situations, and using models to bring students to higher levels of mathematical thinking and reasoning is central to the domain-specific instruction theory of *Realistic Mathematics Education* (Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).

Second, according to Leinhardt (1990), for teachers to be able to build on students' intuitive and informal knowledge of representations and graphing requires "tremendous levels of content-specific knowledge on the part of the teacher because he must be prepared to go in any of the several directions and to construct on the spot several curriculum scenario's" (p. 49) (see also Hill & Ball, 2004). Because each student has their own idiosyncratic ways of interpreting a situation, or a situation represented in a graph, it can be difficult for teachers to tap into students' intuitive notions of how a motion phenomenon is graphically represented, as well as their (evolving) understandings of associated mathematics concepts (e.g., scale, covariation, rate of change, steepness of slope). This requires high levels of mathematical knowledge, and pedagogical content knowledge on the part of the teacher. Yet, based on my own experiences in the classroom, as being that teacher, the activities that we developed in the context of this research project, were met with great enthusiasm. As shown in the excerpt described above, as well as in the teaching episodes described in *Chapter 3*, rich interactions between students occurred over the course of the lessons. This was not only the case in the instruction condition offering students direct embodied support, but in the indirect support condition as well. Students were challenged to ask questions to the other students and the teacher, to express their ideas, to pose hypotheses, and to explore all kinds of alternative solutions, which turned out to be helpful in supporting their reasoning about these graphs.

Third, the findings of this PhD thesis show that technologically enhanced learning environments are ideal to ground students' understanding of motion phenomena, as well as their formal representation as a mathematical graph, in experienced motion. The incorporation of direct physical experiences in mathematics lessons through the use of motion sensor technology in the classroom, is quite different from regular instruction in primary school. Especially those teachers with little preference for these types of activities will need further support on how to successfully use these tools in their own practice (e.g., Lyublinskaya & Zhou, 2008). In order to implement the teaching sequence and to use motion sensor technology in primary mathematics education, an effort should be made to sufficiently support primary school teachers to use these instructional tools and the related materials.

3. Limitations of this PhD thesis

The study design chosen for this PhD thesis (see *Chapters 3 to 5*) allowed us to study the effects of the teaching sequence on graphing motion in a real classroom setting, resulting in high ecological validity. Yet, this particular study design inevitably had some methodological drawbacks. First, we could not apply random assignment of students to instruction conditions. Second, only a relatively small sample of students could be included in our research. Because we performed quite advanced statistical analyses on this relatively small sample of students, we have to be cautious regarding the interpretations of these findings. A lack of power, as a result of the small sample size, might have caused some of the fit measures to be below the conventional cut-off criteria (see *Chapter 5*). We assume that increasing the sample size would result in a more adequate reflection of the found effects. Nevertheless, we are fairly certain that the main results (i.e., intervention effect and condition effect) so far are trustworthy and convincing enough to warrant further investigations.

Another limitation pertains our focus on students' written responses to the graphing tasks (see *Chapter 3 to 5*). Although we think it is important to consider students' written explanations as an insightful alternative to merely applying correctness scores (i.e., right/wrong), or using answers to multiple-choice tests (e.g., Berg & Smith, 2004; Lai et al., 2016), we cannot be sure that the written explanations of the students were a true reflection of their complete understanding. Previous research has shown that this might not always be the case, and that verbal descriptions of students' understanding often suffer from reliability and validity issues (e.g., Fagginger-Auer et al., 2015; Torbeyns et al., 2015). In our study, we provided the students with clear instructions on how to elaborate on their answer by asking them "how do you know" (this instruction was given both written and orally). Further, we asked the student to immediately write down their explanations, without any time delay, which adheres to the guidelines for verbal reports outlined by Ericsson and Simon (1993). Yet, based on the results presented in *Chapter 3* we are inclined to think that the written explanations of the students were actually an underestimation of their full understanding, especially with regard to the tasks measuring students' ability in graph interpretation, and their reasoning in an informal covariational manner therein. Also, our focus on measuring students' understanding verbally, to a certain extent, contradicts our focus on the embodied understanding that was elicited in the lessons, through movements and reflections on those movements. Further research could expand on the use of verbal assessment tasks with non-verbal assessment methodologies to measure students' understanding as embodied cognition. Several non-verbal methodologies to uncover students' strategies when solving mathematics tasks are summarized by Torbeyns et al. (2015), one of which is eye-tracking. Schot et al. (2015) conducted a study in which they investigated children's strategies when solving a number line task (i.e., placing a number on the number line). When solving these tasks, children's eye-movements were captured and compared to both the *given* response, and the *correct* response, for each task separate. As such, a more fine-grained understanding of the different strategies when placing numbers on the number line were obtained. Similarly, in the domain of graphing motion, students eye-movements could provide information regarding the variables students attend to, as well as the simultaneous coordination of those variables, as covariational reasoning, when solving particular graphing tasks. Yet, the use of eye-tracking could not replace regular assessment in this domain. Therefore, another way to capture students' understanding of distance-time graphs (among others) could be to use the motion sensor technology for assessment purposes. For example, students could be asked to "walk" a set of given graphs, to obtain a direct physical measure of their (embodied) understanding of the graph as representing a specific motion situation.

In order to measure students' *macro-development* over the schoolyear we incorporated four tasks, of which three were related to graph interpretation and one to graph construction. Although incorporating a limited number of tasks certainly contributes to the ecological validity of the research (i.e., more aptly reflecting regular classroom practices), and adheres to the ethical guidelines for social research practices, the incorporation of more tasks, especially more graph construction tasks, might have offered a fuller picture of the breadth of students' understanding. Finally, the lessons presented in this PhD thesis were given to the students by the researcher. We cannot readily assume that primary school teachers will feel confident enough to teach this particular mathematics topic to their students and to incorporate motion sensor technology in their lessons (see also Lyublinskaya & Zhou, 2008).

4. Concluding remarks

We can draw a few main conclusions about the research presented in the chapters of this PhD thesis: (1) graphical reasoning and the related HOT can be stimulated in primary school mathematics classrooms, (2) students' graphical reasoning benefits from (physical) experiences offered within a teaching sequence targeting motion graphs, and (3) the HOT that was targeted in our research did not transfer to the domain of linear equation solving. Based on these findings we may infer that direct – and to a lesser extent indirect – physical experiences can be considered a worthwhile entry point to stimulate students' graphical reasoning as domain-specific HOT in primary mathematics education, strengthening the grounding of HOT in embodied cognitions. Carefully chosen lesson activities can give students the opportunity to build upon their informal and intuitive understanding of motion phenomena, in relation to the formal representation of those phenomena, as a mathematical graph. We found that when partaking in these activities, their reasoning about these graphs (e.g., taking into account the variables represented on the axes) was elicited. In the *Beyond Flatland*-project, of which this PhD thesis was a part-project (see *Chapter 1*), possibilities for enriching a "flat" arithmetic-focused mathematics curriculum were explored. This PhD thesis presents a small step in this direction and provides a domain-specific operationalization of this HOT as students' reasoning about motion graphs.

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SAMENVATTING

(summary in Dutch)

In onze huidige samenleving worden we voortdurend geconfronteerd met een overvloed aan grafisch gerepresenteerde informatie. Deze grafieken die we tegenkomen in de krant, op de televisie, of op het internet kunnen informatie bevatten die niet alleen van zichzelf complex is, maar soms ook op vrij complexe wijze weergegeven wordt. Om de informatie in deze grafieken te beschouwen en hieruit de relevante informatie te destilleren is grafiekbegrip essentieel. Grafiekbegrip omvat naast grafiekinterpretatie en grafiekconstructie ook het kritisch kunnen beschouwen van grafieken. Grafieken waarin een continue verandering is weergegeven, zoals temperatuur of beweging, vragen in het bijzonder om het nodige inzicht. Het is dan ook van belang om al op jonge leeftijd inzicht te ontwikkelen in zowel de formele aspecten van grafieken (bijv. de betekenis van de assen, van de weergegeven variabelen in de grafiek, van de helling, samengestelde grootheden) als het ontwikkelen van de wiskundetaal die gebruikt wordt om over deze grafieken te spreken (bijv. stijgen, dalen, constant, helling, horizontale en verticale as). Ook is het van belang om tijdens het beschouwen van grafieken te oefenen met het redeneren over samengestelde grootheden, beschrijven van oorzaak-gevolg relaties, logisch redeneren en het oplossen van problemen gerelateerd aan grafieken. Deze complexe vaardigheden worden ook wel aangeduid met 21ste-eeuwse vaardigheden of hogere-orde vaardigheden (HOV).

Het onderwijzen van HOV wordt gezien als een belangrijk onderdeel van het reken-wiskundeonderwijs in de 21ste eeuw. Terwijl internationaal wordt benadrukt om de fundamenten voor HOV op jonge leeftijd te leggen, wordt in het Nederlandse basisonderwijs aan HOV vrijwel geen aandacht besteed. De meeste tijd wordt besteed aan het uitvoeren van rekenprocedures. Om HOV te introduceren in het reken-wiskundecurriculum op de basisschool, werd het *Beyond Flatland* project geïnitieerd. Dit project heeft als doel het "platte" reken-wiskundecurriculum te verrijken middels het implementeren van hogere-orde wiskundige activiteiten in de rekenles. De onderzoeken beschreven in dit proefschrift dragen hieraan bij vanuit een domein-specifieke benadering van HOV en richten zich op wiskundige HOV binnen het deeldomein grafieken.

Om aan te sluiten bij de behoeften van basisschoolleerlingen is het belangrijk de relevante HOV in te bedden in innovatieve onderwijsarrangementen. Een kansrijke aanpak is het introduceren van activiteiten waarbij ingezet wordt op de actieve rol van het lichaam. Dit idee, dat fysieke ervaringen waardevol zijn voor wiskundig redeneren, vormt de kern van theorieën die uitspraken doen over zogenoemde *embodied cognition* (vertaling: belichaamde cognitie). Namelijk, de lichamelijke ervaringen die we opdoen gedurende basale fysieke activiteiten, zoals lopen door de ruimte, balanceren op een evenwichtsbalk of het beklimmen van een trap, zorgen voor de totstandkoming van bepaalde conceptuele metaforen die behulpzaam kunnen zijn in het begrijpen en duiden van abstracte ideeën, waaronder wiskundige concepten. Binnen theorieën van *embodied cognition* is het handelingsrepertoire van het lichaam, meer specifiek perceptuele-motoractiviteiten, essentieel. Cognitie wordt gevormd in interactie met jezelf, de ander en de omgeving.

Dit promotieonderzoek kent een drietal onderzoeksdoelen. Het eerste doel was te onderzoeken *of*, en *in hoeverre*, wiskundige activiteiten binnen het domein grafieken, meer specifiek afstand-tijdgrafieken, het redeneren van leerlingen over deze grafieken kan stimuleren. We hebben ons gericht op leerlingen uit groep 7, in de leeftijd van 9 tot en met 11 jaar oud. Een tweede doel van dit onderzoek was nagaan wat de rol is van *embodied* ervaringen om het redeneren over de wiskundige concepten gerelateerd aan grafieken te stimuleren. Om de rol van deze lichamelijke ervaringen op het wiskundig redeneren te onderzoeken hebben we (1) een systematische literatuurstudie uitgevoerd en (2) een interventie ontwikkeld, geïmplementeerd en geëvalueerd. De systematische literatuurstudie richtte zich op de werkzame elementen van *embodied* leeromgevingen gericht op grafieken van beweging. De interventie bestond uit een reeks van zes lessen met daarin *embodied* activiteiten. Wiskundige HOV gerelateerd aan afstand-tijd grafieken kunnen gezien worden als domein-specifiek. Hogere niveaus van redeneren worden bereikt als gevolg van een toenemende mate van kennis binnen dit deeldomein. Dit domein-specifieke element van wiskundige HOV maakt gebruik van het redeneren over covariantie. Dit redeneren over covariantie speelt ook een rol binnen andere wiskundige domeinen, waaronder algebra. Het derde en laatste doel van dit promotieonderzoek was onderzoeken in hoeverre een interventie gericht op grafisch redeneren ook HOV kan stimuleren binnen een ander wiskundig domein, namelijk algebra. Om aan te sluiten bij bovenstaande onderzoeksdoelen zijn een aantal deelstudies opgezet waarover achtereenvolgens is gerapporteerd in de *Hoofdstukken 2* tot en met *5* van dit proefschrift.

