Chapter 2
Mathematics in Teams—Developing Thinking Skills in Mathematics Education

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Abstract Mathematics is more than just basic skills. Mathematical thinking should be an important aspect of mathematics education. In the Netherlands, higher-order thinking skills like mathematical problem solving, reasoning, modelling and communicating mathematics have been part of the examination program since 1989. To assess these skills in an authentic and open way, the Mathematics A-lympiad, a competition for teams in upper secondary school, was designed. Shortly hereafter a Mathematics B-day was developed which showed that open-ended tasks for teams can also be designed within the domain of pure, formal mathematics. As a result of the success of the Mathematics A-lympiad, similar activities have been created for lower secondary and for primary school. The Mathematics A-lympiad assignments fulfil specific requirements, such as being accessible for all students, eliciting mathematical thinking and providing opportunity for different strategies and solutions. In the wake of these events more attention is paid to higher-order thinking skills in regular mathematics education as well.

2.1 Introduction

To survive in modern society, the emphasis of education should be on learning what to do with knowledge, rather than on what knowledge to learn—this shift is referred to as the essence of 21st century skills (Silva, 2009). It implies a focus on skills like critical thinking, problem solving, inquiry, creativity, communication and cooperation. These skills are not only related to the 21st century, however. Problem solving and mathematical thinking have been part of mathematics education in several countries around the world for decades (Törner, Schoenfeld, & Reiss, 2007).

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Since 1989, there has been a radical change in thinking about the question 'what mathematics for whom?' in upper secondary pre-university education in the Netherlands. This resulted in two different types of mathematics curricula: Mathematics A and Mathematics B. Mathematics B with calculus as core component, is suitable for students who will attend scientific/technical/mathematical (STEM) studies; Mathematics A with core topics discrete mathematics, statistics and probability and a little bit of calculus, is meant for students who prepare for academic studies in social or economic sciences. More important however than the differences in topics in these two types of mathematics curricula, were the different and new ideas that guided the design of Mathematics A.

> Mathematics A is intended for students who will have little further education in mathematics in their academic studies, but who must be able to use mathematics as an instrument to a certain extent. In particular, we have in mind those who have to prepare themselves for the fact that subjects outside the traditional sciences are more frequently being approached with the use of mathematics. This means that students must learn to be able to assess the value of a mathematically tinged presentation in their education. To do this they must become familiar with the current mathematical use of language, with formulations in formula language, and with divergent forms of mathematical representation. Furthermore, they must learn to work with mathematical models and be able to assess the relevance of these models. (HEWET report, 1980, p. 19)

The emphasis in Mathematics A is on applying mathematics and on the process of modelling and problem solving, more than on the product. This has greatly influenced the type of problems. Instead of just formal mathematical tasks, in Mathematics A real life situations are used as a context for mathematical modelling and problem solving. Some of the contexts also guide students to develop and reinvent mathematical tools and concepts. In Fig. 2.1 the context of the helix of a propeller is used to introduce the concept of the sine graph (Lange, 1982).

Since this shift in 1989, several curriculum changes have been implemented in Dutch education. As a result, a focus on mathematical thinking and reasoning is visible in all standards for mathematics in primary education (Wit, 1997) and in lower secondary education (Bos, Braber, Gademan, & Wijk, 2010) and in the examination syllabi for upper secondary schools.

This focus has been inspired by the view on the teaching and learning of mathematics in the Netherlands which was initiated in the early 1980s and has evolved as the theory of Realistic Mathematics Education (RME) (Heuvel-Panhuizen, 1998, 2000).

