Networking theories in design research: an embodied instrumentation case study in trigonometry

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There is increasing attention for the embodied and extended nature of mathematical cognition but the bodies of literature on embodied and on extended cognition have developed mostly separately. We propose a new step in the tradition of networking theories: design research from two theoretical perspectives to promote integration. Embodied design and instrumental genesis inspired us to elicit embodied instrumentation: learning via techno-physical interaction with digital artifacts. A case study illustrates a design and subsequent problem solving by a student (aged 16), who uses her body and the designed artifact to solve trigonometric equations. We reflect on the benefits of not only analyzing but also designing from different theories to network these theories.

Keywords Mathematics curriculum, schemata (cognition), sensory experience, learning theory, networking theories

Introduction

With the recently foregrounded E-perspectives (embodied, extended, embedded, enactive, enculturated, etc.) on cognition, there is increasing attention for the embodied and extended nature of mathematical learning. So far, however, embodied and extended cognition have their own bodies of literature in mathematics education, for example embodied design (Abrahamson, 2014) and the instrumentation approach to tool use (Artigue, 2002). Both theoretical frameworks shift from a traditional view on cognition to allowing other players, such as tools and physical interaction to be part of, or at least heavily shape, emerging mathematical cognition. Consequently, educational designers carefully design learning experiences with (digital) materials to stimulate schemes that are relevant for the targeted mathematical knowledge: being either sensorimotor dynamics or instrumented techniques (Drijvers, 2019).

In this paper, we propose that using both theoretical lenses can enhance mathematical learning and its study. However, as observed by Bikner-Ahsbahs and Prediger (2014), the synthesis and integration of theories is rare and challenging. Until now the networking of theories in mathematics education has primarily involved two groups analyzing the same dataset collected by one of the research groups using their "home" theory. To promote integration, which we consider fruitful in the case of embodied and extended cognition, we take a possible new step in the tradition of networking theories: conducting design research in which the design is informed by two different theories. To explore the value of integrative design research, we elicited, by means of a newly designed digital tool, a phenomenon for which it made sense to analyze it by the integration of embodied design and the instrumentation approach to tool use: *embodied instrumentation* as learning via techno-physical interaction with digital tools (cf. Drijvers, 2019). Joining forces within design research seems promising, because it requires cycles of joint theorization, designing, implementation, and analysis (Bakker, 2018).

We aim here to explore how design research can facilitate integration of theoretical views that hold promise to be combined, in our case embodied and extended cognition. To guide this exploration, we circle around the question, *What could embodied instrumentation look like?* We applied this question to the domain of trigonometry. A major Achilles heel for students seen throughout mathematics education is that graphic, algebraic, or numerical representations are treated as isolated conceptions instead of as different symbol systems that jointly make up the conception of functions (Drijvers & Gravemeijer, 2005).

In this paper we elaborate the idea of embodied instrumentation, then describe a digital tool based on embodied instrumentation for trigonometry, and provide an empirical illustration of how a case-study student was using her body (sensorimotor) and the tool's elements (techniques) to solve trigonometric equations in the form of $\sin(\theta) = c$. Through this, we hope to shed light on the added value of integrating embodied and extended views for mathematical learning and integrative design research in general.

Embodied instrumentation: techno-physical mathematical learning

Under the foregrounded E-perspectives, cognition does not just reside inside the learner, but distributes across their interactions in the physical and social (cultural and linguistic) world (Smith & Gasser, 2005). These embodied and extended views on cognition shift away from the traditional idea of internal representation and computation that is fed by external environmental sense-information, though the latter is not seen as part of cognition (Wilson & Golonka, 2013). Cognition emerges from embodied multimodal experiences; when people act and perceive in the world, their sensorimotor schemas self-organize, which leads to increased functional acting in and understanding of their environment (Smith & Gasser, 2005). Put more radically, the range and kind of embodied experiences determine the emerging cognition.