Hoofdstuk 2 beschrijft de systematische reviewstudie die werd uitgevoerd om de al bestaande *embodied* leeromgevingen gericht op grafieken van beweging in kaart te brengen. In de onderzoeksliteratuur is veelvuldig gerapporteerd over *embodied* leeromgevingen, al dan niet bewust. Deze leeromgevingen komen voort uit verscheidene cognitieve tradities en invalshoeken. Middels een review zijn deze onderzoeken systematisch bekeken, om zo meer inzicht te genereren in het educatieve potentieel van *embodied* leeromgevingen. De uiteindelijke selectie van artikelen (n = 44) bevatte 62 *embodied* leeromgevingen. Om grip te krijgen op de belangrijkste kenmerken van deze leeromgevingen hebben we deze leeromgevingen gecategoriseerd op twee dimensies: *bodily involvement* (betrokkenheid van het lichaam in de activiteit) en *immediacy* (de onmiddellijkheid van de activiteit). Wat betreft de eerste dimensie is een onderscheid gemaakt tussen het zelf bewegen (directe *embodied* ervaringen) en het observeren van andermans bewegingen of de bewegingen van objecten (indirecte *embodied* ervaringen). Wat betreft de tweede dimensie, *immediacy*, hebben we onderscheid gemaakt tussen *immediate* (onmiddellijk: "online" cognitieve activiteiten) en *non-immediate* (niet-onmiddellijk: "offline" cognitieve activiteiten). Beide dimensies gecombineerd resulteerde in een taxonomie van embodied leeromgevingen met daarin vier afzonderlijke categorieën: Categorie 1 – *Immediate Own Motion*, Categorie 2 – *Immediate Others/Objects' Motion*, Categorie 3 – *Non-immediate Own Motion*, en Categorie 4 – *Non-immediate Others/Objects' Motion*. De *embodied* leeromgevingen behorende tot de eerste categorie waren het vaakst voorkomend.

De systematische review resulteerde in acht karakteristieken, die gezien kunnen worden als specifiek voor *embodied* leeromgevingen waarbinnen het leren over bewegingsgrafieken centraal staan. Deze acht karakteristieken, hierna mediërende factoren, zijn: *real-world context* (realistische context), *multimodality* (multimodaliteit), *linking motion to graph* (koppeling tussen beweging en grafiek), *multiple representations* (meerdere representaties), *semiotics* (semiotiek), *student control* (controle), *attention capturing* (aandacht vangen), en *cognitive conflict* (cognitief conflict). Deze acht mediërende factoren hebben elk hun eigen rol in hoe ze het leren binnen de leeromgeving bevorderen. Bijvoorbeeld, de medierende factor *linking motion to graph* beschrijft hoe leerlingen de directe link tussen beweging en de daarbij behorende grafiek kunnen observeren. Een andere factor *multimodality* houdt in dat door de specifieke kenmerken van een hulpmiddel (bijvoorbeeld een bewegingssensor) of een bepaalde instructie, ten minste twee modaliteiten, zoals zien, horen, aanraken, inbeelden, of motoractie, tegelijkertijd worden geactiveerd. De vier categorieën in combinatie met de acht mediërende factoren zijn kenmerkend voor de complexe natuur van *embodied* leeromgevingen zoals deze kunnen bestaan in onderwijs en onderzoek. Elke categorie kent een bepaalde combinatie van mediërende factoren. Als zodanig is er veel variatie mogelijk tussen *embodied* leeromgevingen. Leeromgevingen die gebruik maken van de eigen beweging welke onmiddellijk gelinkt wordt aan de grafische representatie van die beweging (Categorie 1), zijn, aldus de review, het meest effectief in termen van leeruitkomsten.

In deze specifieke categorie werden de drie mediërende factoren, *multimodality*, *linking motion to graph*, en *multiple representations* het vaakst genoemd.

In de *Hoofdstukken 3* en *4* wordt verslag gedaan van de twee deelstudies waarin de potentie van fysieke ervaringen tijdens het leren over afstand-tijd grafieken is onderzocht. In deze deelstudies is gebruik gemaakt van een *embodied* leeromgeving, bestaande uit een zesdelige lessenserie, waarbij directe of indirecte *embodied* ervaringen een rol speelden. Deze zesdelige lessenserie was gericht op het stimuleren van het redeneren van leerlingen over grafieken, meer specifiek, het weergeven van beweging met afstand als functie van tijd. Vanuit het idee, gebaseerd op theorieën van *embodied cognition*, dat perceptuele-motorervaringen een belangrijk startpunt zijn om hogere niveaus van wiskundig redeneren te bereiken, werden twee parallelle versies van de lessenserie ontworpen. In de ene versie van deze lessenserie kregen de leerlingen directe *embodied* ondersteuning door gebruik te maken van een bewegingssensor. Hiertoe werden bij de bewegingssensor activiteiten ontwikkeld waarin de eigen bewegingen direct gevisualiseerd (bijvoorbeeld geprojecteerd op het digitale schoolbord) werden als een lijn in de afstand-tijdgrafiek. In de andere versie van deze lessenserie kregen de leerlingen indirecte *embodied* ondersteuning door gebruik te maken van pen en papier of projecties op het digitale schoolbord. Leerlingen kwamen in deze conditie ook in aanraking met de context van de bewegingssensor, echter zonder de aanwezigheid van de fysieke tool.

In de studie waarover werd gerapporteerd in *Hoofdstuk 3* is dieper ingegaan op de leeromgeving met daarin directe *embodied* ondersteuning. De focus van deze studie lag op de micro-ontwikkeling van het grafisch redeneren van de leerlingen over de zes lessen. De resultaten van de review gepresenteerd in *Hoofdstuk 2* suggereerden al enigszins dat een leeromgeving waarbij de eigen bewegingen onmiddellijk gekoppeld konden worden aan de grafische representatie van die beweging (bijv. afstand-tijdgrafieken, snelheid-tijdgrafieken) gezien kan worden als meest kansrijk om het begrip van deze grafieken te vergroten. In deze studie hebben we dan ook meer specifiek gekeken naar de rol van *embodied* ervaringen tijdens activiteiten met de bewegingssensor in de ontwikkeling van het redeneren over grafieken. Deze ontwikkeling over de lessen werd gemonitord middels grafiekinterpretatietaken en grafiekconstructietaken na elke les. De resultaten lieten zien dat de leerlingen over de zes lessen van een iconisch begrip van afstand-tijd grafieken naar een dieper begrip gingen. Hierbij redeneerden de leerlingen over de grafieken door de variabelen afstand en tijd, gerepresenteerd op de assen van de grafiek, impliciet of expliciet te benoemen. Op dit hogere niveau van redeneren lieten de leerlingen ook informeel redeneren over covariantie zien (bijv. "het afleggen van meer afstand in minder tijd"). Daarnaast bleek uit een kwalitatieve analyse van twee lesepisoden, waarbinnen de interacties van een leerling met de docent en andere leerlingen centraal stonden, dat deze vooruitgang in redeneren plaatsvond in relatie tot de perceptuele-motorervaringen voor de bewegingssensor. Zo zagen we dat deze leerling meer begrip ontwikkelde over het lopen van een bepaalde grafiek middels de coördinatie van verscheidene modaliteiten (zien, horen, gebaren, bewegen), die door de leerling (al dan niet bewust) gekoppeld werden aan de grafische representatie die zich op het scherm van de computer openbaarde. De bewegingen van deze leerling, evenals bepaalde kenmerken van de grafische weergave die naar voren kwamen tijdens het lopen van de grafiek, kwamen ook terug in het redeneren van de leerling over de grafiek. De technologie die tijdens de lessen werd gebruikt, was een belangrijke facilitator in dit proces. Kortom, bij deze en de andere leerlingen ontdekten we dat de lichamelijke activiteiten voor de bewegingssensor resulteerden in hogere niveaus van wiskundig redeneren in termen van het redeneren over de variabelen afstand, tijd en snelheid (bijv. "meer afstand in minder tijd resulteert in een steilere helling").

In de studie beschreven in *Hoofdstuk 4* gaan we dieper in op het effect van directe *embodied* ervaringen versus indirecte *embodied* ervaringen op het redeneren over afstand-tijdgrafieken. In deze effectstudie maakten we gebruik van een cohort sequentieel longitudinaal ontwerp om de veranderingen in het grafisch redeneren van de leerlingen gedurende het schooljaar te onderzoeken. De ontwikkeling van de leerlingen in hun grafisch redeneren werd vastgesteld middels het analyseren van hun antwoorden op vier grafiekinterpretatie- en grafiekconstructietaken. De lessen in de indirecte *embodied* conditie bevatten vergelijkbare activiteiten als de activiteiten in de directe *embodied* conditie, echter zonder dat de leerlingen de mogelijkheid kregen om bepaalde bewegingen zelf uit te voeren middels het gebruik van een bewegingssensor. In deze indirecte *embodied* conditie werd de beweging van een object (d.w.z. een speelgoedauto) als uitgangspunt genomen. De beweging van dit object werd enerzijds weergegeven op papier, als beschrijving, en anderzijds dynamisch, als projectie op het digitale schoolbord. Een derde groep leerlingen diende als baseline conditie. Deze leerlingen kregen les over een ander wiskundig onderwerp, namelijk kans. De leerlingen in zowel de directe als indirecte *embodied* conditie gingen sterk vooruit in hun grafisch redeneren na het volgen van de lessen, waarbij de resultaten van de leerlingen in de baseline conditie dienden als ijkpunt om deze vooruitgang vast te stellen. Hieruit kunnen we afleiden dat de leerlingen in de baseline conditie niet meer vooruit gingen dan je op basis van een reguliere ontwikkeling over de tijd zou mogen verwachten. Dit geeft aan dat beide versies van de ontwikkelde lessenserie zeer effectief waren. Daarnaast vertoonden de leerlingen in de directe *embodied* conditie een sterkere groei in hun grafisch redeneren dan de leerlingen in de indirecte *embodied* conditie. Hieruit blijkt dat directe *embodied* ervaringen kansrijk zijn in het stimuleren van het grafisch redeneren van leerlingen in groep 5.

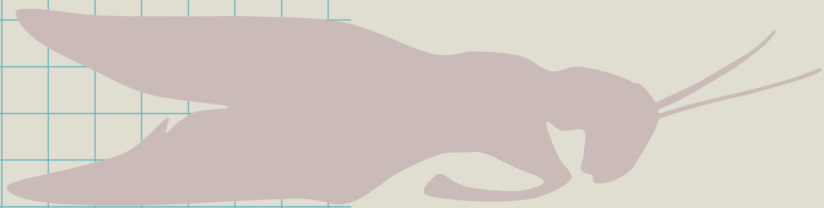
In de literatuur worden HOV veelvuldig geconceptualiseerd als domein-algemeen. Dit houdt in dat HOV gestimuleerd kunnen worden ongeacht de context. Er is echter ook onderzoek dat stelt dat HOV domein-specifiek zijn en juist ontwikkelen door een groei in kennis en vaardigheden binnen een bepaald academisch domein. Wiskundige HOV binnen het deeldomein afstand-tijd grafieken, waaronder het redeneren over deze grafieken, maken onder meer gebruik van het vermogen van een leerling om te redeneren over covariantie. Covariantie is een parameter die de mate van samenhang tussen bepaalde variabelen uitdrukt. Dit redeneren over covariantie is ook relevant binnen andere wiskundige domeinen, waaronder lineaire vergelijkingen. Meer algemeen is het aannemelijk dat bepaalde elementen van HOV die relevant zijn binnen meerdere wiskundige domeinen, versterkt kunnen worden binnen een van deze domeinen en zo kunnen worden overgebracht naar het andere wiskundige domein, op basis van domein-algemeen wiskundig redeneren (zoals het extraheren, gebruiken, en combineren van meerdere bronnen van informatie). In de laatste studie van dit proefschrift, waarover werd gerapporteerd in *Hoofdstuk 5*, werd onderzocht of de domein-specifieke lessenreeks over afstand-tijd grafieken tevens resulteerde in de ontwikkeling van HOV binnen een ander wiskundig domein: het oplossen van lineaire vergelijkingen. Om het algebraïsch redeneren over deze lineaire vergelijkingen te onderzoeken werd gebruik gemaakt van vier taken waarin leerlingen gevraagd werd lineaire vergelijkingen op te lossen. De bevindingen van deze studie wijzen uit dat de domein-specifieke lessenreeks over grafieken resulteerde in een significant hoger gemiddeld groepsniveau in het algebraïsch redeneren van de leerlingen. Dit effect was minder sterk dan het effect op het grafisch redeneren. Echter, op individueel niveau werd geen verband gevonden tussen de groei van leerlingen in hun grafisch redeneren en de groei van leerlingen in hun algebraïsch redeneren. Met andere woorden, de leerlingen die verbeterden in grafisch redeneren, verbeterden niet hun algebraïsch redeneren, en omgekeerd. Op basis hiervan trekken we de voorlopige conclusie dat de HOV gestimuleerd in de lessenreeks over grafieken niet per definitie resulteerde in HOV binnen een ander wiskundig domein. De bevinding dat de leerlingen wel een verbetering in hun algebraïsch redeneren vertoonden na deelname aan de interventie, verdient nader onderzoek.

