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The aim of this study was to investigate the effect of a six-lesson teaching intervention on fifth-grade students' linear equation solving abilities. A hanging mobile, a balance model consisting of a horizontal beam with on each side a number of bags hanging on a chain, played a central role in this intervention. In total, 213 fifth-graders participated in one of the two intervention conditions or in the control condition. The intervention conditions differed with respect to the type of hanging mobile; either a physical hanging mobile that students could manipulate or a static version on paper. Preliminary analyses of the scores on the pre- and post-test seemed to show an improvement in students' linear equation solving abilities for students in both intervention conditions. Students, who worked in an embodied learning environment with a physical hanging mobile, seemed to show more improvement than students who worked with a paper-based mobile.

Keywords: Early algebra, linear equation solving, balance model, physical experiences, embodied cognition theory.

Introduction

The importance of laying the foundation for learning algebra at a young age is increasingly being emphasized. (e.g., Kaput, Carraher, & Blanton, 2008; National Council of Teachers of Mathematics [NCTM], 2000). A large number of studies have provided evidence that, by taking students' natural, intuitive ideas and informal reasoning as an entry point, students of primary school age can already engage in algebraic thinking (e.g., Kaput et al., 2008). Research in this area of *early algebra* has revealed that several algebraic concepts such as equivalence, expressions, equations, inequalities, generalized arithmetic, functional thinking, variable, and proportional reasoning can be taught to young students (Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015). In the current study, we investigated how linear equation solving of fifth-graders can be fostered.

Teaching linear equation solving

Studies on early algebraic thinking often investigated students' linear equation solving ability. These studies generally found positive results. It was for example found that third-grade students can successfully deal with equation-related concepts such as equality and the equal sign, crucial for learning to solve linear equations (e.g., Bush & Karp, 2013), can solve simple equations such as $3 \times n + 2 = 8$, and can use letters to represent unknown quantities (Blanton et al., 2015). Moreover, when starting from a meaningful context, 10-year old students were found to be able to represent, meaningfully discuss, and solve equations with unknowns on both sides of the equal sign (Brizuela & Schliemann, 2004), and, through informal reasoning, sixth- and seventh-grade students have even shown themselves able to solve systems of linear equations (Van Amerom, 2003; Van Reeuwijk, 1995). However, there are only a limited number of studies in which young students had to solve systems of equations and those that were carried out were not set up systematically and contained small samples of students.

A frequently used model to make linear equation solving more accessible to young students is the balance model. This model can support students in different ways when solving linear equations and is particularly deemed suitable for grounding the equality aspect of an equation, as such enhancing the conception of the equal sign as a symbol for representing equality (Otten, Van den Heuvel-Panhuizen, & Veldhuis, 2019). The balance model also can assist students in providing a language base for solving problems, as was shown by Warren and Cooper (2005) in a study involving third-grade students. Furthermore, by means of providing a meaningful context, it can help students to handle problems with unknowns (Gavin & Sheffield, 2015).

Role of physical experiences in learning mathematics

Considering the role of physical experiences for learning has a long tradition. For example, Piaget (Piaget & Inhelder, 1967) already assumed that actions form the basis for learning and are important for understanding abstract ideas. A recent study on the efficacy of teaching mathematics through the use of manipulatives, showed benefits for instruction with manipulatives especially for children aged 7-11 and for the mathematical domain of algebra, compared to children that received abstract symbolic instruction (Carbonneau, Marley, & Selig, 2013). Using manipulatives has also been found to be beneficial for young students to learn to solve simple symbolically presented equations (Sherman & Bisanz, 2009).

Since the emergence of theories of embodied cognition, the attention for physical experiences in learning has been renewed (De Koning & Tabbers, 2011). According to embodied cognition theories, bodily experiences are essential for cognitive learning processes (e.g., Wilson, 2002) and abstract higher-order cognitive processes, such as mathematics, are assumed to be grounded in action and perception (Barsalou, 1999). Hence, embodied learning environments are regarded as important for learning mathematics (Abrahamson & Lindgren, 2014).