Although general mathematical (thinking) skills, and more broadly 21st century skills, are considered important in society and in mathematics education, it is not easy to realise these in educational practice. The teaching and assessing of problem solving, mathematical modelling, communicating and critical thinking requires other types of problems than the regular textbook problems. Furthermore, it needs a teacher facilitating the process, rather than just explaining mathematical concepts. In the following section, we will describe how these other types of problems came into being.
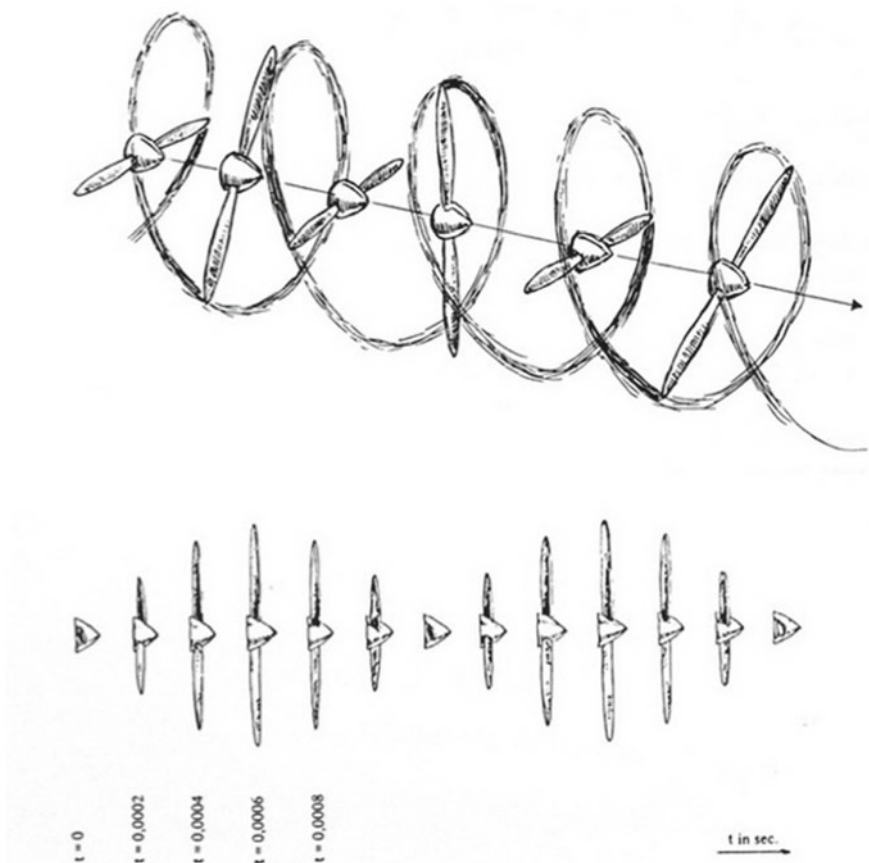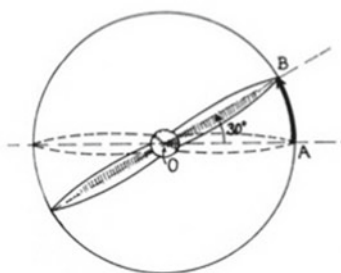
Fig. 2.1 Developing the concept of sine (Lange, 1982)

2.2 The Emergence of Mathematics in Teams to Develop Mathematical Thinking

Although RME can be seen as the leading approach on learning and teaching mathematics in the Netherlands since the 1980s (Heuvel-Panhuizen, 2000), an ever-changing balance exists—especially in assessment—between the emphasis on problem solving and using mathematical thinking skills on the one hand and reproducing basic skills (knowledge and procedures) on the other hand. Being an institute focusing on innovation, the Freudenthal Institute (FI) aims to keep mathematical thinking at the heart of mathematics education and assessment. This needs to be done within the—also changing—constraints of the central examination programs and curricula for higher secondary education and the core standards and curricula for primary and lower secondary education. In this section, we present a brief historical overview of the way in which assessment of problem solving and mathematical thinking has been put into practice in the Netherlands. We focus on secondary education, but when appropriate we discuss similar developments in primary education.