In *Hoofdstuk 6* worden de resultaten van dit promotieonderzoek samengevat en is gekeken naar de implicaties van dit onderzoek voor theorie en praktijk. Hierbij was aandacht voor de beperkingen van het onderzoek en suggesties voor mogelijk vervolgonderzoek. Op basis van de resultaten van het onderzoek beschreven in de afzonderlijke hoofdstukken kunnen we een aantal algemene conclusies opstellen aangaande directe versus indirecte *embodied* ervaringen in wiskundige activiteiten voor het stimuleren van grafisch redeneren. Zo kunnen relatief simpele activiteiten, zoals "het lopen" van grafieken voor de bewegingssensor, ervoor zorgen dat leerlingen op natuurlijke wijze een connectie maken tussen de eigen beweging en de lijn in de grafiek. Hierdoor kunnen hogere niveaus van redeneren over deze afstand-tijd grafieken worden bereikt. Het is aannemelijk dat de eigen bewegingen voor de sensor resulteren in de totstandkoming van nieuwe *embodied* metaforen, welke gekoppeld worden aan de al bestaande intuïtieve ideeën over de grafiek. Ook indirecte *embodied* ervaringen hebben geleid tot een dieper begrip van afstand-tijd grafieken. Het is aannemelijk dat in de indirecte *embodied* leeromgeving lichamelijke ervaringen op een kwalitatief andere wijze werden geactiveerd, meer waarschijnlijk door middel van *embodied* simulatie. Daarnaast kan het stimuleren van het grafisch redeneren van de leerlingen, binnen de leeromgeving die we hebben ontwikkeld in het kader van dit promotieonderzoek, gezien worden als een domein-specifieke operationalisatie van wiskundige HOV op het niveau van de basisschool. Dit is een waardevol resultaat voor de huidige onderwijspraktijk waar de vraag bestaat hoe HOV, als een belangrijk onderdeel van 21ste-eeuwse vaardigheden, gestimuleerd kunnen worden. Om het reken-wiskundecurriculum te verrijken kunnen activiteiten zoals we voor dit onderzoek hebben ontwikkeld, ingezet worden. Hierbij is het van belang rekening te houden met de al bestaande ideeën van leerlingen over grafieken door op deze ideeën voort te bouwen. Daarnaast is het van belang om het gebruik van technologie, waaronder bewegingssensoren, in het basisonderwijs te faciliteren door zowel de leerkracht te ondersteunen in het gebruik van dergelijke tools en de daarbij behorende lesactiviteiten alsmede de leeromgeving zo in te richten dat de bewegingssensor optimaal ingezet kan worden.

DANKWOORD

“En toen stapte hij in bed, trok zijn deken over zich heen, wreef in het donker zijn voelsprieten over elkaar en fluisterde: “Dankjulliewel’”

(Toon Tellegen, in: *Dankjewel: Dierenverhalen om iemand te* bedanken, 2012)



In de periode die voorafging aan de totstandkoming van dit proefschrift hebben een fantastische groep onderzoekers, lieve vrienden en familie menig hoogte- en dieptepunt, lief en leed met mij gedeeld.

Allereerst wil ik mijn (co-)promotoren bedanken voor de mogelijkheid die ze mij hebben gegeven om dit promotieonderzoek te kunnen doen. Marja, je was een kritische supervisor, met ontzettend veel enthousiasme voor – en kennis over – het reken-wiskunde onderwijs in binnen- en buitenland, iets wat de kwaliteit van dit proefschrift zeker ten goede is gekomen. Paul, telkens was je er om met wat scherpe kanttekeningen en juist geplaatste vragen, teksten, artikelen, en uiteindelijk dit proefschrift naar een hoger niveau te tillen. Michiel V., jouw kwaliteiten als onderzoeker zijn bewonderenswaardig, en ik ben blij dat je altijd de tijd en ruimte had (of maakte) om met me mee te denken. Hoe kan ik je toch bedanken, wellicht met een kuipje Philadelphia? Michiel D., jouw rust en kalmte hebben menig stressvol moment verlicht. Je inhoudelijke suggesties waren voor mij altijd even waardevol, evenals de gesprekjes over alle niet promotie gerelateerde zaken.

Jan, je was het Mplus-wonder van ons project. Al ging ik af en toe met lood in mijn schoenen richting je kamer om de zoveelste inconsistentie met je te bespreken, jouw humor en zekere nonchalance maakten van onze bijeenkomsten altijd weer een feestje!

Lieve Mara en Suzan, ik ben blij dat jullie als paranimfen naast mij staan. Mara, de afgelopen jaren waren op vele vlakken onvergetelijk; van slenterend door New York tot laminaat leggend in Utrecht, alle pieken en dalen, we hebben ze doorstaan. Zonder jouw support was dit proefschrift er niet geweest. Suzan ook al was je geografisch gezien niet altijd dichtbij, dat maakte eigenlijk niet uit. Naast alle gezelligheid en gedeelde interesses kan ik zo genieten van je gave om altijd het positieve te zien.

Leerkrachten en leerlingen van basisscholen De Howiblo, CNS Abcoude, de Willibrordschool, St. Ludgerus, Wereldwijs, De Rank, en De Regenboog, we kunnen wel stellen dat jullie bewegingen voor dit onderzoek eigenlijk onmisbaar waren. Dat geldt ook voor de onderzoeksassistenten en stagiaires die hebben geholpen met het verzamelen (en coderen) van de data.

My (former) roommates from H2.08 and F2.14. Roos, als ik terugdenk aan die eerste jaren denk ik met name terug aan je humor en je optimisme, aan de avonturen die we hebben beleefd, aan heel veel lachen en een enkele traan. Dit proefschrift is af mede dankzij jou. Dear Yan, although you cannot be here in person, your warm-heartedness, kindness, and never ending interest in everyone around you including me, is something I cherish. Ali, my modest colleague and roommate, you were the person who introduced saffron to me, and I hope one day I will see the country where that saffron came from. Ilona, dankjewel voor je rust, je heldere blik en je bemoedigende woorden.

Er zijn ook heel veel andere collega's geweest die dit promotietraject de afgelopen jaren leuker, verfrissender, interessanter, en gemakkelijker hebben gemaakt! Collega's van de afdeling Orthopedagogiek, ik kijk met veel plezier terug op alle lunches, borrels, en gesprekjes in de wandelgangen, en meer, die ik de afgelopen jaren met jullie heb gedeeld. Collega's van het Freudenthal Instituut, niet alleen stonden velen van jullie klaar om de telefoon te beantwoorden als de techniek het weer eens liet afweten, ook heb ik genoten van de interessante inhoudelijke besprekingen over allerhande onderwerpen, en de gezellige koffie sessies op een aantal van de hotspots die de Uithof rijk is. Colleagues from IPN, more specifically, Anke and Aiso, your hospitality and kindness have made it a pleasure to be in Kiel during the first years of our project. En tenslotte, mijn (inmiddels niet meer zo) nieuwe collega's van de Marnix Academie, meer in het bijzonder mijn kamergenoten van het MIC, maar ook alle lieve betrokken collega's buiten die vier muren, jullie warmte en interesse in mijn promotieonderzoek en jullie relativeringsvermogen heeft me heel goed gedaan het afgelopen jaar.

Mijn oud-studiegenootjes, the saturated *p*-values, jullie mogen hier niet ontbreken. Veel van ons zaten en zitten in hetzelfde schuitje, een moment met jullie is dan ook een feest van herkenning. Eva, Eveline, Rianne, Ryanne, Marloes, jullie brachten me de nodige afleiding en nieuwe inzichten, werk gerelateerd en persoonlijk, met fijne gesprekken, leuke uitstapjes of beide.

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Terugkijkend op deze jaren ben ik boven alles dankbaar dat ik zoveel mocht ontvangen.

About the author

Carolien Duijzer was born on January 24th, 1989, in Brakel (the Netherlands). After obtaining her secondary school degree, she enrolled at the HKU (Hogeschool voor de Kunsten Utrecht) and obtained her bachelor's degree Visual Art and Design in Education in 2010. She then followed her interest in the visual arts by pursuing a bachelor's degree in Art History and Archeology at Utrecht University. In 2011, she simultaneously enrolled in the pre-master program Educational Sciences at the same university. She completed both programs in 2013. She then joined the research master program Educational Sciences: Learning in Interaction, from which she graduated cum laude in 2015. During this two-year master program her academic interest broadened to mathematics education and embodied cognition. She continued to explore these topics in her PhD research about stimulating fifth-grade students' higher-order thinking as reasoning about motion graphs through embodied mathematical activities. She started this project in August 2015 in collaboration with Marja van den Heuvel-Panhuizen, Paul Leseman, Michiel Veldhuis, and Michiel Doorman at the Faculty of Social and Behavioural Sciences of Utrecht University. This PhD project was part of the larger Beyond Flatland project. Carolien was a PhD council member from 2016 till 2018. She presented her work at various conferences, including ICME, AERA, and CERME. Carolien's PhD research fulfilled all requirements of the Interuniversity Center of Educational Sciences (ICO) Research School in the Netherlands. Since August 2019, Carolien works at the Marnix Academie in Utrecht as a teacher educator and educational researcher.

List of publications related to this thesis

Scholarly publications

Duijzer, C., Van den Heuvel-Panhuizen, M., Veldhuis, M., Boom, J., Doorman, M., & Leseman, P. (2020). Moving towards understanding: Students interpret and construct motion graphs. *Mediterranean Journal for Research in Mathematics Education*, *17*, 25–51.

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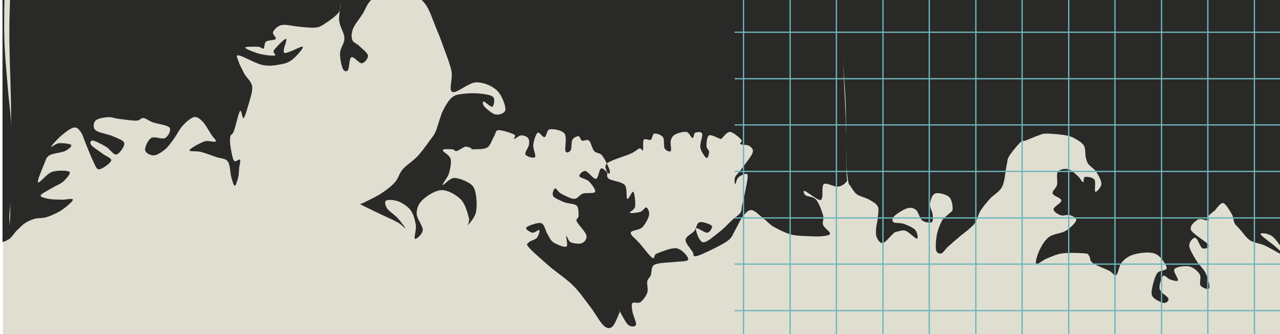
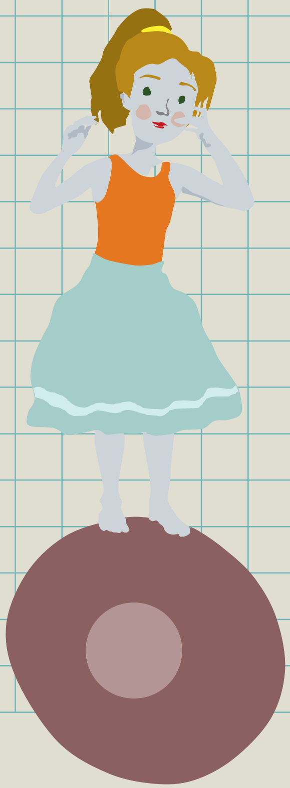


APPENDICES



Appendices 2.1 to 2.3 belong to *Chapter 2* of this PhD thesis

Appendix 2.1

Sample Query and Sample Filters.

Education	(adult* OR child* OR class* OR course* OR curricul* OR education* OR grade* OR instruct* OR kindergarten OR learn* OR lesson* OR pedagogic* OR pupil* OR school* OR student* OR teach*)
	AND
Learning facilitator	("body motion" OR bodily OR "conceptual metaphor*" OR embod* OR "fictive motion" OR haptic OR kinesthetic* OR multimodal* OR "multiple modalities" OR "perceptuo-motor" OR "physical event*" OR "physical experience*" OR "physical participation" OR psychomotor OR sensorimotor OR "sensory-motor" OR "sensuous cognition" OR cbr OR coach OR "data-acquisition probeware" OR GeoGebra OR "graphing calculator*" OR Kinect OR "learning in motion" OR manipulative* OR mbl OR motiondetector OR "motion detector" OR motionsensor OR "motion sensor" OR "motor simulation" OR "physical object*" OR "physical tool*" OR "real-time graphing technology" OR "smartgraph*" OR "Texas instruments" OR "TI-Nspire" OR Vernier OR video* OR "virtual graphing program*" OR Wii)
	AND
Domain	(engineering OR math* OR "physical science" OR physics OR science OR stem)
	AND
Topic	(graph* OR kinematics OR kinetics)
	AND
Variables	(acceleration OR change OR data OR distance OR "dynamic data modelling" OR motion OR time OR speed OR velocity)
	AND
Filter(s)	• In SCOPUS and Web of Science, the limitations were set to journal articles. • In ERIC, the limitations were set to journal articles and peer-reviewed articles.