Current study

In the current study, we investigated the effects of an intervention, consisting of a six-lesson teaching sequence, on fifth-grade students' linear equation solving abilities. The intervention was based on the idea that embodied learning environments can be beneficial for learning mathematics. A hanging mobile, a balance model consisting of a horizontal beam with on each side a number of bags hanging on a chain, played a central role in this intervention. In this study, we formulated the following research questions: *(1) What is the effect of an intervention based on a balance model on* *students' linear equation solving performance?* and (2) *What is the difference between the effect of an intervention based on a balance model with a manipulable physical hanging mobile from that with a static hanging mobile on paper on students' linear equation solving performance?*

Method

Research design

To investigate these questions, we set up a classroom experiment based on a staged comparison design consisting of two intervention conditions and a control condition. In the first intervention condition, a physical hanging mobile was used that students could manipulate. In the second intervention condition, a paper-based hanging mobile was used. For each condition there were three cohorts, which differed in the timing of the intervention. In each cohort the students' performance on linear equation solving was measured four times (see Table 1). Each measurement contained the same test items. Making use of a staged comparison design made it possible that the same teacher taught all the lessons of the interventions.

Condition	Cohort (*n*)	Measurement 1 Oct. 2016	Nov.-Dec. 2016	Measurement 2 Dec. 2016	Feb.-March 2017	Measurement 3 March 2017	May-June 2017	Measurement 4 June 2017
Intervention Condition 1	Cohort 1 (*n*=22)	**M1**	Intervention	**M2**		M3		M4
	Cohort 2 (*n*=18)	M1		**M2**	Intervention	**M3**		M4
	Cohort 3 (*n*=25)	M1		M2		**M3**	Intervention	**M4**
Intervention Condition 2	Cohort 1 (*n*=22)	**M1**	Intervention	**M2**		M3		M4
	Cohort 2 (*n*=21)	M1		**M2**	Intervention	**M3**		M4
	Cohort 3 (*n*=25)	M1		M2		**M3**	Intervention	**M4**
Control Condition	Cohort 1 (*n*=25)	**M1**	Control intervention	**M2**		M3		M4
	Cohort 2 (*n*=30)	M1		**M2**	Control intervention	**M3**		M4
	Cohort 3 (*n*=25)	M1		M2		**M3**	Control intervention	**M4**

Table 1: Research design. For the current study we focus on the measures in bold

Participants

Participants included 213 students (47% boys), with ages ranging from 9 to 11 ($M = 10.04$, $SD = 0.49$) from nine fifth-grade classes in seven schools in the Netherlands. The classes were selected by convenience. Three classes participated in Intervention Condition 1 ($n = 65$), three classes in Intervention Condition 2 ($n = 68$), and the final three classes formed the Control Condition ($n = 80$). Students had received no prior instruction on equation solving or other algebra topics.

Intervention program

In both intervention conditions students were taught linear equation solving by means of the same four-episode (six lessons) teaching sequence (see Figure 1). The lessons were taught by the first author of this paper. The overall aim of this sequence was to elicit algebraic reasoning related to (informal) linear equation solving. In each of the episodes the aim was to provide students with opportunities to develop algebraic strategies related to linear equation solving, such as restructuring strategies (e.g., change the order of terms in an expression, exchange the expressions on both sides of the equation), isolation strategies (i.e., strategies to isolate an unknown, such as taking away similar things on both sides), and substitution strategies (i.e., replacing unknowns by other unknowns or by values). In the first episode, students worked with one hanging mobile to discover relationships between unknowns and were thus reasoning about one equation. Students' main task was to discover all possible ways to maintain the balance of the mobile, for example by exchanging the bags of the left and right side of the mobile, by taking away similar bags from both sides, or by substituting one color of bags by another color. From Episode 2 on, the information from two equations had to be combined to discover relationships between unknowns to solve the problems. Students were thus reasoning about a system of equations, first still in the context of the hanging mobile (Episode 2) and then in new contexts such as a tug-of-war situation (Episode 3). From Episode 3 on, students were additionally challenged to use more symbolic notations. In the final episode, Episode 4, students had to find the values of unknowns in a system of two symbolically notated linear equations.

The two intervention conditions differed as regards whether students gained physical experiences during the teaching sequence (Figure 1). In the first episode of Intervention Condition 1, an embodied learning environment was created in which students worked in small groups (2-3 students) with a physical hanging mobile. While trying to maintain the balance of the mobile, the tilting beam could be in or out of balance, thus providing students real-time feedback on their actions while manipulating the bags. In Episode 2, students in Intervention Condition 1 worked with paper-based hanging mobiles. The physical hanging mobiles were, however, still present in front of the classroom in this episode, but not all students worked on them. Instead, the physical hanging mobiles were used during classroom discussions by the teacher or by some students to make their reasoning processes explicit. Also, in Episodes 3 and 4 the physical hanging mobile was present in front of the classroom.