2.2.1 Secondary Education

When the Mathematics A curriculum was formally introduced in 1989, the need for changes in assessment was felt. The emphasis on higher-order mathematical skills, mathematical modelling and the use of mathematics to solve real world problems had to be reflected in the assessment of Mathematics A as well. Furthermore, cooperating and problem solving in small groups was seen as an important aspect of Mathematics A, since it would contribute to the development of communicating and mathematical reasoning. In the research on the pilot of the small-scale implementation of Mathematics A it was found that working in groups added to the quality of the process as well of the product (Lange, 1987). This is in line with later findings from research by Dekker and Elshout-Mohr (1998), which showed how working in small groups on mathematical problems stimulates mathematical level raising for each individual group member.

The written central examination did not seem the appropriate way to assess these higher-order skills. Although this examination is made up of problems in context, the questions are often closed and focussed on specific mathematical skills. Modelling and problem solving are hardly ever needed, and teamwork is not possible.

In 1989 a pilot was carried out to design a different type of open assignment for teams of students, to assess the new goals of Mathematics A. This resulted in the Mathematics A-lympiad, a mathematical real-world-problem-solving competition for teams, as a way to assess what we now call '21st century skills for mathematics' in an authentic open way. Since the first pilot in 1989 this competition, which consists of two rounds—a qualifying preliminary round in the participating schools and an international final round taking a whole weekend in a conference centre—has

been organised yearly. All assignments are designed by the Mathematics A-lympiad committee, a committee residing at the FI consisting of teachers, teacher educators, mathematicians and educational designers.

Participation has grown from 14 schools in 1989 to over 170 schools in 2007. Since then there is a slow but gradual decline resulting in about 100 participating schools in 2014, which is about 15% of all upper secondary schools in the Netherlands. At each school, an average of 40 students—10 teams of four students—participate.

In 1999 the curriculum for Mathematics B—aimed at students with ambitions to continue in STEM studies—was changed to include more modelling and applications. This was a result of a larger educational reform, in which new standards were formulated for all subjects. Higher-order thinking skills were also included in Mathematics B. The curricular changes implied that these skills should be assessed in school examinations, through big, mostly open-ended tasks or projects, of which at least one should be done by a group of three students. Because schools were already familiar with the assignments of the Mathematics A-lympiad, Mathematics B teachers asked for a similar assignment for their students, and this resulted in the Mathematics B-day. The experiences with the assignments of the Mathematics B-day showed that open-ended tasks for teams can also be designed within the domain of pure, formal mathematics.

Participation in the Mathematics B-day rose fast from 22 schools in 1999 to almost 160 in 2010. Since then we have seen the same phenomenon as for the Mathematics A-lympiad: a slow but gradual decline, resulting in 110 participating schools in 2014. The participation of schools in the Mathematics A-lympiad and in the Mathematics B-day is illustrated in Fig. 2.2.

The decline in participation in both competitions (which have no overlap in participating students) started around 2007 when a new curricular change led to more emphasis on basic algebraic skills—both in Mathematics A and Mathematics B. At the same time the explicit link between the choice for a type of mathematics (A or B) and the overall orientation on future studies was abandoned, as well as the obligation to include at least one big open-ended task in a school examination. Also, the recent renewed emphasis on mathematical thinking in the curricula, as
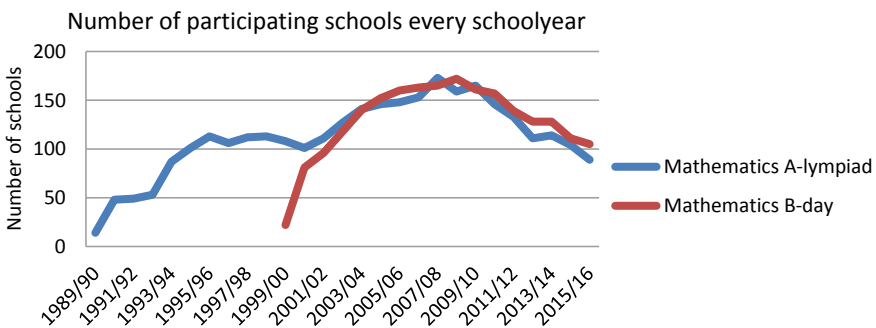
Fig. 2.2 Participation of schools in the Mathematics A-lympiad and in the Mathematics B-day