Appendix 2.2

1. Quality check

We conducted a quality appraisal on the 26 studies making a comparison with a comparison group and/or making pre-posttest comparisons. First, they were coded based on their design, being (quasi) experimental or descriptive (see Barzilai, Zohar, & Hagani, 2018). Articles were coded as (quasi) experimental if a research strategy was used in which certain variables were controlled and actively manipulated by applying an intervention (i.e., including a certain learning environment). Articles were coded as descriptive if the outcomes of an intervention were described (i.e., including a report of a case study analysis of the specific occurrence of teaching and/or learning present within the learning environment). Hereafter, by making use of a coding scheme developed by Jabbar and Felicia (2015), we coded the articles' methodology on seven quality indicators, being: control group, type of data gathering, number of time-points, type of data analysis, sample size, research setting, and quality of reporting. For a description and an illustration of each quality indicator see Table 1. We assumed that an indicator was not met if there was no information provided about the indicator in the article.

Table 1
Quality indicators and scoring procedure

	Criteria	Description	Score
A.	Internal validity (validity)	Preventing or minimizing bias:	
		• Control group (Making a comparison between groups, either receiving an intervention, or not)	2
		• No control group	1
		Data gathering	
		• Collecting both qualitative and quantitative data	1 (**Yes**) 0 (**No**)
		Time-points	
		• Multiple time-points (time series - longitudinal)	3
		• Pre-posttest (before and after study)	2
		• Posttest observation	1
		Data analysis	
		• Correlation/Regression analysis (association between variables)	3
		• Factor analysis (clustering variables)	2
		• Descriptive analysis	1
		• Qualitative analysis	1
B.	External validity (generalizability, applicability, transferability)	Sample size (participants)	
		• 1-99	1
		• 100-199	2
		• 200-299	3
		• 300-399	4
		• 400 and above	5
		Research setting	
		• Related to real-world experiences and context	2
		• Laboratory setting	1
		Quality of reporting	
		• Adequate details of the relation between the embodied learning environment, the activity and the research questions	2 (**Yes**) 1 (**Partly**) 0 (**No**)
		• Adequate details of participants (age, gender, academic background, and sampling decisions)	2 (**Yes**) 1 (**Partly**) 0 (**No**)

1.1 Explanation of each quality indicator

1.1.1 Control group

Articles were coded as with or without a control group (yes=2, no=1). A control group could both be a condition receiving an intervention or a condition not receiving an intervention.

1.1.2 Data gathering

Studies were coded as mixed method data gathering when they adopted a methodology where they included both qualitative and quantitative data and reported results from both data sources (yes=1, no=0).

1.1.3 Measurements

Studies were coded multiple measurements (3) if they included time series data (longitudinal data or retention data), pre-test posttest (2) if they included a before and after study, and posttest observation (1) if they included observational measures over the course of a certain period or at the end of an intervention.

1.1.4 Data analysis

Data analysis was coded based on the statistical methods used (for example, regression/correlation (3), factor analysis (2), descriptive analysis (1), or qualitative analysis (1)).

1.1.5 Sample size

Sample size was coded ranging from very small (1-99) (1) to very large (400 and above) (5), based on the work of Comrey and Lee (2009), cited in Jabbar and Felicia (2015).

1.1.6 Research setting

The data collection was coded as real-world data collection when data-collection took place in a real-world environment (e.g., at school) (2), and laboratory (e.g., outside-school context) (1).

1.1.7 Quality of reporting

Indicating whether the studies provided adequate details about their study, coded either sufficient (2), partly sufficient (1), or insufficient (0). These details included information about the relation between the embodied learning environment, the activity and the research questions, and information about the participants such as age, gender, academic background, and sampling decisions.

1.2 Quality appraisal

Scores for each quality indicator were added such that each article was given a quality score (possible range: 5-20). Figure 1 shows a histogram of the scores allocated to each article in the final selection of 26 articles. The mean rating was 11.77, with a standard deviation of 2.93 (the mean rating of the entire sample was 10.27, with a standard deviation of 2.97). Based on these scores we positioned the articles as being of low (5-8), fair (9-12), high (13-16), or very high (17-20) quality. Every article rated 13 and above, can be considered as providing methodologically high-quality evidence of the effects of embodied learning environments on students understanding of graphing change. Accordingly, 9 articles (35%) were appraised as being of high quality, 14 articles (54%) were appraised as being of fair quality, and 3 articles (12%) were appraised as being of low quality.

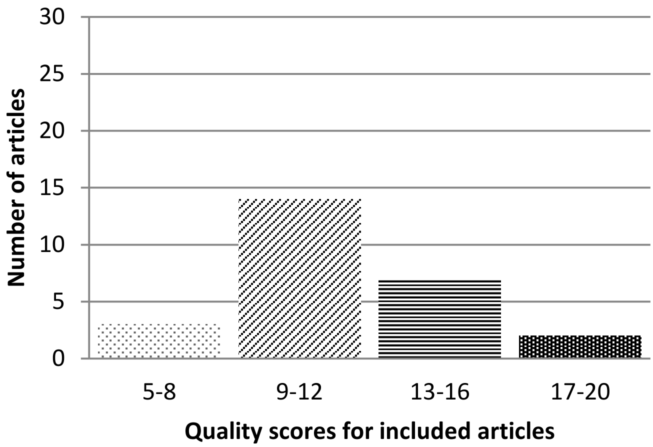


Figure 1. Histogram of quality scores for the 26 articles.

A histogram of the scores allocated to each article for the entire sample of articles (n = 44) are given in Figure 2.

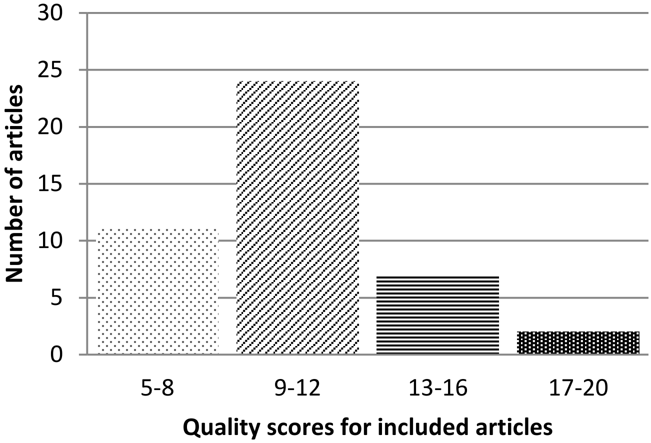


Figure 2. Histogram of quality scores for the 44 articles.

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Appendix 2.3

Learning outcomes for subset of 26 articles comparing multiple groups or pre- and post-tests (part 1)

Article	Intervention				
	Study design	Topic	Tool(s)	Length	Graphing activities
Altiparmak (2014) University students *N* = 58	Intervention (pre-post two group design) *Quasi-experimental*	Derivative, function; *d*-t and *v*-t graphs	Derivative, function; *d*-t and *v*-t graphs	540	Students worked with simulation software, giving them the opportunity to input values that resulted in specific movements of a car. Subsequently, they watched an animation of the movements of the car and its corresponding function graph.
Anastopolou, Sharples, & Baber (2011) University students *N* = 18	Intervention (pre-post two group design) *Quasi-experimental*	Kinematics; *d*-t and *v*-t graphs	Kinematics; *d*-t and *v*-t graphs	50	Students in the first learning environment moved their own hands in front of a motion sensor. The motion of their hands was represented in distance-time and velocity-time graphs. Students in the other learning environment observed hand movements demonstrated by the teacher (whole-class demonstration).
Anderson & Wall (2016) Middle school students *N* = 692	Design based research (interviews) *Descriptive*	Kinematics; displacement, velocity and acceleration	Kinematics; displacement, velocity and acceleration	A set of activities	Article contains four learning environments in total, slightly differing from each other. Students in the first learning environment moved objects in front of the Kinect sensor, or moved in front of the sensor themselves. Students in the second learning environment were building ramps by using a variety of materials. After this they had to choose three objects to roll of the ramp while collecting time and distance data using timers and measuring tapes. Subsequently, the floor was marked at specific distances. Each group was asked to record the time as individual students moved across the floor. Students in the third learning environment walked in front of the Kinect sensor. In the fourth learning environment students were asked to build ramps by using a variety of materials. After this they had to choose three objects to roll of the ramp and collect time and distance data using timers and measuring tapes.
Brasell (1987) Pre-University students *N* = 75	Experimental (four group between group design) *Quasi-experimental*	Kinematics; *d*-t and *v*-t graphs	Kinematics; *d*-t and *v*-t graphs	250	In both learning environments students walked in front of a motion sensor. At first, movements in front of the motion sensor were exploratory, to familiarize their selves with the sensor. After these exploratory movements, students engaged in a prediction activity of what the distance and velocity graph would look like for a complex event, and a reproduction activity for a complex distance and velocity graph. Students either walk in front of the motion sensor while a graph of their movement appeared in real-time (standard MBL) or not (delayed MBL).

Brungardt & Zollman (1995) Secondary education students $N = 30$	Experimental (posttest only, two group between group design) x Interviews *Quasi-experimental*	Kinematics: One- and two-dimensional motion (displacement, velocity, and acceleration)	Kinematics: One- and two-dimensional motion (displacement, velocity, and acceleration)	200	In both learning environments, by making use of video, four scenes of sporting events were presented to the students, ranging from object motion (e.g., basketball), to other person's motion (e.g., sprinter). The students had to observe the motion events presented to them, and collect motion data, which they could extract from the video by overlaying the video with acetate paper. In the one learning environment students saw the kinematics graphs appear simultaneously alongside the motion event. In the other learning environment the production of the kinematics graphs was intentionally delayed.
Deniz & Dulger (2012) Primary education students $N = 39$	Experimental (pre-post two group design) x Interviews *Quasi-experimental*	Science & kinematics; position-time	Science & kinematics; position-time	360	Students in the first learning environment were allowed to walk in front of a motion sensor. These students immediately received a graphical representation of their movement by means of the motion sensor and the graphing software. Students in the other learning environment were also moving, while carrying a bottle of water with a hole in the bottom. When they walked drops of water fell on the floor. These students had to measure their time of travelling, and the distance between the drops of water. As a result, a position-time graph could be constructed by hand.
Espinoza (2015) Undergraduates Study 1: $N = 117$ Study 2: $N = 40$	Experimental (pre-post two group design) (2 studies) *Quasi-experimental*	Kinematics; position-time	Kinematics; position-time	Two tasks for each study	Article consisted of two studies, including one learning environment per study. In the first learning environment students moved in front of a motion sensor. They also used their experiences with the motion sensor to interpret graphs of someone else's movements in front of a motion sensor. In the second learning environment students influenced the movement of a pendulum by setting the pendulum in motion and subsequently saw how the movement of the pendulum was captured by a motion sensor and represented as a graph. The learning environment included both a physical and a simulated pendulum.
Ferrara (2014) Primary education students $N = 1$	Case study (longitudinal design) *Descriptive (pre-post)*	Functions; Two-dimensional motion, position-time	Functions; Two-dimensional motion, position-time	Two short lesson	In the first learning environment the student walked in front of a motion sensor whilst the movements of the student were directly represented in the graph. In the second learning environment the student watched the movements of an animated glove (within a simulation environment), being represented in both horizontal and vertical direction.
Holbert & Wilensky (2014) Out-of-school context $N = 6$	Intervention study (pre- post interviews) *Descriptive (pre-post)*	Kinematics; Acceleration and velocity	Kinematics; Acceleration and velocity	120	Students used a Wii Motion controller to move a racecar around a racetrack. The Wii motion controller could be moved forward and backward by making forward and backward hand movements. Moving the controller forward the car accelerated, moving the controller backward the car decelerated. The bodily movements of the students were related to what happened on the screen by linking the movement of the body to the car's motion. In the third phase of the activity, the movements managed through the Wii, were directly related to the construction of a graphical representation representing those movements as well.

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Kozhevnikov & Thornton (2006) Undergraduates Study 1: *N* = 76 Study 2: *N* = 122	Quasi-experimental (pre-post design) *Quasi-experimental*	Force and motion; Position, velocity, and acceleration	Force and motion; Position, velocity, and acceleration	Study 1: E1: 280 Study 2: E2: 1800 E3: 80	Study 1: students in the first learning environment performed (some) experiments in kinematics laboratories, including experiments where they studied the motion of carts on ramps. The students did not initiate the motion of the carts on the ramps, rather the low friction cart was pushed along by a fan unit. The students had small group discussions about motion, made predictions about what the graphs would look like, and then recorded their final predictions. Study 2 students in in the second and third learning environment (experimental and control condition) followed lectures that contained demonstrations of motion events by the teacher. The data was gathered by the motion sensor equipment and represented on the screen of a computer. Subsequently, data were analysed.
Metcalf & Tinker (2004) Teacher professional development *N* = 30	Intervention study (pre- post multiple group design) x Interviews *Quasi-experimental*	Force and motion; Position and velocity	Force and motion; Position and velocity	Length varies per teacher	The aim of the implemented curriculum was for students to connect motion phenomenon to graphs. Students made use of carts (among other manipulatives) in graphing movements. For example, they received a task containing specific motions, made predictions on what the graphs would look like and had to perform the experiments (i.e., graphing motion events). They did this by rolling their carts out in the hallway, presumably using their whole body since the cars had to keep moving, and generated the specific motions.
Mitnik, Recabarren, Nussbaum, & Soto (2009) Secondary education students *N* = 23	Experimental (pre-post two group design) *Quasi-experimental*	Kinematics; One-dimensional motion (position-time and velocity-time)	Kinematics; One-dimensional motion (position-time and velocity-time)	240	In the experimental condition students were looking at the motion of a robot. Students had no control over the motion of the robot. In the control condition, students were looking at a computer simulation of a robot. Again students did not have control over the robots' motion. When the robot moved through the room or on the computer screen, data gathered by the robot was presented on the handheld computers. Once the motion [of the robot] ended, an editing options appeared, allowing one student to draw, erase, redraw, or ask for motion repetition. Once the student finished editing, he send his graph to the data collectors who were then able to verify that the plot corresponded to the data they gathered. Data collectors, therefore, had two choices: to accept or to reject the received plot.
Mokros & Tinker (1987) Secondary education students *N* = 125	Quasi-experimental study (pre- post design) x Interviews *Quasi-experimental*	Kinematics; Position and velocity	Kinematics; Position and velocity	20 class sessions	Students engaged in two main activities. During the first activity they had to walk in front of the motion sensor. During the second activity the students made use of a toy car, and moved the toy car in front of the motion sensor. A corresponding graph was projected. The students could view the graph of their own motion or the motion of the toy car in real time. They were encouraged to make many predictions (e.g., "Predict the graph you will get if you push the car up the ramp, then have it come back down to the start").