Much of multimodal interaction is mediated through tools, extending the scope and kind of actions people can carry out. A useful lens on the interaction of user and digital tools is the instrumentation approach (e.g., Artigue, 2002). The instrumentation approach considers the careful designing of educational tools to enhance mathematical learning. Under this flag, the bare tool is considered an *artifact*. Central in learning is the process of *instrumental genesis*: The user applies and thereby develops *instrumentation schemes* appropriating the use of the artifact to solve a specific class of tasks (Artigue, 2002; Drijvers & Gravemeijer, 2005; Vérillon & Rabardel, 1995). What students actually do (using instrumented techniques) depends on the opportunities and constraints of the artifact and their knowledge. At the same time, the doing itself contributes to their understanding. Artigue (2002) highlights this pragmatic and epistemic value of instrumentation schemes which co-emerge dependently. However, in most applications of instrumentation approach in mathematics education the embodied nature of these instrumentation schemes is rarely studied. The body seemed to have disappeared from sight when the tools became more complex and calculations more hidden from the user. Yet going back to original sources of the ideas (Vérillon & Rabardel, 1995), there is ample potential to revive the attention for the embodied origin of instrumental genesis.

A neighboring framework to fill this gap is embodied design for mathematics. With the increased availability of technologies, embodied learning for mathematics now includes multimodal learning experiences with various sensors and motion trackers. An emerging embodied design genre in mathematics education is the Mathematics Imagery Trainer (MIT) (Abrahamson, 2014). Within this pedagogical framework, mathematical cognition is promoted through relevant embodied experiences. As such, mathematical cognition is re-defined as grounded in sensorimotor schemes for material interaction. In the activities students learn about co-variation (proportions, functions) by discovering their own bimanual motion patterns (Abrahamson & Bakker, 2016). For example, in the original task, designed to foster reasoning about proportional equivalence such as 1:2 = 2:4 (the MIT-P for proportion), students project two cursors on a screen by manipulating two Wii remotes. When the ratio is set to say 1:2, the screen turns green only when the right hand remote is twice as high as the left hand remote; otherwise it is red. Multimodal learning analytics, including hand- and eye-tracking methods, revealed that while students develop effective ways of moving in or enacting proportional equivalence, their gaze converged to specific locations (where no object is visually present per se). These are shown to have value both pragmatically (improved coordination) and epistemologically (understanding). These gaze points are attentional anchors (AAs), a phenomenon known to occur in a variety of physical activities (juggling, swimming), and other MIT versions designed to promote the learning of multiplicative reasoning (Duijzer, Shayan, Bakker, Van der Schaaf, & Abrahamson, 2017) and parabola functions (Shvarts & Abrahamson, 2019).

Designing an embodied instrumentation tool for trigonometry

We set ourselves the goal to design activities based on embodied instrumentation via techno-physical interaction, which promote learning, understanding and solving of trigonometric equations in the form of $\sin(\theta) = c$. The first and second authors have a background in embodiment theory and the last author in instrumentation approach. Each initially designed a task to meet the learning goal from their theoretical perspectives, which were then discussed. The differences between the two initial artifacts were mainly on a practical level: the embodied design perspective being more informal in the domain of geometry and the instrumentation approach being more algebraic and formal, including a constant and symbolic language. A key realization was that this could be used to integrate the artifacts in a progressively formalizing sequence. These activities were developed in the Digital Mathematics Environment, piloted with four students, and adjusted where needed. We report here on the designs of the second design cycle.

Figures 1 and 2 show elaborations on the tasks, the digital tools and the intended learning schemes. In the initial tool, users drag two points; their left hand moves along a unit circle and their right along a sine graph as illustrated in Figure 1a. The rotation (angles in radians) along the arc of the unit circle and the x-axis of the sine graph were both marked in blue to make salient the equivalence of the angle in both representations. Users are instructed to move two points so that a frame turns green. Consistent green is only achieved when the angles (or the length of the blue marking) are equivalent in the unit circle and the sine graph (as in Figure 1a), otherwise the frame is red (as shown in Figure 1b). The intended relevant sensorimotor solution to this problem was to keep the hands at the same height, representing a grounded scheme of the sine of an angle, and equivalence across the unit circle and the sine graph.