well as in the assessments for upper and lower secondary education (Commissie Toekomst Wiskundeonderwijs, cTWO, 2012) has not yet led to a higher participation rate in both competitions. However, to ensure more continuous attention for the development of students' mathematical thinking during their full secondary education, in 2012 the FI started to design an activity similar to the Mathematics A-lympiad and the Mathematics B-day for lower secondary education (Grade 9): the Lower-Secondary-Mathematics-Day.

2.2.2 Primary Education

For a long time, RME has had a significant influence on mathematics curriculum standards as well as on the textbooks in primary education. However, despite this, the focus in the commercial textbooks is not on mathematical thinking and reasoning (Kolovou, Heuvel-Panhuizen, & Bakker, 2009). To counter this approach and present primary school teachers and students with a different and broader view on mathematics, in 2003 the Grote Rekendag[1] for primary education was initiated. This is a full day with thematic mathematical activities for students in all grades in primary school (for students aged 4–12). The open activities are mostly performed in small groups and ask for inquiry and creativity by students and focus on mathematical thinking, modelling and communicating. In this respect, the activities are comparable with those described for secondary education. For primary education however, the Grote Rekendag is not a competition and instead of one large open assignment it is made up of a number of smaller activities connected by the theme.

2.3 Characteristics of the Mathematics A-lympiad and the Mathematics B-day Assignments

In the previous section, we described how various open-ended assignments to assess mathematical thinking and problem solving came into being. In this section, we will focus on the characteristics of these assignments and the specific requirements they need to fulfil in order to do what they are meant to do: elicit students to think mathematically, to creatively solve open-ended unfamiliar problems, to model, structure and represent problems and solutions, to work collaboratively and to communicate about mathematics.[2]

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[1]Big Mathematics Day.

[2]References to similar characteristics can also be found in Dan Meyer's 'Three acts' problem (http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/) and in Lange (1987).

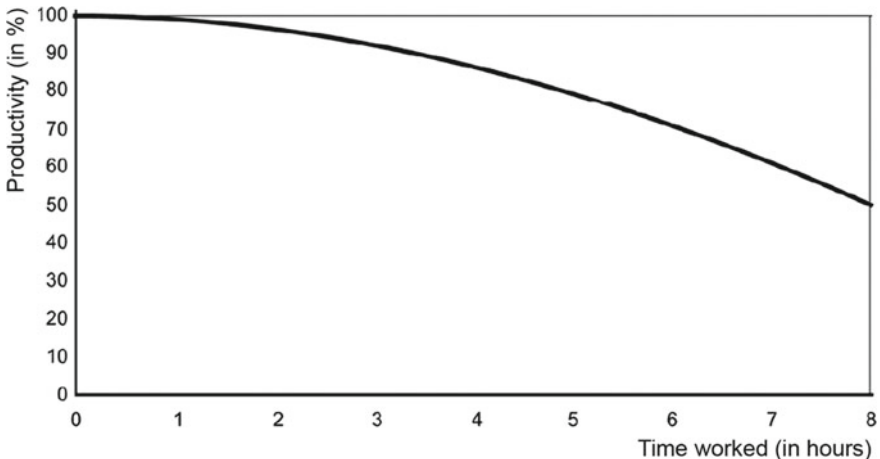### 2.3.1   Example from the Mathematics A-lympiad: 'Working with Breaks'

An example of a Mathematics A-lympiad task is 'Working with breaks'.[3] The complete task is based on one graph only (see Fig. 2.3).