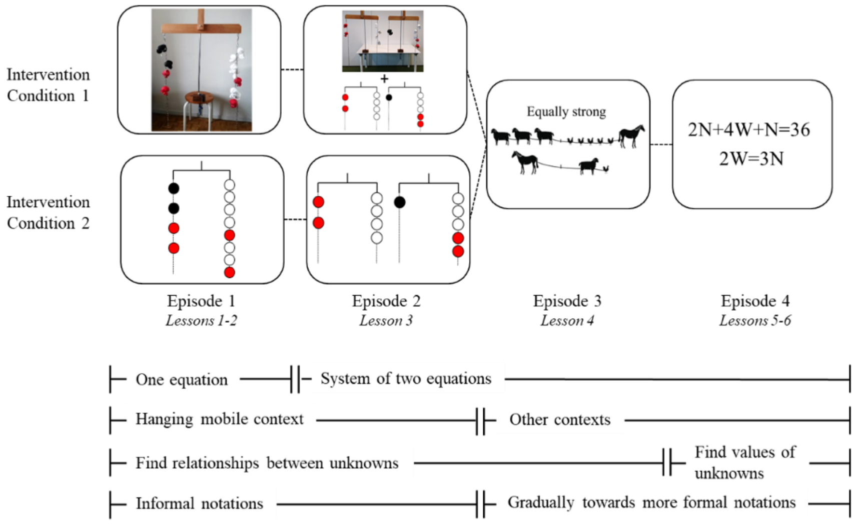

Figure 1: Schematic representation of the intervention (for both intervention conditions)

In Intervention Condition 2, students received the exact same lessons with the exact same assignments; only the physical hanging mobile was replaced by a paper-based hanging mobile so that these students did not gain physical experiences during the lessons. Lastly, students in the Control Condition were not taught any lessons on linear equation solving. Instead they participated in a control intervention consisting of a six-lesson teaching on probability – a topic which is also not taught at primary schools in the Netherlands. Students were taught the probability lessons as a control group, so that possible differences between the intervention and control conditions could not be attributed to the fact that only in the intervention conditions students received additional lessons on a (to them) new mathematical topic.

Test for linear equation solving

Students' performance on linear equation solving was assessed by a paper-and-pencil test, consisting of four items in which students had to solve (a system of) linear equations (see Figure 2). The same test was administered to the students repeatedly before and after the intervention (see Table 1, Measurement 1-4). The items were formulated in such a way that prior instruction on linear equation solving was not essential to solve the problems. The algebraic strategies that students developed during the lessons (i.e., restructuring, isolation, and substitution strategies) could be used to solve the problems. The open-ended questions explicitly invited students to explain their thinking and to reveal their reasoning.

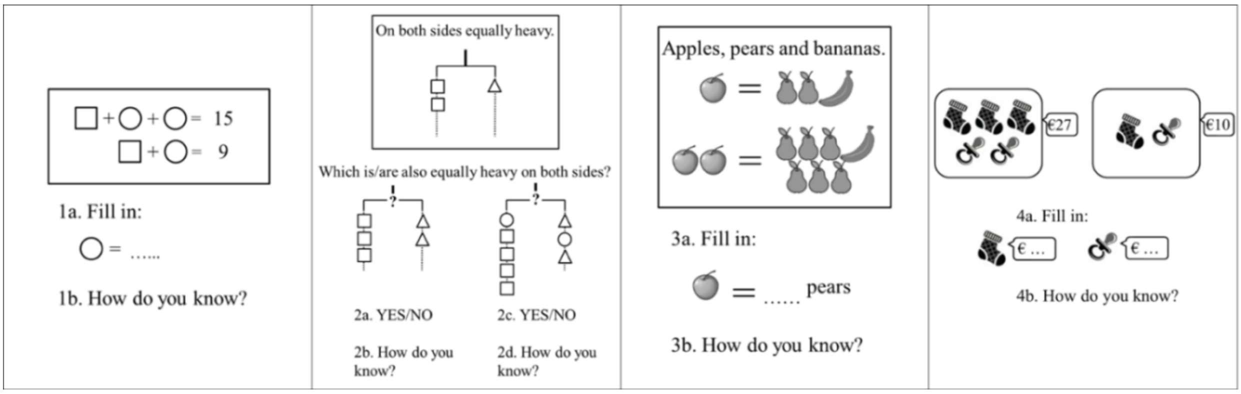

Figure 2: Test items on linear equation solving

Data analysis

Each linear equation solving performance item (Figure 2) was scored as incorrect (0) or correct (1), resulting in a total correctness score with scores ranging from 0 to 4. Items 2 and 4 were only scored as correct when both sub questions were answered correctly. Students' explanations were categorized by means of a coding scheme. In each test item, students had to solve the problem by combining the information of two equations. Taking the information from both equations into account and reasoning on the basis of both equations was crucial for solving these problems. In the coding scheme, we therefore coded students' level of reasoning based on their ability to incorporate information from the different equations in their reasoning process. More specifically, we distinguished between students who did not use any of the equations in the description of their reasoning (Level R0), students who reasoned on the basis of only one of the two given equations

(Level R1), and students who reasoned on the basis of both given equations by combining the information of both of them (Level R2).