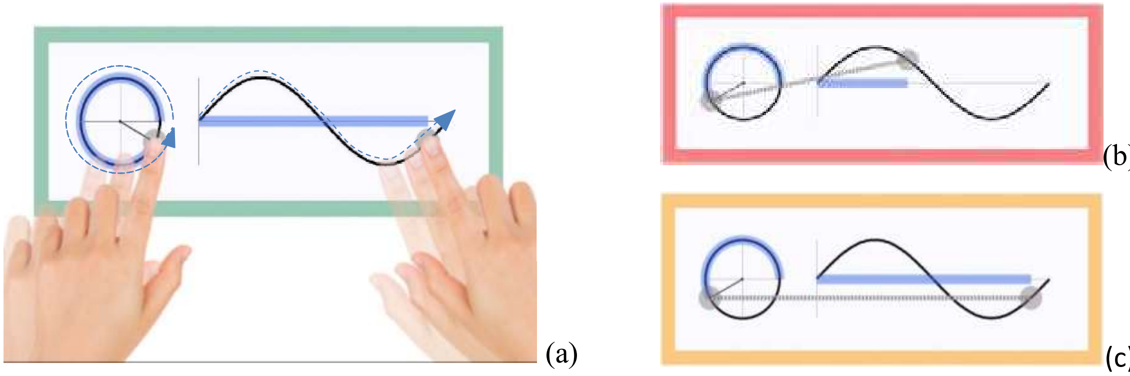

Figure 1: Example of the artifact when (a) enacting equivalence of angle and sine (green color feedback), (b) incorrectly enacting equivalence (red color feedback) with a (slanted) segment, and (c) enacting equivalence of sine for complementary angles (orange color feedback) with the segment

Solving the task is done first without, and then with, a segment connecting the points on the unit circle and sine graph (two examples shown in Figures 1b and c). The idea is that the segment would work as a shortcut for action and perception coordination, as a conspicuous measure of horizontality. This highlights the height of both points, enabling more successful and continuous interaction. Visually, the segment intersects both graphs when orange is achieved, and as such it might prime the relevance of intersections in equation solving (Figure 1c).

The second task involves solving trigonometric equations of the form $\sin(\theta) = c$, solving for both the angle (in radians) and the sine value. Values of the angle θ are shown in both the unit circle and the sine graph. An additional element has been added, a constant that users can move up and down with a touchpoint (Figure 2a). The idea is that the values and constant would invoke a quantification process, and would link the positions of the points to elements and values in the equation. Solving the equation of $\sin(\theta)$ = 0.86 would involve moving the constant up to the y-value of 0.86 (the vertical distance), and then moving the point on either the unit circle or sine graph to intersect this and read the value of the angle rotation (solution is shown in Figure 2b). The constant-line would further make salient that there are multiple solutions for the same sine value.

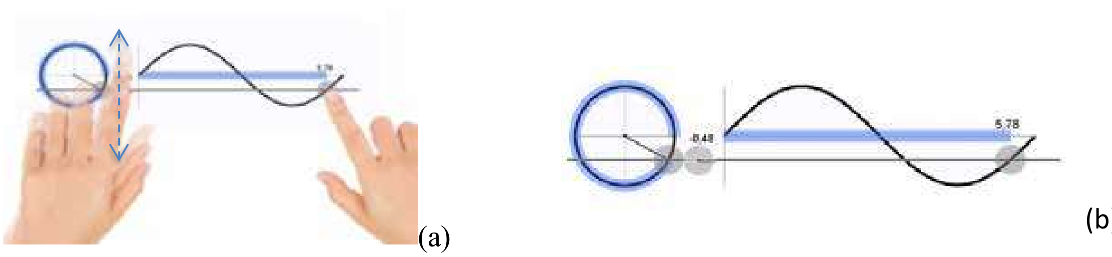

Figure 2: Example of the artifact with constant and values with (a) moving the constant up and down to manipulate the sine value (emphasis added) and (b) the solution to $\sin(\theta)$ = -0.48