Roschelle, Shechtman, Tatar, Hegedus, Hopkins, Empson, Knudsen, & Gallagher (2010) Secondary education students Study 1: $N = 95$ (teachers), $N = 1621$ (students). Study 2: $N = 30$ (teachers), $N = 1048$ (students). Study 3: $N = 56$ (teachers), $N = 825$ (students)	Experimental (two studies; pre-post design); Quasi experimental (one study; pre-post design) *Experimental*	Proportionality and linear functions; Position-time graphs	Proportionality and linear functions; Position-time graphs	Study 1 13.8 days Study 2 12 days Study 3 12.4 days	Two grades, for both grades: The software used in the study made use of animations of motion events. In this respect, students watched the animations. These animations could take many forms, the study showed an example of a running girl. The students controlled the movements of the animations by building and editing mathematical functions in either algebraic or graphical form. After editing the functions, students could press a play button to see the corresponding animation. The graphs in the study plotted position versus time, while the animation above the graph showed the corresponding motion.
Simpson, Hoyles, & Ross (2006) Secondary education students x out-of-school setting $N = 21$	Design study x observations *Descriptive (pre-post)*	Motion; Position, velocity and acceleration (force and mass)	ToonTalk software x Web reports (web-based collaboration tool)	990	The students worked in a software environment, where they had to program the movements of a LunarLander. Successful programming was necessary for successful landing. Students could view the result of their programmed models represented in the movements of the LunarLander. By making use of Microsoft Excel students could simultaneously see the results of their models as position-time and velocity-time graphs. In this way, running the system allowed students to see two different representations: first, the movement of ToonTalk objects; and second, position–time and velocity–time graphs plotted from the exported data.
Skordoulis, Tolias, Stavrou, Karamos, & Gkiolmas (2006) Secondary education students $N = 6$	Case study *Descriptive (multiple measurements)*	Oscillations, determinism, and chaos; Angular displacement, angular velocity	Rotary motion sensor x Chaotic pendulum system x Computer interface x DATA STUDIO software	180	The students collected experimental data on chaotic motion by making use of the pendulum system. Students had to make observations, discuss the phenomena, and draw graphics. During the laboratory sessions students performed the experiments. The motion of the pendulum was captured by the rotary motion sensor. The resulting graphical representations could then be studied by the students.
Solomon, Bevan, Frost, Reynolds, Summers, & Zimmerman (1991) Secondary education students $N = 26$	Observation study *Descriptive (pre-post)*	Motion; Distance-time, velocity-time	Motion sensor x Software (not specified)	40	Students (secondary education) got nine pre-set graphs which they had to emulate by moving in front of the motion sensor. The graphical representation was presented in real-time to the students on a computer by means of the motion sensor.

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Struck & Yerrick (2008) Secondary education students $N = 39$	Quasi-experimental study (split category random assignment) *Quasi-experimental*	Kinematics; One-dimensional motion (one and two objects – Distance, velocity, acceleration)	Motion sensor x PASCO Xplorer GLX hand-held computer x Digital video camera (Canon) x Computer interface x Loggerpro software	(unknown)	All students received two different methods (MBL and DVA), but the order in which they received these methods varied between conditions. During the MBL session students had to walk in front of a motion sensor. During the DVA session students made videos of their own and each other's motions. Students had to create their own graphs by extracting position and time measurements from the videos.
Stylianou, Smith, & Kaput (1996) (Pre-service) Teacher education students $N = 28$	Observational study *Descriptive (pre-post)*	Motion functions; Position, velocity	Calculator-based Ranger (CBR) x Graphing calculator x Mathworlds software	300	Students walked in front of the motion sensor. During the first session, they were asked to "match" a position-time graph consisting of one segment of positive constant slope by walking a similar motion. The movements of the students were represented on the graphing calculator. The software also made it possible to replay the motion, in order to match the walk piece-by-piece to the corresponding graph.
Svec, Boone, & Olmer (1995) (Pre-service) Teacher education students $N = 106$	Intervention (pre-post design) *Descriptive (pre-post)*	Motion; Distance, velocity, acceleration	Motion sensor (sonic ranger) x Computer interface) x manipulatives	840	There are three activities described in the article. Students were first asked to predict how a graph would look for a certain type of motion such as walking toward the sonic ranger at a constant speed. Students then carried out this motion and compare their predictions with the resulting graph. They applied this sequence of predictions to more complicated motion events, involving stops, changes in direction, and changing velocities. Toy cars on horizontal and inclined ramps were employed for the final activities. The control condition received treatment as usual. The movements of the students and the motion of the objects is captured by the motion sensor, and displayed on the screen of a computer.
Svec (1999) Pre-university students $N = 170$	Experimental (pre-post two group design) *Quasi-experimental*	Motion; Distance, velocity, acceleration	Motion sensor (sonic ranger) x Computer interface) x manipulatives	840	There are three activities described in the article. During the first two activities, students were walking in front of the motion sensor. During the last activity, students made use of manipulatives, such as toy cars and bouncing balls. The movements of the students and the motion of the objects were captured by the motion sensor, and displayed on the computer in real time.
Thornton, & Sokoloff (1990) University students Study 1: $N = 262$ Study 2: $N = 249$ Study 3: $N = 524$	Quasi-experimental study (pre-post design) *Quasi-experimental*	Kinematics; One-dimensional motion (Position, constant velocity, changing velocity, acceleration)	Motion sensor x Manipulatives x Computer interface x Software (not specified)	Two kinematics laboratories	Students had to walk in front of the motion sensor, while completing small motion assignments, such as walking quickly and slowly toward and away from the motion sensor. The motions they had to perform ranged from simple to complex. The movements of the students were immediately transferred to a graphical representation and displayed on the screen of a computer.

Wilhelm, & Confrey (2003) Secondary education students $N = 4$	Interviews *Descriptive (multiple measurements)*	Functions and motion; Accumulation, rate of change, changing rate of change, position, velocity, change in velocity	Motion detector x Computer interface x software (not specified) x Bankaccount software	450	The study makes use of a motion and a money context. In the motion context students walked in front of a motion sensor. Students movements ere represented as a graphical representation on the computer in real-time. In the money context, students used their experiences gained in the motion context and had to apply these to the money context. In the money context students watched simulations. The simulations and the accompanying graphical representations were presented simultaneously.
Woolnough (2014) Secondary education students $N = 30$	Intervention study (pre and posttest) x Interviews *Descriptive (pre-post)*	Rectilinear motion, Newton's second law and circular motion; Displacement, velocity, and acceleration	Motion sensor x Computer interface x manipulatives	(unknown)	Students conducted activities involving falling masses and trolleys accelerating along benches in order to understand the relationship between displacement, time, and velocity. Graphs of motion were plotted in an interactive group environment.
Zajkov & Mitrevski (2012) (Pre-service) Teacher education students $N = 10$	Interview study *Descriptive (multiple measurements)*	Motion, free-fall and vertical launch; Position, velocity, acceleration	Camera x Video measurement software x Manipulatives	Two tasks	The study made use of two different methods of data gathering and analysis. In the first method students made videos of a motion event (concerning objects). Students used the video of their motion event and, based on this video, extracted data points, which could be represented graphically. During the second method students watched videos of motion events. Students analysed a video, without capturing the motion event represented in the video.
Zucker, Kay, & Staudt (2014) Secondary education students Study 1: $N = 35$ (teachers), $N =$ (approximately) 2000 (students) Study 2: $N = 29$ (teachers), $N =$ (approximately) 1700 (students)	Study 1: Experimental (pre-post two group design). Study 2: Quasi-experimental ((pre- post four group design) *Quasi-experimental*	Motion; Position, velocity, acceleration	Motion sensor x Computer interface x Smartgraphs software	250	Students conducted five activities using the Smartgraphs software, either using or not using motion sensor technology. Study 1: One condition conducted motion sensor activities (e.g., walking in front of the sensor). One condition did not use the software at all (control condition). Study 2: Two conditions used the motion sensor like in study 1. One condition used the software with one motion sensor per classroom, whereby the teacher demonstrated the motion sensor activities to the students. One condition used the software without scaffolding (but with the motion sensor activities).

A

Learning outcomes for subset of 26 articles comparing multiple groups or pre- and post-tests (part 2)

Article	Dependent measures	*g*	Reported results	QS
Altiparmak (2014)	Conceptual understanding of derivative	0.81[a]	Students in the experimental group scored (sometimes significantly) higher on the posttest than did the students receiving 'traditional' education (ANOVA). This shows that by making use of animations throughout a sequence of tasks students conceptual' understanding of derivative is supported.	12
Anastopolou, Sharples, & Baber (2011)	Interpreting and constructing motion graphs (d-t graphs and v-t graphs)		On the posttest there was a significant difference between learning environments (Mann-Whitney U-test). Especially on the interpretation and construction of v-t graphs, the experimental condition scored better. This indicates that physical manipulation of kinematics graphs has a significantly greater effect on students' ability to relate graphs to movements than observing the graphs being produced by someone else.	13
	Hand movements x Demonstration	0.24[a]		
	Demonstration x Hand movements	-0.24[a]		
Anderson & Wall (2016)	Understanding the concepts of velocity and acceleration	-	Students showed two specific accomplishments, described by the authors as: (1) being able to demonstrate both velocity and acceleration through the visualization of velocity and acceleration in real-time graphs by means of the Kinect, and (2) showing a qualitative understanding of the concepts of velocity and acceleration. However, based on student interviews it was found that even though students demonstrated that connections between the Kinect technology and the involved kinematics were made, they still had some difficulties in unpacking what they observed in the graph (i.e., understanding graphs as being visualizations that extrapolate information from multiple data points). Moreover (and this aspect is motivational) the students were only moderately engaged in making sense of their observations.	13
Brasell (1987)	Translating between a verbal description of a physical event and the graphic representation		There was a significant difference between the the two learning environments and/or the trollrol conditions (ANCOVA). The scores of the students in the condition where the graphical representation was intentionally delayed (delayed MBL) were significantly lower than the scores of the students in which the graphical representations appeared in real-time (standard MBL). When looking more specifically at the differences between distance and velocity sub questions, the standard MBL students scored significantly higher than the other conditions on distance, but not on velocity. For velocity, there was no advantage of standard MBL over the other treatments.	14
	Distance graph			
	Standard MBL x Control	1.22		
	Delayed MBL x Control	0.30		
	Standard MBL x Delayed MBL	1.01		
	Delayed MBL x Standard MBL	-1.01		
	Velocity graph			
	Standard MBL x Control	0.39 (ns)		
	Delayed MBL x Control	-2.31		
	Standard MBL x Delayed MBL	0.48 (ns)		
	Delayed MBL x Standard MBL	-0.48 (ns)		