This graph, from a German study, relates the productivity of workers in a factory to the hours they work without a break. Furthermore, in this model there are some rules of thumb relating productivity to the length of the break:

– After a break within the first five hours of working (that is non-stop working) productivity will be back at the level that the productivity was '3.5 times the length of the break' before the start of the break.
– After a break that is taken after more than five working hours the productivity will be back at the level that the productivity was '3 times the length of the break' before the start of the break.

The main question that students have to answer for the board of directors of the company is: how to get 'maximum productivity' in the factory by scheduling breaks in the most effective way. To make calculations easier, the so-called 'work production-units' (wpu) per hour (per worker) are introduced in the assignment with 600 wpu being the maximum productivity.

In the first part of the assignment, students are asked to use the graph and the two rules of thumb to estimate the productivity for one day, in two conditions: without a break, and with one break. In the middle part of the assignment, a linear approximation of the graph is introduced, and students are asked to investigate a

Fig. 2.3 Productivity graph

few different models (working with one break, with more breaks) and extend their calculations from one day to one week. They also have to work within restraints: the company production must reach a certain (minimum) number of wpu per week, and the workers want to have as much free time as possible. The final part of the assignment asks for at least two well-founded proposals for a daily schedule for the workers. The workers' council and the board of directors together must be able to choose between these proposals, while taking into account:

– The interest of both employer and employee (worker)
– Health and safety rules
– The minimum of 19200 wpu per week.

The health and safety rules are a new, authentic, component in the task, at this stage. Of course, all consequences, choices and assumptions, must be described and justified by the teams in their proposals.

Students work on the assignments in teams for one full day and produce a report. This report is first judged (and sometimes graded as well) by their own teacher. Then, the best ones are judged by a teacher from a different school or by the jury of members of the committee. This evaluation results in a winning team.

As said before, the assignments need to be designed in such a way that they elicit problem solving and mathematical thinking. An important characteristic of the assignments is that they are new to students, which means that the problems are non-routine and non-trivial (Doorman et al., 2007). Schoenfeld (2007) states that these types of problems are needed for problem solving to happen. The absence of a known procedure forces students to come up with new strategies, that need to be tested, compared and evaluated. Other requirements of the assignments are that:

– They should be rich, meaning that there is not only one way to come to a solution, and solutions can vary in mathematical depth
– They should build on knowledge students already have, and extend it
– They should use higher-order questions (how? why?) and encourage reasoning rather than 'answer getting' (Swan, 2005).

Besides general characteristics that elicit problem solving and mathematical thinking, it is important that the assignments are suitable for a competition. An important condition for a competition is that the teacher has a minimal role. He or she facilitates the organisation and the process, but provides no content-related guidance. This asks for a well-structured, but open assignment. All teams must be able to enter the problem without help from a teacher, and on the other hand—in order to determine a winner—the problem has to allow for different approaches and strategies based on decisions by the students and for solutions that differ in quality. Swan (2005), when describing characteristics of rich collaborative tasks, speaks of tasks being 'accessible and extendable'.

The accessibility and extendibility of the assignments for the Mathematics A-lympiad, which are situated in a real-life context, is realised by a more or less fixed structure (Haan & Wijers, 2000):

– The first part is an introduction with some smaller, less open problems to get to understand the context. This ensures that the assignment is accessible for all students.
– The middle part often asks for an analysis of data, of a model, or of a solution that is presented in the assignment.
– The final part asks for creativity in designing, comparing and evaluating a new approach, system, model, solution or product.

The example 'Working with breaks', discussed in this section, illustrates how this structure is concretised in the assignment.

2.3.2 Example from the Mathematics B-day: 'How to Crash a Dot?'

An example of a Mathematics B-day task is 'How to crash a dot?'[4] The assignment is based on one of the first computer games in the 1970s.