A case study

As part of the design research cycle, we asked a case-study student, Spring (pseudonym), to interact with the artifact and solve the two tasks in a clinical interview. Spring (aged 16 years and left handed) had just completed grade 11 at a US high school. She encountered trigonometry in a Pre-Calculus course (Grade 11) and prior to that in an Algebra 2 with Trigonometry course (Grade 10). Spring is a top student with a grade point average of 4.2 (US system). We conducted a short pre-interview to assess her prior knowledge. In some instances the option of sketching and (unrecognized) writing was made available. Spring remembered the main terms from trigonometry (arc length, sine, cosine, radian) fairly well, and remembered several formulas. She was able to identify the sine of an angle in a triangle, and relate certain angles in degrees to radians. We repeated the four pre-interview questions after the session and recorded a debrief session with the student and researchers. During the entire clinical interview we used audio, video and eye-tracking (Tobii X2-60) to monitor closely how Spring was using her body (actions, perceptions, and verbalizations). We present here the findings on how she came to use the newly developed tool in the context of new discoveries and relationships in a clinical interview with two interviewers (second and third author).

Finding and keeping green

Spring easily found places that were green. However, maintaining the green while moving was a more difficult task, and Spring looked back and forth between her two fingers (horizontal gaze direction) while trying to achieve this. Her initial description of her strategy was to keep the speed of both fingers the same. She explains:

Spring: If I move my hands at the same speed I will go the same distance, for each … So I get the same distance in the same time. So, I get to the same points at the same time, with both my hands.

Interviewer: You mean distance on the graph or the blue line?

Spring: Distance on the graph. So, I am following the black, and the gray.

This initial strategy is commonly seen when students begin to interact with MIT artifacts: Keeping the hands at the same speed and looking between them (in a horizontal direction) resonates with familiar intrinsic dynamical tendencies (Duijzer et al., 2017). In doing so in this trigonometric context, the student reasoned that the arc length in the unit circle corresponded with the curve length instead of x value (angle in radians) in the sine graph.

Using the segment

When the segment (Figures 1b–c) was added (after 22 minutes), Spring almost immediately shifted her strategy, and verbalized that she had to keep the segment horizontal. While keeping the segment level, she gazed at the middle of the segment, most likely attending to the vertical displacement.

Spring: When the helper line [segment] is horizontal you know you are at the correct … y value. You are at the correct units for both of them...because...it's the same thing graphed over, in different formats.

Spring: This is definitely easier, its …it feels more like I am using it as a singular thing rather than a…you know like…two dots, and trying to keep them alike. […] So, like it feels more, like the two graphs feel more connected, so it's a more obvious… like it's easier for me to focus on how to keep them green, how to keep them both showing the same thing.

Adding the segment changed the interaction both pragmatically and epistemically. Prime indicators of effective scheme development here seemed to be: (1) a convergence of gaze to the middle of the segment, (2) more successful enactments, (3) reasoning about the vertical distance (sine) of both points, and (4) a sense of coherence between the two representations. These results coincide with previous research in which attentional anchors emerge as a means to improve coordination and understanding (Abrahamson, 2014; Abrahamson & Bakker, 2016; Duijzer et al., 2017).