For the current paper, we focus on the data of the tests directly before and after the interventions (see Table 1, measures in bold), consisting of students' correctness and reasoning scores. Mean values and standard deviations were calculated for both scores on the pre- and post-test. Effect sizes (Cohen, 1988) were also calculated for each condition.

First results

For both intervention conditions, students' correctness scores increased. For Intervention Condition 1, the correctness scores increased from $M = 2.31$ ($SD = 1.17$) on the pre-test to $M = 3.22$ ($SD = 0.91$) on the post-test ($d = 0.87$) For Intervention Condition 2, the correctness scores increased from $M = 2.43$ ($SD = 1.21$) on the pre-test to $M = 3.15$ ($SD = 0.93$) on the post-test ($d = 0.67$). In contrast, students of the Control Condition almost showed no improvement on correctness scores, going from $M = 2.65$ ($SD = 1.26$) on the pre-test to $M = 2.72$ ($SD = 1.33$) on the post-test ($d = 0.05$). Students' reasoning also improved for both intervention conditions (see Figure 3). When comparing pre- and post-tests, a decrease in percentage of the lowest level of reasoning (Level R0) was observable, while there was an increase in the highest level of reasoning (Level R2). The percentages of the intermediate level of reasoning (Level R1) remained more or less stable.

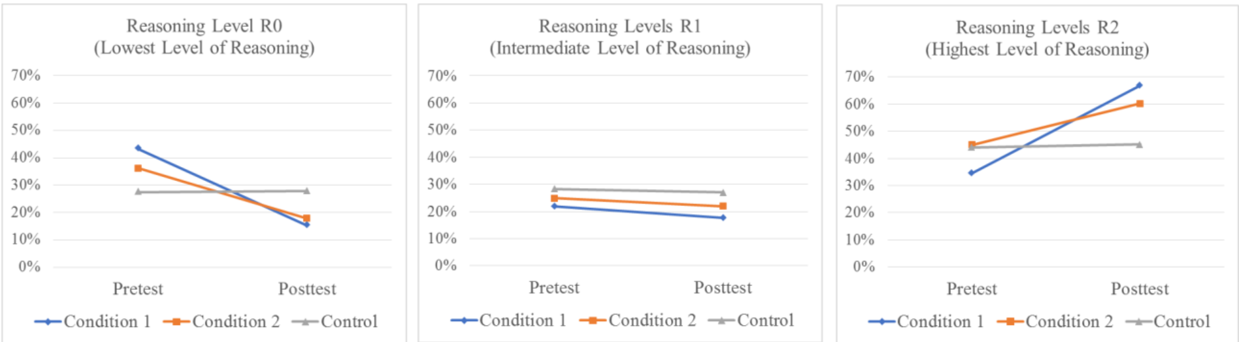

Figure 3: Percentage of students showing each level of reasoning on the pre- and post-test

When comparing both intervention conditions, students in Intervention Condition 1 showed a somewhat larger improvement than students in Intervention Condition 2, both on correctness scores and level of reasoning. For both intervention conditions, there was a decrease in percentage of the lowest level of reasoning (R0) and an increase in the highest level of reasoning (Levels R2), with a somewhat larger decrease of Level R0 and a somewhat larger increase of Level R2 for Intervention Condition 1. The percentages of the intermediate level of reasoning (Levels R1) remained more or less the same for both conditions. For the Control Condition, the percentages of all levels of reasoning (almost) did not change.

To provide an example of an individual learning process of one of the students, we zoom in on the on the pre- and post-test answers to Item 4 of Omar, who participated in Intervention Condition 1. On the pre-test, Omar gave an incorrect answer to this item, with the explanation "I don't know". This response was categorized as Level R0, because he did not use any of the given equations in his explanation. His answer on the post-test is shown in Figure 4. Here, Omar interestingly represented the equations as hanging mobiles. Then he doubled the second equation (S + P = 10) to substitute this in the first equation (S + S + S + P + P = 27), so that unknown S was isolated. In this way he found that S equals 7 and then used the second equation to find the value of P. Omar's solution strategy was categorized as Level R2, because he reasoned on the basis of both given equations by combining the information from both equations.