The route of the dot (see Fig. 2.4) is determined by using buttons that make the dot move in a certain direction (N = north, S = south, E = east, W = west) with a certain increasing speed. Furthermore, there is a button (P = pass), which means that the same direction and speed is kept. For example, when E is used the first time, the dot moves one unit to the east. When you use the button P the next time, the dot

Fig. 2.4 Route of a dot

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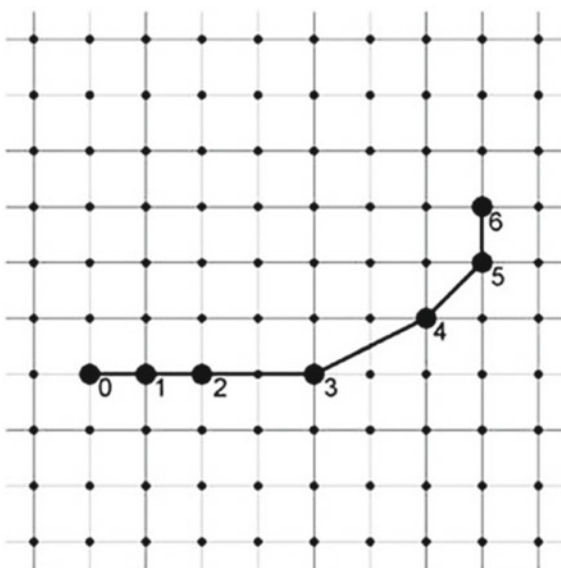
[4]The complete task from schoolyear 2009–2010 can be found at http://www.fisme.science.uu.nl/toepassingen/28174/.

moves in the same direction with the same speed. When you use E again however, the dot moves two units two the east.

This example makes clear that the assignments for the Mathematics B-day are situated within the mathematical world itself. Often new mathematical content is addressed, for which a longer and more guided introduction is needed. Therefore, in the first part of this assignment, the rules of the game that let students move a dot along grid points are formulated and students learn how to use the rules. In the middle part of the assignment students first explore movements in one dimension. They investigate for example how to use the rules to make a dot move along a straight horizontal line. Next, the students study movements in both horizontal and vertical directions. They make use of the results found when exploring movements in one direction. In the last part three different final questions are formulated, letting students make a choice between doing all three with the risk that they can only report superficially on them, or making a wise choice and report fully, carefully and in-depth. This final part asks for mathematical creativity. Here extendibility is realised when some of the teams go further and deeper in designing and investigating their mathematical ideas and hypotheses.

2.4   The Role of the Teacher

The assignments discussed so far are meant mainly for assessment and not primarily for learning. They are not part of the regular mathematics classes. During the competition day, the teacher has a very small role. He or she facilitates the process and keeps the teams going, but has no role in providing help (Dekker & Elshout-Mohr, 1998; Haan & Wijers, 2000). One could argue, as Kirschner, Sweller, and Clark (2006) do, that this minimal guidance does not work, but in this case, the aim is not instruction and the teacher can still give process help.

To prepare teachers for using and grading these open, non-routine large assignments, a workshop is offered each year to all teachers who have students participating in one of the competitions. In this three-hour workshop, teachers get to know part of the assignment that will be used later that year. They can work on it in teams themselves and discuss with colleagues their experiences, findings and the problems they foresee. Members of the committee can use the comments to improve the assignments. An important topic in this workshop is how to evaluate and grade student work. Experienced teachers share their tips and tricks with teachers who are new to the competition. The workshop proved to be useful for both novice and experienced teachers: it is a way of preparing for process-guidance during the competition, and it is a way to get a grip on the core content of the assignments. A teacher said once:

> When I don't know the assignment very well, I tend to 'help' students in giving answers to their questions; attending the workshop helps me in getting a grip on the assignment, so when a student now asks me about the content, I know what guidance question I can ask to help them.

Prior to the competition the teacher has an important role in preparing students for this type of assignments. The preparation can be done in different ways. One way is having students practise with old assignments from previous competitions. Although all assignments are available on the web, this approach is seldom used. The principal objection is that it takes a lot of time at the expense of the time available for teaching. To finish one full assignment takes about one whole day, which is equivalent to about five lessons.