Using the constant and values to solve trigonometric equations

In the second task, the constant was added with the values for θ and sine (after 30 minutes; Figure 2). Although we did not anticipate this, the position Spring gazed at in the middle of the segment coincided with the positioning of the touchpoint that would move the constant in a vertical direction. Spring was quick to use this constant as intended and elaborated on her previous explanations:

Spring: So, it's, mmm, it's measuring the same sine value in both the first and second quadrant I think. So this keeps it level, so there is, we can't switch it, it's always going to be attached to the graph. […] So, the yellow is that it's the same sine value, but not the same distance, or radians, or… So, it's dealing with the different quadrants, or something?

Spring used a well-organized sensorimotor instrumentation scheme to solve each of the equations. For example, she solved $\sin(\theta) = 0.86$ by moving the constant up 0.86, and then moving the point on the unit circle so that it intersects with the constant. She spontaneously showed two solutions, by moving the point first to the intersection with the same angle rotation, and then to the complementary angle. For the second type of equations, she used a similar scheme, but the action sequence was reversed. For example, $\sin(2) = a$ was solved by moving the point on the unit circle to 2, followed by moving the constant to a height that intersected the point to find its sine value. To solve the equation $\sin(2.5\pi) = a$, students are required to rotate beyond one rotation of 2π. As the tool is limited to one rotation, this equation is solved without the tool providing the correct angle markings and values. Spring solved this successfully by moving the point on the unit circle over a full round, while saying, ½ π, π, 2π and then kept moving to "add another half." This might show how the epistemic value of the instrumented sensorimotor schemes can assist to overcome practical constraints of the tool.

Conclusion and Discussion

In this paper, we aimed to contribute to the tradition of networking theories by exploring a possible new approach: design research from two theoretical perspectives to promote integration between these perspectives. Embodied design and instrumentation approach inspired us to elicit embodied instrumentation or learning via techno-physical interaction with digital artifacts. This approach assumes that bodily and instrumental experiences of mathematics are fundamental for learning. Design research requires cycles of joint theorization, designing, implementation, and analysis

(Bakker, 2018), and as a methodological approach provided a structured way to integrate embodied and extended perspectives on each of these components in the research cycle. As such, the frameworks inspired both the design and the analysis.

Bringing back the embodied nature of instrumentation schemes through embodied instrumentation, can, we believe, enhance both students' mathematical learning and our ability to study it. Insights from both embodied design and the instrumentation approach facilitated careful alignment of embodied and instrumental experiences to promote trigonometric problem solving. Multimodal learning analytics allowed us to assess how a student was using her body and the digital tool to solve trigonometric equations, and provided empirical insight into the phenomenon of embodied instrumentation in trigonometry. Spring's repeated and flexible use of a well-organized sensorimotor instrumentation scheme might be indicative of embodied instrumental genesis, though ideally one would like to study longer-term processes to make such claims.

The difficulty of linking symbols to previous and current actions and perceptions requires a more in-depth analysis of our tasks and digital tool. For some of our previous pilot students the gap between enacting and solving trigonometric equations was too large, and they were unable to link the symbols of the equation to their sensorimotor experiences. Coinciding with previous research we found converging gaze patterns that served as an attentional anchor (Abrahamson, 2014; Abrahamson & Bakker, 2016; Duijzer et al., 2017) by improving both coordination and understanding in trigonometry as well as creating a sense of coherence between the two representations. Adding epistemic value from physical interaction (making movements relevant) into current digital tools in mathematics may increase the educational potential of these tools.

Integration is a step in the networking of theories that is driven by the question of what each approach can learn from each other. Theory cultures and their boundaries are dynamically (re)produced and developed by their members' ways of understandings and acting based on a set of principles, related methodologies and paradigmatic questions. The activity of networking theories is a way of renewing theory cultures, and could be sparked by solving a problem that all members share (Bikner-Ahsbahs et al., 2010). Integrative design research can provide a structure to elicit a phenomenon that is interesting for different parties to be involved in. Its cycles of joint design, implementation, and analysis seem to provide a structure for sensible theoretical integration by making explicit the theoretical influence on design choices, measurement tools, and interpretations.

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