Brungardt & Zollman (1995)	Linear motion (including displacement questions, velocity questions, acceleration questions, and mixed questions) Simultaneous time x Delayed time Delayed time x Simultaneous time	 0.29 (ns) -0.29 (ns)	The researchers did not find a significant learning difference between the simultaneous-time learning environment and the delayed-time learning environment (ANOVA). In general, the simultaneous-time learning environmnet scored higher on the posttest, but not significantly higher. Even after splitting the posttest over the different graphing variables (displacement, velocity, and acceleration), no differences were found.	15
Deniz & Dulger (2012)	Interpret graphical representations of distance and time and interpreting graphical representations of temperature and time Technology group x Conventional group Conventional group x Technology group	 0.39[a] -0.39[a]	There was a significant advantage of using the MBLs (motion sensor x real-time graphing software) over not using MBLs, but a more traditional approach (i.e., using conventional laboratory instruments) (multiple *t*-tests). Students ability to interpret graphs improved substantially in the learning environment using motion sensor technology.	13
Espinoza (2015)	Study 1 Conceptual understanding of motion from a kinesthetic perspective Study 2 Conceptual understanding of motion from a kinesthetic perspective	 0.20 0.05(ns)	Study 1: The analyses (Chi-square and gain scores) revealed how students significantly improved in their understanding of motion when being in a learning environment that included kinesthetic experiences. Study 2: The students in the second study received both a real motion event and a simulated motion event (the order in which students received both differed between conditions). No differences were found between conditions. In general, the overall results (of both studies) indicated by the authors was that there was a clear advantage of generating a graphical representation of motion from one's own movements (study 1) or exposure to the physicality of motion (study 2)	11
Ferrara (2014)	Making sense of position-time graphs	-	The student was able to connect his movements with the graph(s) representing his movements. This understanding happened in three phases: recollection, imagination, and interpretation. When taking part in the second learning environment (approximately one year later) the student not only understood the graph, but was also able to communicate his understanding of the graph to others. The other students, whilst not having the same embodied experiences, were able to think multimodal as well, thus showing the importance of imaginary activities.	12
Holbert & Wilensky (2014)	Intuitive motion knowledge and formal graphical representations (velocity-time)	-	During the post-interview participants showed improvement on graph construction (graph construction was not an explicit part of the intervention). Five of the six participants were able to create qualitatively correct velocity-time graphs indicating that students drew on their in-game graph construction resources outside the gaming context.	9

Kozhevnikov & Thornton (2006)	Study 1		A paired sample *t*-test showed a significant increase in students' scores on the post force and motion test, as compared to the students' scores on the pretest. The authors also looked into students' spatial visualization abilities. On the pre-test students' spatial visualization ability was a reliable predictor of students' success in solving the problem. This result disappeared after receiving the kinematics laboratories. Graphing acceleration presented the most difficulties to students with low spatial visualization abilities.	12
	Force and motion conceptual evaluation test for students' conceptual understanding of force and motion concepts (FMCE)	2.09		
	Spatial visualization ability (PFT) related to pre-FMCE			
	Spatial visualization ability (PFT) related to post-FMCE	- (ns)		
	Study 2			
	Spatial visualization ability			
	Experimental 1	0.21		
	Experimental 2	0.62		
	Experimental 3	0.22		
Metcalf & Tinker (2004)	Motion and forces Transfer of energy	-	The outcomes of the study were divided over both teachers and students. In general, the teachers were able to successfully implement the investigations in the classrooms. In general, student learning was enhanced through the use of motion sensors, when comparing measures on the pre- and posttest. The students of the two Australian teachers improved most (12% from pretest to posttest). The authors also looked into specific sub questions, indicating that students especially improved on questions regarding position-time graphs.	10
Mitnik, Recabarren, Nussbaum, & Soto (2009)	TUG-K Understanding graphs of kinematics		An ANCOVA revealed that the students in the experimental condition significantly increased in their graph interpretation skills as compared to the students in the control condition. The real robot, compared with a computer-simulated activity, proved to be almost twice as effective.	12
	Robot x Simulation	0.76[a]		
	Simulation x Robot	-0.76[a]		
	Motivation	-		
	Student collaboration	-		
Mokros & Tinker (1987)	Interpreting and using graphs (distance-time, velocity-time, temperature time)	0.60	Students' scores on the graphing items indicated a significant change in students' ability to interpret and use graphs, from pre- to posttest (*t*-tests). The largest gains were found on the items where the motion phenomenon and its graphical representation did not share a pictorial link.	10

Roschelle, Shechtman, Tatar, Hegedus, Hopkins, Empson, Knudsen, & Gallagher (2010)	Study 1 Rate and proportionality M1 foundational concepts (e.g., basic graph and table reading) M2 Advanced concepts (e.g., reasoning about representations; graphs, tables, and functions) Study 2 (delayed) Rate and proportionality M1 foundational concepts (idem) M2 Advanced concepts (idem) Study 3 Linear function M1 foundational concepts (e.g., categorizing functions; linear vs. non-linear) M2 Advanced concepts (interpreting multiple functions representing change over time)	 0.16[a](ns) 0.77[a] 0.13[a] 0.56[a] 0.17[a](ns) 0.45[a]	Multilevel modelling showed that for all three studies, the main effect was statistically significant, indicating that students who received the SimCalc intervention learned more than the students in the control conditions who received lessons from the regular curriculum.	19
Simpson, Hoyles, & Ross (2006)	Understanding the relationship between initial speed and acceleration	-	Throughout the learning episodes, the students developed new understandings, such as reasoning about the relationship between initial speed and acceleration and the visualization of this relationship in the graph. Students learned that the kinematics graph is an abstract representation of a motion event, by showing that they were able to reason about the motions represented in the graph.	11
Skordoulis, Tolias, Stavrou, Karamos, & Gkiolmas (2006)	Understanding chaotic behavior	-	Based on the students participation in the didactical activities and the activities in the follow-up study a few months later, the authors come to the conclusion that despite the difficulties students experienced at the beginning, they developed a qualitative understanding of chaotic behavior using phase space representations.	11
Solomon, Bevan, Frost, Reynolds, Summers, & Zimmerman (1991)	Sensorimotor learning	-	When students used worksheets to guide their inquiry's, as opposed to only using the motion sensor, a more durable learning process was taking place. Students performed quite well on the questions that referred to the motion sensor context, but performed less on the questions that deviated from the motion sensor context a lot. This indicated that, in most of the students, throughout the learning episode no robust bridging mechanisms between the motion sensor context and the real-world were made.	9

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Struck & Yerrick (2008)	Interpreting graphs of one dimensional motion Interpreting motion situations graphically	-	By making use of the two different methods (DVA and MBL) students improved significantly in their graphing skills and in their interpretations of (graphically represented) motion (authors looked at percentages). These findings hold especially for d-t and v-t graphs, and to a lesser extent for a-t graphs. Large differences between the two methods weren't found except that in a few cases (when splitting the results over graph types) students studying motion by using DVA improved more than students studying motion by using MBL. This was not the case for d-t graphs, students ability to produce d-t graphs increased regardless of the used method.	13
Stylianou, Smith, & Kaput (1996)	Graphical representations of motion functions Development of graph-associated skills	-	The authors state that the pre-service teachers made gains in their ability to use graphs, to such extent that they were able to overcome some of their initial misconceptions about graphs. The teachers also developed pedagogical insights, referring to their own learning experiences as valuable in understanding the difficulties of their students.	9
Svec, Boone, & Olmer (1995)	Interpreting and constructing motion graphs (distance-time, velocity-time, and acceleration-time graphs)	-	Students' posttest scores on their understanding of graphs representing distance-time, velocity-time and acceleration-time were higher than their pretest scores (presented in percentages). Students also scored higher on making connections between distance-time, velocity-time and acceleration-time graphs (i.e., interpreting velocity from a distance-time graph). Students showed the largest gains on their understanding of distance-time and velocity-time graphs.	7
Svec (1999)	Graph interpretation (GIST) Motion content test (MCT) Kinematics graphs Non-graphing motion questions	0.80[a] 1.79[a] 1.03[a]	Learning outcomes are divided over graph interpretations skills, motion graphs, and conceptual understanding of motion. (1) The experimental group had lower pretest scores than the control condition. After receiving the treatment, they were at the same level of achievement. Graph interpretation was not explicitly taught. (2) Students in the MBL condition made the largest gains in their interpretation of motion graphs, being distance-time, velocity-time and acceleration-time graphs. (3) The gains made by the experimental condition were typically larger than found in the control condition, as such these students showed a larger understanding of velocity and acceleration.	8
Thornton, & Sokoloff (1990)	Understanding graphs of distance and velocity	-	By looking at percentages the authors show that students following the kinematics laboratories significantly improved their learning, compared to the students following lecture-based classes. Gains were found for all topics, including velocity. On the distance-graphs students hardly improved, since they already showed a large understanding at the pre-test.	14

Wilhelm, & Confrey (2003)	Understanding rate of change and accumulation (and their appearance) in multiple contexts	-	There were two main outcomes. First, a student did not necessarily had to have a complete understanding between rate of change and accumulation graphs in a single context to apply this knowledge in multiple contexts. Second, being able to differentiate between rate of change and accumulation in a single context did not necessarily mean that the student could apply this differentiation in multiple (different) contexts.	11
Woolnough (2014)	Calculating and interpreting slope Connecting knowledge about mathematics and physics	-	A strong emphasis of the curriculum was on the students' ability to calculate and interpret slope, and apply mathematical knowledge of slope to physics knowledge. No student entering the study assigned units to their calculation of slope. A year later (posttest) many students had learned how to calculate slope and many of the students started to relate slope to physical quantities. However, still not many students assigned units to slope, indicating a compartmentalization of knowledge between mathematics and physics.	8
Zajkov & Mitrevski (2012)	Understanding and interpreting position-time graphs	-	Results showed that using manual video measurement caused slight improvements in students understanding of graphs. When students conducted their own motion experiments and analysed these motion experiments students understanding of position-time graphs increased substantially. Moreover, students' use of vocabulary to explain their answers became more specific, and they showed a greater understanding of the physical quantities involved. Students understanding of velocity-time graphs was still poor.	11
Zucker, Kay, & Staudt (2014)	Understanding direction of motion Understanding rate of change Transferring knowledge to contexts other than motion	-	Students using the Smartgraphs software scored significantly higher than students in the control condition (not using the Smartgraphs software (ANOVA). This result was found in study 1 and confirmed in a follow-up study in study 2. In study 2 more specific comparisons were made. It was found that students who looked at demonstrations of motion sensor activities showed significantly lower gains than students acting out the motion activities themselves. Also comparisons were made between conditions using scaffolding provided by a slope tool and conditions not using this tool. It was found that conditions not using this tool had significantly larger gains than conditions using this tool. Analyses using multilevel modelling underscored these results.	19

Note. ns = non-significant (as reported by the authors); [a] = Adjusted ES

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Appendices 3.1 to 3.3 belong to *Chapter 3* of this PhD thesis

Appendix 3.1 The intruder task

In addition to the tasks as described in the two versions of the teaching sequence, in both versions a problem-solving task "The intruder task", which spanned the entire six-lesson teaching sequence, had to be solved by the students. This addendum provides a complete description of the problem-solving task that we developed. In order to solve the problem, students have to use knowledge and skills related to graphing motion. Students acquired this knowledge and skills throughout the teaching sequence. We will first give a description of the problem-solving task, which was inspired by a task as reported on by Espinoza (2015). We will then show how the problem-solving task was related to the topics of the respective lessons in the teaching sequence.

1. Problem description of the Intruder task

Each student received a booklet with a description of the task. The problem-solving task was introduced as follows:

> In a secret laboratory somewhere in the world new plants are being developed. Seeds of these plants are extremely rare and very valuable. Yet, something terrible has happened! In the middle of the night an intruder has broken into the laboratory and has stolen the seeds of one of the plants. Because of the unique characteristics of each plant it is important to find out from which plant the intruder took some seeds. Can you help the police solve this problem?

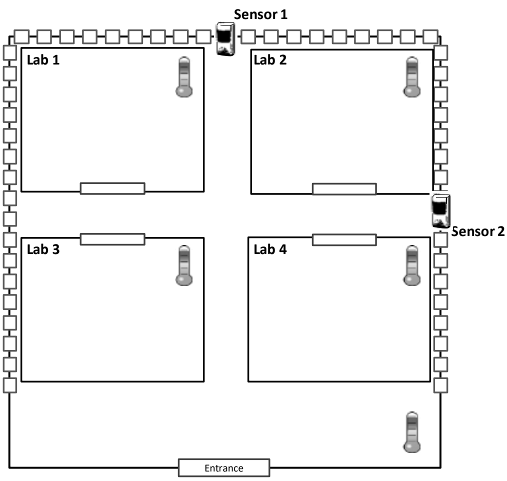

Figure 1. Floorplan of the laboratory.

The students then received the information necessary to solve the problem: the floorplan of the laboratory facility, including the positioning of the motion sensors and the entrance doors to each lab, and seven graphical representations. This information was introduced as follows:

> In this booklet you will find information about the laboratory facility and the burglary. You find a description of the plants, a floorplan of the laboratory facility (including the four labs), and seven graphs. Two motion sensors are positioned in the two main corridors of the facility. The motion sensors track the movements of any person (or object) present within the facility. Using this information, you are going to investigate from which lab (1-4) the intruder took some seeds.
>
> The intruder was in the building from 01:30 till 01:45. During the night several graphs were made. Two graphs represent the movements each motion sensor captured during the time period the intruder was inside, for each sensor. Five graphs represent the temperature in the four labs and the corridor during this same time period.

2. Description of the lessons and the problem-solving task

Table 1 shows how elements of the problem-solving task are related to the topics addressed in the teaching sequence, per lesson. In Lesson 2, the students were asked to more closely inspect the two graphs that belonged to the motion sensors, see Figure 2 and Figure 3. How the information was represented in each of the graphs was discussed with the students. Motion graph 1 closely resembled a continuous graph, whilst Motion graph 2 had longer periods of time in between measurement moments. This distinction between discrete and continuous graphs was already discussed during the first part of Lesson 2, where students had performed various activities related to this distinction. After this discussion of the two motion sensor graphs, students received Graph 2 in the same format as Graph 1.