Often teachers give a form of preparation (Dijk, 2014) in which they present organisational information on how to deal with this type of assignments as a team. They often have students read one assignment as an example and work on it for half an hour and then discuss ways of working and the product requirements that are listed in an addendum to the assignment. Although the assignments are quite different every year, the criteria for assessing the quality of the reports, and more specific the higher-order general mathematical skills, are more or less constant–apart, of course, from the specific mathematical content. The reports are graded based on aspects such as:

– Quality of argumentation and justification of choices being made
– Use of mathematics
– The (mathematical) creativity in strategies and solutions
– Quality and extensiveness of (mathematical) reasoning and modelling
– The presentation: including form, readability, clarity, completeness, structure, use and function of appendices.

Teachers can also prepare their students for this type of open assignments in their regular mathematics classes. They can do so by creating a classroom culture in which students are used to listening to each other, asking each other questions and writing down their own thinking before they share it. Teachers who do so also help students by orchestrating their thinking (Drijvers, 2015) and evoking mathematical discussions in their regular mathematics classes.

2.5 The Student Perspective

For students in upper secondary the assignment in the competition is often their first experience with this type of large open problems for teams. Textbooks rarely offer this type of problems, and if open problems are included in the textbooks they mostly refer to the core content of the lesson or the chapter at hand, which means that part of the strategy is known or obvious. In this case less mathematical thinking is needed and there is no need for creativity and 'real' problem solving in the sense of Schoenfeld (2007).

To illustrate the experiences of the students, some quotes from students, taken from several reports from different assignments, are presented in Fig. 2.5. These quotes show that the students discover the fun of doing mathematics, they are allowed to do their own investigations, and sometimes they surpass themselves and exceed their teachers' expectations!

> "*This was a special day. We learned things and it was fun. We were free to plan the work ourselves. In the introductory task, we tried to explain the methods that were presented. After 'hard thinking' we understood what was going on. In the beginning we were frustrated, but after we found out 'how it worked', it became much more fun.*"
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> "*In the introductory tasks, we were confronted with mathematics we didn't fully understand. We kept to our initial problem-solving strategies throughout the tasks and we believe this led to a very good outcome. By struggling through the introductory tasks, we got more and more familiar with the context and the mathematics. In the end, we had enough knowledge to complete the final part of the assignment.*"
>
> "*At a certain moment, we understood how everything worked out and from that moment on we 'raced' through the tasks. Because we had divided the work efficiently we could finish the tasks fast and at the same time keep up the fun.* [..] *There was a relaxed atmosphere and we were better at math than we thought and that is worth something as well.*"

Fig. 2.5 Quotes from students

As discussed in the previous section, teachers can prepare their students in several ways. A small-scale study on a comparison of schools participating in the Mathematics A-lympiad (Dijk, 2014) showed that students of teachers who put more effort in creating a investigative classroom culture in which mathematical thinking and creativity are stimulated and who prepare students by introducing them to the ideas and goals of the competition are more often among the winning teams of the preliminary round in the Mathematics A-lympiad. In this case the regular classroom teaching lays a foundation for the students' higher-order thinking skills and thus for their successes in the competitions.

The influence can also work the other way around. Experiences with these competitions can prompt changes in teachers' regular teaching. For example, we noticed that students who participate in the competition of the Lower-Secondary-Mathematics-Day (Grade 9), often struggle with the openness of the task. Although the results do show creativity and mathematical thinking, it is clear that a lot of the students lack a structured approach of formulating hypotheses and systematically investigating these by varying the variables, constraints, representations, models or other aspects. For their teachers, this may be a reason to start paying more attention to how to handle unstructured problems and to stimulate modelling and investigations by their students. In this respect, the competitions function as an entrance into a more inquiry-based way of teaching mathematics.