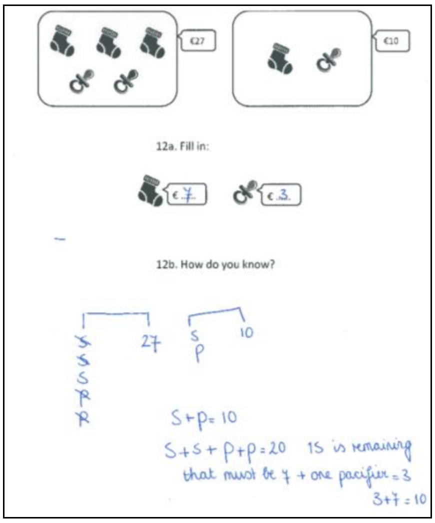

Figure 4: Solution strategy of Omar (translated from Dutch) on the post-test; categorized as Level R2

Discussion

Based on these first results, we can draw the tentative conclusion that our six-lesson intervention in which the balance model in the form of a hanging mobile plays a central role, can improve students' linear equation solving abilities. Moreover, our descriptive data indicate that when this intervention took place in an embodied learning environment in which the students could work with a physical hanging mobile instead of with a version on paper, the performance gain was even larger. This latter finding is in line with the idea that embodied learning environments are beneficial for learning mathematics (e.g., Abrahamson & Lindgren, 2014), and it could add to the use of these environments in whole classroom settings. To be more certain about what we can learn from our study more advanced statistical analyses will be carried out.

References

Abrahamson, D., & Lindgren, R. (2014). Embodiment and embodied design. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (2nd Edition) (pp. 358-376). Cambridge, UK: Cambridge University Press.

Barsalou, L. W. (1999). Perceptions of perceptual symbols. *Behavioral and Brain Sciences*, *22*, 637-660. doi:10.1017/S0140525X99532147

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J. S. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education*, *46*, 39-87. doi:10.5951/jresematheduc.46.1.0039

Brizuela, B., & Schliemann, A. (2004). Ten-year-old students solving linear equations. *For the Learning of Mathematics*, *24*(2), 33-40.

Bush, S. B., & Karp, K. S. (2013). Prerequisite algebra skills and associated misconceptions of middle grade students: A review. *The Journal of Mathematical Behavior*, *32*, 613-632. doi:10.1016/j.jmathb.2013.07.002

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, *105*, 380-400. doi:10.1037/a0031084

Cohen, J., (1988). *Statistical power analysis for the behavioral sciences*. Hillsdale, NJ: Lawrence Erlbaum Associates

De Koning, B. B., & Tabbers H. K. (2011). Facilitating understanding of movements in dynamic visualizations: An embodied perspective. *Educational Psychology Review*, *23*, 501-521. doi:10.1007/s10648-011-9173-8

Gavin, M. K., & Sheffield, L. J. (2015). A balancing act: Making sense of algebra. *Mathematics Teaching in the Middle School*, *20*, 460-466. doi:10.5951/mathteacmiddscho.20.8.0460

Kaput, J. J., Carraher, D. W., & Blanton, M. L. (2008). *Algebra in the early grades*. New York, NY: Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics [NCTM] (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Otten, M., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (2019). *The balance model for teaching linear equations: A literature review*. Manuscript submitted for publication.

Piaget, J., & Inhelder, B. (1967). *The child's conception of space*. New York: W. W. Norton.

Sherman, J., & Bisanz, J. (2009). Equivalence in symbolic and nonsymbolic contexts: Benefits of solving problems with manipulatives. *Journal of Educational Psychology*, *101*, 88-100. doi:10.1037/a0013156

Van Amerom, B. A. (2003). Focusing on informal strategies when linking arithmetic to early algebra. *Educational Studies in Mathematics*, *54,* 63-75. doi:10.1023/B:EDUC.000

Van Reeuwijk, M. (1995). The role of realistic situations in developing tools for solving systems of equations. *Paper presented at the annual meeting of the American Educational Research Association*, San Francisco, CA.

Warren, E., & Cooper, T. J. (2005). Young children’s ability to use the balance strategy to solve for unknowns. *Mathematics Education Research Journal*, *17*, 58-72. doi:10.1007/BF03217409

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, *9*, 625-636. doi:10.3758/BF03196322