Table 1

Elements of the problem-solving task in relation to the topics addressed in the teaching sequence, per lesson.

Lesson	Topic and main activities	Problem-solving task
1. Motion: reflecting and representing	Informal graphical representations *Reason with variables and construct representations of a real-world situation*	
2. From discrete to continuous graphs	Measuring distance *Measure distance in discrete intervals and continuously, and reason about differences between discrete and continuous graphs*	Part 1. Introduction Intruder task *Comparing both motion sensor graphs.* *Graph 1 – Continuous graph* *Graph 2 – Discrete graph* *Measuring points are close together (Graph 1) or further apart (Graph 2).* *After this lesson students receive a continuous graph representing the movements captured by motion sensor 2.*
3. Continuous graphs of “distance to”	Reason with continuous graphs *Coupling specific movements to their representation as a line in the graph* *Coupling a concrete situation to a graphical representation*	

4. Continuous graphs of "distance to"	Reason with continuous graphs *Coupling specific movements to their representation as a line in the graph* *Investigating how speed is represented in the steepness of slope*	Part 2. Solving first part of the Intruder task *Using the knowledge and skills gained in Lesson 3 and Lesson 4 regarding motion captured by a motion sensor and the representation of motion in the graph, to interpret Graph 1 and Graph 2.* *Example question asked:* *"Where did the intruder enter the building?"* *"Which motion sensor captured motion first, which motion sensor second?"* *"How do you know?"* *"What movements are represented in the graphical representations of each motion sensor"* *"Can you explain?"* *"What route can you draw fitting the graphs?"* *At the end of Lesson 4 it should be clear that the Intruder could have been in Lab 2 or Lab 4.*
5. Scaling on the graphs' axes	Reason about the relationship between two variables through scaling *Construct graphs with different scales on the axes*	Part 3. Solving the second part of the Intruder task *The temperature in Lab 1-4 varies. The temperature in the corridor is 15 degrees. The temperature in Lab 2 is 20 degrees in lab 4 is 40 degrees. There are no obvious fluctuations in temperature.* *"The security company can make various adjustments to these graphs, regarding scale. Which adjustments would help us?"* *Based on the knowledge and skills gained in this lesson, students can infer that the scale of the graph should be adjusted. Zooming in on a smaller part of the graph, can make fluctuations in temperature appear more clearly.*

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6. Multiple movements and their graphical representations	Generate, refine, and reason about simultaneous movements and their representation as a graph *Critically evaluate points of intersection and their meaning*	Part 4. Graphing the pursuit of the intruder *“Unfortunately the intruder escaped. Luckily the police did catch the intruder. A newspaper published an article of the intruder’s escape”* *Using the knowledge and skills gained during the first part of Lesson 6, students were asked to graph the movements of both the intruder and the police in a distance-time graph.*

In Lesson 4, the first part of the problem could be solved. Students could deduce the route of the intruder through the laboratory facility by combining the information from both graphs. This resulted in two possible solutions to the task; the intruder was in either Lab 2 or Lab 4.

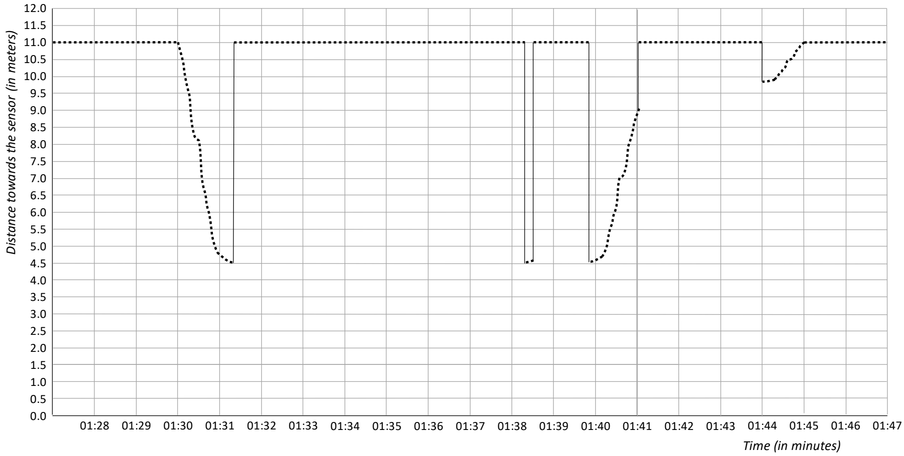

Figure 2. Graph 1 – Motion sensor 1.

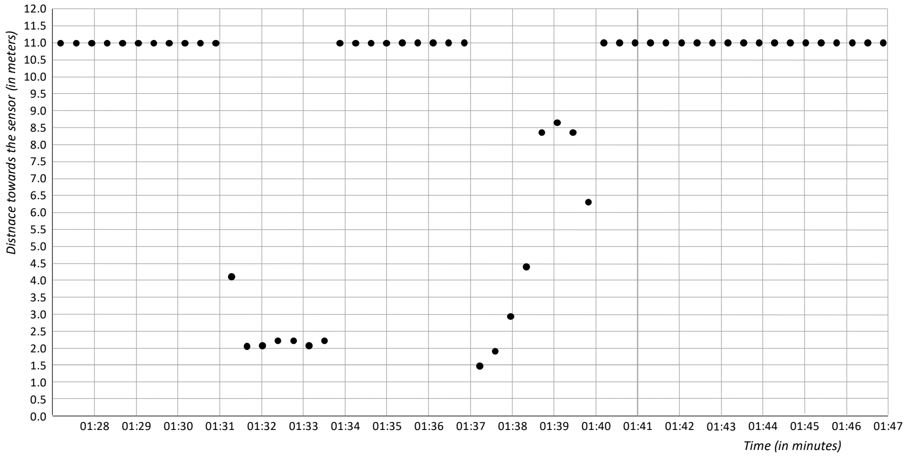

Figure 3. Graph 2 – Motion sensor 2.

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In Lesson 5, the second part of the problem could be solved. In relation to the topic addressed in this lesson, the importance of scaling of the axes was made salient. The problem-solving task showed how each laboratory and the corridor has its own temperature (Lab 1 = 10 °C, Lab 2 = 20 °C, Lab 30 = 30 °C, Lab 4 = 40 °C, corridor: 15 °C). First, students were invited to think about how these temperature graphs could possibly help in solving the problem. The students should infer that opening a door would most likely cause a major fluctuation of the temperature inside the room if the solution would be Lab 4, or to a lesser extent, if the solution would be Lab 2. Thus, in order to know in which laboratory the intruder had been, students should closely inspect the given temperature graphs (see Figures 4 till 8). Yet, the students were given graphs with a similar scale on the *y*-axis leaving not much room for seeing possible fluctuations in the temperature. Therefore, to be absolutely certain about a possible solution it is necessary to change the scaling of the axes, to receive more targeted information. The students, in small groups had to think about what information they would like to receive (change the scale on the *x*-axis, the *y*-axis, or both). After making this decision, the students received a version of the graphs they requested (see Figures 9 till 12). On the basis of the now visible fluctuations students could formulate an answer to the problem.

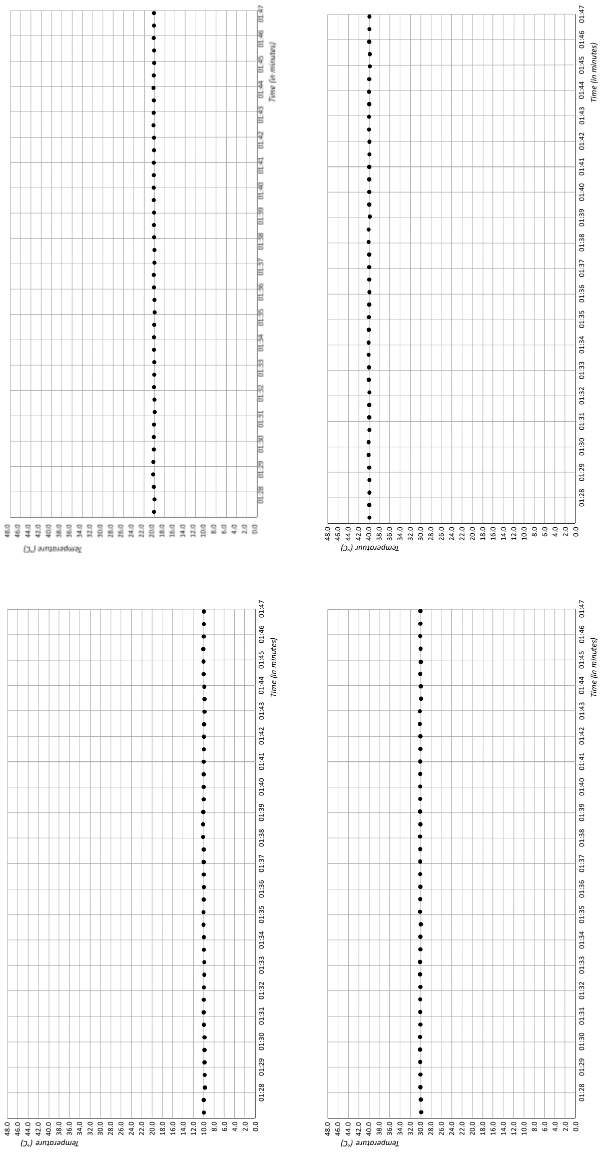

Figure 4 – 7. Temperature graphs representing the temperature in laboratory 1 till 4 (from left to right, top to bottom).

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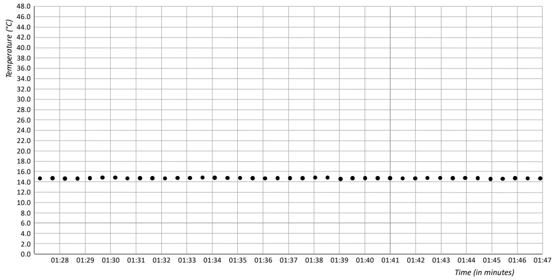

Figure 8. Temperature graph representing the temperature in the corridor.

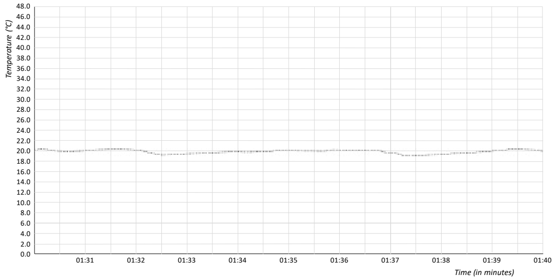

Figure 9. Temperature graph with scaling on the *x*-axes, for Lab 2.

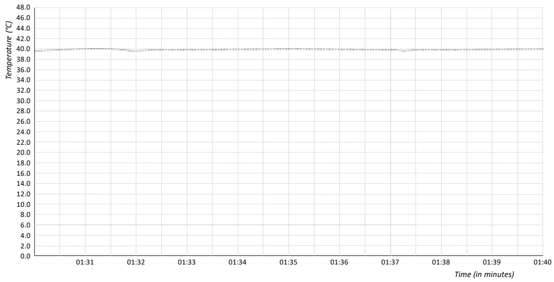

Figure 10. Temperature graph with scaling on the *x*-axes, for Lab 4.

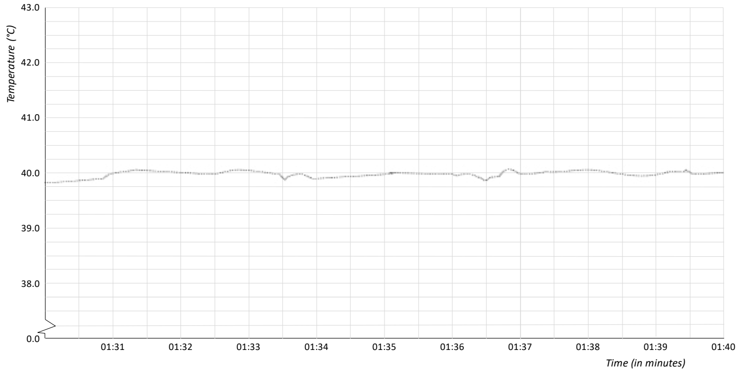

Figure 11. Temperature graph with scaling on both the *x*-axis and the *y*-axis for Lab 4.

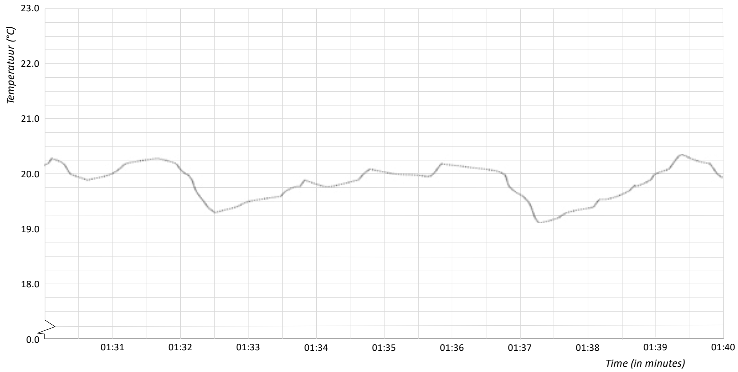

Figure 12. Temperature graph with scaling on both the *x*-axis and the *y*-axis for Lab 2.