2.6 The Future of Mathematical Thinking in Secondary Mathematics Education

As a consequence of the recent curriculum change that was fully implemented in 2017, mathematical thinking activities were embedded in the standards for mathematics in upper secondary (cTWO, 2012). Never before have these higher-order thinking skills been described in the standards in such detail. They are characterised in connection to the content domains and include:

– Modelling and algebraisation
– Ordering and structuring
– Analytical thinking and problem solving
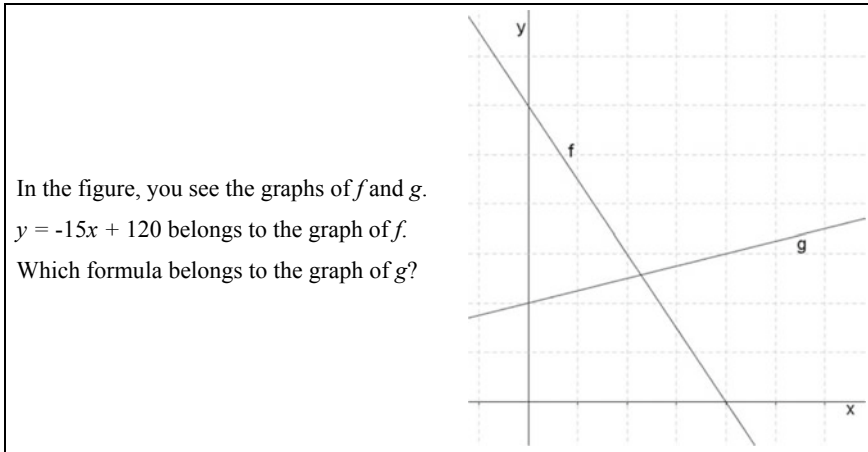– Manipulating formulas
– Abstracting
– Reasoning and proving.

Although the assignments of the mathematical competitions stimulate mathematical thinking, they do not reach all students and teachers. It is not always possible in a school to dedicate a full day to mathematics, in which the content may even be outside the core curriculum. Furthermore, to really implement mathematical thinking for all students and help them develop the appropriate skills this should be part of the regular curriculum, which means that suitable assignments and problems are needed that fit within regular 50-min mathematics lessons. To realise this, two movements are currently ongoing: textbooks authors start inserting so called 'mathematical thinking problems' in their textbooks, but since it takes time until a new generation of textbooks is published, mathematics teachers themselves also design problems, often as a result of professional development on this topic. These problems are often smaller, open problems, that are non-routine and evoke students' mathematical thinking, reasoning and creativity and that help students and teachers to make the shift towards 'relational understanding' of mathematics, instead of keeping the focus on the (more common) 'instrumental understanding' (Skemp, 1976). An example of such a problem, designed by a teacher,[5] is presented in Fig. 2.6.

Usually in the textbooks the scale of the axes is given, and students have to come up with the formula using this information. In this problem, students have to show understanding of the concept of the linear formula, in order to find out the scaling of the axes.

All in all, we may conclude that the time seems right for a shift in mathematics education towards a more inquiry-based 21st century fitting approach. Not all requirements are met, of course, but the necessary conditions seem to be established: standards, examinations, textbooks and teachers are being prepared for such an approach. Professional development of teachers is organised, in which mathematical thinking activities are designed and implemented by teachers in their classrooms. In these courses, they learn how to implement a classroom culture in which they

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[5]Egbert Jan Jonker, mathematics teacher at Roelof van Echten College in Hoogeveen, the Netherlands.

In the figure, you see the graphs of *f* and *g*.

$y = -15x + 120$ belongs to the graph of *f*.

Which formula belongs to the graph of *g*?

Fig. 2.6 Problem that stimulates mathematical thinking

stimulate mathematical thinking and problem solving. Research into mathematical thinking is carried out and is disseminated in journals and in research-meets-practice conferences. Especially dissemination of research through databases with assignments and guidelines for teachers (in text and through videos) are used to increase the incorporation of mathematical thinking activities in the classroom.

The assignments for teams discussed in this chapter will cause the stream of development of mathematical thinking to continue flowing. We hope this stream will grow, supported by classroom environments with the right tasks and the appropriate teacher and student attitudes.

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