In Lesson 6, the final lesson in which the students worked on the intruder task, the students received a written description of how the intruder escaped.

> Unfortunately the intruder escaped! Luckily the police did catch the intruder. A newspaper published a story related to the intruder's escape. Can you draw the graph based on this newspaper article?

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Based on this written description of the pursuit of the intruder, students were asked to graph the simultaneous movements of both the intruder and the police in a distance-time graph.

Appendix 3.2

Coding scheme used to indicate students' level of reasoning on the lesson-specific graph interpretation tasks with examples of student responses

Level	Description	Code	Example of student response to: Task 1	Task 3	Task 5
No reasoning					
Level 0	Students' reasoning is not related to the graphical representation or the motion event	R0	- "I have no idea" - "He walks and waits for the traffic light" - "It is a guess" - "She […] out" - "Then you can trace the height" - "Because you cannot make turns"	- "That is what I think, I do not have a reason for this one." - "Not one […] up and fort down." - "a and b are too small, or is that just me?" - "A hot air balloon is everywhere"	- "No idea" - "Just no" - "He goes up in 2 straight lines" - "Not the measurement"
Iconic reasoning					
Level 1	Student reasons on the basis of the shape of the graphical representation or superficial characteristics of the motion event	R1	- "Because an airplane takes off in a straight line" - "He is not going up" - "The stairs go up" - "In the graph it goes up, and here the plane is also going up" - "I think the graph does not contain a traffic light and it does not go up" - "The staircase has almost the same shape as the graph"	- "It does not go upwards, but straight" - "A hot air balloon can go upwards" - "It does not go diagonally" - "The line goes up and the boat stays at the same height" - "Because a hot air balloon rises, and the line does too" - "A Ferris Wheel makes circles, and does not go up in a straight line"	- "First the line goes diagonally upwards and then straight and then downwards again" - "It has a similar shape" - "This one has to go down as well" - "It"s almost identical, however, this one is higher" - "It is identical to the one above" - "He goes like this [drawing: diagonal line] and then [drawing: horizontal line] and the other goes like this [drawing: example graph] in nothing this is similar"

Single variable reasoning					
Level 2	Student reasons on the basis of a single variable (distance/time/ speed)	R2	- "An airplane usually does not stop" - "Yes, he travels a distance and then continues." - "Its appearance is correct, but the time is not" - "You see that the line above is making a stop, and the plane does not" - "You see him stop, so does the line in the graph" - "She does not stop anywhere, in the graph there is a stopping point, and here there is not"	- "The distance is increasing because the boat is moving" - "…" - "He goes around in circles, so the distance should become bigger and smaller, so bumps actually" - "The boat moves forwards and the graph as well, if the graph goes up it means you go forward." - "Because it goes upwards, and the graph represents an increase in distance" - "Daan moves both forwards and backwards, and in the graph you only see movement forward"	- "The shape is not identical, but if the kilometres are 2 times as much, then the line will be longer as well" - "He does not go until 30 kilometre from the sensor" - "It does not come closer again" - "This graph goes until the 30 kilometre, so does the one above" - "You see that it goes until 15, and the one above till 30" - "It does not take 15 minutes as it does in the graph above"
Multiple variable reasoning					
Level 3	Student reasons on the basis of multiple variables (distance/time/ speed)	R3	- "Then it will be like this" [student made a drawing of a graph] - "It depends on how long he waits" [student made a drawing of a graph] - "No, because the graph represents time and distance (it would if it represents the distance upwards!)"	- "If the sensor is at point A, he goes further away at the same speed" - "Well, he may go straight up, he also travels within a certain amount of time a certain amount of distance" - "He may go around in circles, he travels within a certain amount of time a certain amount of distance"	- "In 45 minutes you walk 30 kilometre, then stand still, and back again" - "In this graph the line in the middle does stand still for 15 minutes, but not at 30 meters" - "You only see half of it, but it is identical in meters and minutes" - "It goes until 30 kilometre and that takes 15 minutes, just like the graph above" - "This one goes 15 kilometres in 45 minutes and that you do not see in the large graph"

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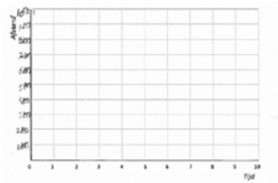
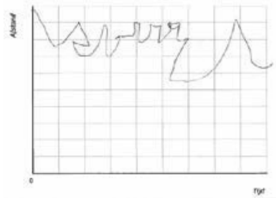
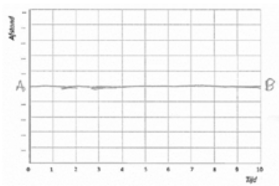
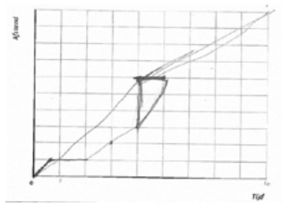
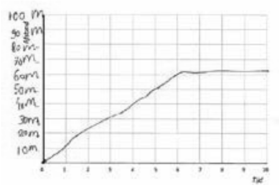
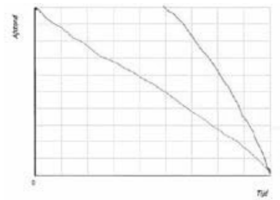
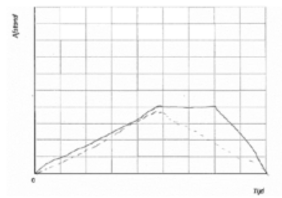
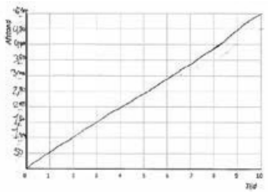
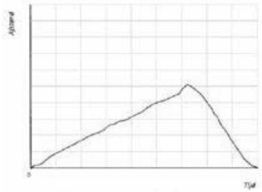
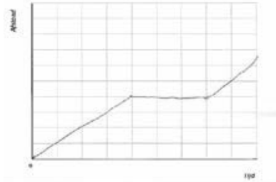
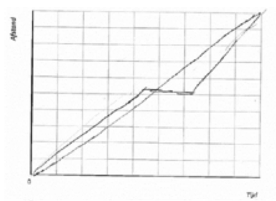
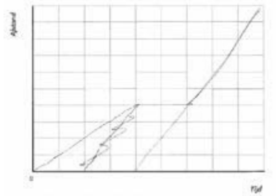
APPENDIX 3.3

Coding scheme used to indicate students' level of reasoning on the lesson-specific graph construction tasks with examples of student responses

Level	Description	Code	Example of student response to Task 2	Example of student response to Task 4	Example of student response to Task 6
No reasoning					
Level 0	Student constructs graph without taking into account the description of the motion event	R0			
Iconic reasoning					
Level 1	Student constructs graph on the basis of superficial characteristics of the description of the motion event	R1			
Single variable reasoning					
Level 2	Student constructs graph taking into consideration a single variable (distance or time or speed)	R2			

Multiple variable reasoning			
Level 3	Student constructs graph taking into consideration multiple variables (distance and/or time and/or speed)	R3	     

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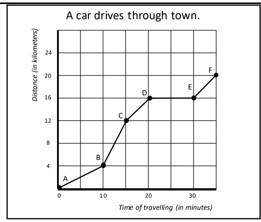
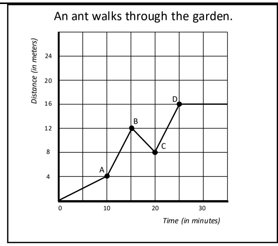
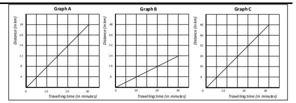
Appendices 4.1 and 4.2 belong to *Chapter 4* of this PhD thesis

Appendix 4.1

Coding scheme used to indicate students' level of reasoning on the graph interpretation tasks with examples of student responses

Level	Description	Code	Example of student response to: Task 1	Task 2	Task 4
			 1a. Between which points does the car go fastest? 1b. How do you know?	 4a. Where does the ant changes direction? 4b. How do you know?	 13a. Which of these graphs depicts the same trip? 13b. How do you know?
No reasoning Level 0	Students' reasons without referring to the graphical representation or the motion event	R0	- "I have no idea" - "There the car travels fastest" - "It is a guess" - "Watch in the table"	- "That is what I think, I do not have a reason" - "This is what I think" - "Not one […] up and fort down."	- "No idea" - "Because these are similar" - "These are exactly the same" - "Thinking about the graph" - "By looking at it closely"

Iconic reasoning					
Level 1	Student reasons on the basis of the shape of the graphical representation or superficial characteristics of the motion event	R1	- [D and E] “Because the line is flat, and then the car can accelerate most” - [E and F] “These are the highest” - [B & C] “There the line is the longest”	- [B] “The ant goes down” - [B and C] “Because the line goes down and then up again”	- [A and C] “They have exactly the same line” - [A and C] “That is exactly the same graph”
Single variable reasoning					
Level 2	Student reasons on the basis of a single variable (distance/time/ speed)	R2	- [B and C] “Because there he travels the most kilometres” - [B and C] “Between these two points there is the distance largest”	- [B] “The distance is decreasing, so the ant turns” - [B] “Because the ant comes back again” - [Nowhere] “Because when an ant walks and changes his direction, the metres only become bigger”	- [A and B] “A and B go until 24 minutes and C goes until 48 minutes” - [A and B] “They both end at 24”
Multiple variable reasoning					
Level 3	Student reasons on the basis of multiple variables (distance/time/speed)	R3	- [B and C] “Because he drives the most kilometres, in the shortest period of time” - [B and C] “The car travels in 10 minutes, 12 kilometre. That is fastest of what is represented in the graph.”	- [C] “He goes from 12 metres in 15 minutes to 8 metres in 20 minutes” - [B and C] “Because you can see 12 metres in 15 minutes and at c you have less meters in 20 minutes”	- [A and B] “Because they both walk 24 kilometre and both take 30 minutes” - [A and B] “Both 24 kilometres in 30 minutes”

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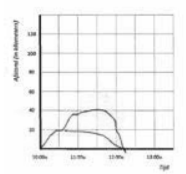
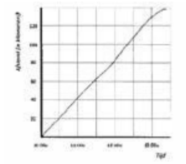
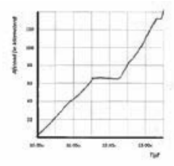
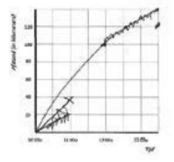
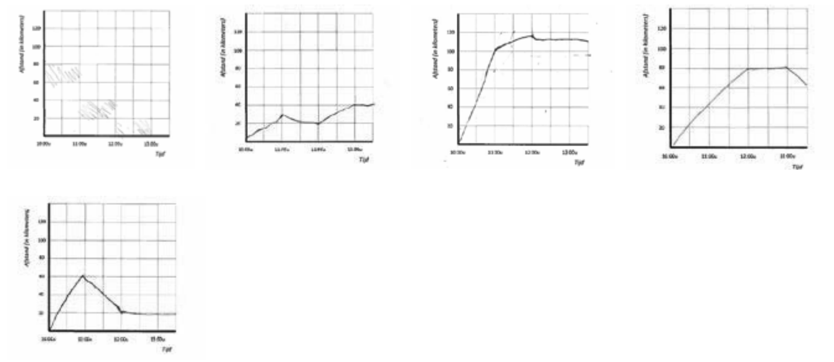
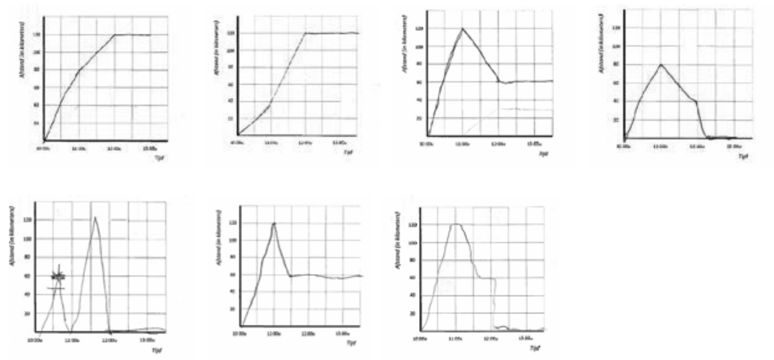
APPENDIX 4.2

Coding scheme used to indicate students' level of reasoning on the graph construction task with examples of student responses

Level	Description	Code	Example of student resonse to Task 3
No reasoning			
Level 0	Student constructs an illogical graph in which the description of the motion event is not taken into account	R0	
Iconic reasoning			
Level 1	Student constructs graph on the basis of superficial characteristics of the description of the motion event	R1	  

Single variable reasoning			
Level 2	Student constructs graph taking into consideration a single variable (distance or time or speed)	R2	
Multiple variable reasoning			
Level 3	Student constructs graph taking into consideration multiple variables (distance and/or time and/or speed)	R3	

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